# Cooperative estimation for feature-based SLAM

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July 31, 2014

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# Outline

- Introduction
- Problem background
- Simulation results
- Summary and future work

# Introduction

Multi-vehicle scenarios

- Academic problems
  - Cooperative control
  - Coalition tasking/planning
  - Mapping
- Examples
  - Search and rescue
  - Geological survey
  - Firefighting

# Cooperative estimation

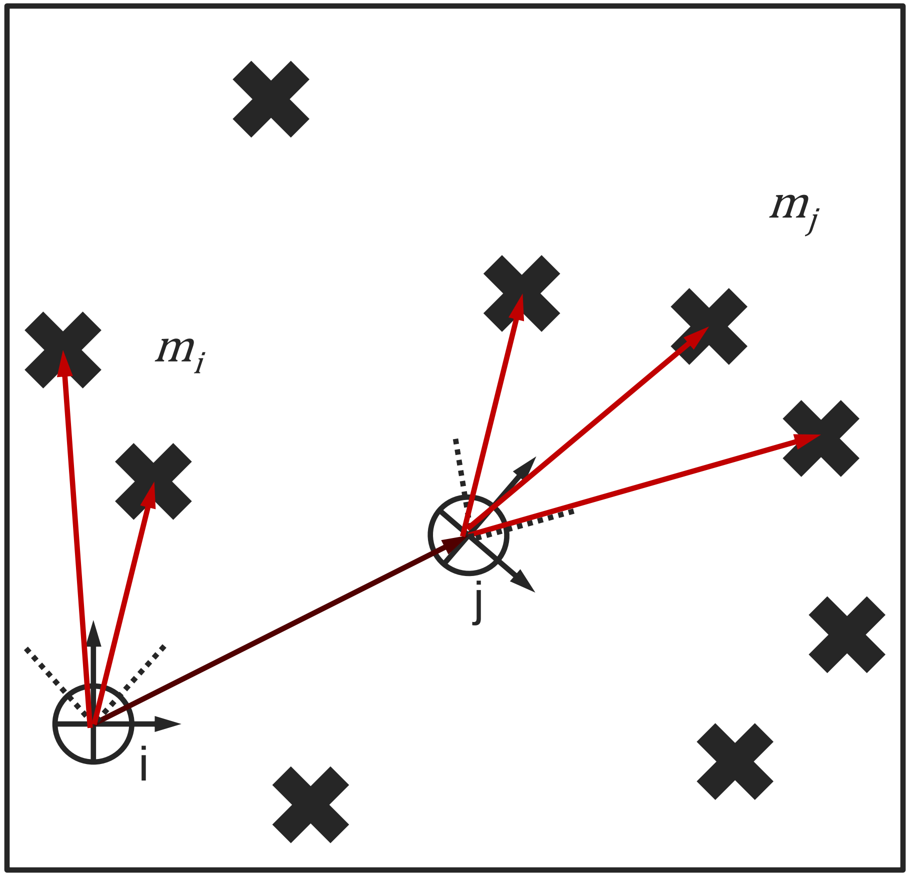
- Challenges
  - GPS-denied/indoor operation
  - Sensor accuracy
  - Comm bandwidth/loss
  - What data to share?
- Approach
  - Simultaneous localization and mapping (SLAM)
  - Agent-agent position sensing
  - Shared feature measurements
  - Decentralized filter

# Pathfinder project

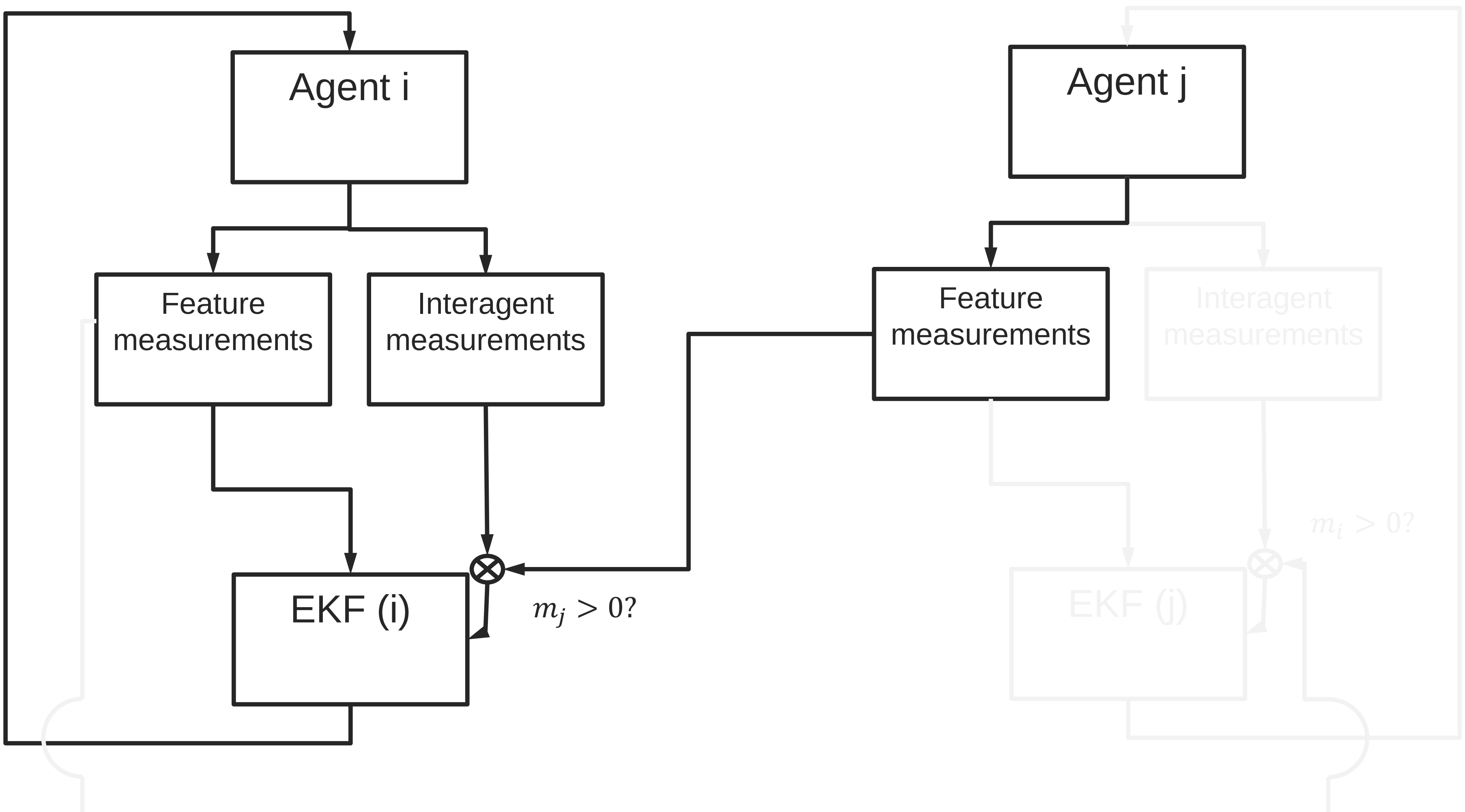
- Interest in project
  - Cooperative/multi-agent hardware problems
    - Game theory
    - Estimation
    - Planning
    - Mapping
  - Collaboration
    - High-level decision-making
    - Low-level sensors
- My contribution
  - Optitrack/PX4 data fusion and processing
  - Parameter identification (drag, sensor variance)

# Problem definition

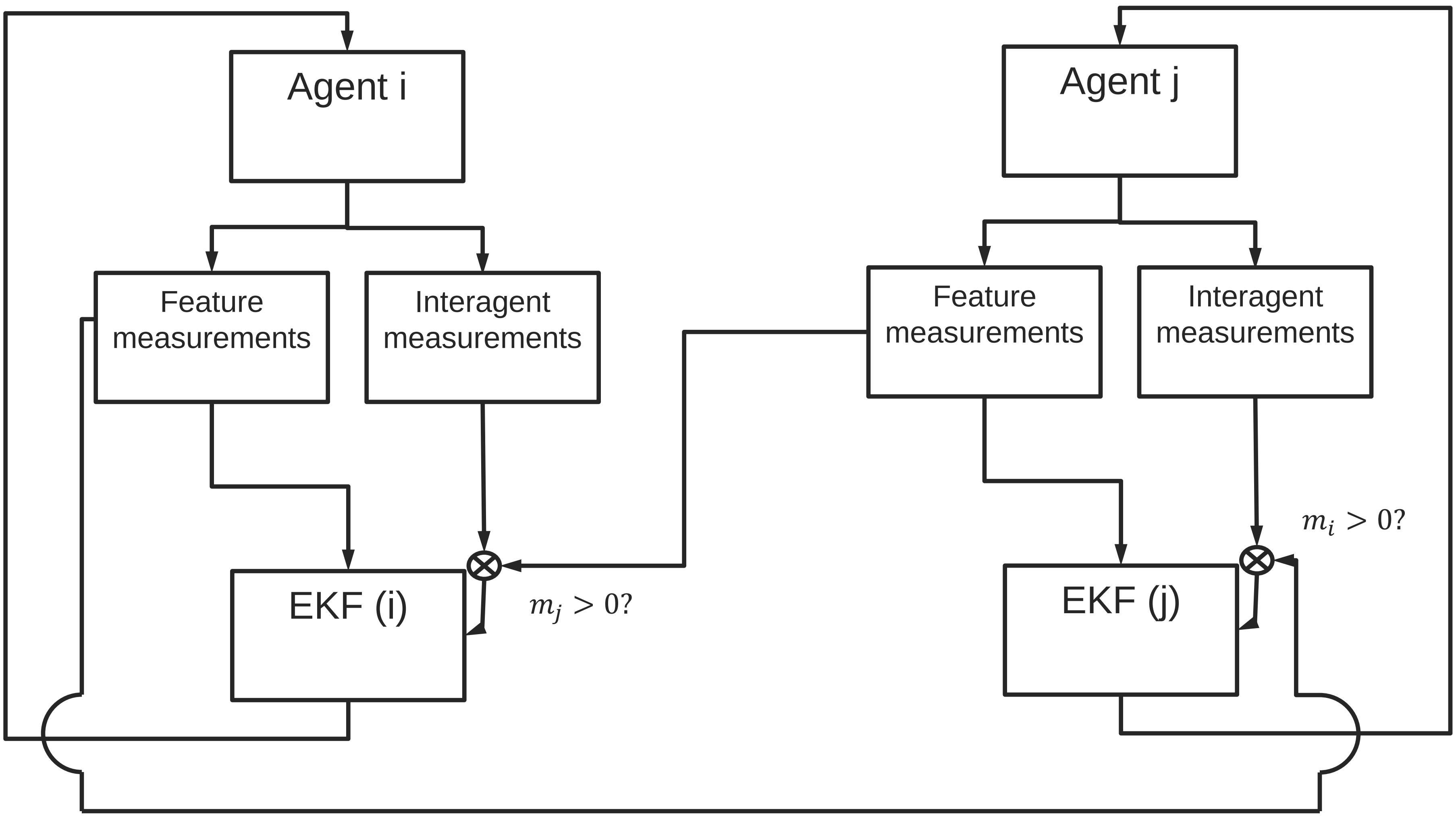
- Two agents, planar
- $M$ features, known
- Interagent range & bearing
- Two cases
  - Feature range and bearing
  - Feature bearing only



# Information flow



# Information flow



# State and measurement model

## Measurements:

- Feature range and bearing:

$$\hat{\rho}_{ki} = \|[\hat{C}_{b/n}][\mathbf{r}_k]_n - [\hat{\mathbf{r}}_i]_b\|$$

$$\hat{\theta}_{ki} = \arctan \frac{\begin{bmatrix} -\sin\psi & \cos\psi \end{bmatrix} [\mathbf{r}_k]_n - \hat{r}_{iy}}{\begin{bmatrix} \cos\psi & \sin\psi \end{bmatrix} [\mathbf{r}_k]_n - \hat{r}_{ix}}$$

- Feature bearing only:

$$\Phi = \arctan \frac{\hat{r}_{ki2} - \tilde{\rho}_{ji} \sin \tilde{\theta}_{ji}}{\hat{r}_{ki1} - \tilde{\rho}_{ji} \cos \tilde{\theta}_{ji}} - \tilde{\theta}_{kj} - \tilde{\Delta}$$

$$\Delta = \pi - \theta_{ij} + \theta_{ji}$$

## States:

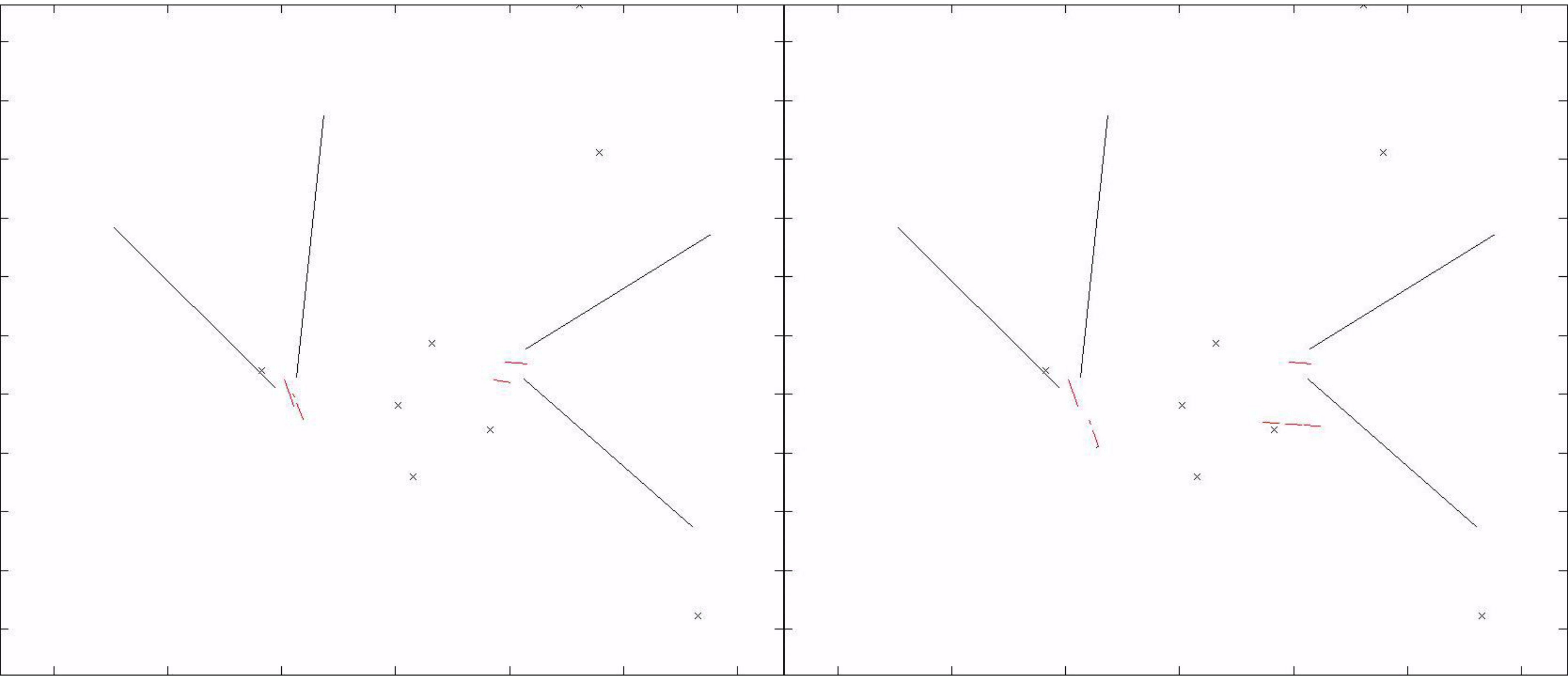
- $\hat{\boldsymbol{x}} = [r_{ix} \quad r_{iy} \quad u \quad v \quad \psi]^T$
- Open-loop trajectories

# Simulation parameters

- 100 Monte Carlo
  - Fixed trajectories, features
  - $\sigma_r = 1, \sigma_\theta = 0.01$
  - M = 15
  - $60^o$ FOV, 1-10 m detection range
- Compare
  - Individual/cooperative
  - Feature number
  - Feature range measured/not
  - Sharing rate

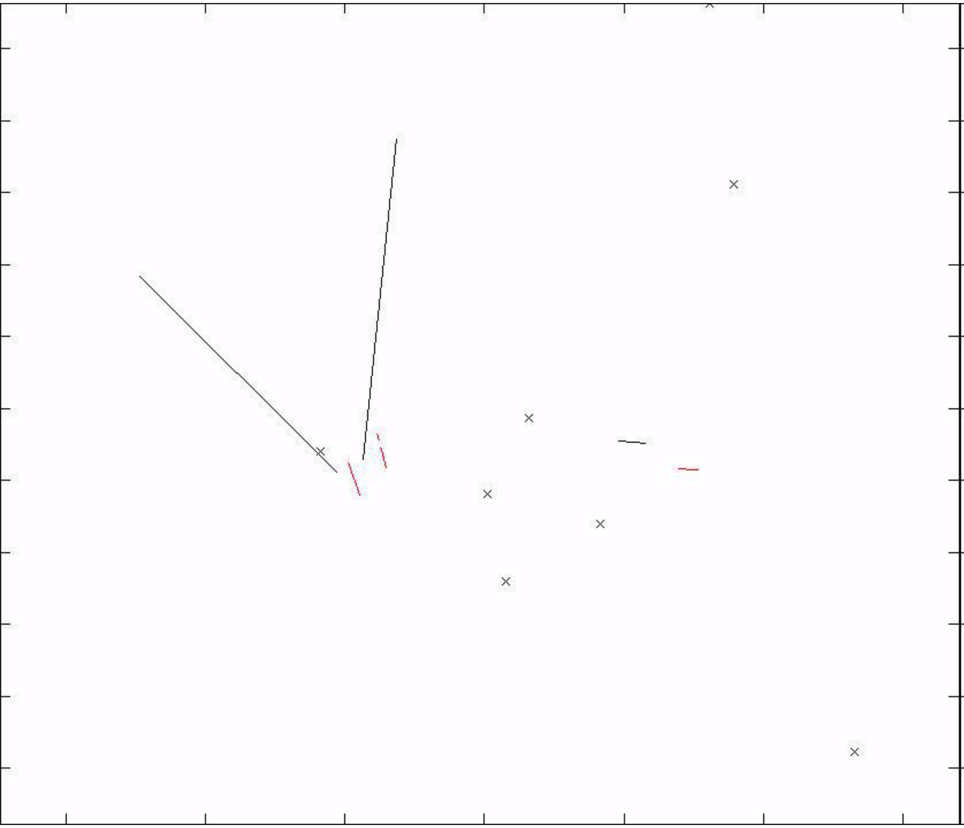
# Animation #1

Individual Cooperative

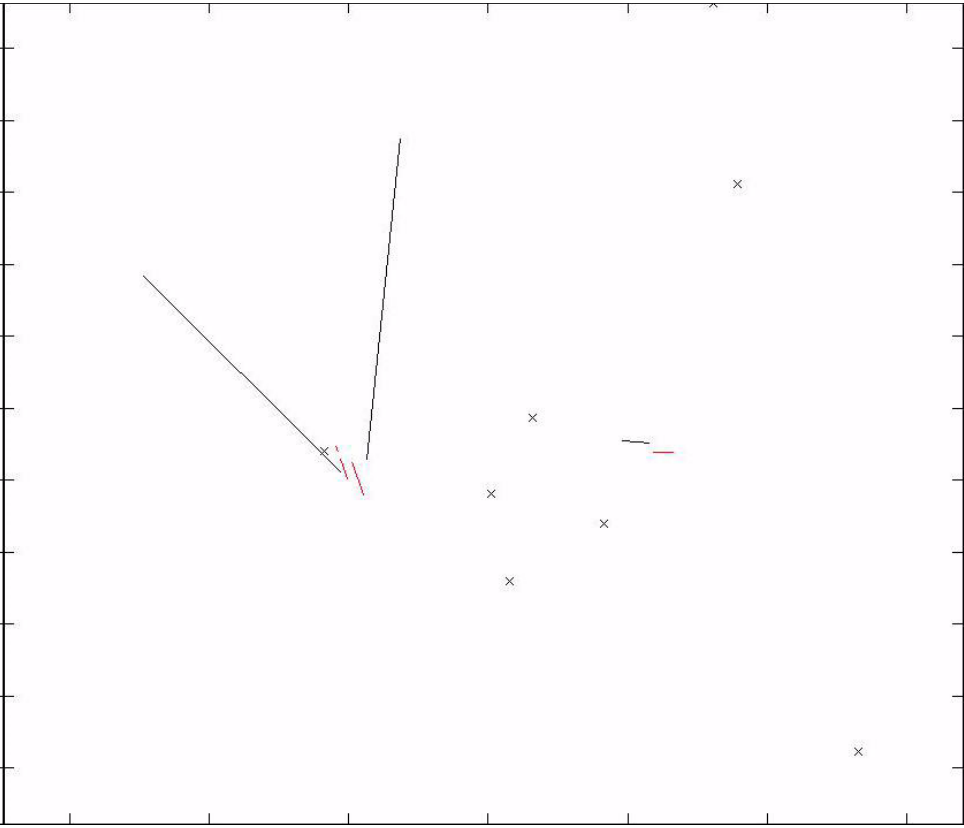


# Animation #2

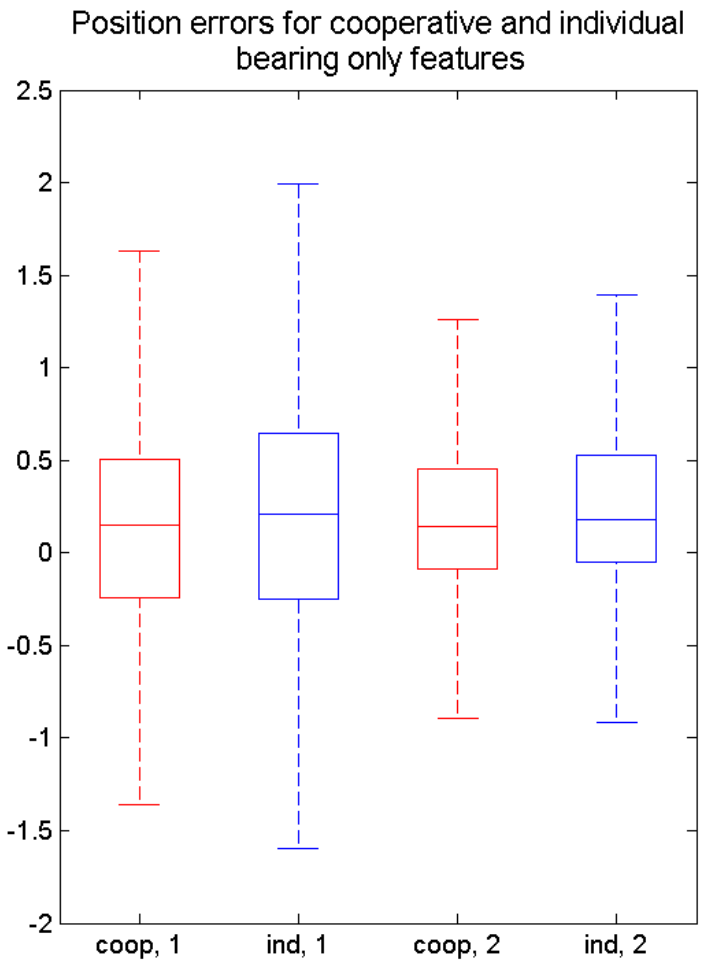
Individual



Cooperative



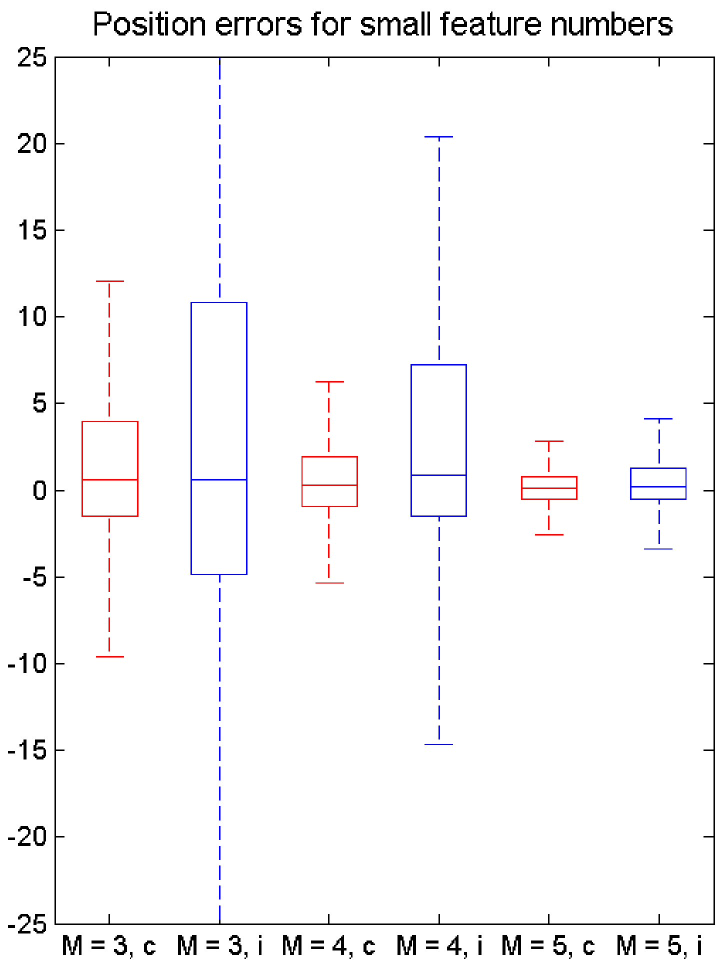
# Sharing vs. Individual




- $\sigma_r = 1, \sigma_\theta = 0.01$
- M = 15

| Case | $S(\varepsilon_X)$ | $S(\varepsilon_Y)$ |
|---|---|---|
| Individual, 1 | 1.63 | 1.37 |
| Coop, 1 | 0.98 | 0.654 |
| Individual, 2 | 0.975 | 0.684 |
| Coop, 2 | 0.713 | 0.668 |

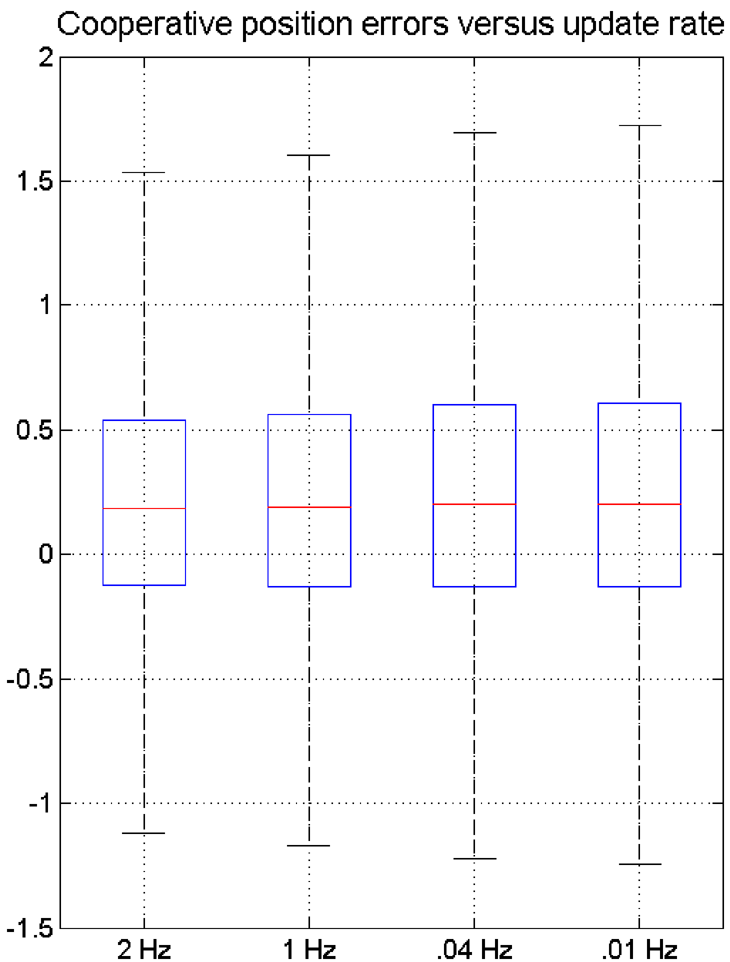
# Sparse features




- $\sigma_r = 1, \sigma_\theta = 0.01$
- M = [3,5]

| Case | $S(\varepsilon_X)$ | $S(\varepsilon_Y)$ |
|---|---|---|
| M = 5, coop | 1.67 | 1.33 |
| M = 4, coop | 7.2 | 7.87 |
| M = 5, ind | 3.54 | 3.29 |
| M = 4, ind | 27.5 | 27.2 |

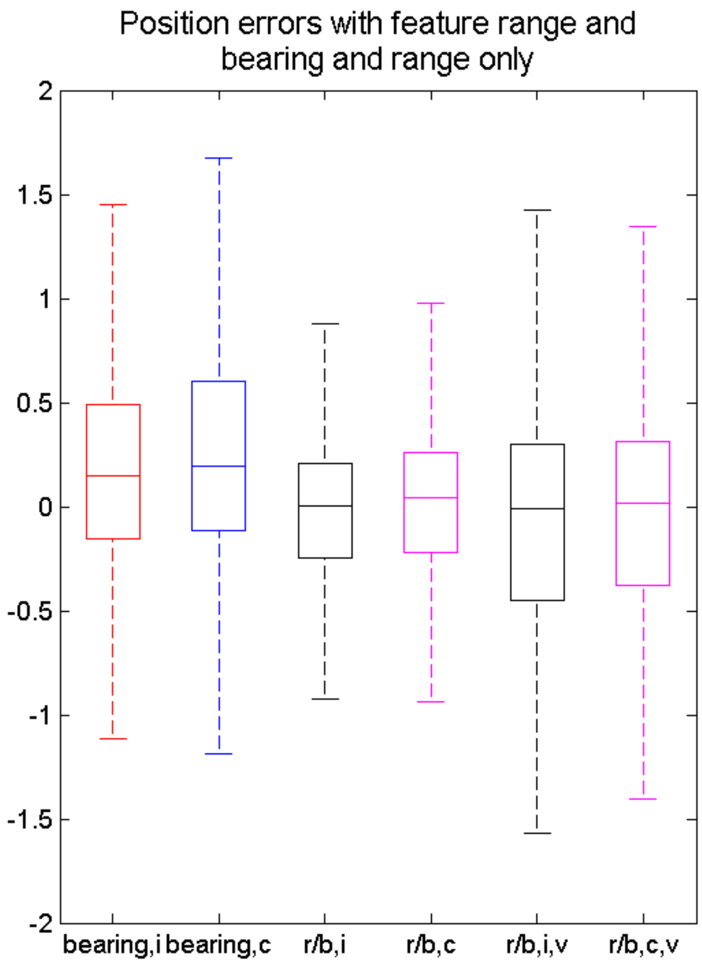
# Sharing rate




- 10 Hz nominal
- M = 15
- $\sigma_r = 1.0$

| Case | $S(\varepsilon_X)$ | $S(\varepsilon_Y)$ |
|---|---|---|
| 2 Hz | 1.08 | 0.801 |
| 1 Hz | 1.24 | 0.943 |
| 0.04 Hz | 1.26 | 0.959 |
| 0.01 Hz | 1.36 | 1.07 |

# Feature range effect




- $\sigma_r$ varies
  - $\sigma_r = \{1.0, 10.0\}$

| Case | S(ε$_X$) | S(ε$_Y$) |
|---|---|---|
| B, ind | 1.63 | 1.37 |
| B, coop | 0.98 | 0.654 |
| R/b, σ = 1 | 0.749 | 0.649 |
| R/b, σ = 10 | 1.33 | 0.973 |

# Conclusions

- Improved estimation accuracy for cooperative
- More robustness to few features
- Sharing effective at ≈10% nominal rate
- Bearing-only vs. range/bearing

# Future work

- Extensions
  - Higher dimensions
  - Hardware experiments
- Planning/tasking
- Cooperative search

http://commons.wikimedia.org/wiki/File:IRobot_Roomba_780.jpg

https://store.3drobotics.com/products/apm-3dr-quad-rtf

# Questions

# Backup

# References


1. Gyorgy, K., Kelemen, A., and David, L., "Unscented Kalman Filters and Particle Filter methods for nonlinear state estimation," in International Conference in Engineering Interdisciplinarity, 2014
2. Crassidis, J.L. and Junkins, J.L., *Optimal Estimation of Dynamic Systems,* 2nd ed., CRC Press, 2011

# Statistical linearization

- Bearing-only constraint: $\Phi\left(\tilde{\rho}_{ji}, \tilde{\theta}_{ji}, \tilde{\theta}_{ij}, \tilde{\theta}_{kj}\right)$
- Measurement covariance numerically[1]:
  - $2n + 1$ sigma vectors $\boldsymbol{\sigma}$, random var $\boldsymbol{x}$

$$\boldsymbol{\sigma}_0 = \mathbf{x}$$

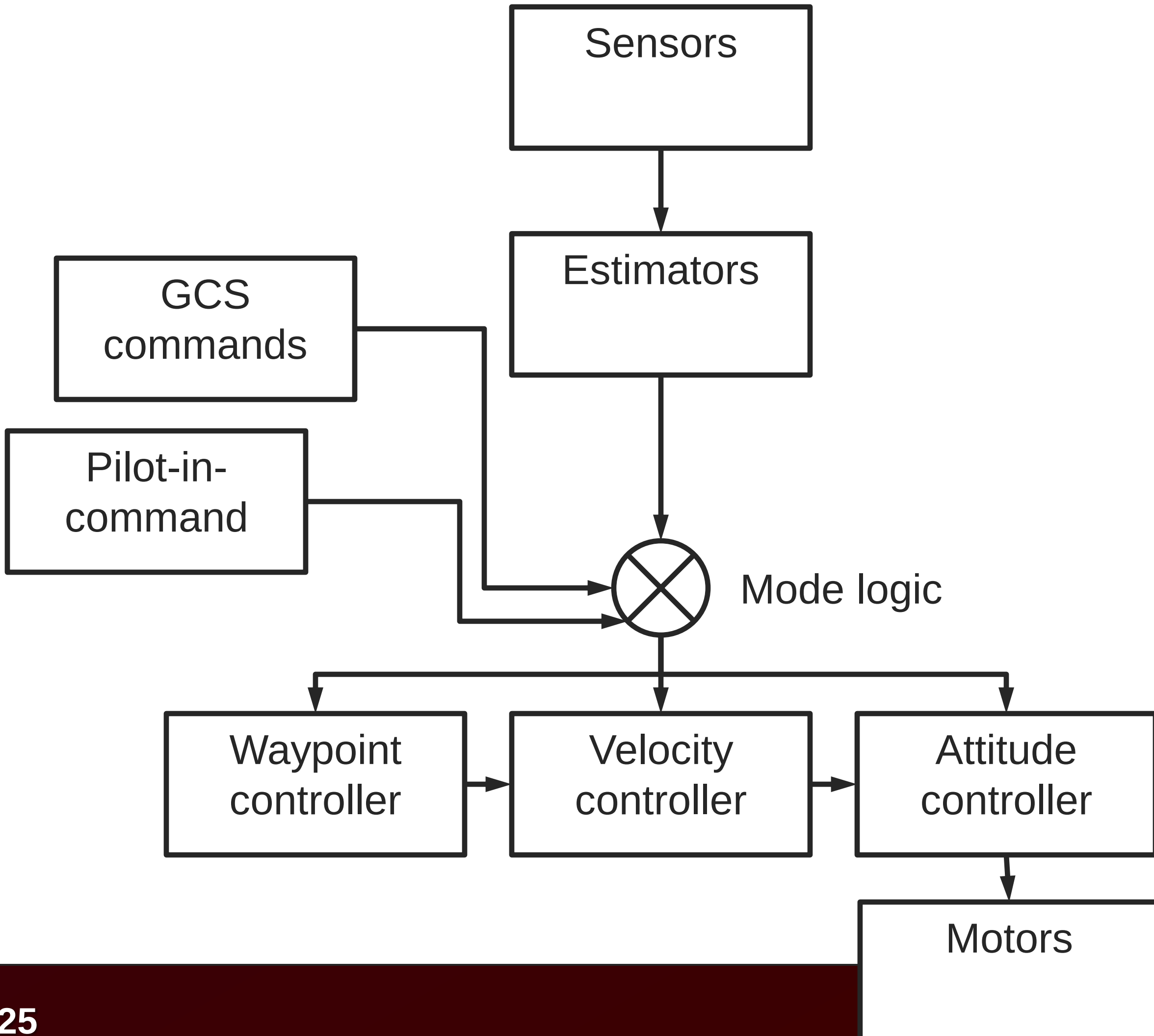
$$\boldsymbol{\sigma}_i = \mathbf{x} + \gamma\sqrt{P_x},\ \ i = 1, \ldots, n$$

$$\boldsymbol{\sigma}_i = \mathbf{x} - \gamma\sqrt{P_x},\ \ i = n+1, \ldots, 2n$$

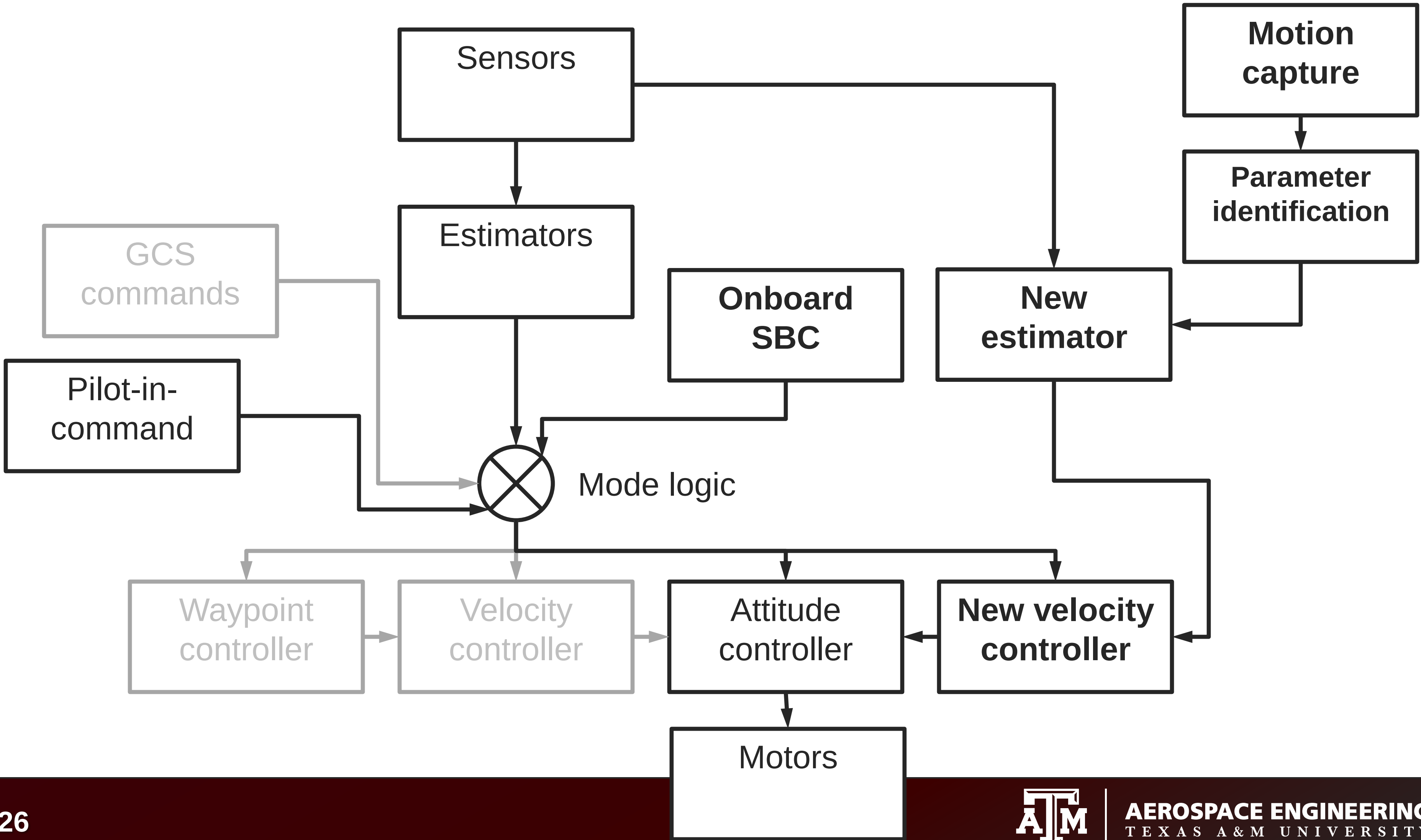
$$\bar{\mathbf{y}} = \sum_{i=0}^{2n} W_i^{(m)} \mathbf{y}_i$$

$$P_y = \sum_{i=0}^{2n} W_i^{(c)} (\mathbf{y}_i - \bar{\mathbf{y}})(\mathbf{y}_i - \bar{\mathbf{y}})^T$$

# PX4 System Overview

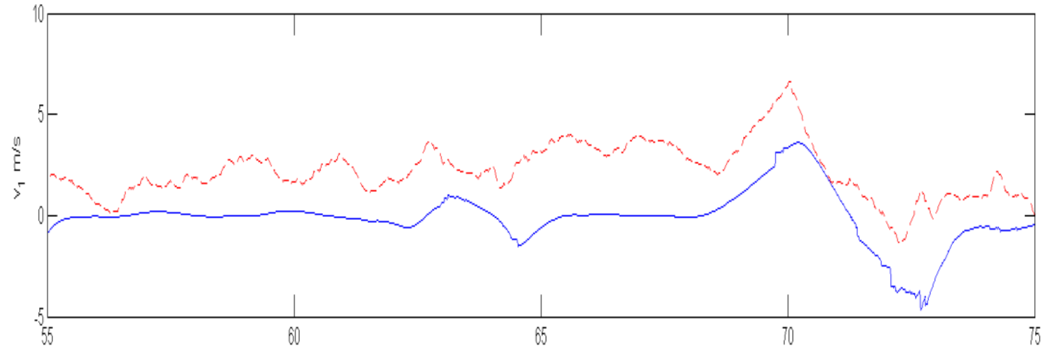


# PX4 System Overview

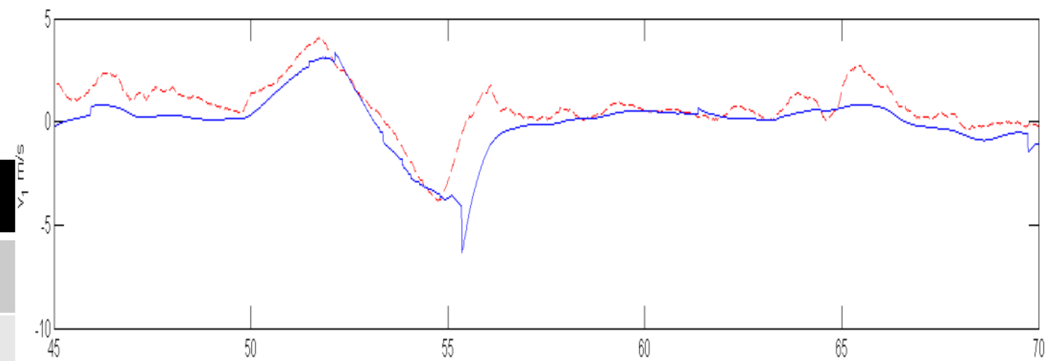


# Motion capture

- Analysis tools
- Sensor characterization
  - Optic flow
  - Sonar
  - IMU
- Drag coefficient




Above: early drag coefficient. Below: current




| μ | $S(v_x)$ (m/s) | $S(v_y)$ (m/s) |
|---|---|---|
| Initial | 1.47 | 1.11 |
| Current | 0.593 | 0.641 |

# Velocity controller

- Two? controllers
  - PID
  - Model inversion

Some kind of simulation result