Signals & Systems PS06 Ryan Louie

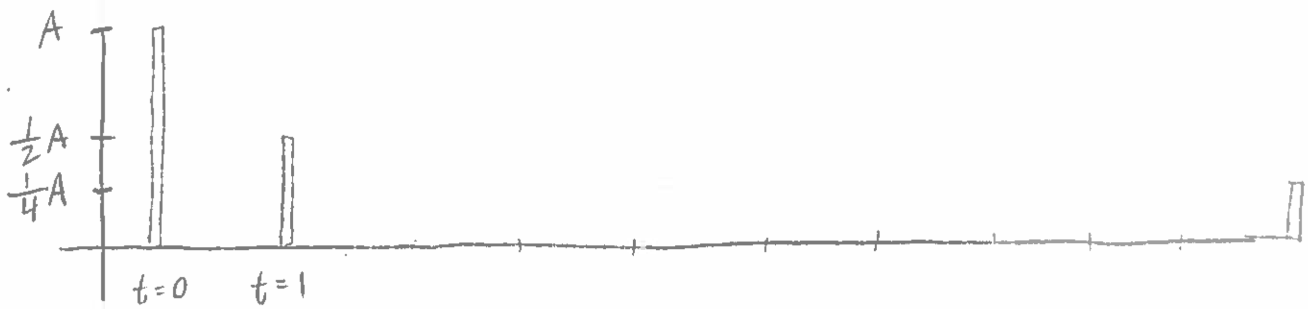
1. When a unit impulse is convolved with a signal, it scales it by a factor of 1, and has no phase shift. This fact is confirmed in the continuous time domain, as "convolving any function with an impulse... scales that function by the area under that impulse" and the area under the unit impulse is 1. In essense the signal is unchanged by this convolution.

However, when the impulse response is convolved with a signal, we hear all the frequency distortions, because a transfer function is applied. We know that the signal (the violin) will sound like it is being played in a gymnasium because, as mentioned previously, the impulse has no effect after convolution, so the impulse response will only have the effect as explained by the system (the gymnasium).

2. Transfer function:

$$y(t) = \frac{1}{2}x(t-1) + \frac{1}{4}x(t-10)$$

I will argue from a graphical standpoint why "echo channel" is an appropriate name.



Echos become lower in amplitude over time, and the echos will come back slower

$h(t) = \frac{1}{2}\delta(t-1) + \frac{1}{4}\delta(t-10)$

because the signal $x(t)$ can be reduced or simplified to $\delta(t)$, the impulse.

3a. In the range $[-\frac{T}{2}, \frac{T}{2}]$, $x(t) = \begin{cases} 1 & [-T/4, T/4] \\ 0 & [-T/2, -T/4] \\ 0 & [T/4, T/2] \end{cases}$

The kth Fourier series coefficient for $x(t)$ can be expressed as

$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi}{T}kt}\, dt$$

$$= \frac{1}{T}\left(\int_{-T/2}^{-T/4} [0] \ldots + \int_{-T/4}^{T/4} [1]\, e^{-j\frac{2\pi}{T}kt} + \int_{T/4}^{T/2} [0] \ldots\right)$$

$$= \frac{1}{T}\int_{-T/4}^{T/4} e^{-j\frac{2\pi}{T}kt}$$

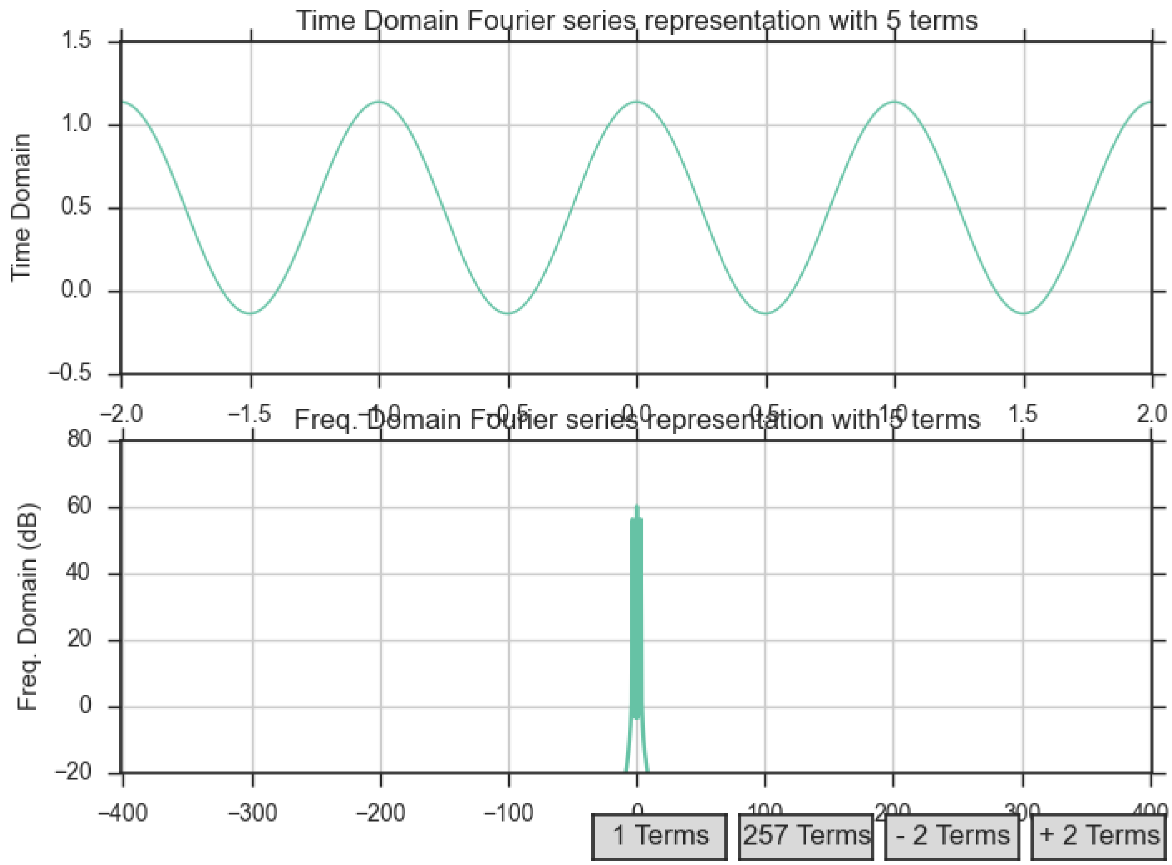
$$= \left(\frac{1}{T}\right)\left(\frac{1}{j\frac{2\pi}{T}k}\left[e^{j\frac{2\pi}{T}kt}\right]_{-T/4}^{T/4}\right)$$

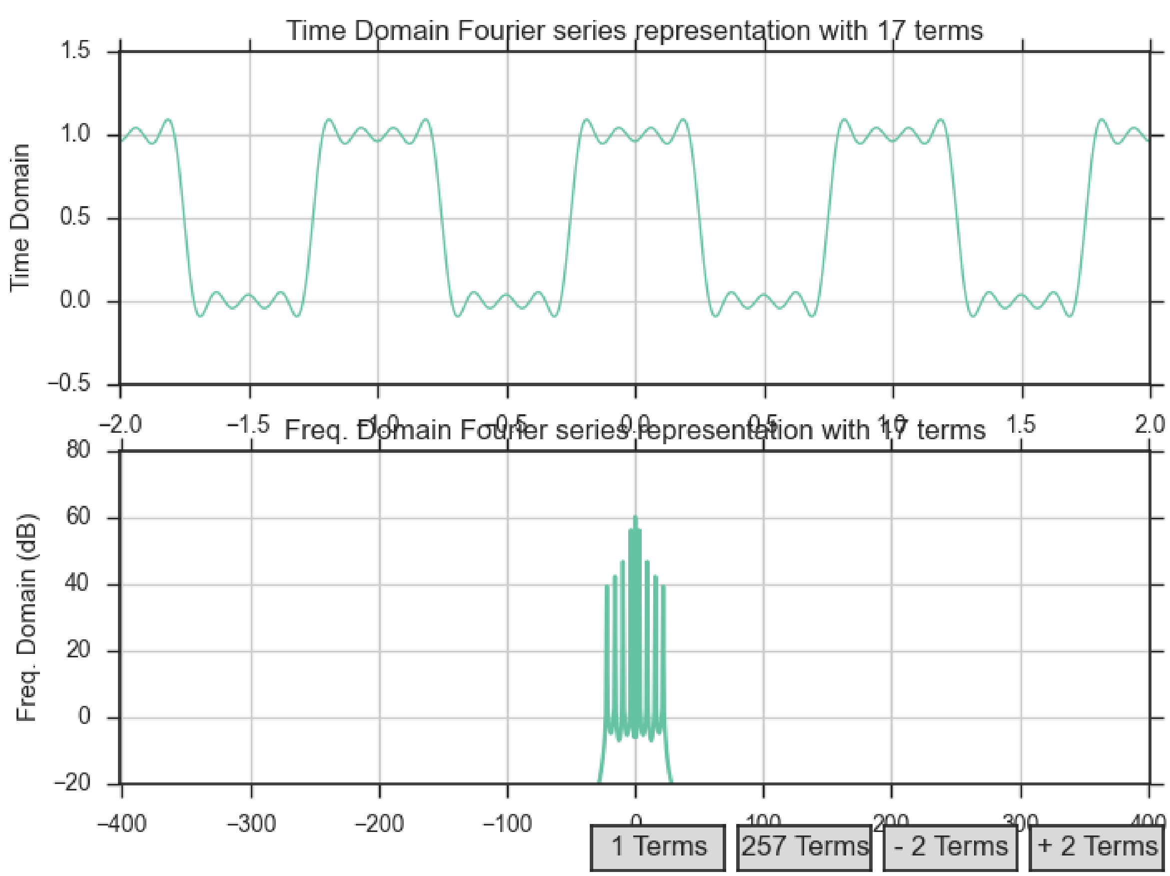
$$= \frac{1}{j2\pi k}\left(e^{j\frac{2\pi}{T}k\left(\frac{T}{4}\right)} - e^{j\frac{2\pi}{T}k\left(-\frac{T}{4}\right)}\right)$$

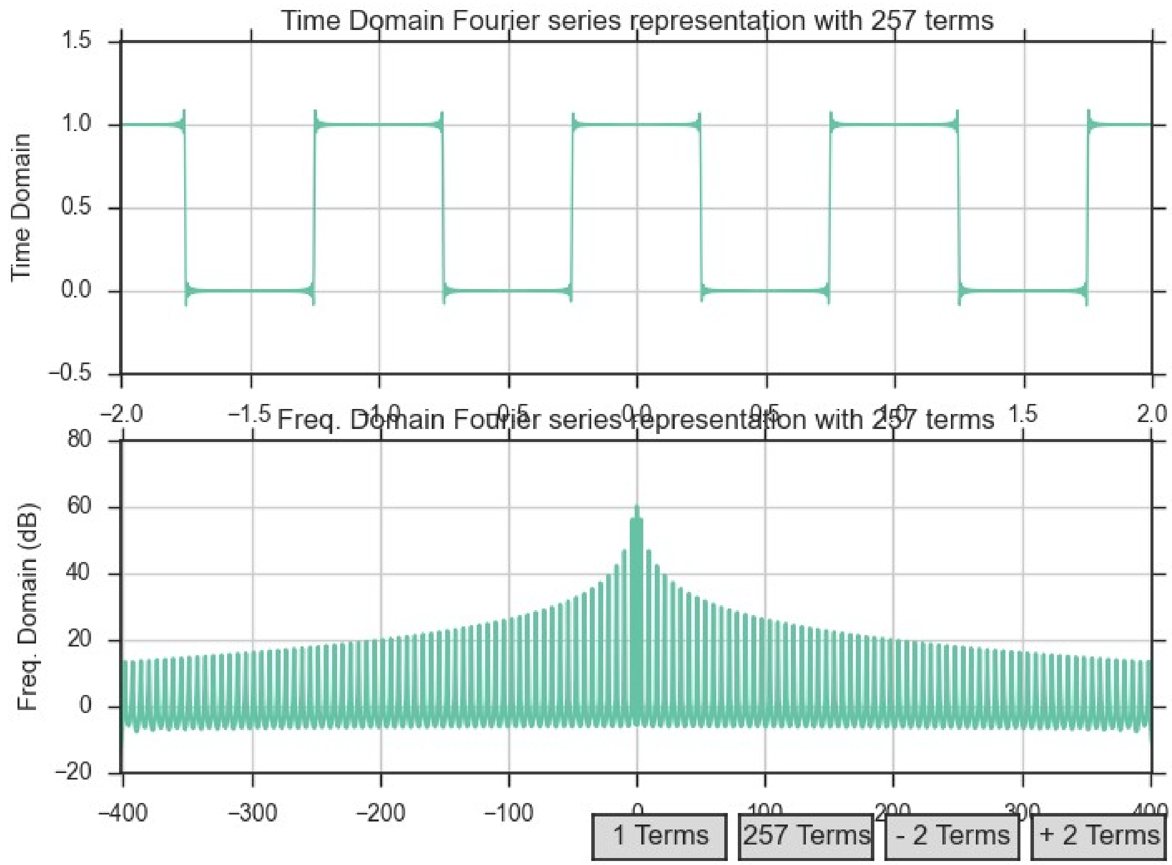
$$= \left(\frac{1}{\pi k}\right)\left(\frac{1}{2j}\right)\left(e^{j\frac{\pi}{2}k} - e^{-j\frac{\pi}{2}k}\right) \qquad \sin(\theta) = \frac{1}{2j}\left[e^{j\theta} - e^{-j\theta}\right]$$

$$= \left(\frac{1}{\pi k}\right)\sin\left(\frac{\pi k}{2}\right)$$

$$C_k = \frac{1}{2\left(\frac{\pi k}{2}\right)}\sin\left(\frac{\pi k}{2}\right) = \boxed{\frac{1}{2}\operatorname{sinc}\left(\frac{k}{2}\right)}$$


Time Domain Fourier series representation with 5 terms
Time Domain
Freq. Domain Fourier series representation with 5 terms
Freq. Domain (dB)
1 Terms
257 Terms
- 2 Terms
+ 2 Terms


Time Domain Fourier series representation with 17 terms
Time Domain
Freq. Domain Fourier series representation with 17 terms
Freq. Domain (dB)
1 Terms
257 Terms
- 2 Terms
+ 2 Terms


Time Domain Fourier series representation with 257 terms
Time Domain
1.5
1.0
0.5
0.0
−0.5
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
Freq. Domain Fourier series representation with 257 terms
Freq. Domain (dB)
80
60
40
20
0
−20
−400
−300
−200
−100
0
100
200
300
400
1 Terms
257 Terms
- 2 Terms
+ 2 Terms

3c. At the points of discontinuity, when the square wave transitions between 0-1 & 1-0. The complex exponentials still exhibit high frequency sinusoidal information at the sudden transitions. As more terms are added, the frequency is increased, and there is more of  these oscillations.

The oscillations still remain the same amplitude, offshooting the 0 & 1 lines.

4a. $C_{ky} = \frac{1}{T}\int_{t=-T/2}^{t=T/2} x(t-T_1)\, e^{-j\frac{2\pi}{T}kt}\, dt$

let $u = t - T_1 \quad t = u + T_1$

$du = dt$

$$C_{ky} = \frac{1}{T}\int_{u=-T/2-T_1}^{u=T/2-T_1} x(u) \cdot e^{-j\frac{2\pi}{T}k(u+T_1)}\, du$$

From (1)

$$= \frac{1}{T}\int_{u=-T/2}^{u=T/2} x(u) \cdot e^{-j\frac{2\pi}{T}k(u+T_1)}\, du$$

a constant

$$= \frac{1}{T}\int_{-T/2}^{T/2} x(u) \cdot e^{-j\frac{2\pi}{T}ku} \cdot e^{-j\frac{2\pi}{T}kT_1}\, du$$

$$= e^{-j\frac{2\pi}{T}kT_1}\left(\frac{1}{T}\int_{-T/2}^{T/2} x(u)\, e^{-j\frac{2\pi}{T}ku}\, du\right)$$

$$\boxed{C_{ky} = \left(e^{-j\frac{2\pi}{T}kT_1}\right)(C_{kx})}$$

4b.

Original Triangle Wave from FourierSeries1.ipynb

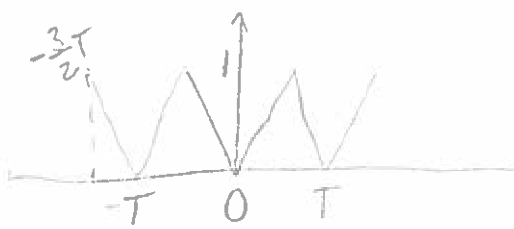
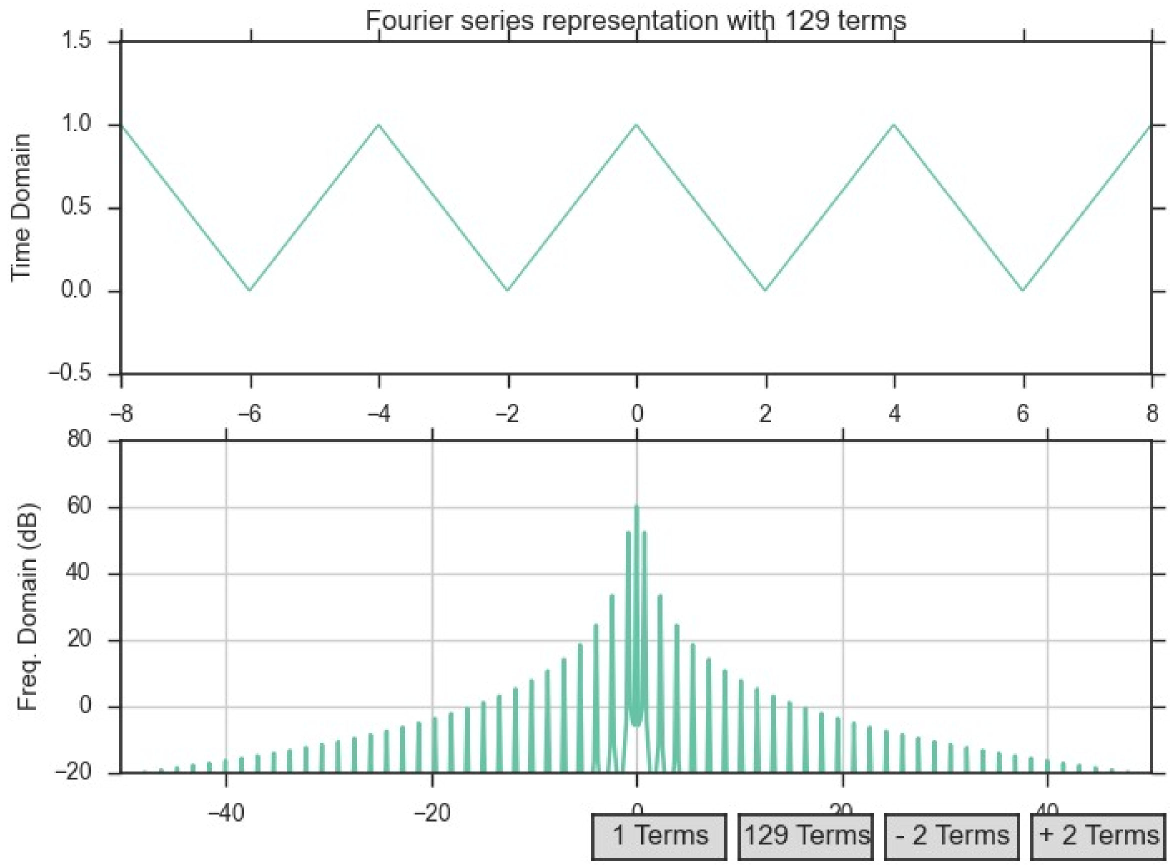


Figure 2 Triangle Wave from PS06



Fourier series representation with 129 terms
Time Domain
Freq. Domain (dB)
1.5
1.0
0.5
0.0
−0.5
−8
−6
−4
−2
0
2
4
6
8
80
60
40
20
0
−20
−40
−20
0
20
40
1 Terms
129 Terms
- 2 Terms
+ 2 Terms