ME-GY 7943
Network Robotics Systems, Cooperative Control and Swarming

Kamal Patel

September 2018

Exercise 1

1. The communication graph is show in the picture below
2. The graph in terms of edges and vertexes are shown below:

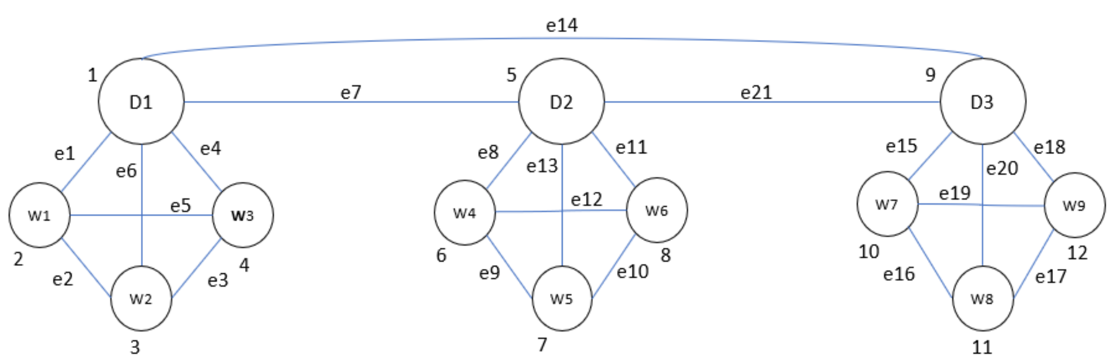

Figure 1:

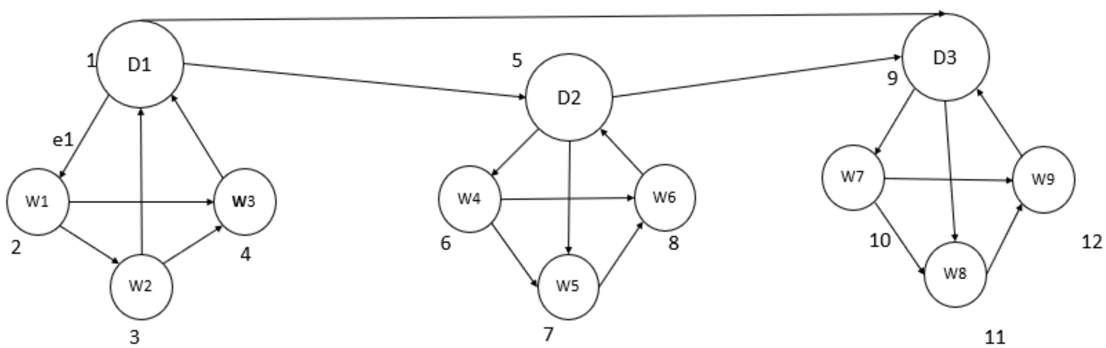

Figure 2:

V = { $\mathrm{d}_1, d_2, d_3, w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8, w_9$}
$E = \{(d_1, w_1), (d_1, w_2), (d_1, w_3), (d_1, d_2), (d_1, d_3), (d_2, w_4), (d_2, w_5), (d_2, w_6), (d_2, w_3), (d_3, w_7),$
$(d_3, w_8), (d_3, w_9), (w_1, w_2), (w_1, w_3), (w_2, w_3), (w_4, w_5), (w_4, w_6), (w_5, w_6), (w_7, w_8), (w_7, w_9),$
$(w_8, w_9)\}$

3. The graph is a undirected graph and connected graph because there is path from one vertex to any other vertex.

4. Yes, there is a path from a mobile robot of the first team to a mobile robot of the second team and third team. The drones can talk to other drones and each drones can talk to their wheel robots so there is path from each wheel robot to other wheel robot.

5. Adjacency, degree and incidence matrices are shown below

   (a) Adjacency Matrix is:

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

(b) Degree Matrix:

$$\begin{bmatrix}5&0&0&0&0&0&0&0&0&0&0&0\\0&3&0&0&0&0&0&0&0&0&0&0\\0&0&3&0&0&0&0&0&0&0&0&0\\0&0&0&3&0&0&0&0&0&0&0&0\\0&0&0&0&5&0&0&0&0&0&0&0\\0&0&0&0&0&3&0&0&0&0&0&0\\0&0&0&0&0&0&3&0&0&0&0&0\\0&0&0&0&0&0&0&3&0&0&0&0\\0&0&0&0&0&0&0&0&5&0&0&0\\0&0&0&0&0&0&0&0&0&3&0&0\\0&0&0&0&0&0&0&0&0&0&3&0\\0&0&0&0&0&0&0&0&0&0&0&3\end{bmatrix}$$

(c) Incidence matrix:

$$\begin{bmatrix}-1&0&0&0&1&0&-1&0&0&0&0&0&0&-1&0&0&0&0&0&0&0\\1&-1&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&1&-1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&1&-1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&1&-1&0&0&1&-1&0&0&-1&0&0&0&0&0&0\\0&0&0&0&0&0&0&1&-1&0&0&0&-1&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&1&-1&0&1&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&1&-1&0&1&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&-1&0&0&1&0&-1\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&-1&0&0&-1&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&-1&0&0&1\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&-1&1&0\end{bmatrix}$$

6. The graph laplacian is:

$$\begin{bmatrix}5&-1&-1&-1&-1&0&0&0&-1&0&0&0\\-1&3&-1&-1&0&0&0&0&0&0&0&0\\-1&-1&3&-1&0&0&0&0&0&0&0&0\\-1&-1&-1&3&0&0&-1&0&0&0&0&0\\-1&0&0&0&5&-1&-1&-1&-1&0&0&0\\0&0&0&0&-1&3&-1&-1&0&0&0&0\\0&0&0&0&-1&-1&3&-1&0&0&0&0\\0&0&0&0&-1&-1&0&3&0&0&0&0\\-1&0&0&0&-1&0&0&0&5&-1&-1&-1\\0&0&0&0&0&0&0&0&-1&3&-1&-1\\0&0&0&0&0&0&0&0&-1&-1&3&-1\\0&0&0&0&0&0&0&0&-1&-1&-1&3\end{bmatrix}$$

The property of the Laplacian for having connected graph is that the second eigen value is greater that zero ($\lambda_2 > 0$)

Exercise 2

1. The Sensing graph is Shown below:

2. The graph in terms of edges and vertexes are shown below.

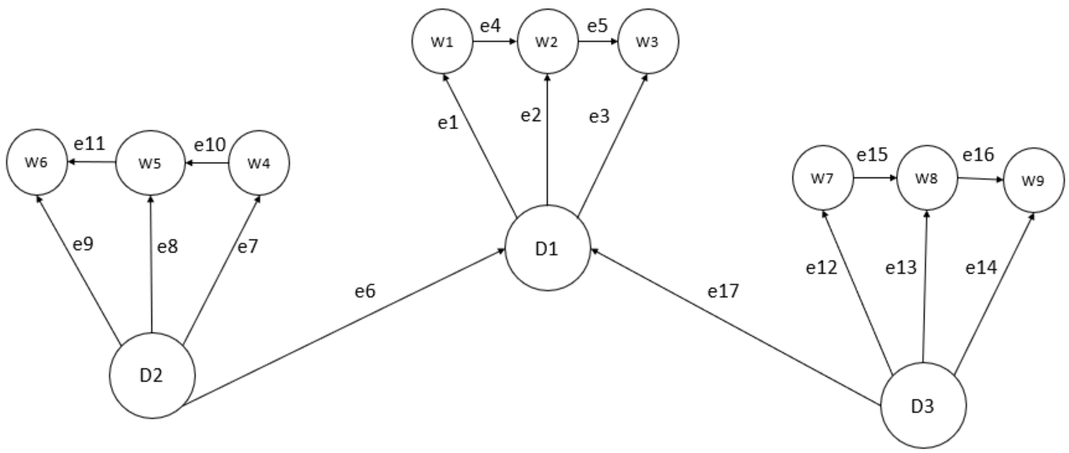

Figure 3:

V = { $\mathrm{d}_1, d_2, d_3, w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8, w_9\}$
$E = \{(d_2, d_1), (d_3, w_1), (d_1, w_1), (d_1, w_2), (d_1, w_3), (d_2, w_4), (d_3, w_5), (d_2, w_6), (d_3, w_7), (d_3, w_8),$
$(d_3, w_9), (w_1, w_2), (w_2, w_3), (w_4, w_5), (w_5, w_6), (w_7, w_8), (w_8, w_9)\}$

3. The graph is a directed graph and weakly connected graph because there is no path from any one vertex to any other vertex.

4. The longest path is:$(d_2- > d_1- > w_1- > w_2- > w_3)$, $(d_3- > d_1- > w_1- > w_2- > w_3)$

5. Adjacency, degree and incidence matrices are shown below

(a) Adjacency Matrix is:

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0
\end{bmatrix}$$

(b) Degree Matrix:

$$\begin{bmatrix}
2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2
\end{bmatrix}$$

(c) Incidence matrix:

$$\begin{bmatrix}
-1 & -1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -1 & -1 & -1 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0
\end{bmatrix}$$

6. The graph laplacian is:

$$\begin{bmatrix}
2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & -1 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & -1 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 2
\end{bmatrix}$$

7) It is not rooted-out branching and there will be two zero eigen values as shown below. And the graph is not balanced as there are no equal in-degree and out-degree at each vertex.

```
Eigen values are [0. 0. 1. 1. 1. 2. 2. 2. 2. 2. 2. 2.]
```

Figure 4:

Exercise 3

1) The graph is undirected and connected so the $\lambda_1 = 0$ and $\lambda_2 > 0$ and other eigen values are greater than zero $\lambda_n > 0$.

2) Since there would be two sub graph there would be two eigenvalues that is $\lambda_1 = 0$ and $\lambda_2 = 0$ and other eigen values would be greater that zero $\lambda_n > 0$.

3)In the graph laplacian for figure 1 the sum of rows and column would be zero because since its is undirected graph there is equal in-degree and out-degree for each of the vertices.

4) The sum of row and column of laplacian matrix is zero.
so the eigenvector and left eigenvector is:
1 is an eigenvector of L with associated eigenvalue 0 (L1 = 0)
1 is an left eigenvector of L with associated eigenvalue 0 ($1^T L = 0$)

```
array([[0.],
       [0.],
       [0.],
       [0.],
       [0.],
       [0.],
       [0.],
       [0.]])
```

[eigenvalue]

```
array([[1.],
       [1.],
       [1.],
       [1.],
       [1.],
       [1.],
       [1.],
       [1.]])
```

[eigenvector]

Figure 5: Figure of eigenvalue and eigenvector

Exercise 4

1) The code file is attached
2) The graph laplacian for Exercise 1 is:

```
Graph laplacian for Exercise 1 is
 [[ 5. -1. -1. -1. -1.  0.  0.  0. -1.  0.  0.  0.]
 [-1.  3. -1. -1.  0.  0.  0.  0.  0.  0.  0.  0.]
 [-1. -1.  3. -1.  0.  0.  0.  0.  0.  0.  0.  0.]
 [-1. -1. -1.  3.  0.  0.  0.  0.  0.  0.  0.  0.]
 [-1.  0.  0.  0.  5. -1. -1. -1. -1.  0.  0.  0.]
 [ 0.  0.  0.  0. -1.  3. -1. -1.  0.  0.  0.  0.]
 [ 0.  0.  0.  0. -1. -1.  3. -1.  0.  0.  0.  0.]
 [ 0.  0.  0.  0. -1. -1. -1.  3.  0.  0.  0.  0.]
 [-1.  0.  0.  0. -1.  0.  0.  0.  5. -1. -1. -1.]
 [ 0.  0.  0.  0.  0.  0.  0.  0. -1.  3. -1. -1.]
 [ 0.  0.  0.  0.  0.  0.  0.  0. -1. -1.  3. -1.]
 [ 0.  0.  0.  0.  0.  0.  0.  0. -1. -1. -1.  3.]]
```

Figure 6: Laplacian for Exercise 1

The graph laplacian for Exercise 2 is:

```
Graph laplacian for Exercise 2 is
 [[ 2.  0.  0.  0. -1.  0.  0.  0. -1.  0.  0.  0.]
 [-1.  1.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [-1. -1.  2.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [-1.  0. -1.  2.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0. -1.  1.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0. -1. -1.  2.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0. -1.  0. -1.  2.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0. -1.  1.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0. -1. -1.  2.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0. -1.  0. -1.  2.]]
```

Figure 7: Laplacian for Exercise 2

The graph laplacian for Exercise 3 is:

```
Graph laplacian for Exercise 3 is
 [[ 3. -1. -1. -1.  0.  0.  0.  0.  0.]
 [-1.  1.  0.  0.  0.  0.  0.  0.  0.]
 [-1.  0.  2. -1.  0.  0.  0.  0.  0.]
 [-1.  0. -1.  3. -1.  0.  0.  0.  0.]
 [ 0.  0.  0. -1.  2. -1.  0.  0.  0.]
 [ 0.  0.  0.  0. -1.  3. -1. -1.  0.]
 [ 0.  0.  0.  0.  0. -1.  2. -1.  0.]
 [ 0.  0.  0.  0.  0. -1. -1.  2.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.]]
```

Figure 8: Laplacian for Exercise 3

3) The eigen value for Cycle graph C4 is :

```
Two smallest eigen values are:  -6.661338147750939e-16 2.0
Two largest eigen values are:  3.999999999999998 2.0000000000000004
```

Figure 9: eigen value for C4

The eigen value for cycle graph C10 is:

```
Two smallest eigen values are:  -6.106226635438361e-16 0.3819660112501048
Two largest eigen values are:  3.9999999999999996 3.6180339887498945
```

Figure 10: eigen value for C10

The eigen value for cycle graph C100 is :

```
Two smallest eigen values are:  3.1086244689504383e-15 0.003946543143458672
Two largest eigen values are:  3.9999999999999942 3.9960534568565484
```

Figure 11: eigen value for C100

Exercise 5

1) The code file is attached
2) The plot for state of the robot as a function of time is shown below:

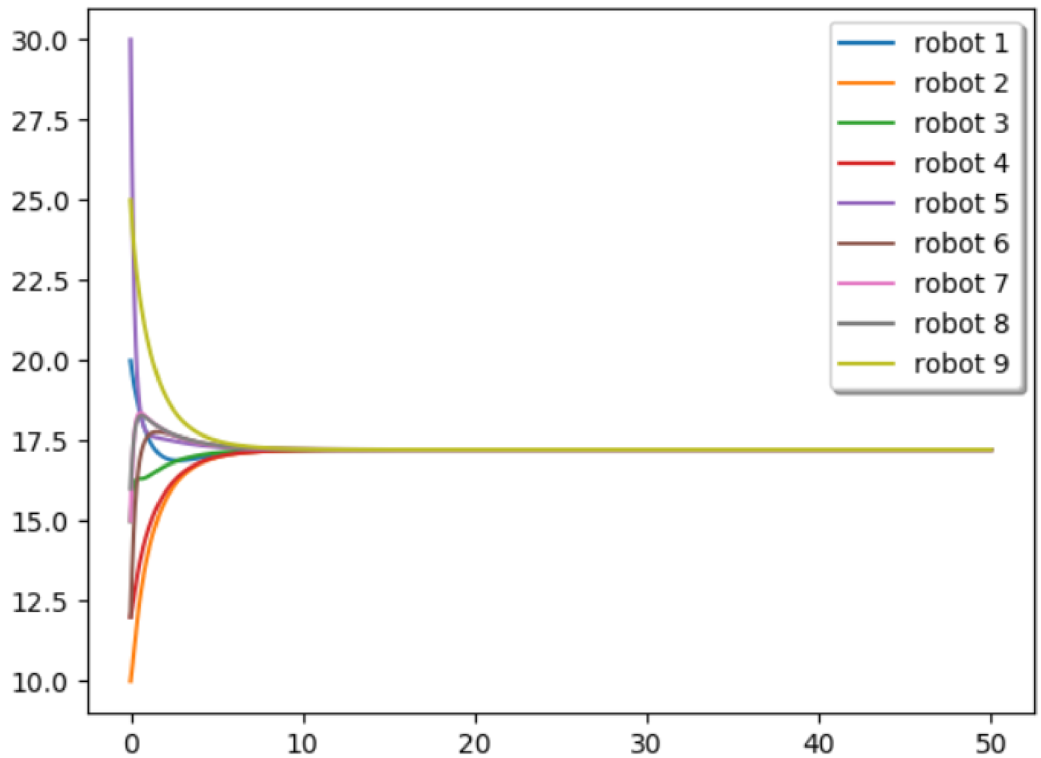

Figure 12: plot for state of the robot

As time goes to infinity the convergence point will remain same.

3) Yes the consensus protocol will converge as the graph is connected but it will converge slowly as the λ_2 (eigen value) is 0.29 which is smaller compared to the previous question(second question) whose eigen value is 0.36.

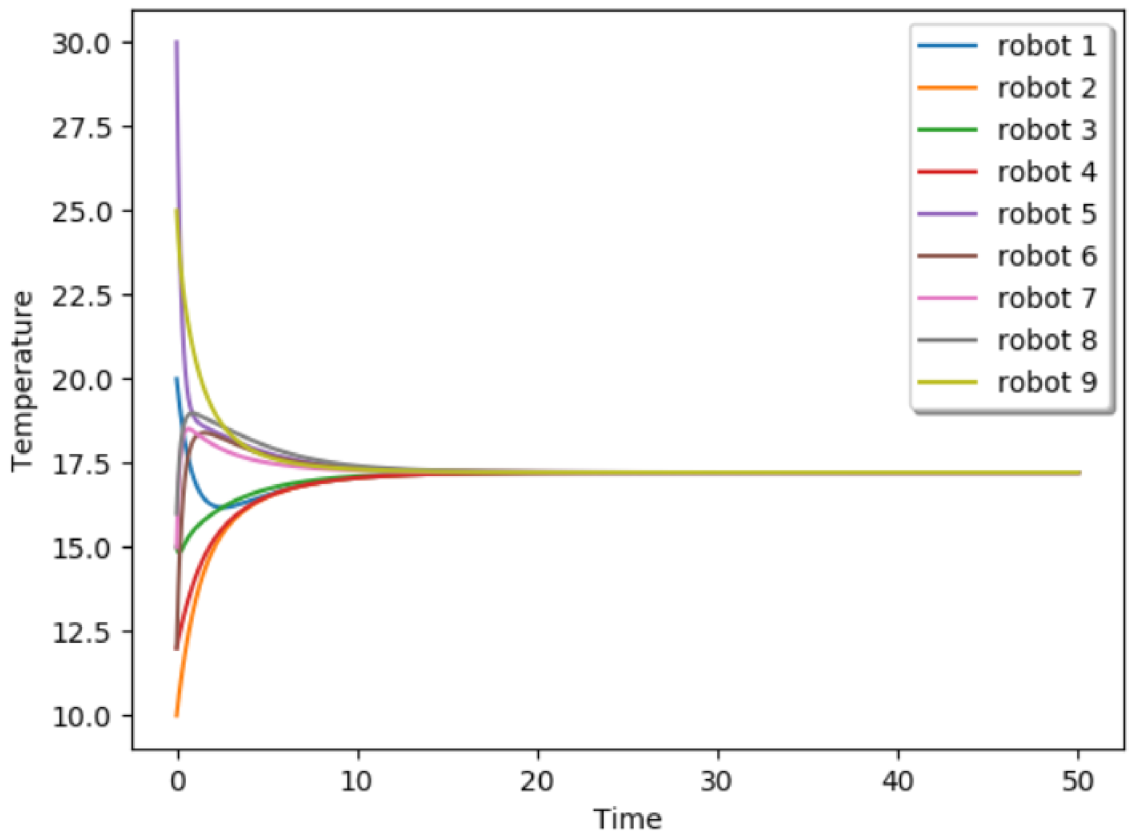

Figure 13: Plot after removing edges between (2, 4) and (6, 7)

4) The consensus protocol will converge faster as the eigen value, $\lambda_2 = 0.6$ value is greater than eigen value of the second question whose eigen value is 0.36.

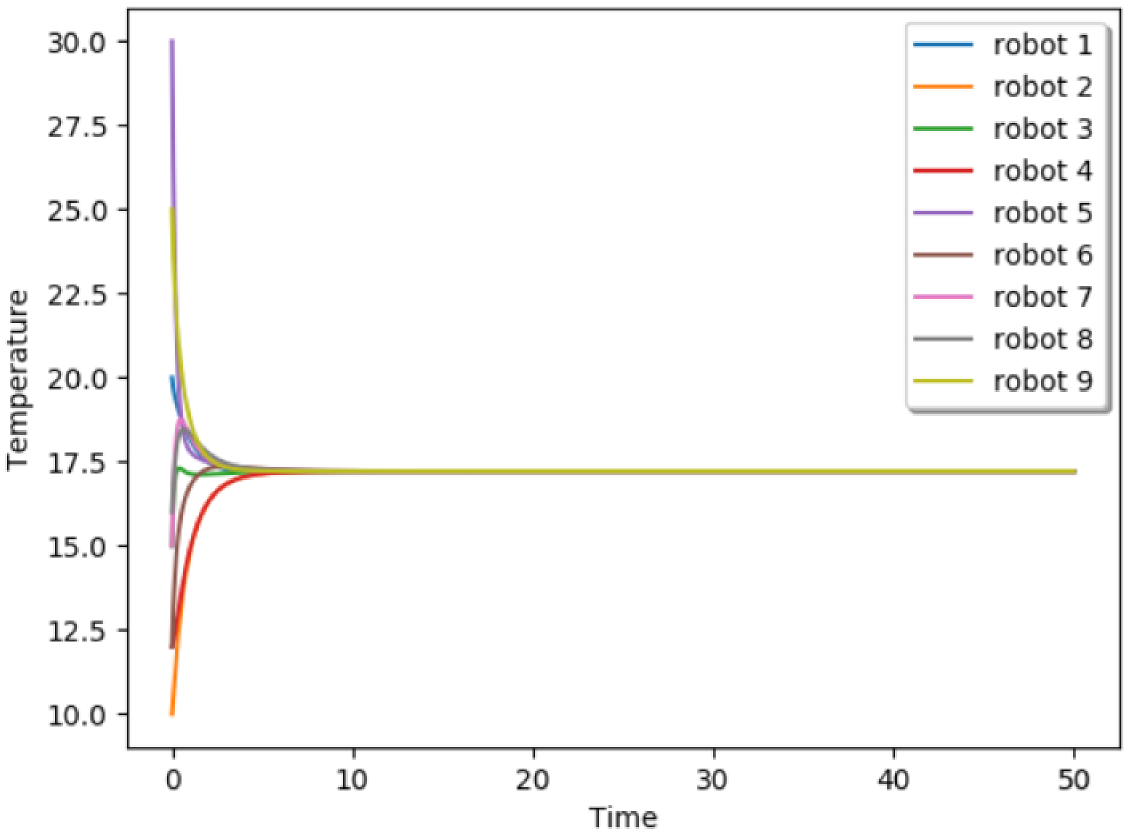

Figure 14: Plot after adding edges between (1, 3), (2, 8) and removing (5, 6)

5) The consensus protocol will converge at two different points that is one at 16.49 and second at 18.66 but we cannot cannot tell about the rate of convergence.

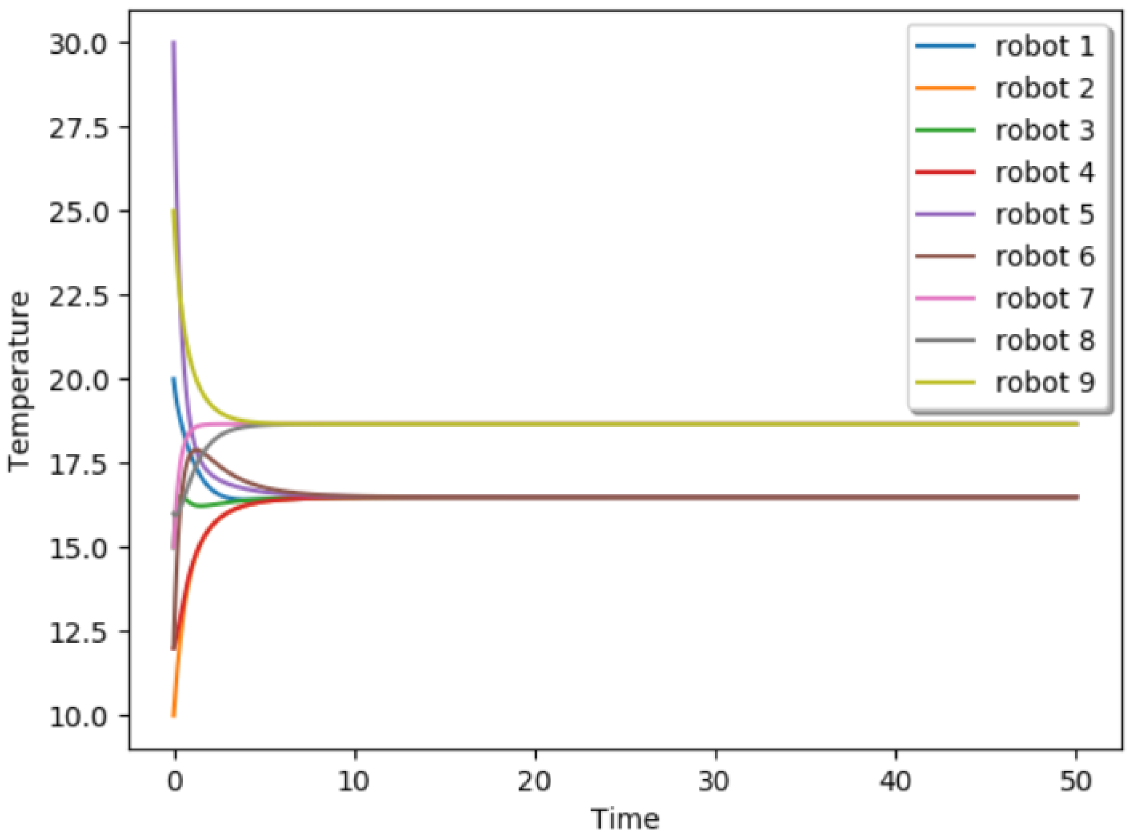

Figure 15: Plot after removing edges between (5, 6), (4, 7),(4, 6), (2, 6)