Grasping [Prattichizzo, 2016]

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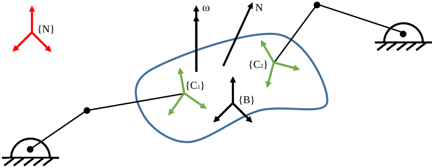
Outline

Motivation

- Mathematical models for robotic grasping
- Object and robot hand behaviour
- Grasping simulations
- Robot hands applications

Grasping Notation

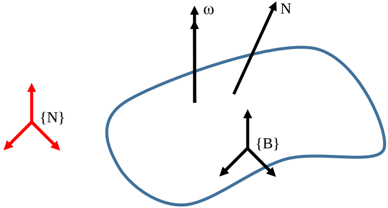
- {N}: inertial frame
- $\{C_i\}$: frame at contact i
- {B}: fixed frame in object (CoM)
- N: object's translational velocity
- ω : object's angular velocity



Twist and Contacts

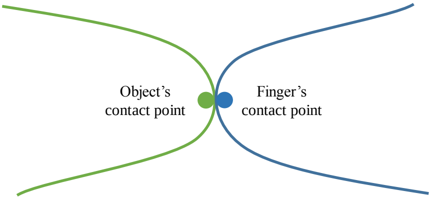
In spatial systems (3D), n_ν=6, n_b=1

$$\nu = \nu^N = \begin{bmatrix} N_x \\ N_y \\ N_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \quad \nu : \textit{twist}$$



Two i-*th* contact points

- object
- hand



Twist of Contact on Object

Goal: Given object's twist ν, compute twist of i-*th* contact on body

$$\nu_{i,obj}^{C} = \tilde{G}_i^{\mathsf{T}} \nu^{N},$$

- $\nu_{i,obj}^{C}$: twist expressed in frame C_i
- ν^{N}: twist expressed in inertial frame N
- $\tilde{G}_i^{\mathsf{T}}$: partial grasp matrix

Translational and Angular Velocity

Object's points have different translational velocity N

$$N_i^N = N^N + \omega^N \times r_i^N = N^N - r_i^N \times \omega^N = N^N - S(r_i^N)\omega^N,$$

S: Skew-symmetric matrix

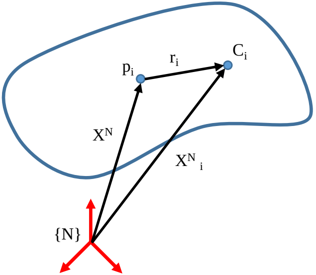
$$S(a) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} = -S(a)^\intercal$$

Angular velocity ω of all points on an object is the same

$$\omega^{C_i} = R_N^{C_i} \omega^N$$

Rigid Body Equation

- Position of contact point i $X_i^N = X^N + r_i$
- Euler theorem: $r_i = Rr_i^N$, with $R^\intercal R = I$
- $X_i^N = X^N + Rr_i^N$
- $N_i^N = N^N + \frac{dR}{dt} r_i^N = N^N + \frac{dR}{dt} R^\intercal R r_i^N = N^N + \frac{dR}{dt} R^\intercal r_i = N^N + \Omega r_i$
- $\Omega r = \omega \times r$, Ω: angular velocity tensor
- $N_i^N = N^N + \omega^N \times r_i^N$



Twist Transformation to Contact Frame

The twist results

$$\begin{bmatrix} N_i^N \\ \omega_i^N \end{bmatrix} = \begin{bmatrix} I_{3x3} & S(r_i^N)^\intercal \\ 0_{3x3} & I_{3x3} \end{bmatrix} \begin{bmatrix} N^N \\ \omega^N \end{bmatrix}$$

Vector expression from contact frame C_i to inertial frame N,
$d^N = R_{C_i}^N d^{C_i} \Rightarrow (R_{C_i}^N)^\intercal d^N = (R_{C_i}^N)^\intercal R_{C_i}^N d^{C_i} \Rightarrow (R_{C_i}^N)^\intercal d^N = I d^{C_i}$

Thus the twist yields

$$\nu_{i,obj} = \begin{bmatrix} N_{i,obj}^{C_i} \\ \omega_{i,obj}^{C_i} \end{bmatrix} = \begin{bmatrix} (R_{C_i}^N)^\intercal & 0 \\ 0 & (R_{C_i}^N)^\intercal \end{bmatrix} \begin{bmatrix} I & S(r_i^N)^\intercal \\ 0 & I \end{bmatrix} \begin{bmatrix} N^N \\ \omega^N \end{bmatrix} = \tilde{G}_i^\intercal \nu^N$$

Twist of Contact in Hand

Goal: Given joint angles $\dot{q}$, compute twist of i-*th* contact in hand

$$\nu^{C}_{i,hand} = \tilde{J}_i \dot{q},$$

- $\nu^{C}_{i,hand}$: twist expressed in frame C_i
- $\dot{q}$: joint angles
- $\tilde{J}_i$: partial hand Jacobian matrix

Translational Velocity

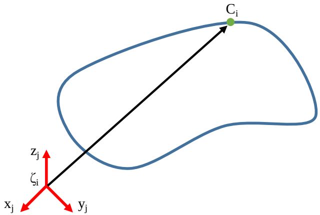
Hand's translational N, similar w/ Jacobian in velocity kinematics

$$N_{ij} = d_{ij}^N \dot{q} = \begin{bmatrix} 0 \\ \hat{z}_j^N \dot{q}_j \\ \hat{z}_j^N \dot{q}_j \times (C_i^N - \zeta_j^N) \end{bmatrix} \begin{matrix} \rightarrow \textit{no contact} \\ \rightarrow \textit{prismatic joint} \\ \rightarrow \textit{revolute joint} \end{matrix}$$

Revolute joint can be written

$$\hat{z}_j^N \dot{q}_j \times (C_i^N - \zeta_j^N) = -(C_i^N - \zeta_j^N) \times \hat{z}_j^N \dot{q}_j = S^{\intercal}(C_i^N - \zeta_j^N)\hat{z}_j^N \dot{q}_j$$

- ζ_j: origin of joint frame
- j: joint



Angular Velocity and Finger Contacts

Angular velocity $\omega_{ij}^N = l_{ij}^N \dot{q}_{ij}$,

$$l_{ij} = \begin{bmatrix} 0 \\ 0 \\ \hat{z}_j^N \end{bmatrix} \begin{array}{l} \rightarrow \textit{no contact} \\ \rightarrow \textit{prismatic joint} \\ \rightarrow \textit{revolute joint} \end{array}$$

which yields

$$\nu_{ij}^N = \begin{bmatrix} d_{ij}^N \\ l_{ij}^N \end{bmatrix} \dot{q}_i$$

For all finger contacts

$$\nu_{i,hand}^N = \begin{bmatrix} d_{i,1}^N \cdots d_{i,n_q}^N \\ l_{i,1}^N \cdots l_{i,n_q}^N \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_{n_q} \end{bmatrix} = Z_i \dot{q}$$

Partial Hand Jacobian

- Transfrom twist to contact frame

$$\nu^{C}_{i,hand} = \overline{R}_i^{\mathsf{T}} Z_i \dot{q} = \tilde{J}_i \dot{q}, \quad \tilde{J}_i \in \mathbb{R}^{6n_c \times n_q}$$

- Partial hand Jacobian $\tilde{J}_i$, maps joint velocities with the contact twists of the hand

Complete Hand Jacobian and Grasp Matrix

For all contacts we get

$$\nu_{C,hand} = \begin{bmatrix} \nu_{1,hand} \\ \vdots \\ \nu_{n_c,hand} \end{bmatrix}, \quad \nu_{C,obj} = \begin{bmatrix} \nu_{1,obj} \\ \vdots \\ \nu_{n_c,obj} \end{bmatrix}$$

Complete hand Jacobian and grasp matrix

$$\tilde{J} = \begin{bmatrix} \tilde{J}_1 \\ \vdots \\ \tilde{J}_{n_c} \end{bmatrix}, \quad \tilde{G}^\intercal = \begin{bmatrix} \tilde{G}_1^\intercal \\ \vdots \\ \tilde{G}_{n_c}^\intercal \end{bmatrix}$$

Models of interest

Three models of interest

- Point contact w/o friction (PwoF)
- Hard finger (HF)
- Soft finger (SF)

Relative twist at i-*th* contact

$$(\tilde{J}_i - \tilde{G}_i^{\intercal}) \begin{pmatrix} \dot{q} \\ \nu \end{pmatrix} = (\nu_{i,hand} - \nu_{i,obj})$$

Particular contact model through homogeneous $H_i \in \mathbb{R}^{l_i \times 6}$

$$H_i(\nu_{i,hand} - \nu_{i,obj}) = 0$$

Homogeneous matrix

- Homogeneous matrix defined as

$$H_i = \begin{bmatrix} H_{iF} & 0 \\ 0 & H_{iM} \end{bmatrix},$$

H_{iF}: Translational components
H_{iM}: Rotational components

- Contact models matrices selection

Model	l_i	H_{iF}	H_{iM}
PwoF	1	$(1\ 0\ 0)$	0
HF	3	I_{3x3}	0
SF	4	I_{3x3}	$(1\ 0\ 0)$

l_i: transmitted twist components

Hand Jacobian and Grasp Matrix

- From the complete hand Jacobian and grasp matrix we get

$$H(\nu_{C,hand} - \nu_{C,obj}) = H(\tilde{J}\dot{q} - \tilde{G}^{\intercal}\nu) = (H\tilde{J} - H\tilde{G}^{\intercal}) \begin{pmatrix} \dot{q} \\ \nu \end{pmatrix} = 0$$

- Hand Jacobian and grasp matrix are

$$J\dot{q} = \nu_{cc,obj} = \nu_{cc,hand} = G^{\intercal}\nu$$

$J = H\tilde{J}$, hand Jacobian matrix
$G^{\intercal} = H\tilde{G}^{\intercal}$, grasp matrix

Conclusions

Robot grasping procedure

- Given: Contact points C_i, hand kinematic structure, object velocity twists ν, and joint velocities $\dot{q}$
- Compute: Velocities of contact points on objects $\nu_{C,hand}$, and velocities of contact points in hand $\nu_{C,onj}$

- Given: Contact model
- Compute: Hand Jacobian matrix, Grasp matrix

References

D.Prattichizzo and J.Trinkle (2016)
Grasping
Springer handbook of robotics, 955–988, 2016.

Thank You!