Master of Science Thesis

Geometric Control of a Quadrotor with a Suspended Load

Subtitle

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Delft Center for Systems and Control
Delft University of Technology

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N.N. Vo

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The undersigned hereby certify that they have read and recommend to the Faculty of Mechanical, Maritime and Materials Engineering for acceptance a thesis entitled **"Geometric Control of a Quadrotor with a Suspended Load"** by **N.N. Vo** in partial fulfillment of the requirements for the degree of **Master of Science**.

Dated: June 11, 2017

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Abstract

A Quadrotor is a type of Unmanned Aerial Vehicle that has received an increasing amount of attention recently with many applications including search and rescue, surveillance, supply of food and medicines as disaster relief and object manipulation in construction and transportation.

An interesting control problem is the Load Position Tracking of a cable suspended load. The system is highly nonlinear and under-actuated. The load cannot be actuated directly and has a natural swing at the end of each Quadrotor movement. A Nonlinear Geometric Control approach allows differential geometric techniques to be applied to systems control, which can be defined on a smooth nonlinear configuration space. This creates a coordinate-free dynamic model, while avoiding the problem of singularities on local charts.

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Intro about GC.. Reasons to consider GC..

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Where simple linear control methods are restricted to small angle movements, nonlinear control methods allow more aggressive and faster movements. The goal of the project is to investigate the effects on load position tracking performance when the system is modeled and controlled via a Nonlinear Geometric Control approach.

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The Quadrotor-Load system is modeled in a compact and coordinate-free fashion which allows the inherent geometric properties of the system to be controlled.

The main goal of this thesis is to research the effects on a cable-suspended load transportation using quadrotors, by involving complex or aggressive maneuvering through implementation of Non-Linear Geometric Control. Where linear control methods are restricted to small angle movement, non-linear control methods allow more aggressive movements.

Furthermore, the studied control techniques are explained and their advantages are addressed. Several trajectory generation approaches and the related optimization techniques are studied. Their applications, with different purposes such as obstacle avoidance, time-optimal and swing-free trajectory planning are explained. The survey is concluded with a discussion

about finding a suitable ************************************
Define suitable

****

control design to achieve the quadrotor-assisted task involving manipulation of a cable-suspended load.

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Acknowledgements

I would like to thank my supervisors dr.ir. T. Keviczky from DCSC and ir. B. van Vliet from Alten Nederland B.V. for their assistance during the writing of this thesis. I would also ...

Delft, University of Technology N.N. Vo
June 11, 2017

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Chapter 1

Introduction

A Quadrotor (QR) is a type of **uav!** (**uav!**) that has received an increasing amount of attention recently with many applications being actively investigated. Possible applications include search and rescue, surveillance, reliable supply of food and medicines as disaster relief and object manipulation in construction and transportation. It has already proven itself useful for many tasks like multi-agent missions, mapping, explorations, transportation and entertainment such as acrobatic performances.

Considering a multi-agent task, one can think of multiple QRs assisting in the transportation of a common load. This cooperation can be executed in many ways, but this literature focuses on QRs with a cable-suspended load in motion. The suspended object naturally continues to swing at the end of every movement. In case a residual motion can result in damage or in order to avoid obstacles and path following, an accurate positioning is required. Reducing the oscillation, or controlling the position of the suspended load might be necessary, but is challenging in the fact that this cable-suspended system is under-actuated.

In this chapter, the motivation for writing this literature study is given. Next, a former research in the scope of this literature survey is discussed. And finally, the organization of the report is presented.

1-1 Aim and Motivation

The inspiration for this literature survey is build upon the idea of creating a multiple autonomous QR system for a cooperative towing task. The advantage of using multiple robots for object manipulation is the possibility to reduce complexity of the individual robot, decrease cost over traditional robotic systems and high reliability. One can think of examples in nature, where individuals coordinate, cooperate and collaborate to perform tasks that they individually can not accomplish. Redundancy makes development of fail safe control methods possible and can extend the capabilities of a single robot.

The aim is to control the position of a suspended load using a Quadrotor. Former work on Quadrotor control regularly rely on linear control such as PID or LQR for a dynamic model linearized about the hover state. A single Quadrotor is considered for the transportation of a cable suspended load, which exerts forces and torques on the Quadrotor.

Numerous control methods have proven useful for QR control. Many are based on a linearized dynamical model of the system, such that the system dynamics are described by a set of linear differential equations.

Optimal control.

PID control.

An Linear Quadratic Regulator (LQR) algorithm is able to find a state feedback controller by linearizing the dynamics around an equilibrium point such as the hover mode.

However, one particular method based on geometric control has been addressed in very few literature.

The control of a QR-Load system is a more specific case, and already is found less in literature.

Optimal control. MPC and LQR. Minimization of the swing.

Globally defined, useful for nonlinear problems
Only few, why so little?
What is its value compared against linear control

This motivates to compare a linear control strategy with a non-linear control strategy. **

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State of the art methods

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1-2 Organization of the Report

- Chapter 1
- Chapter 2
- Chapter 3
- Chapter 4
- Chapter 5
- Chapter ??

****

Hoe de keuze tot stand is gekomen, hoe uitgebreid?
Moet ik nog een stuk wijden aan literature survey?

****

Chapter 2

Dynamic Model

The dynamics of the QR-Load system are described by the laws of kinematics and the application of Newton's laws or Lagrangian mechanics. Opposed to the classical modeling techniques, it is also possible to describe the system's configuration space as a differentiable manifold using the tools of differential geometry.

Considering the properties of the system, the QR is described as a rigid body with six degrees of freedom, driven by forces and moments. The motion of a rigid body can be described by a translation of the Center of Mass (CoM) and a rotation about the CoM. The derived mathematical model is represented by a set of dynamic equations commonly used for rigid body displacements.

To derive the equations of motions traditional modeling methods often parameterize the rotations in a local coordinate system. Euler angles are commonly used, however these coordinates might result in singularities. Furthermore, there are 24 sets of Euler angles, which can lead to ambiguity. In order to avoid these complexities, the dynamics of the QR-Load system can be globally expressed on the Special Orthogonal Group $SO(3)$, *2-sphere* S^2 and Special Euclidean Group $SE(3)$. This leads to a compact notation of the equations of motion, making the large amount of trigonometric functions unnecessary, that Euler angles normally introduce.

An Introduction to Differentiable Manifolds is given by [6]

[2] [3]

****

Computational Geometric Mechanics and Control
Computation algorithms must be developed which preserve the geometric properties of a mechanical system.
Robust and careful numerical implementation of geometric control theory to complex engineering systems.
Provides nontrivial maneuvers that are globally valid on a nonlinear configuration manifold.

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2-1 Modeling Assumptions

A mathematical model of the system needs to be derived in order to simulate and study the effects of Geometric Control. The following assumptions are applied to simplify the model.

The QR model representation is shown in Figure 2-1. Three Cartesian coordinate frames are defined:

- The body-fixed reference frame $\{\mathcal{B}\}$ (Body Frame)

  with unit vectors $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ along the axes
- The ground-fixed reference frame $\{\mathcal{I}\}$ (Inertial Frame)

  with unit vectors $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ along the axes
- The intermediary frame $\{\mathcal{C}\}$, ($\{\mathcal{I}\}$ rotated by the yaw angle ψ)

  with unit vectors $\{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3\}$ along the axes

Figure 2-1: Quadrotor model representation

Figure 2-2: Quadrotor with Load model representation

The position of the body frame is described by a vector evolving on $\mathbb{R}^3$, and is represented with respect to the inertial frame. The orientation, also called attitude, of the body frame with respect to the inertial frame evolves on a nonlinear space, for which several methods exist to describe this, such as *Euler Angles*, quaternions or rotation matrices.

The complex dynamics of the rotors and their interactions with drag and thrust forces are represented by a simplified model. The angular speed ω_i of rotor i, for $i = 1, \ldots, 4$, generates a force F_i parallel to the direction of the rotor axis of rotor i, given by

$$F_i = \left(\frac{K_v K_\tau \sqrt{2\rho A}}{K_t} \omega_i \right)^2 = b\omega_i^2 \tag{2-1}$$

where K_v, K_t are constants related to the motor properties, ρ is the density of the surrounding air, A is the area swept out by the rotor, K_τ is a constant determined by the blade configuration and parameters, and b is the thrust factor.
The torque around the axis of rotor i, for $i = 1, \ldots, 4$, generated due to drag is given by

$$M_i = \frac{1}{2} R\rho C_D A(\omega_i R)^2 = d\omega_i^2 \tag{2-2}$$

where R is the radius of the propeller, C_D is a dimensionless constant, and d is the drag constant.

For given desired total thrust f and total moment $M = \begin{bmatrix} M_\phi & M_\theta & M_\psi \end{bmatrix}^T$, the required rotor speeds can be calculated by solving the following equation

$$\begin{bmatrix} f \\ M_\phi \\ M_\theta \\ M_\psi \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & -Lb & 0 & Lb \\ Lb & 0 & -Lb & 0 \\ -d & d & -d & d \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \tag{2-3}$$

where L is the cable length and M_ϕ, M_θ, M_ψ denote the moments around the x, y, z-axis in $\{\mathcal{B}\}$, resp.

Table 2-1 shows the most common assumptions that are used for modeling the QR, simplifying the complexity of the model.

• The structure of the QR is rigid and symmetric. Elastic deformations and shock (sudden accelerations) of the QR are ignored.
• The mass distribution of the QR is symmetrical in the x-y plane.
• The inertia matrix is time-invariant.
• Aerodynamic effects acting on the QR are neglected. Blade flapping, Turbulence, Ground Effects.
• The air density around the QR is constant.
• The propellers are rigid $\Rightarrow$ The thrust produced by rotor i is parallel to the axis of rotor i.
• Drag factor d and thrust factor b are approximated by a constant. Thrust force F_i and moment M_i of each propeller is proportional to the square of the propeller speed.

Table 2-1: Modeling assumptions Quadrotor model

2-2 Geometric Mechanics

In this section an introduction is given about Geometric Mechanics, which is an approach that describes the classical mechanics from the perspective of Differential Geometry and its fundamental object called a differentiable manifold.

• The cable is modeled as a rigid and massless cable. • The cable is connected to a friction-less joint at the origin of the body-fixed. • The tension in the cable is considered to be non-zero. This implies that the QR-Load subsystem, consisting of a separate QR and Load in free fall, is disregarded. • Aerodynamic effects acting on the load are neglected. reference frame. • Assumption Details Assumption 2

Table 2-2: Modeling assumptions Quadrotor+Load model

In Geometric Mechanics the configuration space of systems is a *group manifold* instead of a Euclidean space. The kinetic and potential energies are expressed in terms of this configuration space and their tangent spaces. It explores the geometric structure of a Lagrangian- or Hamiltonian system through the concepts of vector calculus, linear algebra, differential geometry, and non-linear control theory. Geometric mechanics provides fundamental insights into the non-linear system mechanics and yields useful tools for dynamics and control theory.

****

Mechanics studies the dynamics of physical bodies acting under forces and potential fields. In Lagrangian mechanics, the trajectories are obtained by finding the paths that minimize the integral of a Lagrangian over time, called the action integral. Rigid body dynamics are characterized by Lagrangian/Hamiltonian dynamics. The dynamics of a Lagrangian system has unique geometric properties and these are exploited to obtain Euler-Lagrange equations. The resulting intrinsic form of the Euler-Lagrange equations are more compact than equations expressed in terms of local coordinates.

Problems, singularities with Euler-Angles
Other attitude representations, such as exponential coordinates, quaternions, or Euler angles, can also be used following standard descriptions, but each of the representations has a disadvantage of introducing an ambiguity or singularity. Why charts on $SO(3)$ https://en.wikipedia.org/wiki/Charts_on_SO(3)

Euler Angles
However, the definition of Euler angles is not unique and in the literature many different conventions are used. These conventions depend on the axes about which the rotations are carried out, and their sequence because rotations are not commutative. Therefore, Euler angles are never expressed in terms of the external frame, or in terms of the co-moving rotated body frame, but in a mixture. Other conventions (e.g., rotation matrix or quaternions) are used to avoid this problem.

****

****

When angular errors are large, the difference in Euler angles is no longer a good metric to define the orientation error. Local coordinates often require symbolic computational tools due to complexity of multi-body systems. Hence, the error is rather written as the required 3-D rotation to get from the current to a desired orientation. As a result, the equations of motion and the control systems can be developed on a configuration manifold in a coordinate-free, compact, unambiguous manner, while singularities of local parameterization are avoided to generate agile maneuvers in a uniform way.

Manifolds The fundamental object of differential geometry a manifold. A manifold is a topological space that locally resembles Euclidean space near each point. Manifolds are important objects in mathematics and physics because they allow more complicated structures to be expressed and understood in terms of the relatively well-understood properties of simpler spaces.

A differentiable manifold is a type of manifold that is locally similar enough to a linear space to allow to do calculus. In Figure 2-3 is illustrated that each point of an n-dimensional manifold has a neighbourhood that is homeomorphic to the n-dimensional Euclidean space.

Figure 2-3

****

A manifold is an abstract mathematical space in which every point has a neighbourhood which resembles Euclidean space, but in which the global structure may be more complicated.

For most applications a special kind of topological manifold, a differentiable manifold, is used. One can define directions, tangent spaces, and differentiable functions on that manifold. In particular it is possible to use calculus on a differentiable manifold.

Differentiable manifolds If the local charts on a manifold are compatible in a certain sense, Each point of an n-dimensional differentiable manifold has a tangent space. This is an n-dimensional Euclidean space consisting of the tangent vectors of the curves through the point. Two important classes of differentiable manifolds are smooth and analytic manifolds. For smooth manifolds the transition maps are smooth, that is infinitely differentiable. Analytic manifolds are smooth manifolds with the additional condition that the transition maps are analytic (a technical definition which loosely means that Taylor's theorem holds). The sphere can be given analytic structure, as can most familiar curves and surfaces. A rectifiable set generalizes the idea of a piecewise smooth or rectifiable curve to higher dimensions; however, rectifiable sets are not in general manifolds.

****

An example can be given of an 2-link arm, in Figure 2-4, where the configuration of can be expressed by 2 coordinates. Figure **??** represents the configuration space as a Cartesian space, where the two red dots represent the same configuration. This illustrates that this representation suffers from singularities. In mathematics there are different types of singularity, this case is about the situation where multiple points in one representation are mapped onto a single point in another representation. Figure 2-4c shows the configuration space as a manifold, where every configuration is a unique representation.

(a) (b) (c)

Figure 2-4: Configuration Space of a 2-link arm

Most of nonlinear dynamics and control problems are studied in a linear space.

$$\dot{x} = \mathbf{f}(t, x, u), \quad x \in \mathbb{R}^n, u \in \mathbb{R}^m, \mathbf{f} : \mathbb{R}^{n+m+1} \to \mathbb{R}^n \tag{2-4}$$

Geometric Mechanics and Control is used to understand the structure of the equations of motion of a system in order to allow its analysis and design. The system evolves on a nonlinear manifold and the controllers are designed on this same manifold.

Lie Group Configuration Manifold Lie group is a group that is also a differentiable manifold.

Rotational matrices with determinant 1 is a Lie group: $SO(3)$

and can be represented mathematically by a 3×3 orthonormal matrix.

A manifold is locally diffeomorphic to a Euclidean space, and it also has a group structure with the group action of matrix multiplication. A smooth manifold with a group structure is referred to as a Lie group; the Lie group of 3×3 orthonormal matrices with positive determinant is referred to as the special orthogonal group, $SO(3)$ [5].

The configuration manifold for the attitude dynamics of a rigid body is $SO(3)$, and the configuration manifold for combined translational and rotational motion of a rigid body is the special Euclidean group $SE(3)$, which is a semi-direct product of $SO(3)$ and $\mathbb{R}^3$. A direct product of the Lie groups $SE(3), SO(3),$ and $\mathbb{R}^n$ can represent the configuration of multiple rigid bodies, and it is also a Lie group.

****

Introduction to the basics of Lie group theory and its connections with rigid body kinematics is given in [5].

****

Special Orthogonal group Rotation matrices are used to describe the attitude of the QR. A rotation matrix maps a representation of vectors expressed in $\{\mathcal{B}\}$ to a representation expressed in $\{\mathcal{I}\}$. Rotation matrices provide global representations of the attitude, which is an advantage over other attitude representations, such as exponential coordinates, quaternions, or Euler angles, where each of the representations has a disadvantage of introducing ambiguities or singularities. The configuration of the QR is a rotation matrix R in the Special Orthogonal Group $SO(3)$ defined as

$$SO(3) \triangleq \left\{ R \in \mathbb{R}^{3\times 3} : RR^T = I_{3\times 3}, det(R) = 1 \right\} \tag{2-5}$$

$SO(3)$ is the group of all rotations about origin of three-dimensional Euclidean space, which preserves the origin, Euclidean distance and orientation. Every rotation has a unique inverse rotation and the identity map satisfies the definition of a rotation.

The group operation for $SO(3)$ corresponds to matrix multiplication.
The attitude kinematics equation is given by

$$\dot{R} = R\hat{\Omega} \tag{2-6}$$

where $\Omega \in \mathbb{R}^3$ is the angular velocity represented in the body fixed frame. The hat map $\hat{\cdot} : \mathbb{R}^3 \to \mathfrak{so}(3)$ is an isomorphism between $\mathbb{R}^3$ and the set of 3×3 skew symmetric matrices. The Lie algebra $\mathfrak{so}(3)$ is defined by

$$\hat{\Omega} = \begin{bmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{bmatrix} \tag{2-7}$$

****
Rotation formalisms in three dimensions https://en.wikipedia.org/wiki/Rotation_formalisms_in_three_dimensions#cite_note-5
Combining two successive rotations, each represented by an Euler axis and angle, is not straightforward, and in fact does not satisfy the law of vector addition, which shows that finite rotations are not really vectors at all. It is best to employ the rotation matrix or quaternion notation, calculate the product, and then convert back to Euler axis and angle.

****

This concludes the introduction about Geometric Mechanics, which is used to model the QR-Load system in the following section.

2-3 Quadrotor-Load Model

****
In [4] dynamics and optimal control problems for rigid bodies are studied, incorporating their geometric features. The focus lies on obtaining geometric properties of the dynamics of rigid bodies, how their configuration can be described and how these geometric properties are utilized in control system analysis and design. Computational methods for rigid bodies, that preserve the underlying Lagrangian/Hamiltonian system structure of rigid body dynamics as well as the Lie group structure of the configurations are developed.

****
The orientation of the QR is described on the Special Orthogonal Group $SO(3)$, using rotation matrices, instead of using local charts induced by Euler Angle parameterizations. Whereas the orientation of the Load is described on a two-sphere S^2

The configuration of a rigid body can be described by the location of its mass center and the orientation of the rigid body in a 3-D space. The location of the rigid body can be expressed in Euclidean space, but the attitude evolves in a nonlinear space with a certain geometry. The attitude of a rigid body is defined as the orientation of a body-fixed frame with respect to a reference frame

****

Variations ***************************************

Figure 2-5: Quadrotor-Load model representation

The Quadrotor-Load model is represented in Figure 2-5, where the unit vector q gives the direction from the QR to the Load expressed in $\{\mathcal{B}\}$. The position of the QR and Load, x_Q and x_L resp., are related by

$$x_Q = x_L - Lq \tag{2-8}$$

where L is the length of the cable.

To develop the Euler-Lagrange equations for mechanical systems that evolve on a Lie group, an approach developed by [4, 7, 8, 9] is used, which is based on Hamilton's principle.

The action integral is defined as

$$S = \int_{t_1}^{t_2} \mathcal{L} dt \tag{2-9}$$

where $\mathcal{L} = \mathcal{T} - \mathcal{U}$ is the Lagrangian of the system, where $\mathcal{T}$,$\mathcal{U}$ are the kinetic and potential energy, respectively. Hamilton's principle of least action states that the path a conservative mechanical system takes between two configurations at time t_1 and t_2, is the one for which Equation 2-9 is an extremum, stated as

$$\delta S = \int_{t_1}^{t_2} \delta\mathcal{L} dt = 0 \tag{2-10}$$

where $\delta\mathcal{L}$ is the variation of the Lagrangian. For systems with non-conservative forces and moments, Equation 2-10 is extended to

$$\delta S = \int_{t_1}^{t_2} (\delta W + \delta\mathcal{L}) dt = 0 \tag{2-11}$$

where δW is the virtual work. Equation 2-11 is applied to the QR-Load system, where the configuration manifold is $\mathbb{R}^3 \times S^2 \times SO(3)$. With the following states

$$\mathbf{x} = \begin{bmatrix} x_L & \dot{x}_L & q & \omega & R & \Omega \end{bmatrix}^T \tag{2-12}$$

****

****
Rigid Body Attitude Dynamics evolve on $SE(3)$.

$$J\dot{\Omega} + \Omega \times J\Omega = mg\rho \times R^T e_3 + u \tag{2-13}$$
$$\dot{R} = R\hat{\Omega} \tag{2-14}$$

****

****
The equations of motion for a rigid body with configuration $SE(3)$ are given by the *Newton-Euler equations* [5]:

$$\begin{bmatrix} mI & 0 \\ 0 & \mathcal{I} \end{bmatrix} \begin{bmatrix} \dot{v}^b \\ \dot{\omega}^b \end{bmatrix} + \begin{bmatrix} \omega^b \times mv^b \\ \omega^b \times \mathcal{I}\omega^b \end{bmatrix} = F^b \tag{2-15}$$

where m is the mass of the body, $\mathcal{I}$ is the inertia tensor, and $V^b = (v^b, \omega^b)$ and F^b represent the instantaneous body velocity and applied body wrench.
****

****
The load dynamics evolve on S^2. Based on [9].
****

2-4 Classical Modeling

This section describes the derivation of the model by using classical modeling techniques.

****
Reference A-1-1
****

When assuming small angle maneuvers, *Euler-angles* can be used to locally parameterize the orientation of the body-fixed reference coordinate frame with respect to the inertial reference coordinate frame. Simple linear controllers are often based on a linearized dynamical model, applying this small angles assumption.

From Newton's law follows

$$\begin{aligned} \dot{x}_Q &= v_Q \\ m_Q \dot{v}_Q &= fRe_3 - m_Q g e_3 - Tq \\ \dot{x}_L &= v_L \\ m_L \dot{v}_L &= -m_L g e_3 + Tq \end{aligned} \tag{2-16}$$

Because the Euler-Angles are used, a function is required that maps a vector of the Z-X-Y Euler angles to its rotation matrix $R \in SO(3)$, which is denoted as [10]

$$R_{312}(\phi, \theta, \psi) = \begin{bmatrix} c_\psi c_\theta - s_\phi s_\psi s_\theta & -c_\phi s_\psi & c_\psi s_\theta + c_\theta s_\phi s_\psi \\ c_\theta s_\psi + c_\psi s_\phi s_\theta & c_\phi c_\psi & s_\psi s_\theta - c_\psi c_\theta s_\phi \\ -c_\phi s_\theta & s_\phi & c_\phi c_\theta \end{bmatrix} \tag{2-17}$$

The Z-X-Y Euler angles to model the rotation can be seen in Figure 2-6. The first rotation by yaw angle ψ is around the z-axis of $\{\mathcal{I}\}$. Next is the rotation by roll angle ϕ, and the last rotation is by pitch angle θ.

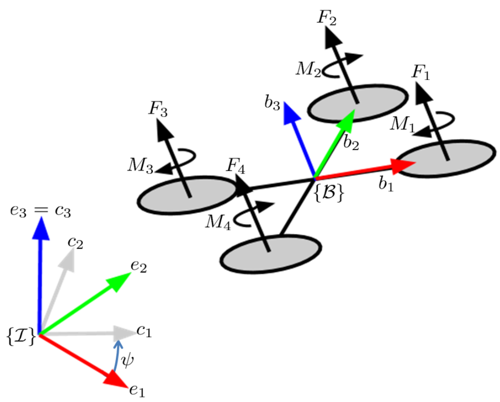

Figure 2-6: Quadrotor-Load model representation

The unit vector q from the QR to the load is represented in $\{\mathcal{B}\}$. Define ϕ_L as the yaw-rotation of the load around the z-axis of $\{\mathcal{B}\}$ and θ_L as the angle between the cable and the z-axis of $\{\mathcal{B}\}$, see Figure 2-7.

DCSC

Figure 2-7

$$q = \begin{bmatrix} s_{\theta_L} c_{\phi_L} \\ s_{\theta_L} s_{\phi_L} \\ c_{\theta_L} \end{bmatrix} \tag{2-18}$$

Differentiating Equation (2-8) and (2-18) gives

$$\ddot{x}_L = \ddot{x}_Q - \ddot{q}L$$
$$\ddot{q} = \begin{bmatrix} \ddot{\theta}_L c_{\theta_L} c_{\phi_L} - \ddot{\phi}_L s_{\theta_L} s_{\phi_L} - \dot{\phi}_L^2 s_{\theta_L} c_{\phi_L} - \dot{\theta}_L^2 s_{\theta_L} c_{\phi_L} - 2\dot{\theta}_L \dot{\phi}_L c_{\theta_L} s_{\phi_L} \\ \ddot{\theta}_L c_{\theta_L} s_{\phi_L} + \ddot{\phi}_L s_{\theta_L} c_{\phi_L} - \dot{\phi}_L^2 s_{\theta_L} s_{\phi_L} - \dot{\theta}_L^2 s_{\theta_L} s_{\phi_L} + 2\dot{\theta}_L \dot{\phi}_L c_{\theta_L} c_{\phi_L} \\ -\ddot{\theta}_L s_{\theta_L} - \dot{\theta}_L^2 c_{\theta_L} \end{bmatrix} \tag{2-19}$$

$$\begin{aligned} \ddot{x}_Q &= \frac{1}{m_Q}(f(c_\psi s_\theta + c_\theta s_\phi s_\psi) - T s_{\theta_L} c_{\psi_L}) \\ \ddot{y}_Q &= \frac{1}{m_Q}(f(s_\psi s_\theta - c_\psi c_\theta s_\phi) - T s_{\theta_L} s_{\psi_L}) \\ \ddot{z}_Q &= \frac{1}{m_Q}(f(c_\phi c_\theta) - T c_{\theta_L}) - g \end{aligned} \tag{2-20}$$

$$\ddot{\psi} = \tilde{\tau}_\psi \tag{2-21}$$
$$\ddot{\theta} = \tilde{\tau}_\theta \tag{2-22}$$
$$\ddot{\phi} = \tilde{\tau}_\phi \tag{2-23}$$

2-5 Summary

****

Compact, unambiguous, globally defined,

Pro/Cons of Classical Modeling Techniques vs Geometric Modeling

Linearized model/State Space model vs. Geometric modeling

Geometric Mechanics/Lie Groups/Lie Algebra is used in order to represent the dynamics of the system onto the nonlinear configuration manifold $SE(3)$
Advantage of this method is
Enables to model on
That type of control is discussed in the next chapter

****

Chapter 3

Geometric Control Design

Geometric Control Theory explores the application of differential geometric techniques to systems control. The objective is to express both the dynamics and its control inputs on manifolds instead of on local charts.

Geometric Control is based on a coordinate-free representation of the dynamics, where the equations of motion are compact, unambiguous and singularity free.

Attitude control systems naturally evolve on non-linear configurations such as S^2 and $SO(3)$.

Global nonlinear dynamics of various classes of closed loop attitude control systems have been studied in recent years [**?**].

****
Existing Control Systems for Quadrotor UAVs:
Based on the linearized dynamics of a quadrotor UAV
Singularities in representing complex maneuvers
Fundamental restriction in tracking nontrivial trajectories

In contrast to hybrid control systems [11], **complicated reachability set analysis is not required** to guarantee safe switching between different flight modes, as the region of attraction for each flight mode covers the configuration space almost globally. Tracking control system can be developed on $SO(3)$, therefore it avoids singularities of Euler-Angles.

Benchmark with Linear Control

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In control systems engineering, the underlying geometric features of a dynamic system are often not considered carefully. For example, many control systems are developed for the standard form of ordinary differential equations, namely $\dot{x} = f(x, u)$, where the state and the control input are denoted by x and u. It is assumed that the state and the control input lie in

Euclidean spaces, and the system equations are defined in terms of smooth functions between Euclidean spaces. However, for many interesting mechanical systems, the configuration space cannot be expressed globally as a Euclidean space.
****

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Geometric control theory is the study of how the geometry of the state space influences controls problems. This includes local properties like curvature, and global properties like the number of ‘holes’ in the space (sphere vs doughnut).

Intuition: For example, I want to write out the equations of motion of a 3D pendulum and also control its orientation directly on the Special Orthogonal Group (using rotation matrices) instead of using local charts induced by Euler Angle Parametrizations (theta, phi and psi - the three famous angles of dynamics!). Pros:
-¿ Compact expressions that are also intuitive. -¿ Globally defined (no singularities!). Therefore, one can build almost globally attractive controllers.
Cons:
-¿ The math gets very involved very quickly. (This could be subjective)
****

Control Schemes
Nonlinear Geometric Control. Error functions

Attitude Controller

Define errors associated with the attitude dynamics of the QR. The attitude and angular velocity tracking error should be carefully chosen as they evolve on the tangent bundle of $SO(3)$. [**?**]

Constants [12] or matrices; enabling unique gains for roll/pitch/yaw [13]

3-1 Backstepping Control

****

Nonlinear control: Backstepping control is common.

Every loop creates a virtual control input for the next loop in order to stabilize the origin?

Definition backstepping: A technique for designing stabilizing controls for a special recursive class of nonlinear dynamical systems.

In a backstepping control approach, the control law is designed by using states as virtual control signals. Each control loop outputs a commanded tracking signal for the underlaying control loop.

****

****
http://www.control.lth.se/media/Education/EngineeringProgram/FRTN05/2013/lec09_2013eight.pdf We want to design a state feedback u=u(x) that stabilizes

$$\begin{aligned} \dot{x}_1 &= f(x_1) + g(x_1)x_2 \\ \dot{x}_2 &= u \end{aligned} \tag{3-1}$$

Idea is to see system as a cascade connection. Design controller first for inner loop, then for the outer.

****

****
Back-Stepping Lemma **Lemma**: Let $z = \left(x_1, \cdots, x_{k-1}\right)^T$ and

$$\begin{aligned} \dot{z} &= f(z) + g(z)x_k \\ \dot{x}_k &= u \end{aligned} \tag{3-2}$$

Assume $\phi(0) = 0$, $f(0) = 0$,

$$\dot{z} = f(z) + g(z)\phi(z) \tag{3-3}$$

stable, and $V(z)$ a Lyapunov function (with $\dot{V} \leq -W$). Then,

$$u = \frac{d\phi}{dz}\left(f(z) + g(z)x_k\right) - \frac{dV}{dz}g(z) - (x_k - \phi(z)) \tag{3-4}$$

stabilizes $x = 0$ with $V(z) + (x_k - \phi(z))^2/2$ begin a Lyapunov function.

****

A backstepping control approach is common for QRs and can be seen in Figure 3-1. The lowest level has the highest bandwidth and is in control of the rotor rotational speeds. The next level controls the QR attitude, and the top level controls the QR position.

Figure 3-1: Backstepping Control representation

****
Because the QR has only four actuators, it is not possible to control all DOFs. A nested feedback loop allows different flight modes to be controlled.

Three Flight Modes
QR Attitude Controlled Mode: track a QR attitude command $R_d(t)$ and a heading direction

$b_{1_d}(t)$
Load Attitude Controlled Mode: track a load attitude command $q_d(t)$
Load Position Controlled Mode: track a load position $x_{L,d}(t)$

****

The earlier mentioned backstepping approach is also used for the control of these different flight modes, see Figure **??**

Figure 3-2: Geometric Control loopfig:con.gcloop

****
Explain how f and $\vec{b_{1_d}}$ is obtained from $x_d(t)$?

****

Configuration Errors Before the controllers are described, the configuration functions are defined as in [2]. These functions describe the error on the manifolds $SO(3)$ and S^2, which describe the configuration spaces for the QR attitude and the load attitude, respectively.

The error function on $SO(3)$ is chosen to be [12]

$$\Psi_R(R, R_d) = \frac{1}{2} tr \left[I - R_d^T R \right] \tag{3-5}$$

The error function on S^2 is chosen to be

$$\Psi_q \tag{3-6}$$

The tracking error functions on $TSO(3)$ are defined as

$$e_R \tag{3-7}$$

$$e_\Omega \tag{3-8}$$

The tracking error functions on TS^2 are defined as

$$e_q \tag{3-9}$$

$$e_{\dot{q}} \tag{3-10}$$

The tracking errors for position and velocity are defined as

$$e_x = x - x_d \tag{3-11}$$
$$e_v = v - v_d \tag{3-12}$$
$$\text{where, } v_d = \dot{x}_d \tag{3-13}$$

where $x_d(t) \in \mathbb{R}^3$ is a smooth desired load position.

3-1-1 Quadrotor Attitude Tracking

This control problem has been addressed in various works *****

The most inner loop controls the attitude of the QR.

Figure 3-3

Attitude control problem is the inner loop of the control design. This inner loop must guarantee stability of the QR.

Must track reference attitude.

The attitude and angular velocity tracking error should be carefully chosen as the evolve on the tangent bundle of the nonlinear space $SO(3)$. [12]
Why? Appendix[12], and [2]?

The moment consists of a proportional term, a derivative term and a canceling term, and is defined as follows

$$M = \frac{1}{\epsilon^2} k_R e R - \frac{1}{\epsilon} k_\Omega e_\Omega + \Omega \times J_Q \Omega - J_Q(\hat{\Omega} R^T R_d \Omega_d - R^T R_d \dot{\Omega}_d) \tag{3-14}$$

****
Asymptotic tracking of the quadrotor attitude does not require specification of the thrust magnitude. As an auxiliary problem, the thrust magnitude can be chosen in many different ways to achieve an additional translational motion objective. For example, it can be used to asymptotically track a quadrotor altitude command [28]. Since the translational motion of the quadrotor UAV can only be partially controlled; this flight mode is most suitable for short time periods where an attitude maneuver is to be completed. [14]

****

Control input [9]

$$u = -k_R e_R - k_\Omega \Omega - mg\rho \times R^T e_3 \tag{3-15}$$

Insert into Equation 2-13; closed loop dynamics are given by

$$J\dot{\Omega} = -\Omega \times J\Omega - k_R e_R - k_\Omega \Omega \tag{3-16}$$

$$\dot{R} = R\hat{\Omega} \tag{3-17}$$

3-1-2 Load Attitude Tracking

Tracks load attitude reference. Outputs attitude reference to attitude controller.

How is the controller built.

Dependent of what values? How to choose parameters.

Figure 3-4

$$R = \tag{3-18}$$

3-1-3 Load Position Tracking

Tracks load position reference. Outputs load attitude reference.

$$q = \tag{3-19}$$

Figure 3-5

3-2 Parameter- and State Estimation

How to choose parameters and how to select gains for errors

How to estimate states?

What parameters must be

Refer to Lyapunov stability analysis [2]

3-3 Summary

****
What is Geometric Control?
Why Geometric Control?
Control design will be based on Nonlinear Geometric Control

The proposed control system is robust to switching conditions since each flight mode has almost global stability properties, and it is straightforward to design a complex maneuver of a QR. [15] Where are the Error functions based on?

Form bridge between Geometric Control and Hybrid Control
Why Hybrid Control?

Parameter Estimation can be done by
State Estimation can be done by

****

Chapter 4

Experiments and Results

4-1 Experiments

Simulations were developed using Matlab and Simulink.

4-1-1 LQR Control

LQR as benchmark.
LQR based on small angle assumption.
Derivation on linearized model in appendix A-1-1

Figure 4-1: LQR control design

4-1-2 Performance Criteria

****
Performance that can be evaluated for the cases in Figure 4-2. Performance can be specified as the following items.

- Step Response
    - Settling time (if swing minimization is important)

  - Rising time (important if time critical)
  - Overshoot (if max swing is critical)
  - Steady state error / swing of load (if accuracy is important)
- Disturbance Rejection
- Trajectory tracking
  - Can we minimize time, while minimizing position error (All Cases)
  - Minimum position error (All Cases)
  - Maximum amplitude/frequency of wave with respect to stability (Case B)
- Computational Effort (?)

Explain cases, why interesting and what can be expected?

4-1-3 Case A

4-1-4 Case B

4-1-5 Case C

Figure 4-2: Cases of which the performance could be evaluated

4-2 Command Filtering

Consequence of backstepping control is that inner control loops depend on the the output of outer control loops. The controllers are functions of these generated outputs and their derivatives. This can be calculated analytically, which can be tedious or by estimating with the use of a Command Filter as explained by [**?**]

A command filter is implemented to compute $\dot{R}_c, \ddot{R}_c, \dot{q}_c, \ddot{q}_c$, the virtual control command to stabilize the loop within. [**?**]

Examples from [16] and [17].

****

Easy implementation. Less computational effort.

Less accurate, because filters high frequency signals.

****

The load attitude controller generates a commanded QR attitude R_c and its derivative $\dot{R}_c$. In the same fashion, the load position controller generates a commanded load attitude q_c and its derivative $\dot{q}_c$. The controllers are functions of these commanded signals and their derivatives. Instead of analytic differentiation of these signals, they are obtained by integration by applying a third order low pass filter to the original signals R_c^o and q_c^o. The transfer function of the original commanded input signal X_c^o and the filtered output X_c has the form

$$\frac{X_c(s)}{X_c^o(s)} = H(s) = \frac{\omega_{n1}}{s+\omega_{n1}} \cdot \frac{\omega_{n2}^2}{s^2 + 2\zeta\omega_{n2}s + \omega_{n2}^2} \tag{4-1}$$

Where x_c is the filtered signal, ζ the damping ratio and ω_n the undamped natural frequency. See Figure 4-3. The state space implementation of this third order filter is [17]

$$\dot{x}_1 = x_2 \tag{4-2}$$
$$\dot{x}_2 = x_3 \tag{4-3}$$
$$\dot{x}_3 = -(2\zeta\omega_{n2} + \omega_{n1})x_3 - (2\zeta\omega_{n1}\omega_{n2} + \omega_{n2}^2)x_2 - (\omega_{n1}\omega_{n2}^2)(x_1 - x_c^o) \tag{4-4}$$

where $x_1 = x_c$, $x_2 = \dot{x}_c$ and $x_3 = \ddot{x}_c$.

Figure 4-3: Representation of the command filter

$$\frac{x_c}{x_c^o} = \frac{\omega_{n1}}{s+\omega_{n1}} \cdot \frac{\omega_{n2}^2}{s^2 + 2\zeta\omega_{n2}s + \omega_{n2}^2} \tag{4-5}$$
$$\Rightarrow x_c^{'''} = -(2\zeta\omega_{n2} + \omega_{n1})x_c^{''} - (2\zeta\omega_{n1}\omega_{n2} + \omega_{n2}^2)x_c^{'} - (\omega_{n1}\omega_{n2}^2)(x_c - x_c^o) \tag{4-6}$$

4-3 Results

4-3-1 Case A

4-3-2 Case B

4-3-3 Case C

4-4 Conclusion

Chapter 5

Conclusions and Future Work

5-1 Summary and Conclusions

5-2 Recommendations for Future Work

****

Model Validation; now it's estimation/copied from other work
System Identification

****

5-2-1 Modeling Constraints

There are several techniques to handle input saturation, the most popular ones are anti-windup techniques. Back-calculation is such a method for PID to activate the integrator, is this possible for NL control?

****

[18] includes uncertainties in the translational dynamics and rotational dynamics. Out of the scope, might be interesting.
****

5-2-2 Hybrid Modeling

Switching between several flight modes yields autonomous acrobatic maneuvers. Robust to switching conditions ***why?
[19]

5-2-3 Trajectory Generation

Minimum Snap Trajectory Generation

Trajectory can be generated by solving a **QP!** via minimum snap generation.

Problem in smaller 4-D space instead of 12-D, with help of differential flatness. Explain differential flatness and its usefulness.

Is able to include constraints in **QP!**.

[13]

Obstacle avoidance

Appendix A

Appendix

A-1 Derivation of LQR controller

A-1-1 Modeling

****
A traditional modeling method might represent the rotation matrix with a local coordinate system, for example with a Euler Angle parameterization.
****

****
A commonly used method for modeling a system is via Newton's law and Lagrangian mechanics. Based on Euler-Lagrange? $\rightarrow$ Geometric Mechanics

****

A-1-2 Controller

$$\mathbf{x} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix} \tag{A-1}$$

$$\mathbf{q} = \begin{bmatrix} x & y & z & \phi & \theta & \psi & \theta_L & \psi_L \end{bmatrix}^T \tag{A-2}$$

$$\dot{\mathbf{q}} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{z} & \dot{\phi} & \dot{\theta} & \dot{\psi} & \dot{\theta}_L & \dot{\psi}_L \end{bmatrix}^T \tag{A-3}$$

$$u = \begin{bmatrix} f & \tau_\phi & \tau_\theta & \tau_\psi \end{bmatrix}^T \tag{A-4}$$

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu \tag{A-5}$$
$$y = C\mathbf{x} + Du \tag{A-6}$$

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \tag{A-7}$$

From Equation follows

$$\ddot{x} = -\frac{f}{m} \tag{A-8}$$
$$\ddot{y} = \tag{A-9}$$
$$\ddot{z} = \tag{A-10}$$

The mathematical model is linearized around the following operating points

$$\bar{\mathbf{x}} = \begin{bmatrix} \bar{x} & \bar{y} & \bar{z} & \mathbf{0}_{1\times 13} \end{bmatrix}^T \tag{A-11}$$
$$\bar{\mathbf{u}} = \begin{bmatrix} (m_Q + m_L)g & 0 & 0 & 0 \end{bmatrix}^T \tag{A-12}$$

Assuming small angles, the following holds

$$\text{for } \gamma = \phi, \theta, \psi, \theta_L, \psi_L \tag{A-13}$$
$$sin(\gamma) \simeq \gamma \tag{A-14}$$
$$cos(\gamma) \simeq 1 \tag{A-15}$$
$$\dot{\gamma} \simeq 0 \tag{A-16}$$
$$F \simeq (m_Q + m_L)g \tag{A-17}$$

$$A = \frac{\partial \mathbf{f}(x,u)}{\partial x} \,|_{x=\bar{x},u=\bar{u}} \tag{A-18}$$

$$B = \frac{\partial \mathbf{f}(x,u)}{\partial u} \,|_{x=\bar{x},u=\bar{u}} \tag{A-19}$$

$$u = -K\left[\mathbf{x}_{des}(t) - \mathbf{x}(t)\right] \tag{A-20}$$

A-2 Derivation of Equations of motion

A-2-1 Load Dynamics

Let x_{CM} denote the position of the center of mass of the combined Quadrotor-Load system, expressed in $\{\mathcal{I}\}$. Which can be found by

$$\begin{aligned} m_Q(x_Q - x_{CM}) + m_L(x_L - x_{CM}) = 0 \\ (m_Q + m_L)x_{CM} = m_Q x_Q + m_L x_L \end{aligned} \tag{A-21}$$

Applying the laws of motion to (A-21) and inserting (2-8) gives the

$$\begin{aligned}(m_Q + m_L)\ddot{x}_{CM} &= fRe_3 - (m_Q + m_L)ge_3 \\ (m_Q + m_L)(\ddot{x}_L + ge_3) &= fRe_3 + m_Q L\ddot{q}\end{aligned} \tag{A-22}$$

A-3 An appendix section

$$Testequation \tag{A-23}$$

A-3-1 A MatlabListing

```
%
% Comment
%
n=10;
for i=1:n
    disp('Ok');
end
```

Bibliography

[1] T. Lee, "Geometric Control of an Autonomous Quadrotor UAV," *Control.*

[2] F. Bullo and A. D. Lewis, *Geometric control of mechanical systems: modeling, analysis, and design for simple mechanical control systems.* Springer, 2005.

[3] V. Jurdjevic, *Geometric Control Theory.* Cambridge: Cambridge University Press, 1997.

[4] T. Lee, *Computational Geometric Mechanics and Control of Rigid Bodies.* PhD thesis, The University of Michigan, 2008.

[5] R. M. Murray, Z. Li, and S. S. Sastry, *A Mathematical Introduction to Robotic Manipulation*, vol. 29. 1994.

[6] W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry.* 2003.

[7] T. L. T. Lee, N. McClamroch, and M. Leok, "A lie group variational integrator for the attitude dynamics of a rigid body with applications to the 3D pendulum," *Proceedings of 2005 IEEE Conference on Control Applications, 2005. CCA 2005.*, pp. 962–967, 2005.

[8] T. Lee, M. Leok, and N. H. McClamroch, "Lagrangian mechanics and variational integrators on two-spheres," *International Journal for Numerical Methods in Engineering*, vol. 79, no. 9, pp. 1147–1174, 2009.

[9] T. Lee, M. Leok, and N. H. McClamroch, "Stable Manifolds of Saddle Points for Pendulum Dynamics on Sˆ2 and SO(3)," p. 9, 2011.

[10] R. Mahony, V. Kumar, and P. Corke, "Multirotor Aerial Vehicles: Modeling, Estimation, and Control of Quadrotor," *IEEE Robotics & Automation Magazine*, vol. 19, no. 3, pp. 20–32, 2012.

[11] J. H. Gillula, H. Huang, M. P. Vitus, and C. J. Tomlin, "Design of guaranteed safe maneuvers using reachable sets: Autonomous quadrotor aerobatics in theory and practice," in *Proceedings - IEEE International Conference on Robotics and Automation*, pp. 1649–1654, 2010.

[12] T. Lee, M. Leok, and N. H. Mcclamroch, "Geometric Tracking Control of a Quadrotor UAV on SE(3)," *49th IEEE Conference on Decision and Control*, no. 3, pp. 5420–5425, 2010.

[13] D. Mellinger and V. Kumar, "Minimum snap trajectory generation and control for quadrotors," in *Proceedings - IEEE International Conference on Robotics and Automation*, pp. 2520–2525, 2011.

[14] F. A. Goodarzi, "Geometric Nonlinear Controls for Multiple Cooperative Quadrotor UAVs Transporting a Rigid Body," no. December 2009, 2015.

[15] T. Lee, M. Leok, and N. McClamroch, "Control of complex maneuvers for a quadrotor UAV using geometric methods on SE (3)," *arXiv*, 2010.

[16] J. A. Farrell, M. Polycarpou, M. Sharma, and W. Dong, "Command Filtered Backstepping," pp. 1923–1928, 2008.

[17] V. Djapic, J. Farrell, and W. Dong, "Land vehicle control using a command filtered backstepping approach," *Proceedings of the American Control Conference*, pp. 2461–2466, 2008.

[18] F. Goodarzi, D. Lee, and T. Lee, "Geometric nonlinear PID control of a quadrotor UAV on SE (3)," *Control Conference (ECC), 2013 European*, pp. 3845–3850, 2013.

[19] S. Tang, "Aggressive Maneuvering of a Quadrotor with a Cable-Suspended Payload," tech. rep., University of Pennsylvania Philadelphia, Pennsylvania, 2014.

Bibliography

[1] T. Lee, "Geometric Control of an Autonomous Quadrotor UAV," *Control*.

[2] F. Bullo and A. D. Lewis, *Geometric control of mechanical systems: modeling, analysis, and design for simple mechanical control systems.* Springer, 2005.

[3] V. Jurdjevic, *Geometric Control Theory.* Cambridge: Cambridge University Press, 1997.

[4] T. Lee, *Computational Geometric Mechanics and Control of Rigid Bodies.* PhD thesis, The University of Michigan, 2008.

[5] R. M. Murray, Z. Li, and S. S. Sastry, *A Mathematical Introduction to Robotic Manipulation*, vol. 29. 1994.

[6] W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry.* 2003.

[7] T. L. T. Lee, N. McClamroch, and M. Leok, "A lie group variational integrator for the attitude dynamics of a rigid body with applications to the 3D pendulum," *Proceedings of 2005 IEEE Conference on Control Applications, 2005. CCA 2005.*, pp. 962–967, 2005.

[8] T. Lee, M. Leok, and N. H. McClamroch, "Lagrangian mechanics and variational integrators on two-spheres," *International Journal for Numerical Methods in Engineering*, vol. 79, no. 9, pp. 1147–1174, 2009.

[9] T. Lee, M. Leok, and N. H. McClamroch, "Stable Manifolds of Saddle Points for Pendulum Dynamics on Sˆ2 and SO(3)," p. 9, 2011.

[10] R. Mahony, V. Kumar, and P. Corke, "Multirotor Aerial Vehicles: Modeling, Estimation, and Control of Quadrotor," *IEEE Robotics & Automation Magazine*, vol. 19, no. 3, pp. 20–32, 2012.

[11] J. H. Gillula, H. Huang, M. P. Vitus, and C. J. Tomlin, "Design of guaranteed safe maneuvers using reachable sets: Autonomous quadrotor aerobatics in theory and practice," in *Proceedings - IEEE International Conference on Robotics and Automation*, pp. 1649–1654, 2010.

[12] T. Lee, M. Leok, and N. H. Mcclamroch, "Geometric Tracking Control of a Quadrotor UAV on SE(3)," *49th IEEE Conference on Decision and Control*, no. 3, pp. 5420–5425, 2010.

[13] D. Mellinger and V. Kumar, "Minimum snap trajectory generation and control for quadrotors," in *Proceedings - IEEE International Conference on Robotics and Automation*, pp. 2520–2525, 2011.

[14] F. A. Goodarzi, "Geometric Nonlinear Controls for Multiple Cooperative Quadrotor UAVs Transporting a Rigid Body," no. December 2009, 2015.

[15] T. Lee, M. Leok, and N. McClamroch, "Control of complex maneuvers for a quadrotor UAV using geometric methods on SE (3)," *arXiv*, 2010.

[16] J. A. Farrell, M. Polycarpou, M. Sharma, and W. Dong, "Command Filtered Backstepping," pp. 1923–1928, 2008.

[17] V. Djapic, J. Farrell, and W. Dong, "Land vehicle control using a command filtered backstepping approach," *Proceedings of the American Control Conference*, pp. 2461–2466, 2008.

[18] F. Goodarzi, D. Lee, and T. Lee, "Geometric nonlinear PID control of a quadrotor UAV on SE (3)," *Control Conference (ECC), 2013 European*, pp. 3845–3850, 2013.

[19] S. Tang, "Aggressive Maneuvering of a Quadrotor with a Cable-Suspended Payload," tech. rep., University of Pennsylvania Philadelphia, Pennsylvania, 2014.

Nomenclature

ω_i	Angular velocity of rotor i around its axis, $i = \{1, 2, 3, 4\}$
$\tau_{drag,i}$	Drag moment generated by each propellor
$\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$	Unit vectors along the axes of $\{\mathcal{B}\}$
$\{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3\}$	Unit vectors along the axes of $\{\mathcal{C}\}$
$\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$	Unit vectors along the axes of $\{\mathcal{I}\}$
$\{\mathcal{B}\}$	Body Frame
$\{\mathcal{C}\}$	Intermediary Frame
$\{\mathcal{I}\}$	Inertial World Frame
b	Thrust factor
d	Drag factor
L	Length of the cable
q	Unit vector from Quadrotor to Load
x_L	Position of the of the Quadrotor CM
x_Q	Position of the of the Quadrotor CM
x_{CM}	Position CM of Quadrotor-Load system

Acronyms

QR	Quadrotor
CoM	Center of Mass
DOF	Degree of Freedom
MPC	Model Predictive Control
LQR	Linear Quadratic Regulator
QP	Quadratic Programming