Master of Science Thesis

Geometric Control of a Quadrotor with a Suspended Load

Subtitle

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June 6, 2017

Delft Center for Systems and Control

Delft University of Technology

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N.N. Vo

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The undersigned hereby certify that they have read and recommend to the Faculty of Mechanical, Maritime and Materials Engineering for acceptance a thesis entitled **"Geometric Control of a Quadrotor with a Suspended Load"** by **N.N. Vo** in partial fulfillment of the requirements for the degree of **Master of Science**.

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Abstract

A Quadrotor is a type of Unmanned Aerial Vehicle that has received an increasing amount of attention recently with many applications including search and rescue, surveillance, supply of food and medicines as disaster relief and object manipulation in construction and transportation.

An interesting control problem is the Load Position Tracking of a cable suspended load. The system is highly nonlinear and under-actuated. The load cannot be actuated directly and has a natural swing at the end of each Quadrotor movement. A Nonlinear Geometric Control approach allows differential geometric techniques to be applied to systems control, which can be defined on a smooth nonlinear configuration space. This creates a coordinate-free dynamic model, while avoiding the problem of singularities on local charts.

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Intro about GC.. Reasons to consider GC..

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Where simple linear control methods are restricted to small angle movements, nonlinear control methods allow more aggressive and faster movements. The goal of the project is to investigate the effects on load position tracking performance when the system is modeled and controlled via a Nonlinear Geometric Control approach.

****

The Quadrotor-Load system is modeled in a compact and coordinate-free fashion which allows the inherent geometric properties of the system to be controlled.

The main goal of this thesis is to research the effects on a cable-suspended load transportation using quadrotors, by involving complex or aggressive maneuvering through implementation of Non-Linear Geometric Control. Where linear control methods are restricted to small angle movement, non-linear control methods allow more aggressive movements.

Furthermore, the studied control techniques are explained and their advantages are addressed. Several trajectory generation approaches and the related optimization techniques are studied. Their applications, with different purposes such as obstacle avoidance, time-optimal and swing-free trajectory planning are explained. The survey is concluded with a discussion

about finding a suitable *************************************
Define suitable

****

control design to achieve the quadrotor-assisted task involving manipulation of a cable-suspended load.

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Delft, University of Technology N.N. Vo
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Chapter 1

Introduction

****

A Quadrotor (QR) is a type of **uav!** (**uav!**) that has received an increasing amount of attention recently with many applications being actively investigated. Possible applications include search and rescue, surveillance, reliable supply of food and medicines as disaster relief and object manipulation in construction and transportation. It has already proven itself useful for many tasks like multi-agent missions, mapping, explorations and even acrobatic performances.

Considering a multi-agent task, one can think of multiple QRs assisting in the transportation of a load. This cooperation can be executed in many ways, but this literature focuses on QRs with a cable-suspended load in motion. The suspended object naturally continues to swing at the end of this movement. It could be the case that the residual motion results in damage or in order to avoid obstacles, a accurate positioning is required. Reducing the oscillation, or controlling the position of the suspended load might be necessary, but challenging in the fact that a cable-suspended system is under-actuated.

In this chapter, the motivation for writing this literature study is given. Next, a former research in the scope of this literature survey is discussed. And finally, the organization of the report is presented.

1-1 Aim and Motivation

The inspiration for this literature survey is build upon the idea of creating a multiple autonomous QR system for a cooperative towing task. The advantage of using multiple robots for object manipulation is the possibility to reduce complexity of the individual robot, decrease cost over traditional robotic systems and high reliability. One can think of examples in nature, where individuals coordinate, cooperate and collaborate to perform tasks that they individually can not accomplish. Redundancy makes development of fail safe control methods possible and can extend the capabilities of a single robot.

The aim is to control the position of a suspended load using a Quadrotor. Former work on Quadrotor control regularly rely on linear control such as PID or LQR for a dynamic model linearized about the hover state. A single Quadrotor is considered for the transportation of a cable suspended load, which exerts forces and torques on the Quadrotor. This motivates to compare a linear control strategy with a non-linear control strategy. ***************************************

****

State of the art methods
Research and Engineering goals
Organization of the report
****

1-2 Organization of the Report

- Chapter 1
- Chapter 2
- Chapter 3
- Chapter 4
- Chapter 5
- Chapter **??**

****
Hoe de keuze tot stand is gekomen, hoe uitgebreid?
Moet ik nog een stuk wijden aan literature survey?
****

Chapter 2

Dynamic Model

****

The dynamics of the QR-Load system are described by the laws of kinematics and the application of Newton's laws or Lagrangian mechanics. Opposed to the classical modeling techniques, it is also possible to describe the system's configuration space as a differentiable manifold with the tools of differential geometry.

Considering the properties of the system, the QR is described as a rigid body with six degrees of freedom, driven by forces and moments. The motion of a rigid body can be described by a translation of the center of mass and a rotation about the center of mass. The derived mathematical model is represented by a set of dynamic equations commonly used for rigid body transformations.

To derive the equations of motions traditional modeling methods often parameterize the rotations in a local coordinate system. Euler angles are commonly used, however these coordinates might result in singularities. Furthermore, there are 24 sets of Euler angles, which can lead to ambiguity. In order to avoid these complexities, the attitude dynamics of the QR-Load system can be globally expressed on the Special Orthogonal Group $SO(3)$, *2-sphere* S^2 and Special Euclidean Group $SE(3)$. This leads to a compact notation of the equations of motion, making all the trigonometric functions that Euler angles introduce unnecessary. ***************************************

****

A mathematical model of the system needs to be derived in order to study the effects of Geometric Control.

****

2-1 Modeling Assumptions

The following assumptions are applied to simplify the model.

The QR model representation is shown in Figure 2-1. Three Cartesian coordinate frames are defined:

- The body-fixed reference frame $\{\mathcal{B}\}$ (Body Frame)

  with unit vectors $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ along the axes
- The ground-fixed reference frame $\{\mathcal{I}\}$ (Inertial Frame)

  with unit vectors $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ along the axes
- The intermediary frame $\{\mathcal{C}\}$, ($\{\mathcal{I}\}$ rotated by the yaw angle ψ)

  with unit vectors $\{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3\}$ along the axes

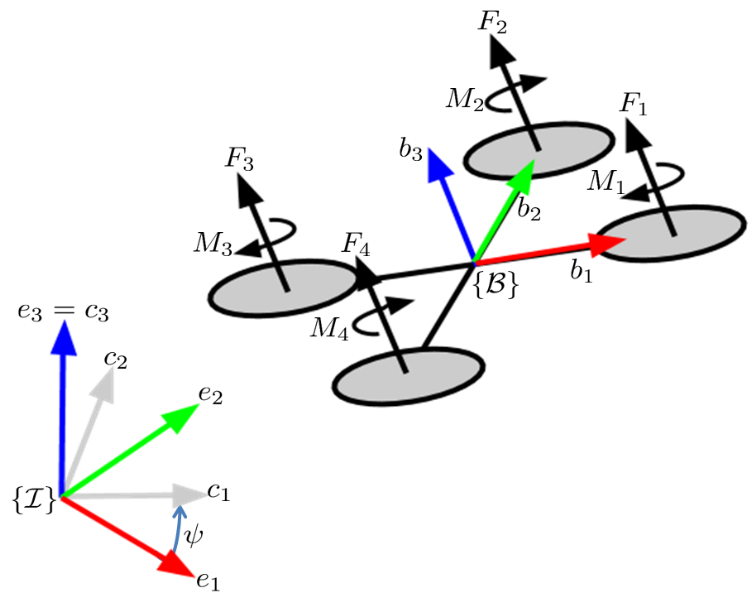

Figure 2-1: Quadrotor model representation

Figure 2-2: Quadrotor with Load model representation

The position of the body frame is described by a vector evolving on $\mathbb{R}^3$, and is represented with respect to the inertial frame. The orientation of the body frame with respect to the inertial frame evolves on a nonlinear space, for which several methods exist to describe this, such as *Euler Angles*, quaternions or rotation matrices.

The complex dynamics of the rotors and their interactions with drag and thrust forces are represented by a simplified model. The angular speed ω_i of rotor i, for $i = 1, \ldots, 4$, generates a force F_i parallel to the direction of the rotor axis of rotor i, given by

$$F_i = \left(\frac{K_v K_\tau \sqrt{2\rho A}}{K_t} \omega_i \right)^2 = b\omega_i^2 \tag{2-1}$$

where K_v, K_t are constants related to the motor properties, ρ is the density of the surrounding air, A is the area swept out by the rotor, K_τ is a constant determined by the blade configuration and parameters, and b is the thrust factor.
The torque around the axis of rotor i generated due to drag is given by

$$M_i = \frac{1}{2} R\rho C_D A(\omega_i R)^2 = d\omega_i^2 \tag{2-2}$$

where R is the radius of the propeller, C_D is a dimensionless constant, and d is the drag constant.

For given desired total thrust f and total moment $M = \begin{bmatrix} M_\phi & M_\theta & M_\psi \end{bmatrix}^T$, the required rotor speeds can be calculated by solving the following equation

$$\begin{bmatrix} f \\ M_\phi \\ M_\theta \\ M_\psi \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & -Lb & 0 & Lb \\ Lb & 0 & -Lb & 0 \\ -d & d & -d & d \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \tag{2-3}$$

where L is the cable length and M_ϕ, M_θ, M_ψ denote the moments around the x, y, z-axis in $\{\mathcal{B}\}$, resp.

Table 2-1 shows the most common assumptions that are used for modeling the QR, simplifying the complexity of the model.

- The structure of the QR is rigid and symmetric.
  Elastic deformations and shock (sudden accelerations) of the QR are ignored.
- The mass distribution of the QR is symmetrical in the x-y plane.
- The inertia matrix is time-invariant.
- Aerodynamic effects acting on the QR are neglected.
  Blade flapping, Turbulence, Ground Effects.
- The air density around the QR is constant.
- The propellers are rigid $\Rightarrow$ The thrust produced by rotor i is parallel to the axis of rotor i.
- Drag factor d and thrust factor b are approximated by a constant.
  Thrust force F_i and moment M_i of each propeller is proportional to the square of the propeller speed.

Table 2-1: Modeling assumptions Quadrotor model

• The cable is modeled as a rigid and massless cable. • The cable is connected to a friction-less joint at the origin of the body-fixed. • The tension in the cable is considered to be non-zero. This implies that the QR-Load subsystem, consisting of a separate QR and Load in free fall, is disregarded. • Aerodynamic effects acting on the load are neglected. reference frame. • Assumption Details Assumption 2

Table 2-2: Modeling assumptions Quadrotor+Load model

2-2 Geometric Modeling

Differential geometry is used to analyze the underlying geometric properties of a system. In geometric modeling the configuration space is a group manifold instead of a Euclidean space. The kinetic and potential energies are expressed in terms of this configuration space and their tangent spaces.

****

The orientation of the QR is described on the Special Orthogonal Group $SO(3)$, using rotation matrices, instead of using local charts induced by Euler Angle parameterizations. Whereas the orientation of the Load is described on a two-sphere S^2

****

****

Problems, singularities with Euler-Angles
Other attitude representations, such as exponential coordinates, quaternions, or Euler angles, can also be used following standard descriptions, but each of the representations has a disadvantage of introducing an ambiguity or singularity. Why charts on $SO(3)$ https://en.wikipedia.org/wiki/Charts_on_SO(3)

****

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[1]

Mechanics studies the dynamics of physical bodies acting under forces and potential fields. In Lagrangian mechanics, the trajectories are obtained by finding the paths that minimize the integral of a Lagrangian over time, called the action integral. Rigid body dynamics are characterized by Lagrangian/Hamiltonian dynamics. The dynamics of a Lagrangian system has unique geometric properties and these are exploited to obtain Euler-Lagrange equations. The resulting intrinsic form of the Euler-Lagrange equations are more compact than equations expressed in terms of local coordinates.

When angular errors are large, the difference in Euler angles is no longer a good metric to define the orientation error. Local coordinates often require symbolic computational tools due to complexity of multi-body systems. Hence, the error is rather written as the required 3-D rotation to get from the current to a desired orientation. As a result, the equations of motion and the control systems can be developed on a configuration manifold in a coordinate-free, compact, unambiguous manner, while singularities of local parameterization are avoided to generate agile maneuvers in a uniform way.

Most of nonlinear dynamics and control problems are studied in a linear space.

$$\dot{x} = \mathbf{f}(t, x, u), \quad x \in \mathbb{R}^n, u \in \mathbb{R}^m, \mathbf{f} : \mathbb{R}^{n+m+1} \rightarrow \mathbb{R}^n \tag{2-4}$$

Geometric Mechanics and Control is used to understand the structure of the equations of motion of a system in order to facilitate its analysis and design. The evolution of the system and the design of controllers is done on a nonlinear manifold.

In mathematics there are different types of singularity, in these cases we are talking about the situation where:
Many points in one representation are mapped onto a single point in another representation.
Infinitesimal changes close to the singularity in one representation may cause large changes in the other representation.

Computational Geometric Mechanics and Control
Computation algorithms must be developed which preserve the geometric properties of a mechanical system.
Robust and careful numerical implementation of geometric control theory to complex engineering systems.
Provides nontrivial maneuvers that are globally valid on a nonlinear configuration manifold.

$$SO(3) \triangleq \left\{ R \in \mathbb{R}^{3\times3} : RR^T = I_{3\times3}, det(R) = 1 \right\} \tag{2-5}$$

Rotational matrices with determinant 1 is a Lie group: $SO(3)$
$SO(3)$ is the group of all rotations about origin of three-dimensional Euclidean space
Rotation about the origin is a transformation that preserves the origin, Euclidean distance and orientation.
Every rotation has a unique inverse rotation and the identity map satisfies the definition of a rotation.
Possible ways to represent rotations: orthogonal matrices with determinant 1, axis and rotation angle, geometric algebra as a rotor, sequence of three rotations about three fixed axes; Euler Angles
Lie group is a group that is also a differentiable manifold.
Differentiable manifold is a type of manifold that is locally similar enough to a linear space to allow to do calculus.
Manifold is a topological space that locally resembles Euclidean space near each point. Each point of an n-dimensional manifold has a neighbourhood that is homeomorphic to the n-dimensional Euclidean space.
Homeomorphism is a continuous function between topological spaces that has a continuous

inverse function. (mug transforms to a torus)

Special Orthogonal group

$$SO(3) = \left\{R \in \mathbb{R}^{3\times 3} | R^T R = I, \quad detR = 1\right\} \tag{2-6}$$

The group operation for $SO(3)$ corresponds to matrix multiplication.
The attitude kinematics equation is given by

$$\dot{R} = R\hat{\Omega} \tag{2-7}$$

where $\Omega \in \mathbb{R}^3$ is the angular velocity represented in the body fixed frame. The hat map $\hat{\cdot} : \mathbb{R}^3 \to \mathfrak{so}(3)$ is an isomorphism between $\mathbb{R}^3$ and the set of 3×3 skew symmetric matrices. The Lie algebra $\mathfrak{so}(3)$ is defined by

$$\hat{\Omega} = \begin{bmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{bmatrix} \tag{2-8}$$

Rotation formalisms in three dimensions https://en.wikipedia.org/wiki/Rotation_formalisms_in_three_dimensions#cite_note-5
Combining two successive rotations, each represented by an Euler axis and angle, is not straightforward, and in fact does not satisfy the law of vector addition, which shows that finite rotations are not really vectors at all. It is best to employ the rotation matrix or quaternion notation, calculate the product, and then convert back to Euler axis and angle.

Euler Angles
However, the definition of Euler angles is not unique and in the literature many different conventions are used. These conventions depend on the axes about which the rotations are carried out, and their sequence because rotations are not commutative. Therefore, Euler angles are never expressed in terms of the external frame, or in terms of the co-moving rotated body frame, but in a mixture. Other conventions (e.g., rotation matrix or quaternions) are used to avoid this problem.

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Geometric mechanics is a modern description of classical mechanics from the perspective of differential geometry [2, 3]. It explores the geometric structure of a Lagrangian or Hamiltonian system through the concept of vector fields, symplectic geometry, and symmetry techniques. Geometric mechanics provides fundamental insights into mechanics and yields useful tools for dynamics and control theory.

In control systems engineering, the underlying geometric features of a dynamic system are often not considered carefully. For example, many control systems are developed for the standard form of ordinary differential equations, namely $\dot{x} = f(x, u)$, where the state and the control input are denoted by x and u. It is assumed that the state and the control input lie in Euclidean spaces, and the system equations are defined in terms of smooth functions between Euclidean spaces. However, for many interesting mechanical systems, the configuration space cannot be expressed globally as a Euclidean space.

In [4] dynamics and optimal control problems for rigid bodies are studied, incorporating their geometric features. The focus lies on obtaining geometric properties of the dynamics of rigid bodies, how their configuration can be described and how these geometric properties are utilized in control system analysis and design. Computational methods for rigid bodies, that preserve the underlying Lagrangian/Hamiltonian system structure of rigid body dynamics as well as the Lie group structure of the configurations are developed.

Lie Group Configuration Manifold The configuration of a rigid body can be described by the location of its mass center and the orientation of the rigid body in a 3-D space. The location of the rigid body can be expressed in Euclidean space, but the attitude evolves in a nonlinear space with a certain geometry. The attitude of a rigid body is defined as the orientation of a body-fixed frame with respect to a reference frame and can be represented mathematically by a 3×3 orthonormal matrix.

A manifold is locally diffeomorphic to a Euclidean space, and it also has a group structure with the group action of matrix multiplication. A smooth manifold with a group structure is referred to as a Lie group; the Lie group of 3×3 orthonormal matrices with positive determinant is referred to as the special orthogonal group, $SO(3)$ [5].

The configuration manifold for the attitude dynamics of a rigid body is $SO(3)$, and the configuration manifold for combined translational and rotational motion of a rigid body is the special Euclidean group $SE(3)$, which is a semi-direct product of $SO(3)$ and $\mathbb{R}^3$. A direct product of the Lie groups $SE(3), SO(3),$ and $\mathbb{R}^n$ can represent the configuration of multiple rigid bodies, and it is also a Lie group.

****

Introduction to the basics of Lie group theory and its connections with rigid body kinematics is given in [5].

An Introduction to Differentiable Manifolds [6]

Geometric Control by [2].

****

Differentiable Manifolds

2-2-1 Quadrotor-Load Modeling

Figure 2-3: Quadrotor-Load model representation

The Quadrotor-Load model is represented in Figure 2-3, where the unit vector q gives the direction from the QR to the Load expressed in $\{\mathcal{B}\}$. The position of the QR and Load, x_Q and x_L resp., are related by

$$x_Q = x_L - Lq \tag{2-9}$$

where L is the length of the cable.

Load Attitude Dynamics To develop the Euler-Lagrange equations for mechanical systems that evolve on a Lie group, an approach developed by [7, 4, 8, 9] is used, which is based on Hamilton's principle.

The action integral is defined as

$$S = \int_{t_1}^{t_2} \mathcal{L} dt \tag{2-10}$$

where $\mathcal{L} = \mathcal{T} - \mathcal{U}$ is the Lagrangian of the system, where $\mathcal{T}$,$\mathcal{U}$ are the kinetic and potential energy, respectively. Hamilton's principle of least action states that the path a conservative mechanical system takes between two configurations at time t_1 and t_2, is the one for which Equation 2-10 is an extremum, stated as

$$\delta S = \int_{t_1}^{t_2} \delta \mathcal{L} dt = 0 \tag{2-11}$$

where $\delta\mathcal{L}$ is the variation of the Lagrangian. For systems with non-conservative forces and moments, Equation 2-11 is extended to

$$\delta S = \int_{t_1}^{t_2} (\delta W + \delta \mathcal{L}) dt = 0 \tag{2-12}$$

where δW is the virtual work. Equation 2-12 is applied to the QR-Load system, where the configuration manifold is $\mathbb{R}^3 \times S^2 \times SO(3)$. With the following states

$$\mathbf{x} = \begin{bmatrix} x_L & \dot{x}_L & q & \omega & R & \Omega \end{bmatrix}^T \tag{2-13}$$

****

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Rigid Body Attitude Dynamics evolve on $SE(3)$.

$$J\dot{\Omega} + \Omega \times J\Omega = mg\rho \times R^T e_3 + u \tag{2-14}$$

$$\dot{R} = R\hat{\Omega} \tag{2-15}$$

****

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The equations of motion for a rigid body with configuration $SE(3)$ are given by the *Newton-Euler equations* [5]:

$$\begin{bmatrix} mI & 0 \\ 0 & \mathcal{I} \end{bmatrix} \begin{bmatrix} \dot{v}^b \\ \dot{\omega}^b \end{bmatrix} + \begin{bmatrix} \omega^b \times mv^b \\ \omega^b \times \mathcal{I}\omega^b \end{bmatrix} = F^b \tag{2-16}$$

where m is the mass of the body, $\mathcal{I}$ is the inertia tensor, and $V^b = (v^b, \omega^b)$ and F^b represent the instantaneous body velocity and applied body wrench. ***************************************

****
The load dynamics evolve on S^2. Based on [9].
****

2-3 Classical Modeling

****
Reference A-1-1
****

From Newton's law

$$\begin{aligned} \dot{x}_Q &= v_Q \\ m\dot{v}_Q &= fRe_3 - m_Q g e_3 - Tq \\ \dot{x}_L &= v_L \\ m\dot{v}_L &= -m_L g e_3 + Tq \end{aligned} \tag{2-17}$$

Where the function that maps a vector of the Z-X-Y Euler angles to its rotation matrix $R \in SO(3)$ is denoted as [10]

$$R_{312}(\phi, \theta, \psi) = \begin{bmatrix} c_\psi c_\theta - s_\phi s_\psi s_\theta & -c_\phi s_\psi & c_\psi s_\theta + c_\theta s_\phi s_\psi \\ c_\theta s_\psi + c_\psi s_\phi s_\theta & c_\phi c_\psi & s_\psi s_\theta - c_\psi c_\theta s_\phi \\ -c_\phi s_\theta & s_\phi & c_\phi c_\theta \end{bmatrix} \tag{2-18}$$

The unit vector q from the QR to the load is represented in $\{\mathcal{B}\}$. Define ψ_L as the yaw-rotation of the load around the z-axis of $\{\mathcal{B}\}$and θ_L as the pitch-rotation of the load around the new y-axis of $\{\mathcal{B}\}$, see Figure 2-4.

$$q = \begin{bmatrix} s_{\theta_L} c_{\psi_L} \\ s_{\theta_L} s_{\psi_L} \\ c_{\theta_L} \end{bmatrix} \tag{2-19}$$

$$\begin{aligned} \ddot{x}_Q &= \frac{1}{m_Q}(f(c_\psi s_\theta + c_\theta s_\phi s_\psi) - T s_{\theta_L} c_{\psi_L}) \\ \ddot{y}_Q &= \frac{1}{m_Q}(f(s_\psi s_\theta - c_\psi c_\theta s_\phi) - T s_{\theta_L} s_{\psi_L}) \\ \ddot{z}_Q &= \frac{1}{m_Q}(f(c_\phi c_\theta) - T c_{\theta_L}) - g \end{aligned} \tag{2-20}$$

Figure 2-4: Quadrotor-Load model representation

$$\ddot{\psi} = \tilde{\tau}_\psi \tag{2-21}$$
$$\ddot{\theta} = \tilde{\tau}_\theta \tag{2-22}$$
$$\ddot{\phi} = \tilde{\tau}_\phi \tag{2-23}$$

2-4 Conclusion

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Compact, unambiguous, globally defined,

Pro/Cons of Classical Modeling Techniques vs Geometric Modeling

Linearized model/State Space model vs. Geometric modeling

Geometric Mechanics/Lie Groups/Lie Algebra is used in order to represent the dynamics of the system onto the nonlinear configuration manifold $SE(3)$
Advantage of this method is
Enables to model on
That type of control is discussed in the next chapter

****

Chapter 3

Control Design

Geometric Control Theory explores the application of differential geometric techniques to systems control. The objective is to have both the dynamics and its control input to flow along their inherent manifolds instead of local charts.

Geometric Control is based on a coordinate-free representation of the dynamics. The equations of motion are compact, unambiguous and singularity free.

Attitude control systems naturally evolve on non-linear configurations such as S^2 and $SO(3)$.

Global nonlinear dynamics of various classes of closed loop attitude control systems have been studied in recent years [**?**].

****

Existing Control Systems for Quadrotor UAVs:
Based on the linearized dynamics of a quadrotor UAV
Singularities in representing complex maneuvers
Fundamental restriction in tracking nontrivial trajectories

In contrast to hybrid control systems [11], **complicated reachability set analysis is not required** to guarantee safe switching between different flight modes, as the region of attraction for each flight mode covers the configuration space almost globally. Tracking control system can be developed on $SO(3)$, therefore it avoids singularities of Euler-Angles.

Benchmark with Linear Control

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Therefore, the prerequisites for its study are linear algebra, vector calculus, differential geometry and non-linear control theory. Geometric control theory is the study of how the geometry of the state space influences controls problems. This includes local properties like curvature, and global properties like the number of 'holes' in the space (sphere vs doughnut).

Intuition: The objective is to have both the dynamics and it's control input to directly flow along their inherent manifolds instead of on local charts. For example, I want to write out the equations of motion of a 3D pendulum and also control its orientation directly on the Special Orthogonal Group (using rotation matrices) instead of using local charts induced by Euler Angle Parametrizations (theta, phi and psi - the three famous angles of dynamics!).
Pros:
-¿ Compact expressions that are also intuitive. -¿ Globally defined (no singularities!). Therefore, one can build almost globally attractive controllers.
Cons:
-¿ The math gets very involved very quickly. (This could be subjective)
****

Control Schemes
Nonlinear Geometric Control. Error functions. Tracking controllers.

Attitude Controller

Define errors associated with the attitude dynamics of the QR. The attitude and angular velocity tracking error should be carefully chosen as they evolve on the tangent bundle of $SO(3)$. [**?**]

How to select gains for errors
Constants [12] or matrices; enabling unique gains for roll/pitch/yaw [13]

Parameter Estimation
State Estimation

3-1 Backstepping Control

****
Nonlinear control: Backstepping control is common. Definition backstepping: A technique for designing stabilizing controls for a special recursive class of nonlinear dynamical systems.

In a backstepping control approach, the control law is designed by using states as virtual control signals. At each step, a

****

3-2 Geometric Control of QR system

Control input [9]

$$u = -k_R e_R - k_\Omega \Omega - mg\rho \times R^T e_3 \tag{3-1}$$

Insert into Equation 2-14; closed loop dynamics are given by

$$J\dot{\Omega} = -\Omega \times J\Omega - k_R e_R - k_\Omega \Omega \tag{3-2}$$

$$\dot{R} = R\hat{\Omega} \tag{3-3}$$

****
where is e_Ω?
****

Three Flight Modes [1, 14]
Attitude Controlled Mode: track an attitude command $R_d(t)$
Position Controlled Mode: track a position command $x_d(t)$ and a heading direction $b_{1_d}(t)$
Velocity Controlled Mode: track a velocity command $v_d(t)$ and a heading direction $b_{1_d}(t)$

3-2-1 Attitude Tracking

****
Asymptotic tracking of the quadrotor attitude does not require specification of the thrust magnitude. As an auxiliary problem, the thrust magnitude can be chosen in many different ways to achieve an additional translational mo- tion objective. For example, it can be used to asymptotically track a quadrotor altitude command [28]. Since the translational motion of the quadrotor UAV can only be partially controlled; this flight mode is most suitable for short time periods where an attitude maneuver is to be completed. [15]
****

3-2-2 Position Tracking

Explain how f and $\vec{b_{1_d}}$ is obtained from $x_d(t)$?

The attitude and angular velocity tracking error should be carefully chosen as the evolve on the tangent bundle of the nonlinear space $SO(3)$. [12]
Why? Appendix[12], and [2]?

The error function on $SO(3)$ is chosen to be [12]

$$\Psi(R, R_d) = \frac{1}{2} tr\left[I - R_d^T R\right] \tag{3-4}$$

3-3 Geometric Control of QR-Load system

Three Flight Modes
QR Attitude Controlled Mode: track a QR attitude command $R_d(t)$ and a heading direction $b_{1_d}(t)$
Load Attitude Controlled Mode: track a load attitude command $q_d(t)$
Load Position Controlled Mode: track a load position $x_{L,d}(t)$

3-3-1 Quadrotor Attitude Tracking

3-3-2 Load Attitude Tracking

3-3-3 Load Position Tracking

3-4 Parameter - and State Estimation

How to choose parameters and how to estimate states?

3-5 Conclusion

****
What is Geometric Control?
Why Geometric Control?
Control design will be based on Nonlinear Geometric Control

The proposed control system is robust to switching conditions since each flight mode has almost global stability properties, and it is straightforward to design a complex maneuver of a QR. [14] Where are the Error functions based on?

Form bridge between Geometric Control and Hybrid Control
Why Hybrid Control?

Parameter Estimation can be done by
State Estimation can be done by

****

Chapter 4

Experiments and Results

4-1 Experiments

Simulations were developed using Matlab and Simulink.

4-1-1 LQR Control

****
Benchmark can be a simple control design
Benchmark can be Praveen's MPC controller to compare results
Benchmark can be open loop response Figure 4-1.

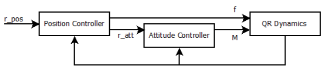

Figure 4-1: Open Loop control QR

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Benchmark can be an **LQR!** controller, A-1-1

Figure 4-2: LQR control design

4-1-2 Performance Criteria

****
Performance that can be evaluated for the cases in Figure 4-3. Performance can be specified as the following items.

- Step Response
  - Settling time (if swing minimization is important)
  - Rising time (important if time critical)
  - Overshoot (if max swing is critical)
  - Steady state error / swing of load (if accuracy is important)
- Disturbance Rejection
- Trajectory tracking
  - Can we minimize time, while minimizing position error (All Cases)
  - Minimum position error (All Cases)
  - Maximum amplitude/frequency of wave with respect to stability (Case B)
- Computational Effort (?)

Explain cases, why interesting and what can be expected?

4-1-3 Case A

4-1-4 Case B

4-1-5 Case C

Figure 4-3: Cases of which the performance could be evaluated

4-2 Command Filtering

Consequence of backstepping control is that inner control loops depend on the the output of outer control loops. The controllers are functions of these generated outputs and their derivatives. This can be calculated analytically, which can be tedious or by estimating with the use of a Command Filter as explained by [?]

A command filter is implemented to compute $\dot{R}_c, \ddot{R}_c, \dot{q}_c, \ddot{q}_c$, the virtual control command to stabilize the loop within. [?]

Examples from [16] and [17].

****

Easy implementation. Less computational effort.

Less accurate, because filters high frequency signals.

****

The load attitude controller generates a commanded QR attitude R_c and its derivative $\dot{R}_c$. In the same fashion, the load position controller generates a commanded load attitude q_c and its derivative $\dot{q}_c$. The controllers are functions of these commanded signals and their derivatives. Instead of analytic differentiation of these signals, they are obtained by integration by applying a third order low pass filter to the original signals R_c^o and q_c^o. The transfer function of the original commanded input signal X_c^o and the filtered output X_c has the form

$$\frac{X_c(s)}{X_c^o(s)} = H(s) = \frac{\omega_{n1}}{s+\omega_{n1}} \cdot \frac{\omega_{n2}^2}{s^2 + 2\zeta\omega_{n2}s + \omega_{n2}^2} \tag{4-1}$$

Where x_c is the filtered signal, ζ the damping ratio and ω_n the undamped natural frequency. See Figure 4-4. The state space implementation of this third order filter is [17]

$$\dot{x}_1 = x_2 \tag{4-2}$$
$$\dot{x}_2 = x_3 \tag{4-3}$$
$$\dot{x}_3 = -(2\zeta\omega_{n2} + \omega_{n1})x_3 - (2\zeta\omega_{n1}\omega_{n2} + \omega_{n2}^2)x_2 - (\omega_{n1}\omega_{n2}^2)(x_1 - x_c^o) \tag{4-4}$$

where $x_1 = x_c$, $x_2 = \dot{x}_c$ and $x_3 = \ddot{x}_c$.

Figure 4-4: Representation of the command filter

$$\frac{x_c}{x_c^o} = \frac{\omega_{n1}}{s+\omega_{n1}} \cdot \frac{\omega_{n2}^2}{s^2 + 2\zeta\omega_{n2}s + \omega_{n2}^2} \tag{4-5}$$
$$\Rightarrow x_c^{'''} = -(2\zeta\omega_{n2} + \omega_{n1})x_c^{''} - (2\zeta\omega_{n1}\omega_{n2} + \omega_{n2}^2)x_c^{'} - (\omega_{n1}\omega_{n2}^2)(x_c - x_c^o) \tag{4-6}$$

4-3 Results

4-3-1 Case A

4-3-2 Case B

4-3-3 Case C

4-4 Conclusion

Chapter 5

Conclusions and Future Work

5-1 Summary

5-2 Thesis Contribution

5-3 Recommendations for Future Work

****
Model Validation; now it's estimation/copied from other work
System Identification
****

5-3-1 Modeling Constraints

There are several techniques to handle input saturation, the most popular ones are anti-windup techniques. Back-calculation is such a method for PID to activate the integrator, is this possible for NL control?

****

[18] includes uncertainties in the translational dynamics and rotational dynamics. Out of the scope, might be interesting.

****

5-3-2 Hybrid Modeling

Switching between several flight modes yields autonomous acrobatic maneuvers. Robust to switching conditions ***why?

5-3-3 Trajectory Generation

Minimum Snap Trajectory Generation

Trajectory can be generated by solving a **QP!** via minimum snap generation.

Problem in smaller space with help of differential flatness.

Is able to include constraints.

Appendix A

Appendix

A-1 Derivation of LQR controller

A-1-1 Modeling

****
A traditional modeling method might represent the rotation matrix with a local coordinate system, for example with a Euler Angle parameterization.
****

When assuming small angle maneuvers, *Euler-angles* can be used to locally parameterize the orientation of the body-fixed reference coordinate frame with respect to the inertial reference coordinate frame.
****
A commonly used method for modeling a system is via Newton's law and Lagrangian mechanics. Based on Euler-Lagrange? $\rightarrow$ Geometric Mechanics

****

A-1-2 Controller

$$\mathbf{x} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix} \tag{A-1}$$

$$\mathbf{q} = \begin{bmatrix} x & y & z & \phi & \theta & \psi & \theta_L & \psi_L \end{bmatrix}^T \tag{A-2}$$

$$\dot{\mathbf{q}} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{z} & \dot{\phi} & \dot{\theta} & \dot{\psi} & \dot{\theta}_L & \dot{\psi}_L \end{bmatrix}^T \tag{A-3}$$

$$u = \begin{bmatrix} f & \tau_\phi & \tau_\theta & \tau_\psi \end{bmatrix}^T \tag{A-4}$$

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu \quad \text{(A-5)}$$
$$y = C\mathbf{x} + Du \quad \text{(A-6)}$$

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \quad \text{(A-7)}$$

From Equation follows

$$\ddot{x} = -\frac{f}{m} \quad \text{(A-8)}$$
$$\ddot{y} = \quad \text{(A-9)}$$
$$\ddot{z} = \quad \text{(A-10)}$$

The mathematical model is linearized around the following operating points

$$\bar{\mathbf{x}} = \begin{bmatrix} \bar{x} & \bar{y} & \bar{z} & \mathbf{0}_{1\times 13} \end{bmatrix}^T \quad \text{(A-11)}$$
$$\bar{\mathbf{u}} = \begin{bmatrix} (m_Q + m_L)g & 0 & 0 & 0 \end{bmatrix}^T \quad \text{(A-12)}$$

Assuming small angles, the following holds

$$\text{for } \gamma = \phi, \theta, \psi, \theta_L, \psi_L \quad \text{(A-13)}$$
$$sin(\gamma) \simeq \gamma \quad \text{(A-14)}$$
$$cos(\gamma) \simeq 1 \quad \text{(A-15)}$$
$$\dot{\gamma} \simeq 0 \quad \text{(A-16)}$$
$$F \simeq (m_Q + m_L)g \quad \text{(A-17)}$$

$$A = \frac{\partial \mathbf{f}(x, u)}{\partial x} \mid_{x=\bar{x}, u=\bar{u}} \quad \text{(A-18)}$$

$$B = \frac{\partial \mathbf{f}(x, u)}{\partial u} \mid_{x=\bar{x}, u=\bar{u}} \quad \text{(A-19)}$$

$$u = -K \left[\mathbf{x}_{des}(t) - \mathbf{x}(t)\right] \quad \text{(A-20)}$$

A-2 Derivation of Equations of motion

A-2-1 Load Dynamics

****

Let x_{CM} denote the position of the center of mass of the combined Quadrotor-Load system, expressed in $\{\mathcal{I}\}$. Which can be found by

$$\begin{aligned} m_Q(x_Q - x_{CM}) + m_L(x_L - x_{CM}) &= 0 \\ (m_Q + m_L)x_{CM} &= m_Q x_Q + m_L x_L \end{aligned} \tag{A-21}$$

Applying the laws of motion to (A-21) and inserting (2-9) gives the

$$\begin{aligned} (m_Q + m_L)\ddot{x}_{CM} &= fRe_3 - (m_Q + m_L)ge_3 \\ (m_Q + m_L)(\ddot{x}_L + ge_3) &= fRe_3 + m_Q L\ddot{q} \end{aligned} \tag{A-22}$$

****

A-3 An appendix section

$$Testequation \tag{A-23}$$

A-3-1 A MatlabListing

```
%
% Comment
%
n=10;
for i=1:n
    disp('Ok');
end
```

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Nomenclature

ω_i	Angular velocity of rotor i around its axis, $i = \{1, 2, 3, 4\}$
$\tau_{drag,i}$	Drag moment generated by each propellor
$\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$	Unit vectors along the axes of $\{\mathcal{B}\}$
$\{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3\}$	Unit vectors along the axes of $\{\mathcal{C}\}$
$\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$	Unit vectors along the axes of $\{\mathcal{I}\}$
$\{\mathcal{B}\}$	Body Frame
$\{\mathcal{C}\}$	Intermediary Frame
$\{\mathcal{I}\}$	Inertial World Frame
b	Thrust factor
d	Drag factor
L	Length of the cable
q	Unit vector from Quadrotor to Load
x_L	Position of the of the Quadrotor CM
x_Q	Position of the of the Quadrotor CM
x_{CM}	Position CM of Quadrotor-Load system

Acronyms

QR	Quadrotor
CM	Center of Mass
LQR	Linear Quadratic Regulator