Homework 3

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Exercise 1. Formation control with desired position offset using double integrator dynamics.

Solution. For the double integrator dynamics

$$\dot{x}_i = v_i$$

$$\dot{v}_i = u_i, \;\; i = 0, 1, \ldots, N$$

where $x_i, v_i, u_i \in \mathbb{R}^2$ and the 0 node is the leader agent. The given formation control is

$$u_i = cK_p(\sum_{j \in N_i} a_{ij}(x_j - \Delta_j - x_i + \Delta_i) + g_i(x_0 - x_i + \Delta_i)) +$$

$$+ cK_d(\sum_{j \in N_i} a_{ij}(v_j - v_i) + g_i(x_0 - x_i))$$

for a spanning tree, where $c > 0$, $K_p = I_2$, $K_d = \gamma I_2$ and the leader has initial conditions and control $x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, v_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, u_o = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

The spanning tree that we employed is depicted in figure 1.

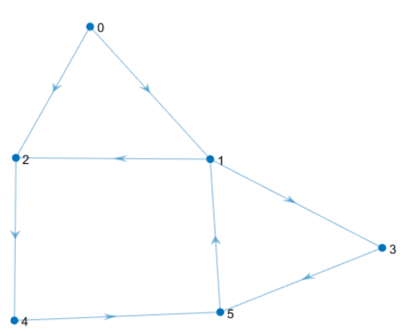

Figure 1: Spanning tree with 5 agents and a leader. The node 0 is the leader.

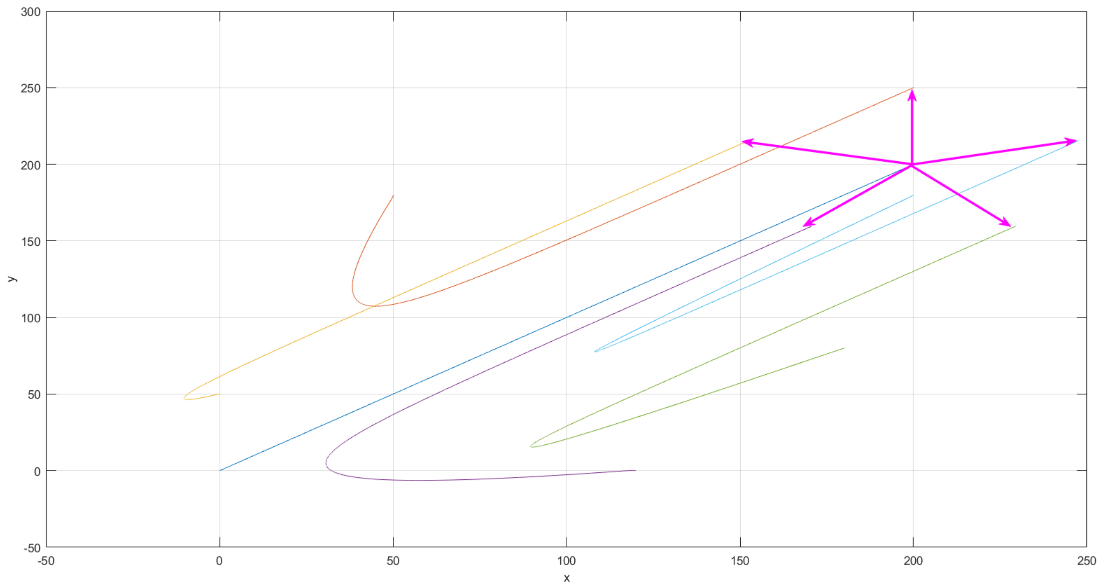

Figure 2: Star formation of 6 agents.

The first formation of the agents was a star. The motion of the agents in the cartesian plane is presented in figure 2. The initial positions were different for all agents, while the initial velocities were zero. Notice that the leader's motion followed a straight line, because its initial velocity was $v_x = 1$, $v_y = 1$.

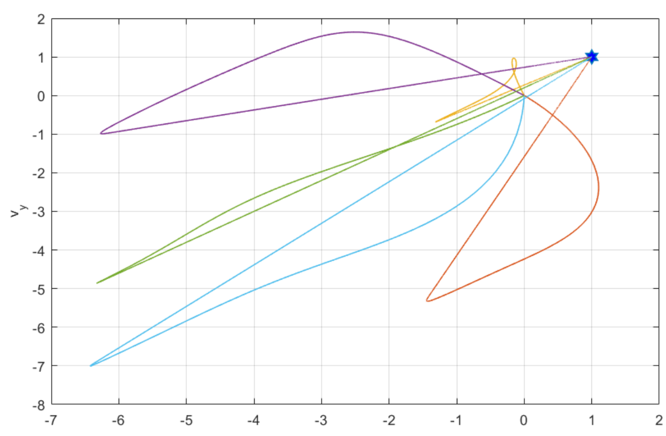

Figure 3: Agent velocities v_x, v_y for star formation.

An important aspect of the agent dynamics is the velocities. The velocities v_x, v_y are shown in figure 3. All initial velocities of the agents were set to zero and they converged with the leader's velocity. The agents velocities performance in time is presented in figure 4. All agents converged with leader's velocity $v_x = 1$, $v_y = 1$.

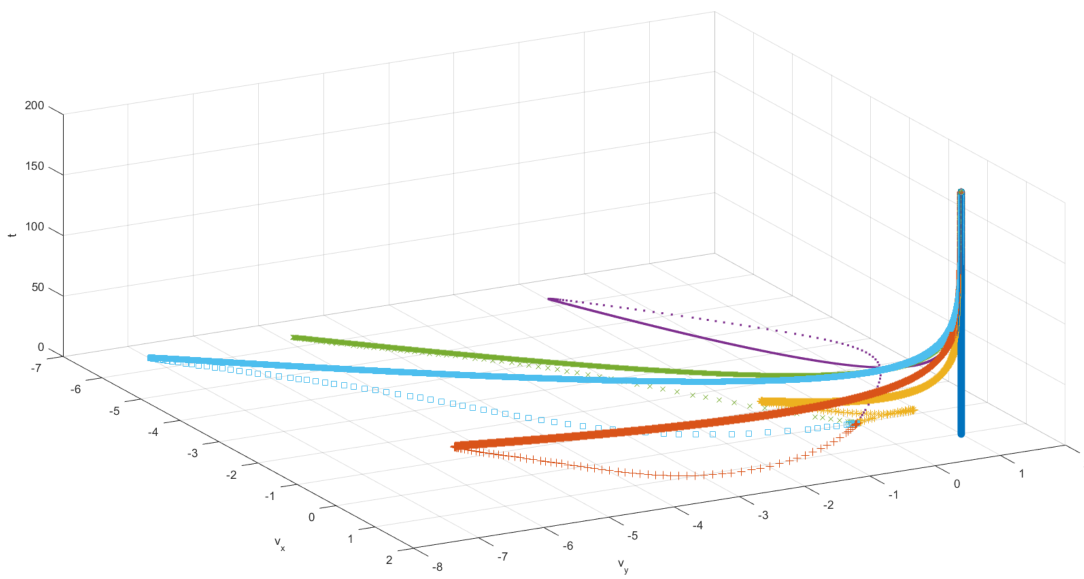

Figure 4: Agent velocities v_x, v_y performance in time for star formation.

The next formation that we want to achieve is a polygon. Initial states remain as previously and the leader's final point in cartesian space is $(200, 200)$. The polygon formation is depicted in figure 5.

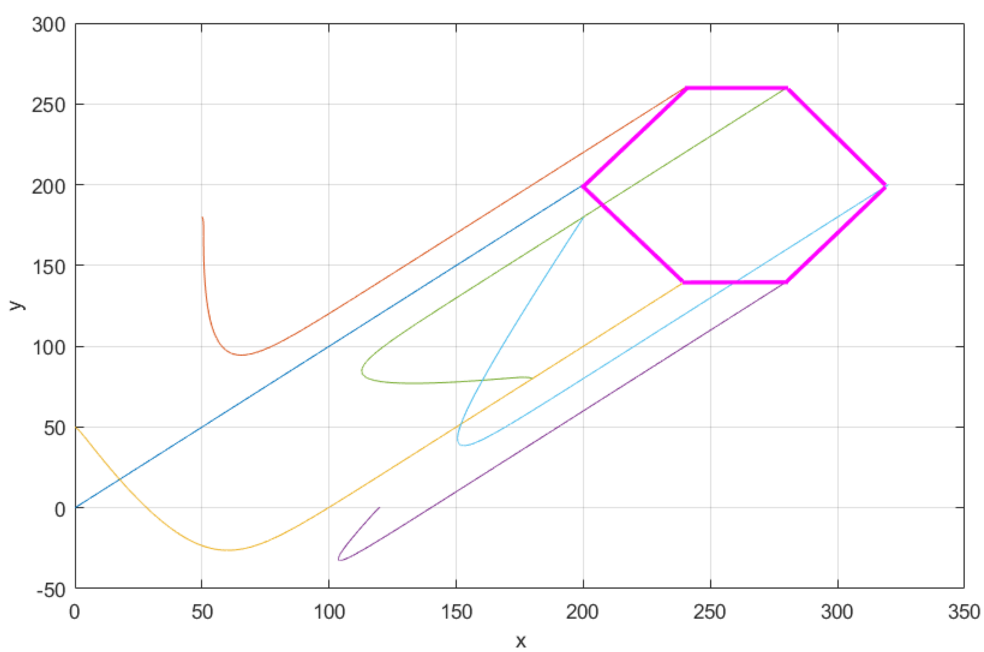

Figure 5: Polygon formation of 6 agents.

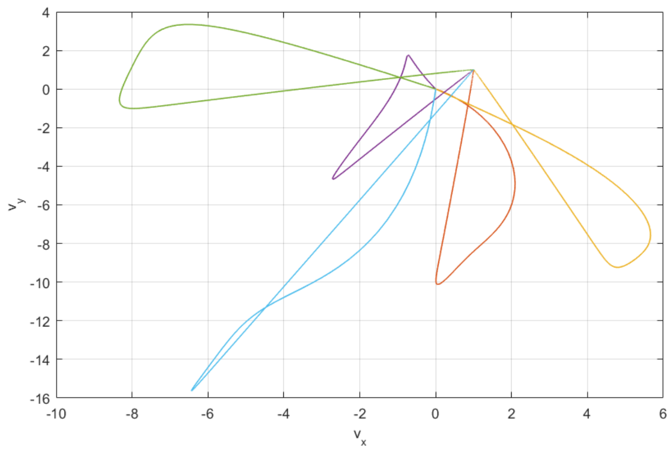

Figure 6: Agent velocities v_x, v_y for polygon formation.

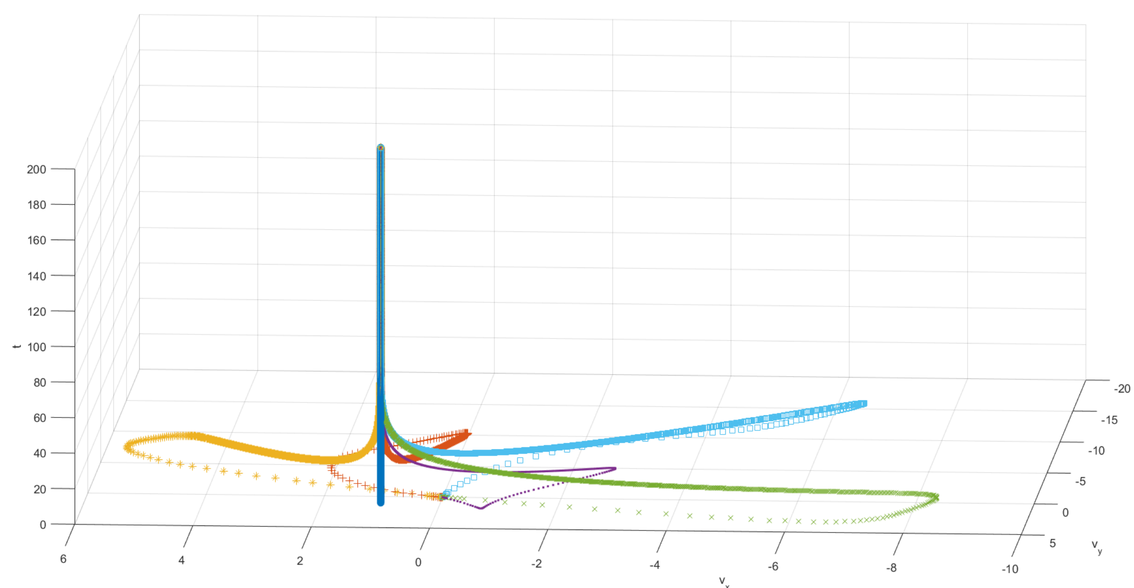

Figure 7: Agent velocities v_x, v_y performance in time for polygon formation.

Velocities of all agents converged with leader's initial velocity $v_x = 1$, $v_y = 1$ as presented in figure 6. Time performance of agent's velocities for polygon formation is shown in figure 7.

The last formation that we want to achieve is a bracket. Initial states states remain as previously and the leader's final point in cartesian space is $(200, 200)$. The bracket formation is depicted in figure 8.

Velocities of all agents converged with leader's initial velocity $v_x = 1$, $v_y = 1$ as presented in figure 9. Time performance of agent's velocities for bracket formation is shown in figure 10.

$\square$

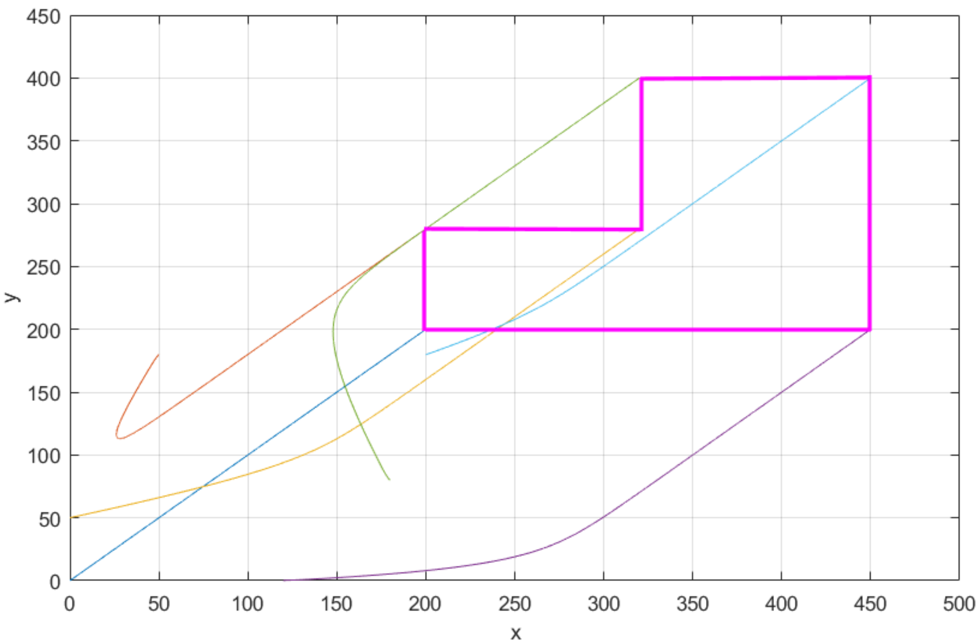

Figure 8: Bracket formation of 6 agents.

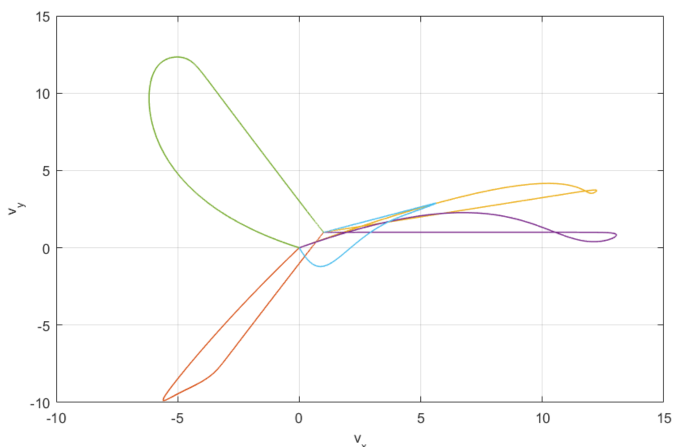

Figure 9: Agent velocities v_x, v_y for bracket formation.

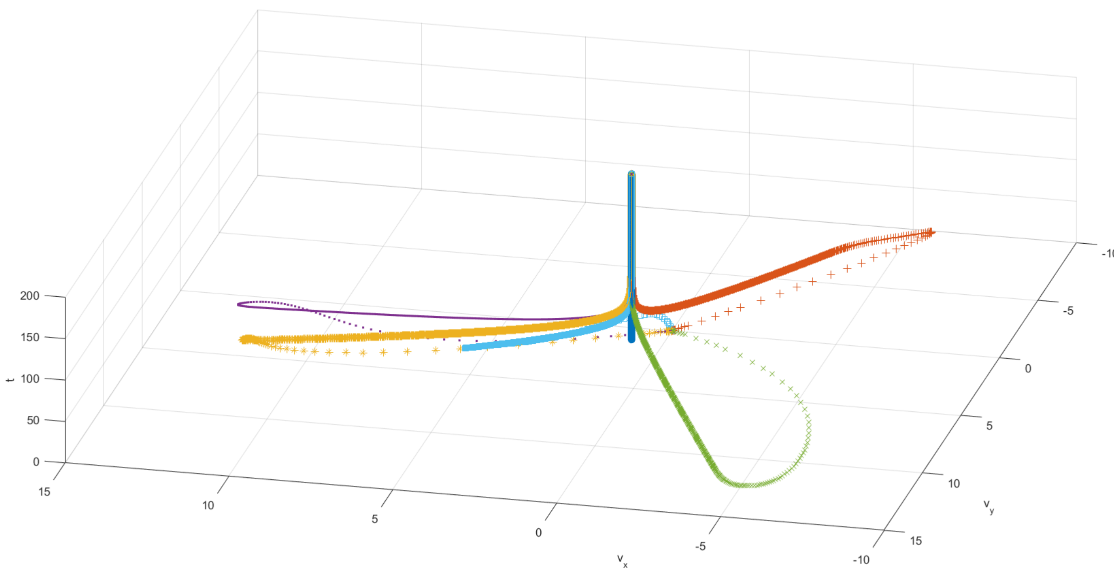

Figure 10: Agent velocities v_x, v_y performance in time for bracket formation.