## A Complex Systems Approach to the Location Behaviour of Firms

By Jonas K. Sekamane, 28. august 2015.

Hotelling (1929) introduced a model analysing the location problem (or product differentiation) of duopolies. Eaton and Lipsey (1975) have proven analytically that no equilibrium solution exists when three firms have similar price and locate on the line market. Eaton and Lipsey (1975, p. 40) note that it is not obvious how three or more firms will choose to locate in two-dimensional space, and write that "using conventional analytical techniques the problem is very complex, perhaps intractable". Instead they use simulation techniques. They suspect that no equilibrium solution exists for three or more firms. Using Agent-based modelling Laver and Sergenti (2011) constructs a multi-party model that investigates how political parties choose to locate in two-dimensional space. They assume that the location of voters/customers are drawn from a bivariate normal distribution. Along one of their dimensions they assume two separate subpopulations of customers, which allows them to account for the effect from asymmetric and multimodal distributions. They investigate the location behaviour of political parties using five different decision rules. In addition their model includes entry and exit of political parties.

My paper will continue the line of inquiry into the competitive location behaviour of firms. I will investigate how multiple firms choose to locate in 2D space. I will use the bivariate normal distribution with two subpopulations. The model will include entry and exit in the market. I will use the Agent-based modelling approach laid out by Laver and Sergenti (2011), but modify the model such that the assumptions captures how firms choose to locate, rather than how political parties choose policy position. Fore instance the simplest decision rule in Laver and Sergenti (2011) is one where the party leader is purely ideological, and thus the party never strays from its ideal policy point. Similarly entry is based on the idea of "citizen candidate" – ie. it is disgruntled voters that raise to political prominence by founding new political parties. These assumptions seem reasonable in a political setting, but much less likely if one considers how firms choose location or product differentiation, or if one considers how new firms come into existence.

## Location behaviour of firms (agents)

Each firm chooses the location that maximises its market share, given the location of the other firms. In this multi-agent choice problem all firms choose location simultaneously. Thus when a firm has to choose its own location, the location of other firms is unknown. The firm can instead use the predicted location of the other firms. But the location outcome that each firm is trying to predict, depends on predictions that the firm and other firms form. "Predictions are forming a world those predictions are trying to forecast" (Arthur 2014, p.175). This self-referential loop leads to logical indeterminacy, and thus the maximisation problem is ill defined and cannot be solved deductively.

Nonetheless firms still make decisions even when faced with an ill defined problem. Firms possibly use heuristics or rules of thumb. A firm could choose:

- to move towards the center of its current market area.
- to move towards its most profitable competitor.
- to continue in the same direction, if the the previous move proved fruitful, and otherwise head in the oppersite direction.
- to survey several directions and move along the most profitable.

The latter two decision rules attempts to maximise market share directly, while in the former two the market share may be maximised, but only indirectly. Laver and Sergenti (2011) use decision rules similar to these. Further note that none of these decision rules considers the simultaneous move of competing firms. They implicitly assume that other firms stay at their current location. And thus the decision rules are stripped of any and all strategic considerations.

I will concentrate on a decision rule with foresight – firms with inductive rationality. A firm with this decision rule forms hypotheses on how other firms choose their location. The firm holds several hypotheses at once, and acts on the hypothesis that worked best in the past. Each hypothesis attempts to predict the movement of the other agents based on their movement history. The firm still chooses its own location such that it maximises market share, but subject to the predicted movements of other firms generated by its most reliable hypothesis. In addition the firm gradually discards poorly performing hypotheses and forms new hypotheses.

I assume that firms are endowed with a decision rule and only use this rule to choose location. I will compare the relative performance of the different decision rules. Do firms with *inductive rationality* stay in the market for longer periods than fims with other decision rules? Do firms with *inductive rationality* capture a greater share of the market? Can all the decision rules coexists in the same market, or are some rules out-competed?

## Limiting the scope of the paper

The paper will be purely theoretical and the analysis framed in terms of geographical location or product differentiation.

The paper will abstract from price competition, and exclusively consider location differentiation. Price competition greatly increases the complexity of the model, and will therefore be beyond the scope of this paper. Furthermore firms can and do change prices more frequently than they change location. So price completion and location competition work on different time scales. And it is not clear how the different time scales should be incorporated in a unified model.

## Time schedule

• 24. Sep.: Finished literature review.

• 11. Dec.: Finalised the model.

• 23. Dec.: Finished sections on methodology, model, result.

• 29. Jan.: Finished writing/paper

• 8. Feb.: Start proof reading final paper

• 19. Feb.: Hand-in final paper