International Journal of Information Acquisition
Vol. 8, No. 2 (2011) 117–132

DOI: 10.1142/S0219878911002380

SIMULTANEOUS COOPERATIVE LOCALIZATION FOR AUVs USING RANGE-ONLY SENSORS

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Received 15 May 2011
Accepted 7 July 2011

The absence of GPS underwater makes navigation for autonomous underwater vehicles (AUVs) a challenge. Moreover, the use of static beacons in the form of a long baseline (LBL) array limits the operation area to a few square kilometers and requires substantial deployment effort before operations. In this paper, an algorithm for cooperative localization of AUVs is proposed. We describe a form of cooperative Simultaneous Localization and Mapping (SLAM). Each of the robots in the group is equipped with an Inertial Measurement Unit (IMU) and some of them have a range-only sonar sensor that can determine the relative distance to the others. Two estimators, in the form of a Kalman filter, process the available position information from all the members of the team and produce a pose estimate for every one of them. Simulation results are presented for a typical localization example of three AUVs formation in a large environment and indirect trajectory. The results show that our proposed method offers good localization accuracy, although a small number of low-cost sensors are needed for each vehicle, which validates that it is an economical and practical localization approach.

Keywords: Autonomous underwater vehicle (AUV); cooperative localization; Kalman filter; range-only sensor.

1. Introduction

In the last 20 years, research in autonomous marine platforms has led to a large and ever growing number of different submarine vehicles. Due to the increasing availability and reliability of AUVs, they are now used for a wide range of applications. The earliest and most widespread current application for AUVs is mapping. To cover larger areas, an AUV typically takes several thousand pictures during a mission. Sophisticated mosaicking techniques are then used to combine the individual frames to a complete picture of the seafloor [Singh & Howland, 2004]. A special case of a mapping mission is the inspection scenario. Here, the AUV is required to map one or several features such as oil rigs, harbor structures or ship hulls. These features are mostly man-made and have very complex shape. The inspection requires the vehicle to be very close to the feature so that it can be mapped in detail with underwater imaging or a high resolution sonar. Thus, the vehicle needs to adapt its trajectory to the feature's shape to provide full sensor coverage [Walter, 2008]. Another application for AUVs is tracking. Unlike mapping applications in which the area of interest is usually static and the vehicle trajectory can often be entirely preprogrammed, tracking applications require a higher level of vehicle autonomy. A tracking application for AUVs is described in German & Yoerger [2008].

Self-localization of an underwater vehicle is particularly challenging due to the absence of Global Positioning System (GPS) reception or features at known positions that could otherwise have been used for position computation. Inertial navigation system (INS) gives accurate position information for a short period of time, while the bias error accumulates with time. This accumulation leads to a very large position error. Thus, AUV applications typically require the pre-deployment of a set of beacons but the use of static beacons in the form of a LBL array limits the operation area to a few square kilometers and requires a substantial deployment effort before operations, especially in deep water [Bahr & Leonard, 2006].

The concept of underwater navigation using range measurements is motivated by the needs of small autonomous underwater vehicles, such as those described in Hobson & Schulz [2001]. Traditional navigation technologies, such as LBL and high accuracy inertial navigation systems, are prohibitively difficult to use for small vehicles due to power and volume constraints. Navigation based on range measurements has the potential to reduce both hardware complexity and deployment time, and it may enable other navigation objectives, such as the navigation of small AUVs, relative to a larger AUV. Underwater navigation based on range measurements has also been considered in Vaganay & Baccou [2000] and Baccou & Jouvencel [2003], in which other potential uses are discussed, such as multiple-vehicle operations, including leader-follower formations and large area surveys.

In this paper, we propose two scenarios in which the members of a group of AUVs exchange navigation information with one another so as to improve their individual position estimates. We describe how cooperation can improve the performance of self-localization. Intra-vehicle communication is crucial to cooperation, so all robots should be equipped with communication devices that allow the exchange of information within the group.

In the following section, we outline previous approaches to the group localization problem. In Sec. 3, we specify the cooperative navigation problem for AUVs. Section 4 presents the kinematic model of an AUV. In Sec. 5, we precisely formulate the Range-only cooperative localization problem. In Secs. 6 and 7, we design a communication channel and use a modulation technique for data transmission. Section 8 presents the simulation results from the application of the proposed algorithm to the case of three AUVs. In Sec. 9, we compare the efficiency of proposed scenarios. Finally, Sec. 10 derives the conclusions from this cooperative localization approach.

2. Related Works

A significant amount of research has been done in cooperative localizing for robots. Kurazume and Nagata [Kurazume & Hirose, 1994–1996] acknowledge that dead reckoning (DR) method is not reliable when a robot travels long distances due to the error accumulation. They proposed a method for cooperative positioning (estimation of position only, no heading).

A set of ground robots was divided into two groups. One group remains stationary and acts as a landmark while the other group moves and performs positioning based on relative position measurement from the stationary group. After a set amount of time, the groups exchange roles. By repeating this process of movement and stoppage, the robot group works towards its destination.

Another variation of cooperative localization is described in Rekleitis & Dudek [1997–1998]. The authors deal with the multi-agent exploration of an unknown environment that improves the quality of the map by reducing the inaccuracies that occur over time from dead reckoning errors. They use two mobile robots, one of which is equipped with a camera tracking system that allows it to determine its relative position and orientation with respect to a second robot carrying a helix target pattern and acting as a portable landmark.

A similar realization is presented in Grabowski & Navarro [2000]. A team of heterogeneous, centimeter-scale robots collaborate to map and explore unknown environments. Sonar-based distance measurements have been used to determine the positions of all the robots in the group. An implementation of a collaborative multirobot localization schema is presented in Fox & Burgard [1999]. The authors have extended the Monte Carlo localization algorithm to the case of two robots when a map of the area is available to both robots.

Finally, a distributed kalman filter implementation of collaborative localization schema is described in Roumeliotis & Bekey [2002]. A group of mobile robots are capable of sensing one another. Each of the robots collects sensor data regarding its own motion and shares this information with the rest of the team during the update cycles. A single kalman filter estimator processes the available positioning information from all the members of the team and produces a pose estimate for every one of them. The equations for this centralized estimator is written in a decentralized form, thereby allowing this single Kalman filter to be decomposed into a number of smaller communicating filters. This distributed form of the filter has reduced computational and communication requirements and it can be realized in the case of large teams of robots.

Attained accuracy by cooperative localization may not be sufficient for certain applications. An alternative approach is for the robots to localize while concurrently building a map of the environment, in which case the uncertainty in their position estimates remains bounded [Fenwick & Newman, 2002].

This introduces the problem of Cooperative Simultaneous Localization And Mapping (C-SLAM). An implementation of C-SLAM is described in Mourikis & Roumeliotis [2006]. A heterogeneous team of mobile robots navigating within a 2D environment are populated with point features. Another variation of this approach is described in Mauro & Garulli [2003], Bryson & Sukkarieh [2006] and Tong & Yalou [2008]. With SLAM as a well-established methodology, the main question is how to compute or approximate the estimate efficiently. The treemap SLAM algorithm proposed in Frese [2006] works by dividing the map into a hierarchy of regions represented as a binary tree. With this data structure, the computations necessary for integrating a measurement are limited essentially.

Pose graph is another approach for solving SLAM problem. A pose graph is a set of robot poses connected by nonlinear constraints. In 2006, Olson *et al.*, presented a novel approach to solve the graph-based SLAM problem by applying stochastic gradient descent to minimize the error introduced by constraints. Tree-based network Optimizer (TORO) is an extension of Olson's algorithm. It applies a tree parameterization of the nodes in the graph that significantly improves the performance and enables a robot to cope with arbitrary network topologies [Grisetti & Stachniss, 2009].

Sparse Pose Adjustment (SPA2d) by Konolige *et al.* provides a fast implementation of the graph-based SLAM [Kurt & Giorgio, 2010]. HOG-Man by Grisetti *et al.* focuses computation on affected regions using a hierarchical approach, which provides an approximate solution [Grisetti & Kummerle, 2010]. In Kim & Kaess [2010], the authors handle incremental smoothing and mapping (iSAM) to get an exact incremental solution to the SLAM problem by solving a full nonlinear optimization problem in real-time.

In the case of underwater navigation, several forms of cooperative behaviors for AUVs have been proposed. The GPS signals do not penetrate sub-sea, underground, or even indoors. Therefore, typical methods for underwater navigation have focused on beacon-based navigation networks, such as the LBL acoustic systems, which offer bounded error position measurements, but require the pre-deployment and calibration of the beacon network. A formation-flying control algorithm for AUVs is described in Edwards & Bean [2004]. The algorithm employs a variant of the leader-follower type strategy to maintain a fixed geometrical formation while navigating mission waypoints. A leader vehicle navigates the mission waypoints using acoustic LBL measurements of position. Each follower vehicle maintains its place in formation using acoustic LBL measurements of inertial position and knowledge of the leader vehicle position.

Another implementation of LBL-based cooperative navigation is described in Bradley & Douglas [2005]. A leader vehicle performs tasks with a fleet of follower vehicles. The lead vehicle would navigate conventionally using acoustic LBL transponders. Each follower vehicle would intercept acoustic navigation signals from the leader and the LBL transponders with a two-hydrophone sensor. The problem of localization for AUVs can be viewed in the more general context of SLAM. A large body of literature addresses the SLAM Problem for cooperative navigation of AUVs. In Eustice & Pizarro [2004], a methodology is described for incorporating camera-based relative pose measurements with AUV navigation data in a SLAM based context. A similar realization is presented in Walter & Leonard [2004] and Diosdado & Ruiz [2007] that use feature-based SLAM for cooperative navigation.

3. Problem Statement

In this section, we specify the AUV group localization problem which this paper addresses. First, we state the following assumptions:

(i) A group of independent AUVs move in an underwater environment by specified trajectories. The motion of each robot is described by its own nonlinear equations of motion.
(ii) Each AUV is assumed to have an inertial measurement unit (IMU) and a sonar sensor (a range-only sensor) that allow it to detect and identify other moving robots and determining their relative distance. A typical IMU contains accelerometers and gyroscopes that provide information about the motion of a moving body. In addition, we assume that AUVs have a depth sensor which allows them to determine their absolute depth.
(iii) It is also assumed that there exists a communications channel over which sensor data is shared.

Our approach to cooperative localization utilizes a heterogeneous team of AUVs to achieve improved localization accuracy without requiring an LBL net (or landmarks in C-SLAM method) or each vehicle is equipped with extra sensors. Here, we consider a group of three AUVs that are moving in their own trajectories with different acceleration and speed. The robots are equipped with a cheap and limited set of sensors, both for IMU as well as sonar. Additionally, their dynamics may be poorly understood resulting in models which are relatively inaccurate. As a result of these deficiencies, robots are prone to error when navigating individually in large environments. Through the use of an Extended Kalman Filter (EKF), we consider a state estimation scheme based upon inter-vehicle measurements and communication. With two AUVs capable of performing data fusion by EKF, the corresponding localization precision can then be exploited to obtain more accurate estimates of pose for each vehicle in the group.

In this paper, we consider two scenarios for cooperative localization of three AUVs. In the first scenario, we divide the robots into two groups. One of the robots, say LR, remains stationary and other two robots use it as landmark. After a set amount of distance (relate to the capability of sonar sensors), moving robots stop and LR moves. The position of LR is estimated by stationary robots. The rear thruster and the through-body thrusters are used to maintain a stationary position for AUVs [Evans & Nahon, 2004]. In the second scenario, it is not necessary that robot LR be stationary and that this robot acts as a portable landmark. Other robots use

LR as landmark and they estimate the position of LR simultaneously.

4. Kinematic Model of AUV

Since the z-axis of an underwater vehicle is easily instrumented using a depth sensor, we simplify the notation and consider motion only in the horizontal plane. As is the standard for navigation algorithms, the system dynamics utilized herein are based on a kinematic model of the AUV. So we use a simplified two-dimensional strapdown navigation System for AUVs. Consider the scenario presented in Fig. 1. The vehicle contains two accelerometers and a single axis rate gyroscope, all of which are attached rigidly to the body of the vehicle. The full set of two-dimensional strapdown navigation system equations are [Titterton & Weston, 2004]:

$$\begin{aligned} \dot{\psi} &= \omega_{zb} + w_z, \\ \dot{v}_{xi} &= (f_{xb} + w_x)\cos\psi - (f_{yb} + w_y)\sin\psi, \\ \dot{v}_{yi} &= (f_{xb} + w_x)\sin\psi - (f_{yb} + w_y)\cos\psi, \\ \dot{x}_i &= v_{xi}, \\ \dot{y}_i &= v_{yi}, \end{aligned} \tag{1}$$

where the measurements in the body axes of the vehicle in the plane of motion are denoted by the subscript b. It is assumed that navigation is required to take place with respect to a space-fixed reference frame. The measurements in this frame are denoted by the subscript i. ω_{zb} is the measured angular rate and f_{xb} and f_{yb} are the measured specific forces. w_x, w_y and w_z are dynamic noise of sensor measurements. v_{xi} and v_{yi} are the vehicle velocity and x_i and y_i are the position of vehicle. We describe a discrete time state model for vehicle using Eq. (1).

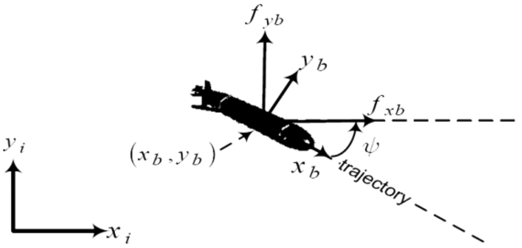

Fig. 1. The AUV states involving in strapdown navigation system.

Assuming that the control inputs at time k are $\mathrm{u}(k) = [\omega_{zb}(k) f_{xb}(k) f_{yb}(k)]$ and the states of system are $\mathrm{x}(k) = [\psi(k) v_{xi}(k) v_{yi}(k) x(k) y(k)]$, the discrete equivalent of Eq. (1) in general form can be written as:

$$\mathrm{x}(k) = \mathrm{f}(\mathrm{x}(k-1), \mathrm{u}(k), w(k)). \tag{2}$$

This model is used for all robots.

5. Range-Only Cooperative Localization

The problem which we consider in this paper is a group of AUVs operating with different acceleration, speed and constant depth. The key to any successful cooperative navigation strategy is the incorporation of the relative positioning between the vehicles.

5.1. *Cooperative positioning*

In this section we propose a cooperative positioning technique with multiple robots. Instead of using landmarks in the environment, this technique uses one of the robots as a landmark. We name this robot LR. Consider the scenario presented in Fig. 2. R1 and R2 use LR as landmark to obtain improved estimates of their location and extract useful navigation information from the data returned by their sensors. Using the position (x_1, y_1) and (x_2, y_2) of robots R1 and R2 as well as measured range r_1 and r_2 of these robots, Eq. (3) and (4) give the position of robot LR:

$$(x_{LR} - x_1)^2 + (y_{LR} - y_1)^2 = r_1^2, \tag{3}$$

$$(x_{LR} - x_2)^2 + (y_{LR} - y_2)^2 = r_2^2. \tag{4}$$

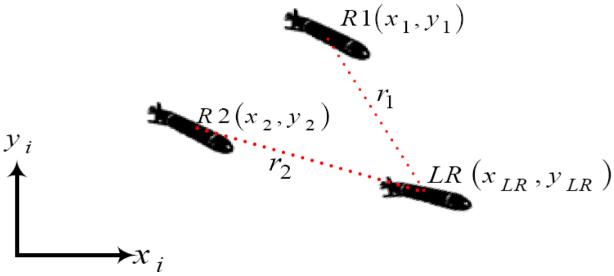

Fig. 2. Positioning in plane using range only measurements.

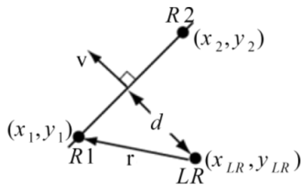

Fig. 3. The distance from LR to the line between R1 and R2.

We should solve Eqs. (3) and (4) concurrently, but this solution provides two answers. Here, we propose a method to select one of these answers.

Consider the line that is specified by the position of R1 and R2. Now, consider the distance from a robot LR to the line such as presented in Fig. 3.

A vector perpendicular to the line is given by:

$$v = \begin{bmatrix} y_2 - y_1 \\ -(x_2 - x_1) \end{bmatrix}. \tag{5}$$

Let r be a vector from the LR position (x_{LR}, y_{LR}) to the R1 position (x_1, y_1):

$$r = \begin{bmatrix} x_1 - x_{LR} \\ -(y_1 - y_{LR}) \end{bmatrix}. \tag{6}$$

Then, the distance from (x_{LR}, y_{LR}) to the line is given by projecting r onto v:

$$d = |\mathrm{proj_v r}|. \tag{7}$$

We use absolute value of d because it could not take a negative value. The relative position between LR and the line determines the sign of d. Based on the sign of d, we can find that the LR is located in which specific side of the line. This means that we can select one of the answers that we have by solving Eqs. (3) and (4).

We execute an estimator in the form of extended kalman filter (EKF) on R1. The EKF has been used as a mechanism by which the specified position of LR and the measurements gathered by Inertial Navigation System (INS) of R1 and R2 are consistently fused to yield bounded estimate of the robots location. So, we have done an accurate navigation for R1 and R2. Since R1 and R2 know their exact location, they can act as an artificial GPS reference and determine the position of LR. In our algorithm, LR did not have any external sensor and its position is determined by other robots.

5.2. *Two EKFs in cooperative localization*

Again, consider the presented scenario in Fig. 2. Range-only cooperative localization process can be accomplished using two Extended Kalman Filters. First EKF is used in R1. In this case, observations are measured distance of LR relative to R1 and R2. The observations for second EKF that is used in LR are the calculated position of LR using measurements r_1 and r_2. This calculation is accomplished in R1 and subsequently sent to LR.

5.3. *First EKF equations*

The goal of first EKF is to treat the centralized Kalman filter equations on R1 so as to distribute the estimation process information among robots. Here, we will derive the equations for a group of three robots. The robots travel through the environment using their sensors to observe each other. The state of the system at time k can therefore be represented by the augmented state vector $\mathrm{X}(k)$, consisting of the states $\mathrm{x}_{R1} = [\psi_1 v_{x_1} v_{y_1} x_1 y_1]$ representing R1, states $\mathrm{x}_{R2} = [\psi_2\, v_{x_2}\, v_{y_2}\, x_2\, y_2]$ representing R2 and states $\mathrm{x}_{LR} = [x_{LR}\, y_{LR}]$ representing the position of LR:

$$\mathrm{X}(k) = \begin{bmatrix} \mathrm{x}_{R1}(k) \\ \mathrm{x}_{R2}(k) \\ \mathrm{x}_{LR}(k) \end{bmatrix}. \tag{8}$$

The augmented state vector gives rise to the augmented state estimate of the current vehicle state estimate $\hat{\mathrm{X}}^+(k)$. The covariance matrix for this state estimate is defined through $\mathrm{P}^+(k)$. For the case of first EKF, the covariance matrix takes on the following form:

$$\mathrm{P}^+(k) = \begin{bmatrix} \mathrm{P}^+_{R1R1}(k) & \mathrm{P}^+_{R1R2}(k) & \mathrm{P}^+_{R1LR}(k) \\ \mathrm{P}^+_{R1R2}(k) & \mathrm{P}^+_{R2R2}(k) & \mathrm{P}^+_{R2LR}(k) \\ \mathrm{P}^+_{R1LR}(k) & \mathrm{P}^+_{R2LR}(k) & \mathrm{P}^+_{LRLR}(k) \end{bmatrix}, \tag{9}$$

where $P^+_{R1R1}(k)$, $P^+_{R2R2}(k)$ and $P^+_{LRLR}(k)$ represent each of robots covariance and $P^+_{R1R2}(k)$, $P^+_{R1LR}(k)$ and $P^+_{R2LR}(k)$ are the cross-covariance between robots.

5.3.1. *Prediction*

The prediction stage of the filter uses the model of the motion of R1 and R2 robots defined in Eq. (2) to generate an estimate of these robots position:

$$\begin{bmatrix} \hat{x}^-_{R1}(k) \\ \hat{x}^-_{R2}(k) \end{bmatrix} = \begin{bmatrix} f_{R1}\left(\hat{x}^+_{R1}(k-1), \quad u_{R1}(k), w(k)\right) \\ f_{R2}\left(\hat{x}^+_{R2}(k-1), \quad u_{R2}(k), w(k)\right) \end{bmatrix}. \tag{10}$$

Since LR remains stationary when R1 and R2 are moving in first scenario, the prediction model for LR is given by:

$$\hat{x}^-_{LR}(k) = \hat{x}^+_{LR}(k-1). \tag{11}$$

Together, these two models result in the propagation of the augmented state matrix during the prediction cycle of the filter:

$$\hat{X}^-(k) = F(\hat{X}^+(k-1), U(k), w(k)), \tag{12}$$

where $\hat{X}(k)$ is defined in Eq. (8) and $U(k) = [u_{R1}(k)\ u_{R2}(k)]^T$.

The covariance matrix must also be propagated through the robots model as part of the prediction:

$$\begin{aligned} P^-(k) = {} & \nabla_X F(k) P^+(k-1) \nabla_X F^T(k) \\ & + \nabla_U F(k) Q(k) \nabla_U F^T(k), \end{aligned} \tag{13}$$

where $\nabla_X F(k)$ and $\nabla_U F(k)$ are Jacobian of F evaluated around $\hat{X}^+(k-1)$ and current input vector $U(k)$, respectively. $Q(k)$ is the covariance of input vector (dynamic noise of IMU measurements).

5.3.2. *Observation*

The fusion of the observation into the state estimate is accomplished by first calculating a predict observation, $\hat{z}^-(k)$, using the observation model, h as:

$$\hat{z}^-(k) = h(\hat{X}^-(k)). \tag{14}$$

Innovation and its covariance in EKF are computed from the current state estimate and its covariance:

$$v(k) = z(k) - \hat{z}^-(k)), \tag{15}$$

$$S(k) = \nabla_X h(k) P^-(k) \nabla_X h^T(k) + R(k), \tag{16}$$

where $\nabla_x h(k)$ is Jacobian of h evaluated around $\hat{X}^+(k-1)$. $R(k)$ is the covariance of observation noise (dynamic noise of sonar measurements).

5.3.3. *Update*

The state estimate can be updated using the following equations:

$$\hat{X}^+(k) = \hat{X}^-(k) + K(k)v(k), \tag{17}$$

$$P^+(k) = P^-(k) - K(k)S(k)K^T(k), \tag{18}$$

where

$$K(k) = P^-(k)\nabla_X h^T(k) S^{-1}(k). \tag{19}$$

5.3.4. *LR position initialization*

Position estimate of LR and its initial covariance must be properly initialized and added to the state vector when LR is observed by R1 and R2 initially. Given an initial position of R1 and R2, a relative observation between the R1 and R2 and LR is done. A feature initialization model, $g(k)$, maps the current robot state estimate and observation to a LR estimate. The initial estimate of the LR position is:

$$\hat{X}^+_{LR}(k) = g(\hat{X}^-_{R1}(k), \hat{X}^-_{R2}(k), z(k)). \tag{20}$$

And its initial covariance is:

$$P0_{LR} = MP0_{R1,R2}M^T + NRN^T, \tag{21}$$

where $P0_{R1,R2}$ is the initial covariance of R1 and R2 states, M and N are Jacobian of g evaluated around augmented state vector and observations, respectively. We do LR position initialization only at the beginning of navigation. While continuing in motion, the LR submits its position to R1 and R2.

The essential steps of filtering are presented in Fig. 4.

5.4. ***Second EKF***

Using the first EKF as described in previous section, we will have an accurate navigation for

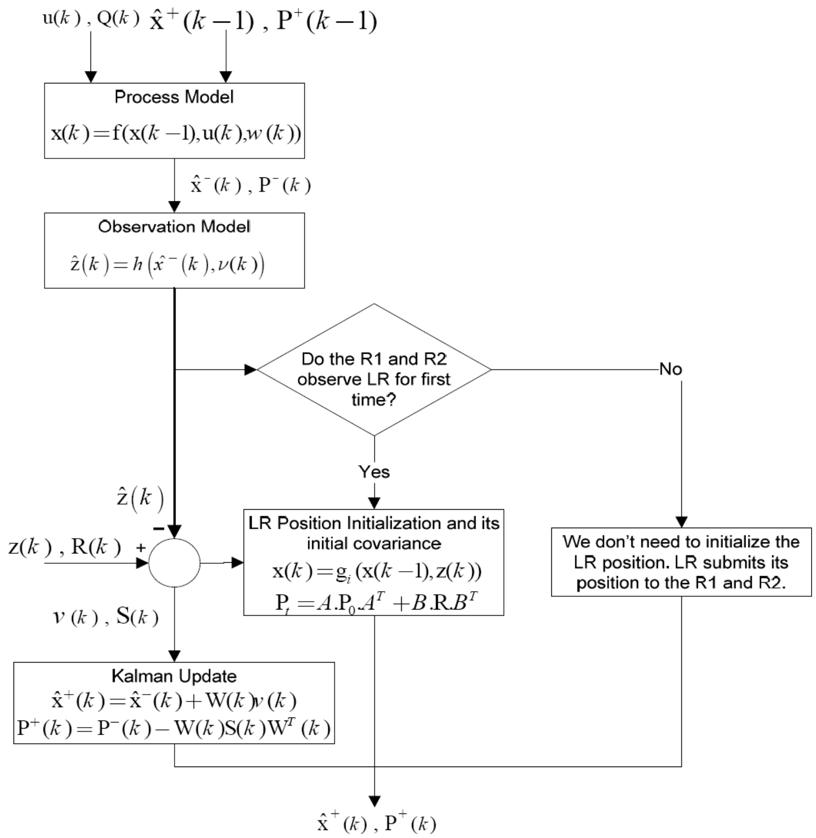

Fig. 4. The essential steps of the filtering process.

R1 and R2. Therefore, these robots have an exact estimate of their position. So we can use them as an artificial GPS to determine the position of LR. Using the cooperative positioning approach described in Sec. 5.1, the position of LR is calculated by R1 and is submitted to LR. A second EKF is executed on the R1 to fuse the INS data of LR and the position measurement that LR receives from R1. This procedure is represented in Fig. 5. In a similar manner, this filter is accomplished in three stages.

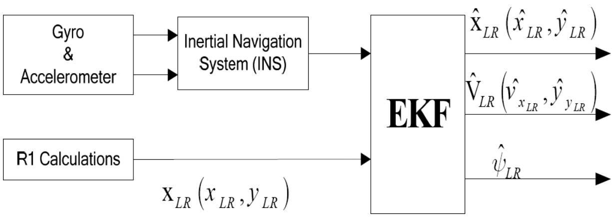

Fig. 5. Second EKF operation.

6. The Underwater Communication Channel

The major constraint in cooperative localization of submarines and AUVs is the extremely complex and continuous variation of the sea. The underwater communication channel is limited by two well define interfaces, the bottom of the sea and the sea surface. Figure 6 depicts direct and major reflected paths between a transmitter and receiver.

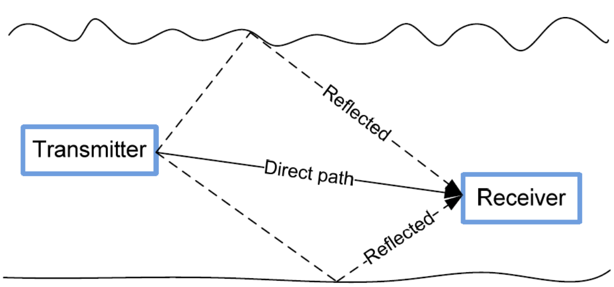

Fig. 6. Multipath in the underwater communication channel.

Rayleigh and Rician fading channels are useful models of real-world phenomena in wireless communications. These phenomena include multipath scattering effects, time dispersion and Doppler shifts that arise from relative motion between the transmitter and receiver. The underwater communication channel model we will use in our simulations is the Rayleigh multipath fading. MATLAB communication toolbox is used for designing the channel. We construct a frequency-selective (multi-path) fading channel that each discrete path in this channel is an independent Rayleigh fading process. Parameters of the Rayleigh channel are the maximum Doppler shift f_d in hertz, path delay t_m and average path gain p_{db} specified in dB. The measurement examples in Grythe & Hakegard [2009] are used to describe the properties of the channel. Selected values are, $f_d = 1\,\mathrm{Hz}$, $t_m = 0.4\,\mathrm{ms}$ and $p_{db} = 1$.

Communication using acoustic modems such as the WHOI Micromodem is at a very low rate (as low as 32 bytes per 10 seconds) with a range of several kilometers in open water [Fallon & Leonard, 2010]. Our approach will utilize 72 bit per second bit rate for communication between robots which uses Differential Phase Shift Keying (DPSK) to transmit small packets of information.

The value of a channel object's t_m property is the number of samples by which the output of the channel lags the input. Compare the input and output data sets directly, we must take the delay into account by using appropriate approaches such as truncating operation. In this approach, we omit the last delayed symbols of the input data and the first delayed symbols of the demodulated signal. Therefore, we can compare the input and output data sets directly.

7. DPSK Modulation Technique

Ordinarily, the transmission of a message signal over an underwater communication channel requires a shift of frequencies contained in the signal into other frequency ranges suitable for transmission (modulation), and a corresponding shift back to the original frequency range after reception (demodulation). In our simulations, we will use DPSK digital modulation technique. In DPSK, we change the phase of the sinusoidal carrier to indicate the information. We will combine an additive white Gaussian noise (AWGN) with fading channel. In AWGN, we specify signal-to-noise ratio (SNR). The SNR expresses the relative importance of the power contribution of the expected signal and the perturbing noise.

8. Results

In this section we present simulation results to validate the theory developed in the previous sections. For our simulation, we have placed three AUVs in the large environment. The three AUVs start from three different locations and move within the same area. An example of the trajectories followed by three robots is shown in Fig. 7.

We evaluate two different scenarios that are discussed in this paper. Simulation is performed using the MATLAB software. The filter parameters for the simulation and characteristics of the sensors are shown in Tables 1 and 2 respectively.

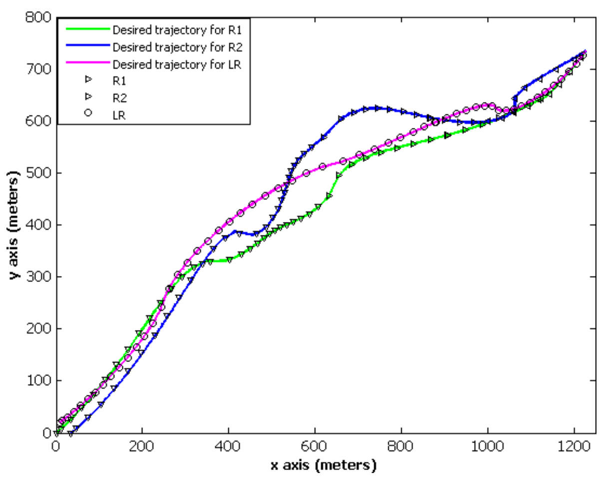

Fig. 7. Trajectories of three robots moving within the same area.

Table 1. Simulation filter parameters.

Sampling period	$\Delta t(k)$	0.01 sec
Initial position std dev of AUVs	σ_x, σ_y	1 m
Initial Velocity std dev of AUVs	$\sigma_{V_x}, \sigma_{V_y}$	0.3 m/s
Initial Heading std dev of AUVs	σ_ψ	0.02 rad

Table 2. Sensor errors.

Sensor	Noise std dev
Rate Gyro	0.1 rad/s
Accelerometer	$0.6\,\mathrm{m/s^2}$
sonar	0.3 m

We suppose that all of the robots are carrying the same sensors. The sonar can scan continuously through 360° of rotation and its maximum range is 100 m. The initial heading of the AUVs is chosen as 30°. The same simulator and trajectory are used for two scenarios.

Scenario 1

As we discussed in Sec. 3, robots track their position by repeating the move-and-stop actions in this scenario. Our results consist of two main figures. Both graphs are error plots in x, y and ψ for three AUVs.

Figure 8 presents the recorded error along the x coordinate for R1, R2 and LR and the 3σ region of confidence for this error.

Similarly, Fig. 9 presents the recorded error along the y coordinate for robots.

Finally, the error in the estimate of the robots heading angle is shown in Fig. 10.

Navigation errors are reduced when the LR remains stationary and other robots use it as landmark. For example, as it is shown in Fig. 11, in the part of trajectory, the R1 reduces the

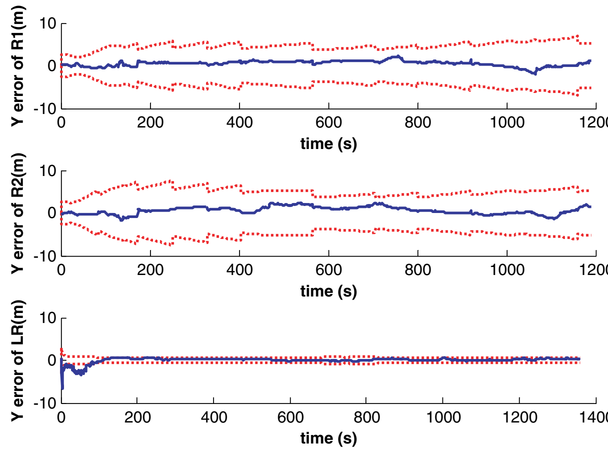

Fig. 9. Position y error of AUVs.

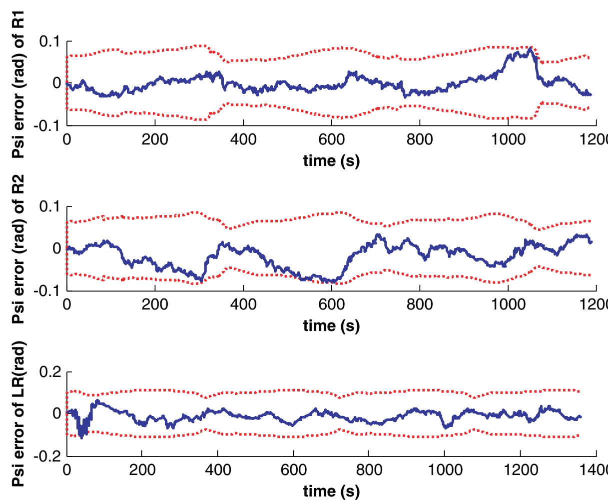

Fig. 10. Heading ψ error of AUVs.

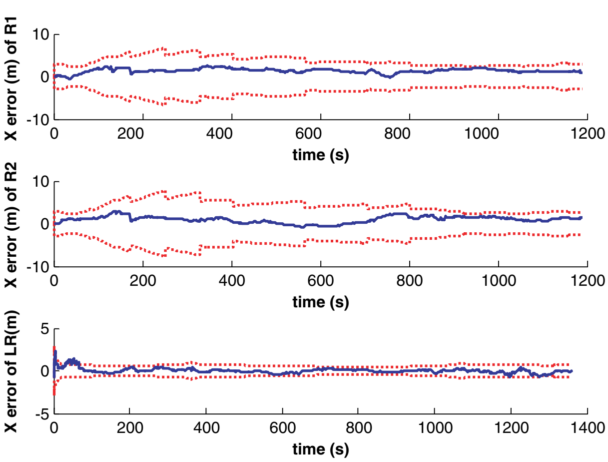

Fig. 8. Position x error of AUVs.

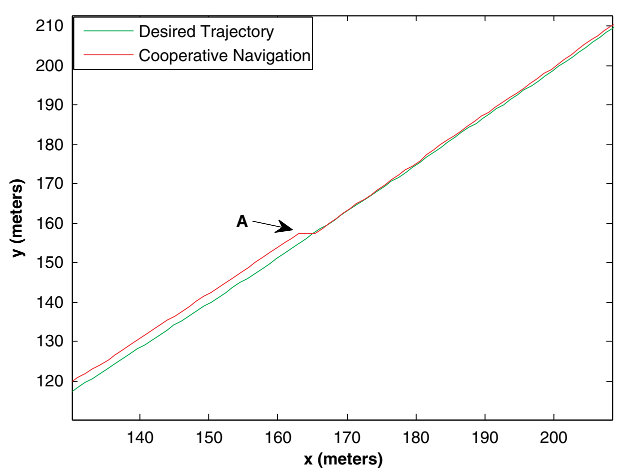

Fig. 11. Part of the trajectory of R1. LR remains stationary when R1 is at location A, B and C.

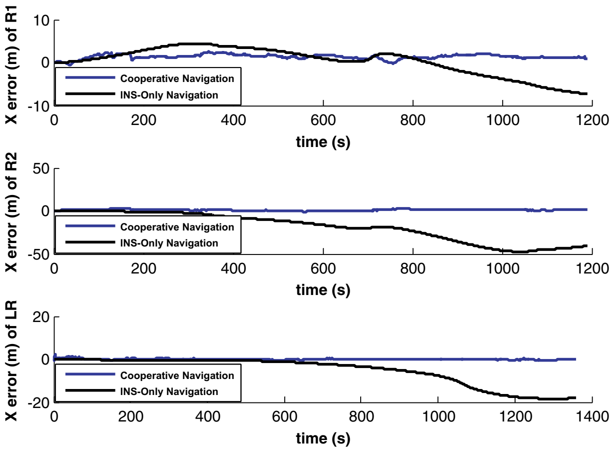

Fig. 12. Comparison between cooperative navigation and INS-Only navigation.

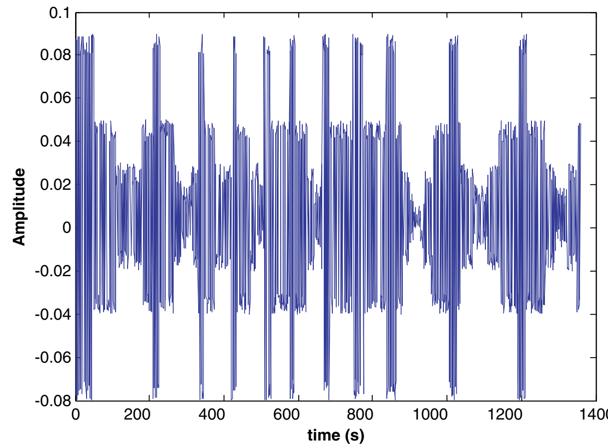

Fig. 14. The effect of communication channel on transmitted signal in the time domain for bit rate of 72 bps.

rate of increase in its position uncertainty. At A($t = 165$ s), R1 measures its relative position with respect to the LR to produce better pose estimate for itself.

Figure 12 establishes the improvement of the cooperative navigation by three robots over INS-only navigation. The innovation sequences can be checked to verify that the filter is well-tuned. In a well-tuned filter, innovation should be a zero-mean white signal. Figure 13 shows the autocorrelation of innovation signal in the first EKF. We have two measurements, r_1 and r_2. Therefore, the innovation signal is a $2 \times \mathrm{N}$ matrix.

Figure 14 shows the effect of the communication channel on the transmitted signal. In this figure, we have plotted the difference of transmitted signal with and without considering the communication channel. This signal is the measured distance r_2 that R2 transmit to R1. The effect of channel on other signals can be shown in the same manner.

The bit error ratio (BER) is a very important measure for communications systems. This is because it tells us something about how a certain signal-to-noise ratio (SNR) affects the received data. In Fig. 15, the calculated bit error ratio for the system is plotted against the different values of the SNR.

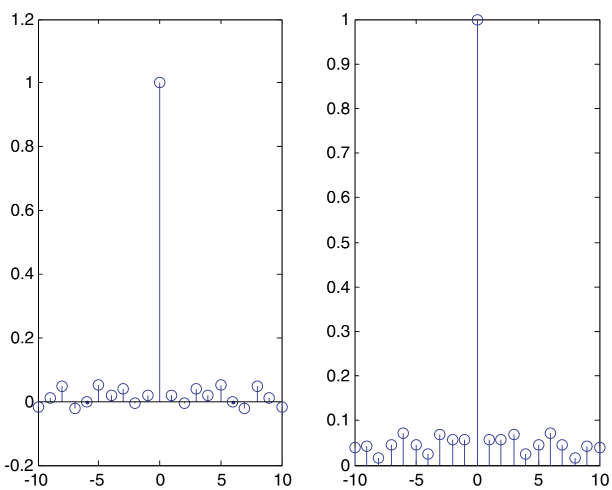

Fig. 13. The innovation sequence. It appears to be zero-mean and white with approximately 90%.

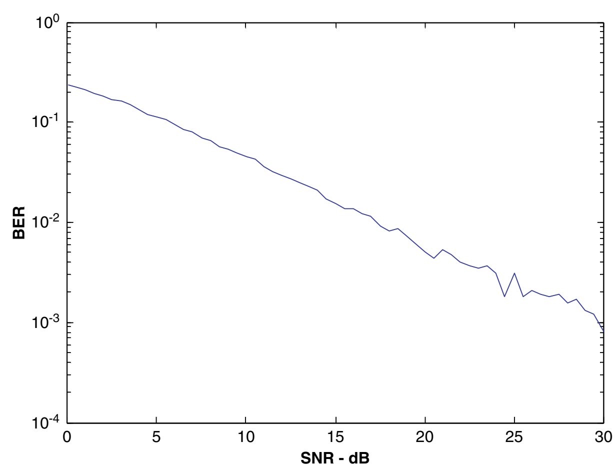

Fig. 15. The bit error ratio as a function of SNR.

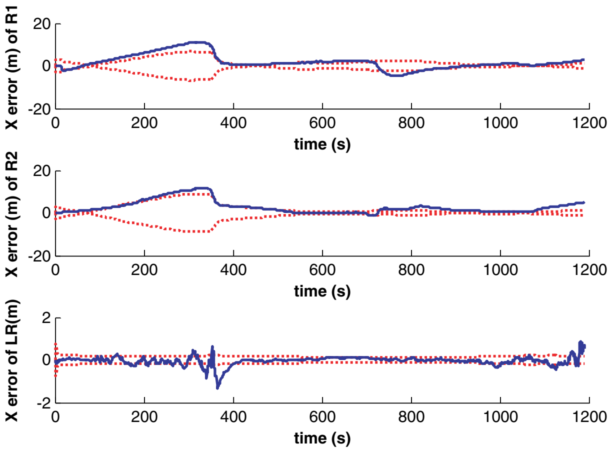

Fig. 16. Position x error of AUVs in scenario 2.

Scenario 2

In this case, R1 and R2 robots, continuously measure the position of LR while moving and simultaneously submit position information to it. When R1 and R2 keep track of the LR, the rate that the position uncertainty grows is higher, as compared with the previous case, where errors exceed the 3σ confidence bounds. Figure 16 shows the error in the estimate of position x of the three robots. Compare Fig. 16 with Fig. 8.

9. Discussion

9.1. *Comparison between Scenario* 1 *and Scenario* 2

At this point, it is worth mentioning why the robots pose estimate in scenario 1 are precise as compared with scenario 2. We can prove that the determinant of the covariance matrix of LR, $\mathrm{P_{LR}}$, is a non-increasing function of the time step [Csorba, 1997]. Each diagonal entry of $\mathrm{P_{LR}}$ is the variance of the position estimate of LR. The variance in each dimension of the estimate of the LR position cannot increase during an update. Further, since the LR acts as a stationary landmark in scenario 1, the prediction stage does not affect the LR estimates. Therefore, the variance of the estimate in each dimension of LR position is a monotonically non-increasing function of the time step:

$$(\sigma^2_{LR\ \mathrm{estimates}}(k))^+ \leq (\sigma^2_{LR\ \mathrm{estimates}}(k))^-. \quad (22)$$

This equation states that in scenario 1, the estimates of LR position are accurate as compared with scenario 2. Therefore, the overall navigation accuracy will be improved.

9.2. *Relevancy of using two separate Kalman filters*

In our architecture, the R1 robot keeps track the state of the R2 only. R1 and R2 robots communicate the range measurements that they measure by their external sensors. So, in the first EKF, we have an augmented vector that is composed of states of these two robots. Therefore, there is a correlation between these robots' positions. If we use a single EKF for cooperative localization, we will have a strong correlation between the states of all AUVs. In order to cope with this issue, we used two separated EKF. The observations for second EKF that is used in LR are the position of this robot that is calculated by absolute position of other robots with known variances and the measured relative distance between them with known uncertainty. Therefore, the correlation between LR's states and other robots' states is being neglected resulting in overconfident position estimates. Figure 17 shows the error in the position estimate of LR when we use single kalman filter for cooperative localization. Compare this figure with Figs. 8, 9 and 16. Note the difference in scale between these figures. Position error and its variance are

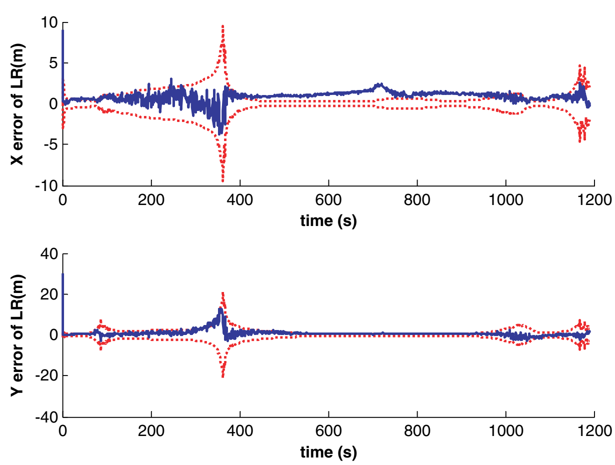

Fig. 17. Position error of LR when we perform cooperative localization with single kalman filter.

higher as compared to the case where we used two separate kalman filter. Therefore, every time robots R1 send out a broadcast to LR. By maintaining a state x^1_{R1} on R1 which is continuously propagated and has not been updated with information from LR, we make sure that a future broadcast from R1 received by LR contains a state which is not cross-correlated with LR and can be used by LR for an update.

For more explanation, we use the correlation coefficient definition to show the high correlation between the robots' states when we use a single EKF for cooperative localization. The correlation coefficient of two random variables is the normalized quantity:

$$\rho_{12} = \frac{\sigma^2_{x_1 x_2}}{\sigma_{x_1}\sigma_{x_2}}, \tag{23}$$

where σ_{x_i} is the standard deviation of x_i. Due to the normalization, the magnitude of the correlation coefficient of any two random variables obeys the following inequality:

$$|\rho_{12}| \leq 1. \tag{24}$$

Two random variables whose correlation coefficient is zero are said to be uncorrelated. At the other extreme, if the correlation coefficient of two random variables has magnitude unity, then it can be shown that they are linearly dependent. Figure 18 depicts the value of correlation coefficient for position x of three AUVs during the mission. It shows a high correlation between them.

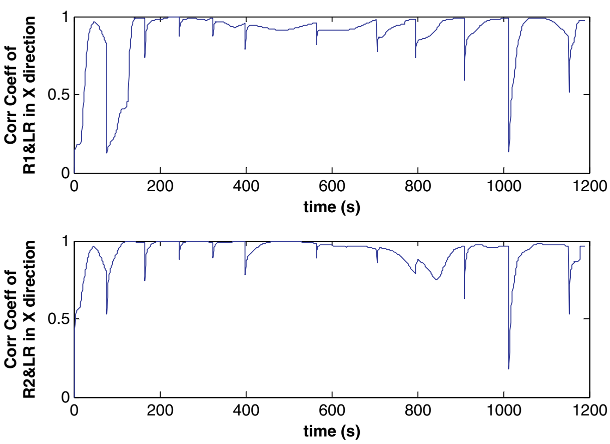

Fig. 18. The value of correlation coefficient for position x of AUVs. It shows the high correlation between robots' states when we use a single EKF for cooperative localization.

9.3. *Comparison with existing methods*

To compare with existing localization methods, the results presented in Walter & Leonard [2004] and Bahr & Walter [2009] are performed under similar conditions, but the dimensions of the operated area is smaller than the area that we considered for our simulation. The proposed method in Walter & Leonard [2004] is performed using a team of three land-based vehicles, and the existing constraints in underwater communication are ignored. Moreover, authors used both range and bearing measurements as observations. Compared with the cooperative localization algorithm in Walter & Leonard [2004], the proposed localization method shows similar performance, although we considered the limitation imposed by the underwater environment. We also used a small number of low-cost sensors.

The cooperative localization method in Bahr & Walter [2009] is an EKF-based approach that only uses range measurements between three underwater vehicles for its update, and cross-correlations are ignored. Simulation result for this approach is shown in Fig. 19.

As we can see in Fig. 19, the error in the position estimate is outside the 3σ bound and the position error grows slowly. Compared with this approach, our proposed method shows better performance. In Sec. 8.1, we prove that position error is reduced when the LR remains stationary and when other robots use it as stationary landmark. Moreover, we considered the strong cross-correlation effects between the states of all

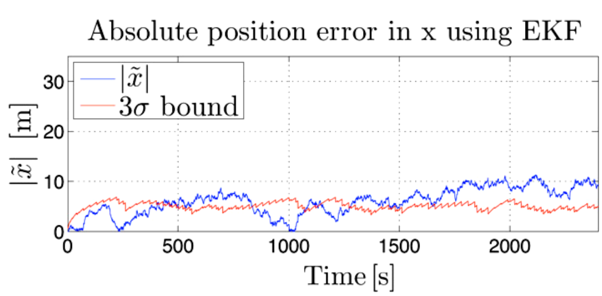

Fig. 19. Simulation result using the method in Bahr & Walter [2009].

AUVs and reduced it by using two separate EKF that is discussed in Sec. 9.2.

10. Conclusion

In this paper, we have proposed an approach for performing cooperative localization with a team of vehicles. All the vehicles are equipped with a low-cost inertial measurement unit. By requiring that only some of the vehicles in the group have an external range-only sensor, it is shown that a combination of inter-vehicle measurements together with data sharing allow each vehicle in the group to navigate with improved accuracy. As range measurements don't determine the state of the robots uniquely, we used point-line distance approach to determine the motion strategies to avoid this ambiguity.

In our approach, position calculations are accomplished in one of the vehicles. We have supposed an Intra-vehicle communication that enables these vehicles to communicate together. Two kalman filter estimators were introduced that combine information collected by group of vehicles. Our approach contains two scenarios for cooperative localization of three AUVs.

We have presented simulation results showing that when the AUVs share their observations, the error in the location of the AUVs are significantly smaller as compared to the errors in the location when AUVs do not share their information. A comparison with the results from robots performing scenario 1 and scenario 2 reveals that the pose estimation error for vehicles when using the first scenario is lower as compared with the second scenario. Moreover, we discussed the relevancy of using two separate Kalman Filter in our approach.

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