Consensus and cooperation in networked multi-agent systems [Olfati, 2007]

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Outline

Consensus & Cooperation

This paper provides a framework for analysis of consensus algorithms for multi-agent network systems

- Consensus is defined as reaching an agreement regarding a certain quantity of interest that depends on the state of all agents
- A protocol, also called consensus algorithm, is an interaction rule that specifies the exchange information between an agent and its neighbors on the network
- Networked systems, that are included in agents, are equipped with sensing, computing, and communicating devices

Consensus in Networks

- For a directed graph $G = (V, E)$, with a set of nodes $V = 1, 2, ..., n$ and edges $E \subseteq V \times V$ a simple consensus algorithm of a nth order linear system on a graph is

$$\dot{x}_i = \sum_{j \in N_i} (x_j(t) - x_i(t)) + b_i(t), \;\; x_i(0) = z_i \in \mathbb{R}, b_i(t) = 0$$

  with collective dynamics $\dot{x} = -Lx$
- Since all row-sums of the Laplacian are zero, L has always a zero eigenvalue $\lambda_1 = 0$
- The consensus value is the avg of the initial states $a = \frac{1}{n} \sum_i z_i$

The f-Consensus problem & Cooperation

Differences between constrained and unconstrained problems

- In unconstrained problems the state of all agents asymptotically become the same
- In constrained problems (f-consensus problems) the state of all agents asymptotically become $f(z)$

To solve the f-consensus problem we need

- Cooperation from all agents
- Willingness to participate from all agents

cooperation and willing to participate from all agents

Applications (1/2)

Common consensus problems for multi-agent systems

- Synchronization of coupled oscillators which has dynamics

$$\dot{\theta}_i = \kappa \sum_{j \in N_i} sin(\theta_j - \theta_i) + \omega_i,$$

  where ω_i is the frequency and θ_i the phase of the *i*th oscillator
- Flocking theory of mobile agents with sensing and communication devices, using proximity graphs
- Fast consensus in small-worlds deals with network design problem. The problem is addressed with either design of weights or design of topology.

Applications (2/2)

- Rendezvous in space which reaches a consensus in position by a number of agents
- Distributed sensor fusion in sensor networks to implement or approximate a Kalman-filter, or estimate linear least-squares
- Distributed formation control

Information Consensus in Networked Systems

- Consider the dynamics $\dot{x}_i = u_i$ of a graph $G = (V, E)$, that reaches consensus asymptotically
- The adjacency matrix is $A = [a_{ij}]$, and the set of neighbors $N_i = j \in V : a_{ij} \neq 0$
- A dynamic graph is time-varying $G(t) = (V, E(t))$ with the $A(t)$ and the linear system is a distributed consensus algorithm

$$\dot{x}_i(t) = \sum_{j \in N - i} a_{ij}(x_j(t) - x_i(t))$$

- For undirected graphs $(a_{ij} = a_{ji})$ as $t = \infty$ results the avg of the initial states $a = \frac{1}{n}\sum_i x_i(0)$

Laplacian expression

- A Laplacian representation of the dynamics is given by $\dot{x} = -Lx$, where $L = D - A$
- For undirected graphs the Laplcian satisfies the SoS property $x^\intercal Lx = \frac{1}{2}\sum_{(i,j)\in E} a_{ij}(x_j - x_i)^2$
- By setting $\frac{1}{2}x^\intercal Lx = \phi(x)$ we get the gradient-descent algorithm $\dot{x} = -\nabla\phi(x)$
- For an undirected graph the algorithm converges asymptotically for all initial values

Algebraic Connectivity & Spectral Properties

Convergence Analysis for Directed Networks

Convergence in Discrete-Time and Matrix Theory

Performance of Consensus Algorithms

Alternative Forms of Consensus Algorithms

Weighted-Average Consensus

Consensus under Communication Time-Delays

Consensus in Dynamic Networks

Cooperation in Networked Control Systems

Collective Dynamics of Multi-Vehicle Formations

Stability of Relative Dynamics of Formations

Consensus in Complex Networks

Multi-vehicle Formation Control

References

R. Olfati-Saber, J. Alex Fax, R. M. Murray (2007)
Consensus and cooperation in networked multi-agent systems
Proceedings of the IEEE, 95.1 215-233, 2007.

Thank You!