Summary of 2D cooperative estimation

Agent-centric coordinates with feature range and bearing

Tim Woodbury

I. Problem formulation

To simplify the problem measurement model, the problem is posed a body-fixed coordinate frame. An inertial vector to agent i is labelled $\mathbf{r}_i$, and the vector from agent i to feature k is $\mathbf{r}_{ki}$. When feature coordinates are known, they are assumed to be given in an inertial reference frame. In the unknown feature case, the agent's task is to estimate the relative position vector $\mathbf{r}_{ki}$; since an inertial reference frame may not necessarily be defined, it is convenient to use the body reference frame for all coordinatizations.

In the known feature case, the agent's task is to estimate its own position vector $\mathbf{r}_i$, its own inertial velocity $\mathbf{v}_i$, and its heading angle ψ. It is preferred to coordinatize the states in a rectangular coordinate system rather than a polar system to simplify the propagation model for the system states. It is simple to write propagation models for the time rate of change of these states in the body frame.

$$\mathbf{r}_i \equiv \begin{bmatrix} r_{ix} & r_{iy} \end{bmatrix}^T \tag{1}$$

$$\mathbf{v}_i \equiv \begin{bmatrix} u & v \end{bmatrix}^T \tag{2}$$

$$\mathbf{a}_i \equiv \begin{bmatrix} a_1 & a_2 \end{bmatrix}^T \tag{3}$$

$$\frac{d \begin{bmatrix} r_{ix} \\ r_{iy} \end{bmatrix}}{dt} = \begin{bmatrix} u + \omega r_{iy} \\ v - \omega r_{ix} \end{bmatrix} \tag{4}$$

$$\frac{d \begin{bmatrix} u \\ v \end{bmatrix}}{dt} = \begin{bmatrix} a_1 + \omega v \\ a_2 - \omega u \end{bmatrix} \tag{5}$$

$$\dot{\psi} = \omega \tag{6}$$

The body-frame relative position vector $\mathbf{r}_{ki}$ can be written in terms of the known inertial coordinates of the landmark $[\mathbf{r}_k]_n$:

$$[\mathbf{r}_{ki}]_b = [C_{b/n}][\mathbf{r}_k]_n - [\mathbf{r}_i]_b \tag{7}$$

$$[C_{b/n}] = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \tag{8}$$

$$[\mathbf{r}_i]_b = \begin{bmatrix} r_{ix} \\ r_{iy} \end{bmatrix} \tag{9}$$

We turn to the measurement model for the agent's own measurements. The expectation of landmark range measurements in terms of the estimated states is simply

$$\hat{\rho}_{ki} = \|[\hat{C}_{b/n}][\mathbf{r}_k]_n - [\hat{\mathbf{r}}_i]_b\| \tag{10}$$

The expectation of the bearing measurements is

$$\hat{\theta}_{ki} = \arctan \frac{\begin{bmatrix} -\sin\psi & \cos\psi \end{bmatrix} [\mathbf{r}_k]_n - \hat{r}_{iy}}{\begin{bmatrix} \cos\psi & \sin\psi \end{bmatrix} [\mathbf{r}_k]_n - \hat{r}_{ix}} \tag{11}$$

I.A. Interagent dynamics

Brief attention is drawn to the interagent dynamics to highlight one of the core challenges of this cooperative estimation problem. Defining the position vector of agents i and j as $\mathbf{r}_i$ and $\mathbf{r}_j$, the relative position vector is simply:

$$\mathbf{r}_{ji} = \mathbf{r}_j - \mathbf{r}_i \tag{12}$$

The time rate of change of the relative position vector in agent i's frame, denoted as frame b^{i+}, is:

$$^{b^i}\frac{d\mathbf{r}_{ji}}{dt} = \mathbf{v}_j - \mathbf{v}_i - \boldsymbol{\omega}_{b^i/n} \times \mathbf{r}_{ji} \tag{13}$$

Ideally, agent i would estimate the position of agent j using relative interagent measurements to update that estimate. Using its onboard sensors, i can estimate its own translational and angular velocity, but cannot propagate the estimated interagent position vector $\mathbf{r}_{ji}$ without knowledge of agent j's velocity. In the case where the agents share IMU data, the velocity $\mathbf{v}_j$ can be estimated and $\mathbf{r}_{ji}$ can be propagated; otherwise, some sort of batch estimation of the time rate of $\mathbf{r}_{ji}$ can be achieved and used to compute the effective velocity of agent j. In the strict case where only sequential estimates are performed without sharing IMU data, $\mathbf{r}_{ji}$ cannot be well estimated. The next section addresses the treatment of shared measurements in the latter case.

I.B. Treatment of cooperative agent measurements without IMU sharing

Measurements made by agent j of feature k are provided to agent i and utilized to improve i's state estimate. It is desired to write the measurement expectation in terms of estimated states only, for simplicity; therefore, we treat the measured states as ρ_{ki} and θ_{ki}, and can write the expectation of these measurements in the same fashion as Eqs. 10 and 11.

For the case of planar motion, trigonometric relationships are used to derive equations relating the measured states, $\tilde{\rho}_{ji}$, $\tilde{\theta}_{ji}$, $\tilde{\rho}_{kj}$, and $\tilde{\theta}_{kj}$. Defining angle $\beta = \theta_{ij} - \theta_{kj}$. ρ_{ki} can be found in terms of a trigonometric relationship:

$$\rho_{ki}^2 = \rho_{ji}^2 + \rho_{kj}^2 - 2\rho_{kj}\rho_{ji}\cos\beta \tag{14}$$

Another relationship yields a solution for θ_{ki}:

$$\theta_{ki} = \pi + \theta_{ji} - \theta_{ij} + \theta_{kj} - \arctan \frac{\rho_{ji}\sin\beta/\rho_{ki}}{\rho_{ji}^2 - \rho_{ki}^2 - \rho_{kj}^2/(-2\rho_{kj}\rho_{ki})} \tag{15}$$

When agents measure both range and bearing to landmarks, Eqs. 14 and 15 are nonlinear functions of the feature and inter-agent measurements and the estimated agent headings. The effective measurements for the cooperative case are nonlinear functions of ρ_{ji}, θ_{ji}, θ_{ij}, ρ_{kj}, and θ_{kj}. It is assumed that agent j provides its bearing measurement of agent i, θ_{ij}, and that the agents have equal bearing variance.

Both Eqs. 14 and 15 depend on the measured range and bearing of feature k from agent j. When feature ranges are not measured, an additional challenge is introduced. For now, both range and bearing are assumed available.

I.B.1. Feature range and bearing measured

When both range and bearing are measured, then the desired output equations are nonlinear functions of the four measured states (two ranges and two bearings) and the estimated headings. The computed measurement $\tilde{\mathbf{y}}$ and its expectation are readily determined. To estimate the effective measurement covariance, the method of statistical linearization employed in unscented Kalman filtering is implemented. For the output vector $\mathbf{y} = f(\mathbf{x}),\ \mathbf{x} \in R^n,\ \mathbf{x} \sim N(0, P_x)$, $2n+1$ sigma vectors $\boldsymbol{\sigma}_i$ are computed as

$$\boldsymbol{\sigma}_0 = \mathbf{x} \tag{16}$$

$$\boldsymbol{\sigma}_i = \mathbf{x} + \gamma\sqrt{P_x},\ \ i = 1, \ldots, n \tag{17}$$

$$\boldsymbol{\sigma}_i = \mathbf{x} - \gamma\sqrt{P_x},\ \ i = n+1, \ldots, 2n \tag{18}$$

$\sqrt{P_x}$ is the ith column of the matrix square root. Outputs corresponding to each sigma vector $\mathbf{y}_i = f(\boldsymbol{\sigma}_i)$ are computed, and the mean and covariance are given by:

$$\bar{\mathbf{y}} = \sum_{i=0}^{2n} W_i^{(m)} \mathbf{y}_i \tag{19}$$

$$P_y = \sum_{i=0}^{2n} W_i^{(c)} (\mathbf{y}_i - \bar{\mathbf{y}})(\mathbf{y}_i - \bar{\mathbf{y}})^T \tag{20}$$

The weights and scaling factors are defined as:

$$\gamma = \sqrt{n + \lambda} \tag{21}$$

$$\lambda = \alpha^2(n + k_f) - n \tag{22}$$

$$W_0^{(m)} = \lambda/(n + \lambda) \tag{23}$$

$$W_0^{(c)} = \lambda/(n + \lambda) + (1 - \alpha^2 + \beta) \tag{24}$$

$$W_i^{(m)} = W_i^{(c)} = 1/(2(n + \lambda)), i > 0 \tag{25}$$

Here, $\alpha = .01$ is used. $\beta = 2$ and $k_f = 0$ are given as conventional values for estimation with Gaussian-distributed $\mathbf{x}$.[1]

P_x is populated using the known sensor errors for the interagent and feature range/bearing, and using the estimate covariance for ψ_i and ψ_j. The matrix is diagonal in this formulation.

I.B.2. Additionally

The measurement gradients are computed in MATLAB using symbolic variables, and the resulting expressions are used in the EKF. For speed, the EKF is implemented as fully discrete, using a first-order difference approximation for the derivative of each state.

II. Simulation results

Two sets of batch simulations are conducted. In the first, the effect of using shared measurements is contrasted with taking additional feature measurements. Performance is compared as sensor uncertainty is decreased. In the second set, a larger set of agents with a sparse set of known features is considered.

Case	Agent	$S(\epsilon_{rix})$	$S(\epsilon_{riy})$	$S(\epsilon_u)$	$S(\epsilon_v)$	$S(\epsilon_\psi)$	$MSE(r_{ix})$	$MSE(r_{iy})$	$MSE(u)$	$MSE(v)$	$MSE(\psi)$
Individual	1	0.794	0.681	0.183	0.181	0.0266	0.644	0.498	0.0365	0.0327	0.000796
Cooperative	1	0.967	0.819	0.201	0.192	0.035	0.952	0.739	0.0445	0.0371	0.00123

Table 1. Sharing versus additional measurements with larger sensor variance.

Case	Agent	$S(\epsilon_{rix})$	$S(\epsilon_{riy})$	$S(\epsilon_u)$	$S(\epsilon_v)$	$S(\epsilon_\psi)$	$MSE(r_{ix})$	$MSE(r_{iy})$	$MSE(u)$	$MSE(v)$	$MSE(\psi)$
Individual	1	0.488	0.456	0.191	0.154	0.0144	0.238	0.209	0.0363	0.0239	0.000207
Cooperative	1	0.528	0.478	0.167	0.149	0.0162	0.279	0.231	0.028	0.0226	0.000261

Table 2. Sharing versus additional measurements with smaller sensor variance.

Case	Agent	$S(\epsilon_{rix})$	$S(\epsilon_{riy})$	$S(\epsilon_u)$	$S(\epsilon_v)$	$S(\epsilon_\psi)$	$MSE(r_{ix})$	$MSE(r_{iy})$	$MSE(u)$	$MSE(v)$	$MSE(\psi)$
Individual	1	2.9	3	0.305	0.307	0.602	8.42	9.02	0.0933	0.0942	0.378
Cooperative	1	1.13	1.07	0.213	0.217	0.0841	1.27	1.16	0.0453	0.0472	0.0185
Individual	2	2.24	2.25	0.292	0.293	0.205	5.03	5.09	0.0852	0.086	0.109
Cooperative	2	0.88	0.806	0.211	0.202	0.11	0.783	0.665	0.0448	0.0412	0.0122
Individual	3	1.98	1.88	0.281	0.278	0.11	3.9	3.63	0.0792	0.0788	0.0242
Cooperative	3	0.83	0.739	0.222	0.21	0.119	0.698	0.546	0.0499	0.0443	0.0156
Individual	4	1.73	1.69	0.277	0.272	0.0968	2.98	2.91	0.0768	0.0746	0.0239
Cooperative	4	0.857	0.829	0.217	0.212	0.118	0.737	0.691	0.0472	0.0455	0.0139
Individual	5	6.32	6.13	0.462	0.46	0.124	40	37.6	0.215	0.211	0.0228
Cooperative	5	0.953	1.15	0.3	0.325	0.119	0.915	1.37	0.0903	0.107	0.0336
Individual	6	2.15	2.07	0.293	0.293	0.112	4.7	4.3	0.0864	0.0862	0.0274
Cooperative	6	0.763	0.792	0.196	0.202	0.0977	0.593	0.629	0.0384	0.0407	0.0102

Table 3. Sharing versus additional measurements with range variance of 1.

II.A. Shared versus additional measurements

To validate results, the case of two agents sharing feature measurements is compared against one agent observing twice as many features. 100 Monte Carlo simulations of 100 seconds each are performed. Two scenarios with different sensor variance levels are considered. In the first, both feature and interagent range have variance 1, and feature and interagent bearing measurements have variance 0.01. Performance is presented in terms of the estimate standard errors S and mean squared errors MSE. Results for the first case are presented in Table 1.

In the second scenario, range variance is reduced to 0.1 and bearing variance to .0001. Results for this case are in Table 2. As would be expected, the single agent with more features outperforms the sharing agents. However, in the case with the lower sensor variance, the accuracy between the two cases is more similar, and it appears that the multi-agent case is converging to the single agent case in the limit as sensor uncertainty is zero.

II.B. Six agents, one features

In the second set of Monte Carlo simulations half a dozen agents with only one visible feature are considered. Performance is compared with range variances all of 1 (Table 3) and all of 10 (Table 4). There is no obvious benefit of sharing at range variances of 10, so this appears to be the upper limit at which feature measurements are usable, unless many more features are present.

References

[1]Gyorgy, K., Kelemen, A., and David, L., "Unscented Kalman filters and Particle Filter methods for nonlinear state estimation," No. 7 in International Conference Interdisciplinarity in Engineering, 2014.

Case	Agent	$S(\epsilon_{rix})$	$S(\epsilon_{riy})$	$S(\epsilon_u)$	$S(\epsilon_v)$	$S(\epsilon_\psi)$	$MSE(r_{ix})$	$MSE(r_{iy})$	$MSE(u)$	$MSE(v)$	$MSE(\psi)$
Individual	1	3.42	3.61	0.317	0.324	0.25	11.7	13.1	0.101	0.106	0.0765
Cooperative	1	3.78	3.81	0.369	0.371	0.379	14.3	14.7	0.138	0.14	0.61
Individual	2	2.91	3.03	0.317	0.322	0.148	8.51	9.23	0.101	0.105	0.0427
Cooperative	2	3.1	3.25	0.379	0.379	0.233	9.59	11	0.144	0.148	0.0788
Individual	3	2.39	2.39	0.309	0.313	0.168	5.75	5.71	0.0954	0.0982	0.0711
Cooperative	3	2.32	2.35	0.325	0.327	0.243	5.41	5.53	0.105	0.107	0.0602
Individual	4	2.48	2.36	0.298	0.293	0.159	6.22	5.65	0.0887	0.0869	0.0324
Cooperative	4	2.72	2.7	0.336	0.339	0.169	7.41	7.44	0.113	0.117	0.0516
Individual	5	4.42	4.43	0.405	0.398	0.144	19.6	19.9	0.164	0.16	0.0214
Cooperative	5	2.02	2.04	0.351	0.362	0.186	4.09	4.15	0.124	0.131	0.0577
Individual	6	3.02	2.93	0.323	0.319	0.111	9.23	8.75	0.104	0.103	0.0269
Cooperative	6	3.17	2.98	0.38	0.381	0.237	10.6	8.91	0.15	0.146	0.0591

Table 4. Sharing versus additional measurements with range variance of 10.