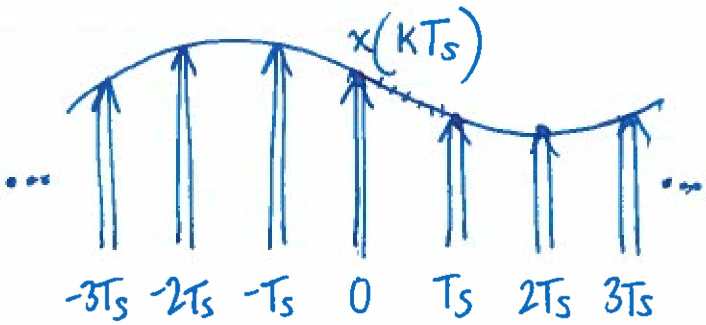
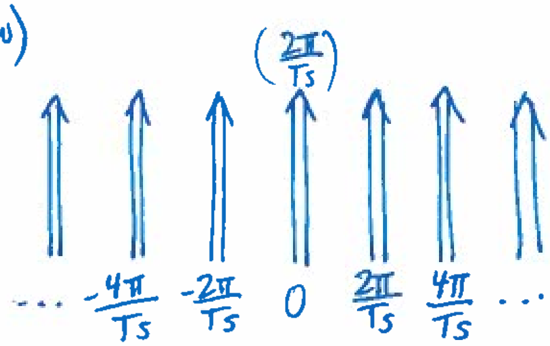
Signals and Systems PS08 Ryan Louie

1.

a) $x_p(t)$. Sample $x(t)$ at each $p(t)$



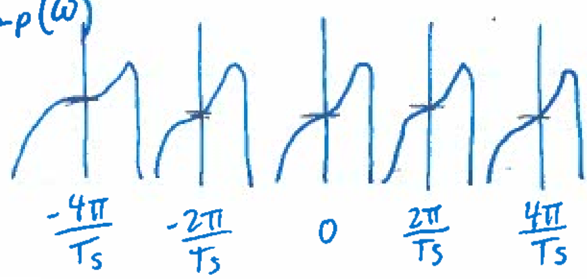
b) $P(\omega)$



time	freq
$\delta(t-t_0)$	$e^{-j\omega t_0}$

$\omega_s = \frac{2\pi}{T_s}$?

c) $X_p(\omega)$

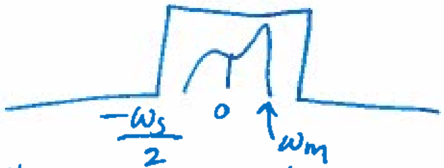


$X(\omega_\circ\ P(\omega) \cdot \frac{1}{2\pi}$

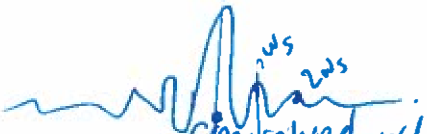
d) $\frac{2\pi}{T_s} - \omega_m > \omega_m$

$\frac{2\pi}{T_s} > 2\omega_m$

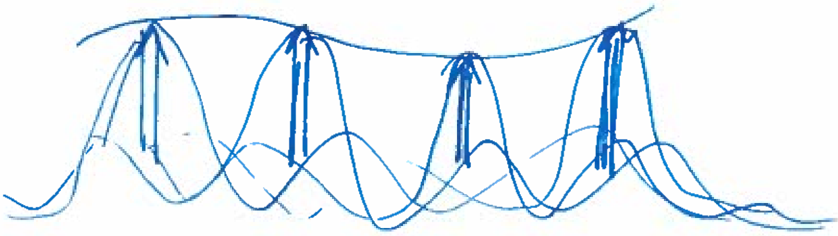
$2\omega_m < \omega_s$



e) Multiplying by an ideal low pass filter, corresponds to convolution of a sinc function in the time domain.



Sinc functions ~~multiplied~~ ~~by~~ convolved w/ each impulse in the impulse train

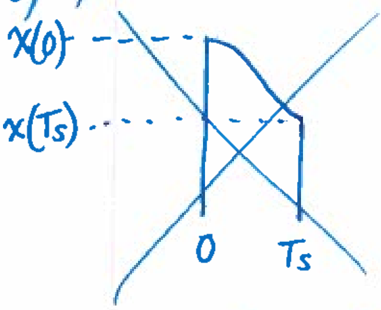


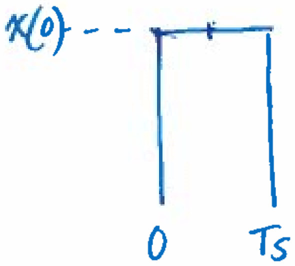
$x(t)\cos(\omega_c t)\cos(\omega_c t) \longrightarrow$

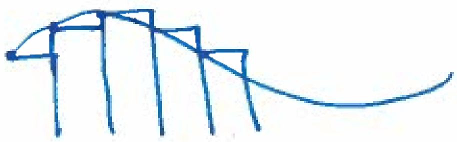
LPF $\longrightarrow$

$\approx x(t)$

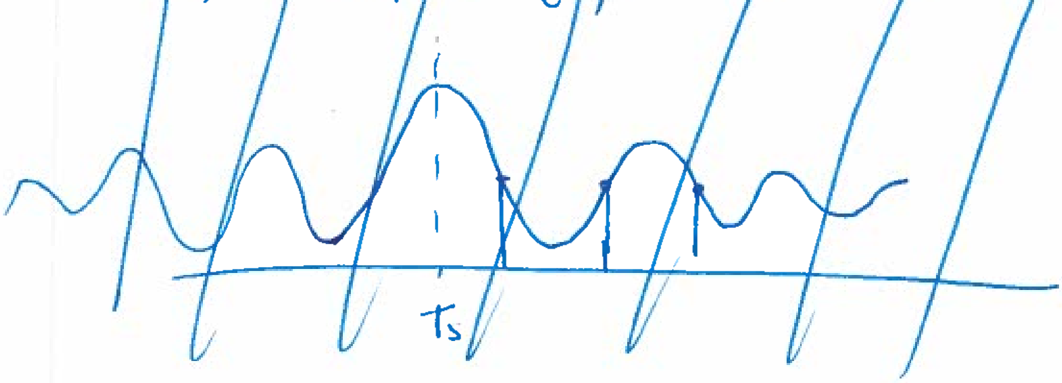
g) $x_z(t) = x_p * z(t)$ zero order hold construction







h) $X_z(\omega) = X_p(\omega)\, Z(\omega)$



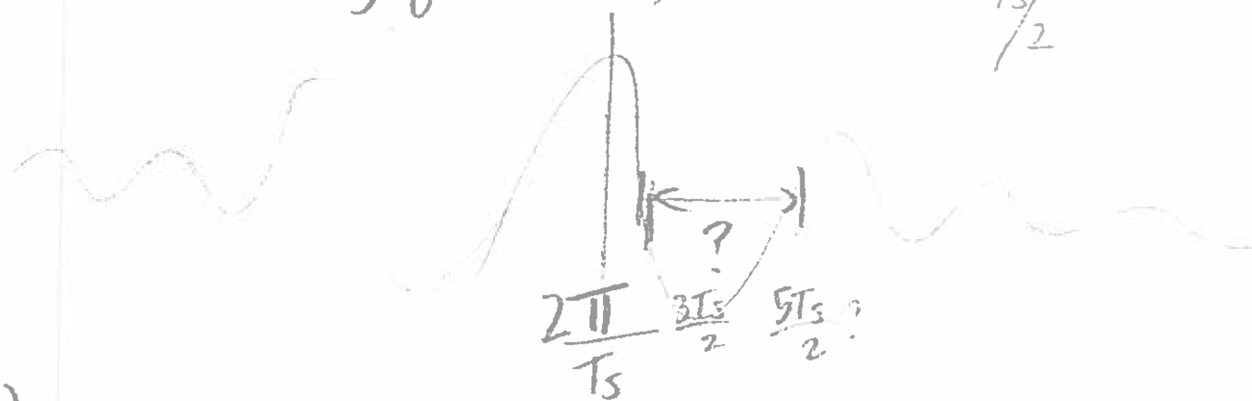
h) $X_z(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

$$= \int_{-T/2}^{T/2} x(t)\, e^{-j\omega t}\, dt$$

let $u = t + T/2$, $t = u - T/2$

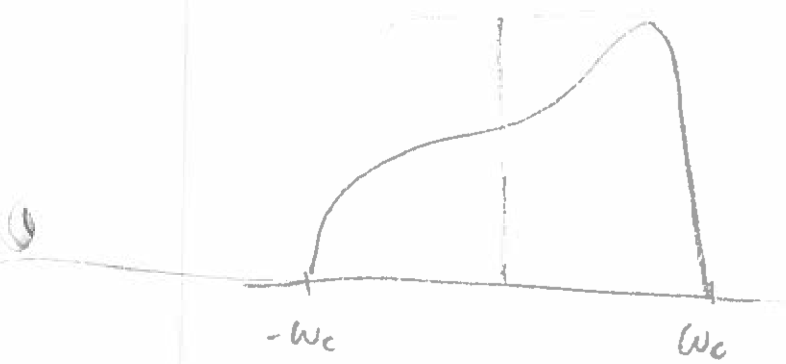
$$= \int_{0}^{T} x(u - T/2)$$

$T_s/2$



i)

$\hat{X}(\omega) = X_p(\omega)\, H(\omega)$

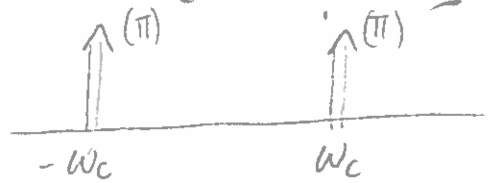


From the table of canonical forms

2. a) $y(t) = x_1(t)\cos(\omega_1 t) + x_2(t)\cos(\omega_2 t)$

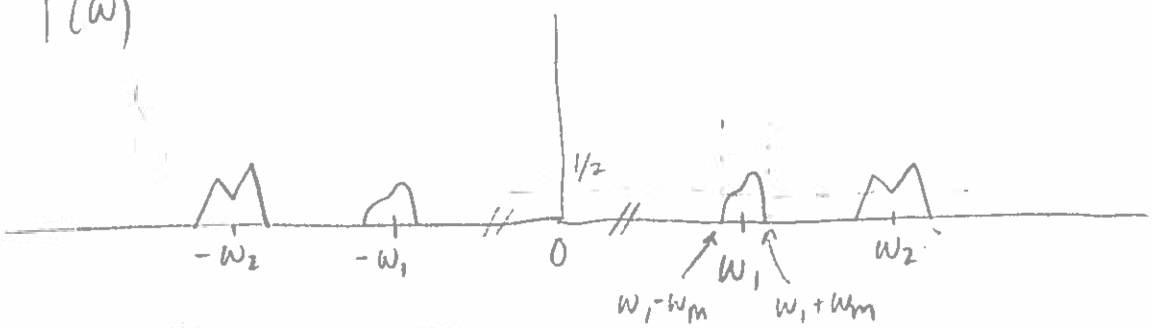
$$Y(\omega) = \frac{1}{2\pi}\Big[X_1(\omega)\cdot FT\{\cos(\omega_1 t)\} + X_2(\omega)\cdot FT\{\cos(\omega_2 t)\}\Big]$$

$FT\{\cos(\omega_c t)\}$



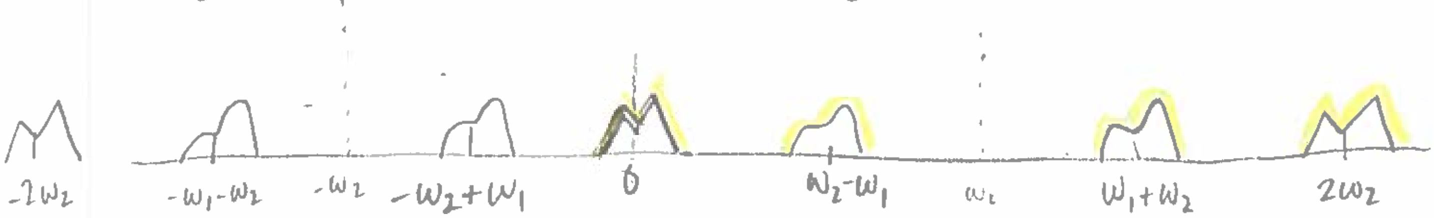
$\omega_1 \gg \omega_m$
$\omega_2 \gg \omega_m$
$\omega_1 + 2\omega_m < \omega_2$

$Y(\omega)$



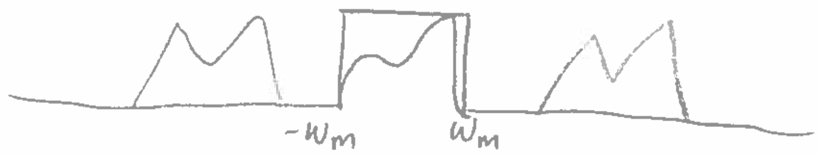
b) $FT\{y(t)\cos(\omega_1 t)\} = Y(\omega)\cdot FT\{\cos(\omega_1 t)\}$

$FT\{y(t)\cos(\omega_2 t)\} = Y(\omega)\cdot FT\{\cos(\omega_2 t)\}$

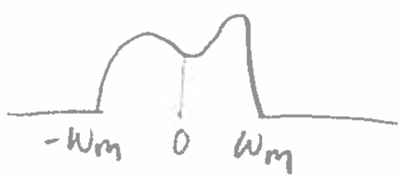


c) To recover $x_1(t)$ from $y(t)$, we can convolve $y(t)$ w/ $\cos(\omega_1 t)$. This results in the wave which, in the frequency domain, looks like 2bI:

If we then apply an ideal low pass filter, with cutoff frequency ω_m,



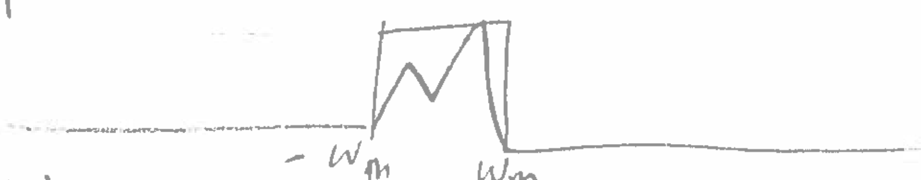
we are left w/ $X_1(t)$



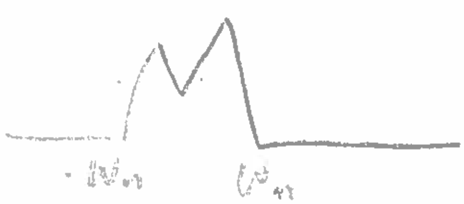
which can easily be turned into $x_1(t)$.

Similarly, $y(t)\cdot\cos(\omega_2 t)$ looks like 2bII in frequency domain

Low pass

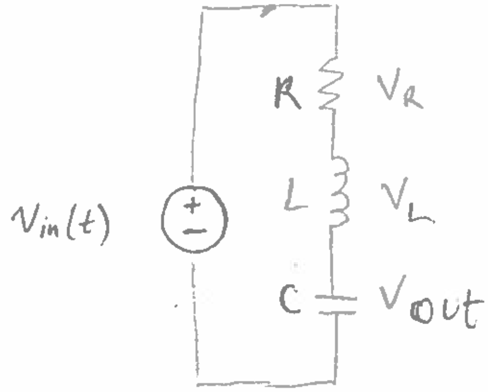


$X_2(t)$



which can give us $x_2(t)$

3.
a)



$$i(t) = C\frac{d}{dt}V_{out}(t)$$

$$V_L = L\frac{d}{dt}i(t)$$

$$V_{in}(t) = V_R + V_L + V_{out}(t)$$

$$V_{in}(t) = R\,i(t) + L\frac{d}{dt}i(t) + V_{out}$$

$$\boxed{V_{in}(t) = RC\frac{d}{dt}V_{out}(t) + LC\frac{d^2}{d^2t}V_{out}(t) + V_{out}(t)}$$

b) $H(\omega) \overset{?}{=} \dfrac{V_{out}(t)}{V_{in}(t)}$ ~~i.e. continuing w/ 3a...~~

$$V_{in}(t) = e^{j\omega t}$$

$$V_{out}(t) = H(\omega_1)e^{j\omega t}$$

$$e^{j\omega t} = \left[RCj\omega e^{j\omega t} + LC\overset{-1}{\cancel{j^2}}\omega^2 e^{j\omega t} + e^{j\omega t}\right]H(\omega_1)$$

$$\boxed{H(\omega) = \frac{1}{RCj\omega - LC\omega^2 + 1}}$$

c) $$\|H(\omega)\| = \sqrt{\frac{2}{C^2L^2\omega^4 + j(2CR\omega - 2C^2LR\omega^3) - C^2R^2\omega^2 - 2CL\omega^2 + 1}}$$

thanks Wolfram!

$$= \sqrt{2}\cdot\frac{1}{-1 + C\omega(L\omega - iR)}$$

thanks Wolfram again!

thanks Wolfram

d) $\frac{\partial}{\partial \omega}\left(\left|H(\omega)\right|\right) \stackrel{\downarrow}{=} \frac{-\sqrt{2}C(R+2iL\omega)}{\left[C\omega(R+iL\omega)-i\right]\left[-1+C\omega(L\omega-iR)\right]} = 0$

$$-\sqrt{2}C(R+2iL\omega) = 0$$

$$2iL\omega = -R$$

$$\boxed{\omega = \frac{-R}{i2L}}$$

uh oh, something went wrong Wolfram lied :(
I will try again w/ C.

c) $|H(\omega)| = \frac{1}{|j\omega RC - \omega^2 LC + 1|} = \boxed{\frac{1}{\sqrt{\omega^2 R^2 C^2 + (1-\omega^2 LC)^2}}}$

d) minimize denom of $|H(\omega)|$ to maximize $|H(\omega)|$

$$0 = \frac{d}{d\omega}\left(\omega^2 R^2 C^2 + (1-\omega^2 LC)^2\right)$$

$$0 = [2R^2C^2]\omega + 2(1-\omega^2 LC)\cdot(2\omega)$$

$$0 = [2R^2C^2]\omega - 4\omega + 4LC\omega^2$$

$$\omega = \frac{4 \pm \sqrt{16 - 4(4LC)(2R^2C^2)}}{2(4LC)}$$

$$\omega = \frac{4 \pm \sqrt{16 - 16(2LR^2C^3)}}{8LC}$$

cal pts.

$$\rightarrow \omega = \frac{1 \pm 4\sqrt{1-2LR^2C^3}}{2LC}$$

when $\omega = \dfrac{1 + 4\sqrt{1-2LR^2C^3}}{2LC}$

denom of $|H(\omega)|$ is [thanks wolfram]

Intermediate helpers

$$\omega^2 = \frac{1 + 2\left[4\sqrt{1-2LR^2C^3}\right] + 16(1-2LR^2C^3)}{4L^2C^2}$$

$$\omega^2 = \frac{1 + 8\sqrt{1-2LR^2C^3} + 16(1-2LR^2C^3)}{4L^2C^2}$$

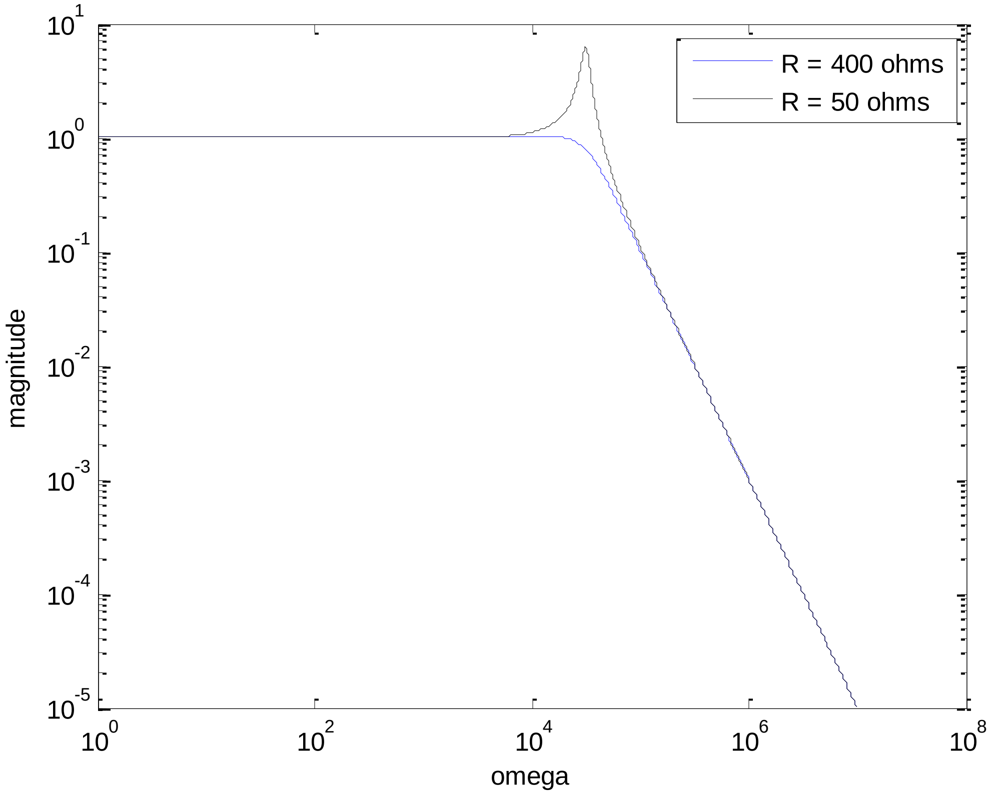
let $u = 1-2LR^2C^3$

$$\omega^4 = 1 + 8\sqrt{u} + 16u + 64u + 256u\sqrt{u} + 16u + 16^2u^2$$
$$= 1 + 8\sqrt{u} + 96u + 256u\sqrt{u} + 256u^2$$

These pluses are going to make denom large; thus, its the other critical point that we want.

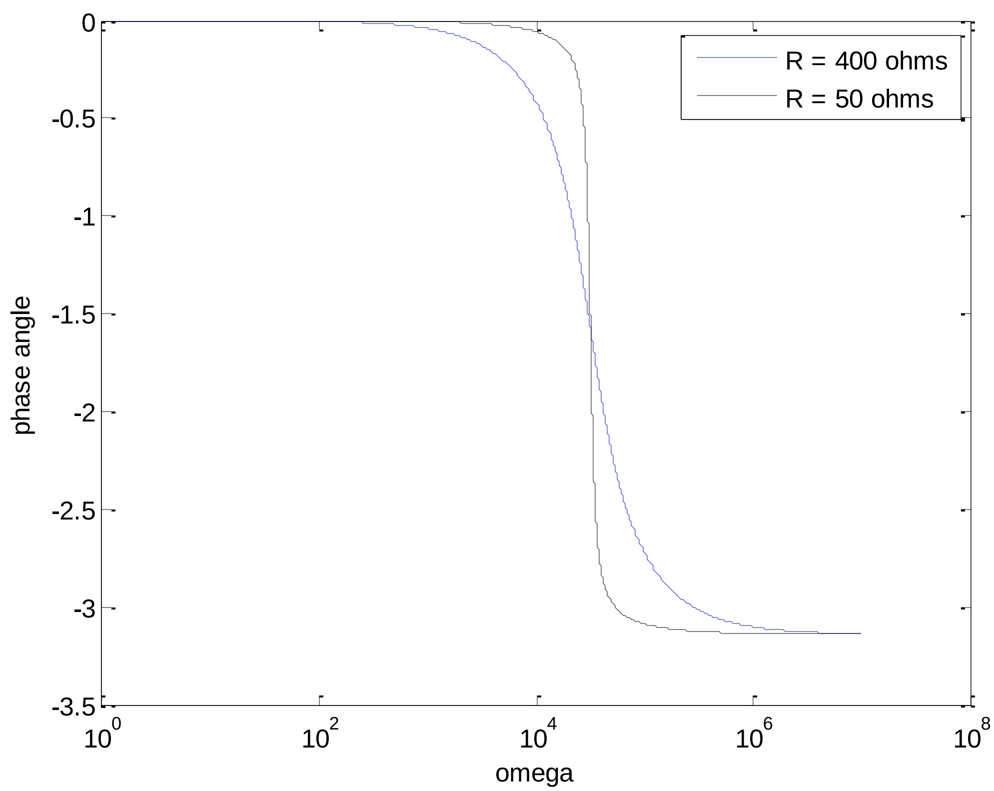
$$\boxed{\omega = \frac{1 - 4\sqrt{1-2LR^2C^3}}{2LC}}$$

NOTE: This was done as a team by Ryan and Jennifer




```
C = 10^-7; %F
L = 10^-2; %H
R1 = 400; %ohms
R2 = 50; %ohms


w = 1:1:10000000;
Mag1 = (1.0./( (w*R1*C).^2 + (1-(w.^2)*L*C).^2 ).^(0.5));
Mag2 = (1.0./( (w*R2*C).^2 + (1-(w.^2)*L*C).^2 ).^(0.5));
loglog(w,Mag1,'b',w,Mag2,'k');
legend('R = 400 ohms', 'R = 50 ohms');
xlabel('omega');
ylabel('magnitude');
```




```
figure;
Freq1 = 1.0./(1+(R1.*C.*j.*w)-((w.^2).*C.*L));
Freq2 = 1.0./(1+(R2.*C.*j.*w)-((w.^2).*C.*L));
phase1= angle(Freq1);
phase2 = angle(Freq2);
%phase1 = atan((j.*w.*R1.*C)./(-(w.^2).*L.*C).^2).*(360/2*pi);
%phase2 = atan((j.*w.*R2.*C)./(-(w.^2).*L.*C).^2).*(360/2*pi);
semilogx(w,phase1,'b',w,phase2,'k');
legend('R = 400 ohms', 'R = 50 ohms');
xlabel('omega');
ylabel('phase angle');
```