6 -DOF EQUATIONS FOR A VTOL AIRCRAFT

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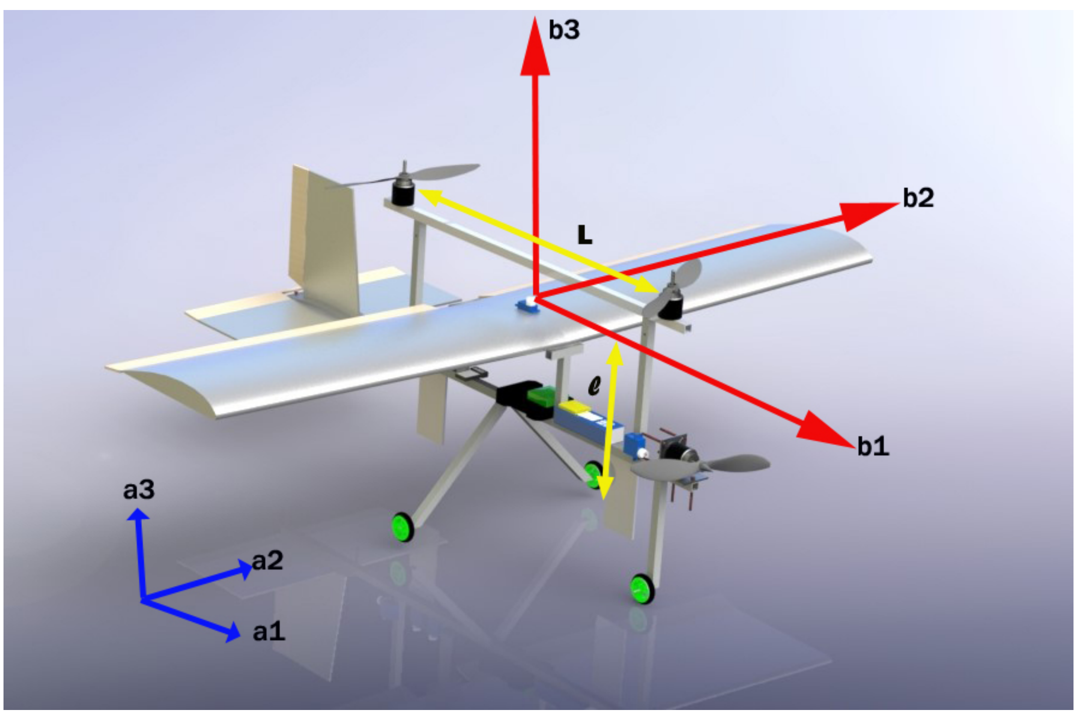
Abstract

This paper presents the equations of motion during vertical take-off/landing of a fixed wing VTOL aircraft and proposes a PI controller for controlling the hovering motion.

List of symbols used

- $\boldsymbol{T}$ *: Thrust force due to vertical rotor*
- $\boldsymbol{\alpha}$*: Deflection of keel*
- $\boldsymbol{F(T,,\alpha)}$ *: Force produced by keel*
- $^{a}\boldsymbol{R_b}$ *: Rotation Matrix for rotaion from body axes to inertial axes.*
- $\boldsymbol{I}$ *: Moment of Inertia tensor referred to the body axes b1, b2, b3.*
- $\boldsymbol{\phi}$*= roll angle,* $\boldsymbol{\theta}$*= pitch angle,* $\boldsymbol{\psi}$*= yaw angle,* $\boldsymbol{p}$ *= roll rate,* $\boldsymbol{q}$ *= pitch rate,* $\boldsymbol{r}$ *= yaw rate*

1 Equations of motion



1.1 Force equations

Moments acting along the positive directions of the body axes are taken positive. α is taken positive if it produces a positive moment.

$$\begin{bmatrix} m\ddot{x}_1 \\ m\ddot{y}_2 \\ m\ddot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + {}^a\boldsymbol{R}_b \begin{bmatrix} 0 \\ F(T_1,\alpha_1)cos\alpha_1 + F(T_2,\alpha_2)cos\alpha_2 \\ T_1 + T_2 - F(T_1,\alpha_1)sin\alpha_1 - F(T_2,\alpha_2)sin\alpha_2 \end{bmatrix}$$

1.2 Moment equations

l is the distance between CG of the aircraft and COP of the keel. **L** is the distance beteen the two vertical rotors.

$$\boldsymbol{I}\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \{F(T_1,\alpha_1)cos\alpha_1 + F(T_2,\alpha_2)cos\alpha_2\}l \\ (T_2 - T_1)L - \{F(T_2,\alpha_2)sin\alpha_2 - F(T_1,\alpha_1)sin\alpha_1\}L \\ \{F(T_1,\alpha_1)cos\alpha_1 - F(T_2,\alpha_2)cos\alpha_2\}L + M_2 - M_1 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \boldsymbol{I}\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

1.3 Relation between ϕ, θ, ψ *and* p , q, r

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} c\theta & 0 & -c\phi s\theta \\ 0 & 1 & s\phi \\ s\theta & 0 & c\phi c\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

2 Controller for attitude control during hover

A Proportional+Integral controller is proposed for maintaining the roll, pitch and yaw angles at 0 during hovering. This would be crucial for transitioning from VTOL phase to cruise phase.

2.1 Control inputs

The control inputs are defined as follows

$u_1 = F(T_1,\alpha_1)cos\alpha_1 + F(T_2,\alpha_2)cos\alpha_2$

$u_2 = (T_2 - T_1) - \{F(T_2,\alpha_2)sin\alpha_2 - F(T_1,\alpha_1)sin\alpha_1\}$

$u_3 = \{F(T_1,\alpha_1)cos\alpha_1 - F(T_2,\alpha_2)cos\alpha_2\} + \frac{(M_2-M_1)}{L}$

2.2 Linearisation of moment equations about hover

In hover phase, the following conditions will hold at equilibrium

- $T_{10} = T_{20} = \frac{mg}{2}$
- $\phi_0 = \theta_0 = \psi_0 = 0$, $p_0 = q_0 = r_0 = 0$, $\dot{p_0} = \dot{q_0} = \dot{r_0} = 0$

In addition, the following assumptions are made:

- $M = \gamma T$
- $I_{xy} = I_{zy} = 0$

The linearised moment equations are given by:

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \{F(T_1,\alpha_1)cos\alpha_1 + F(T_2,\alpha_2)cos\alpha_2\}l/I_{xx} \\ (T_2 - T_1)L/I_{yy} - \{F(T_2,\alpha_2)sin\alpha_2 - F(T_1,\alpha_1)sin\alpha_1\}L/I_{yy} \\ \{F(T_1,\alpha_1)cos\alpha_1 - F(T_2,\alpha_2)cos\alpha_2\}\frac{L}{I_{zz}} + \frac{\gamma}{I_{zz}}(T_2 - T_1) \end{bmatrix} = \begin{bmatrix} \frac{u_1 l}{I_{xx}} \\ \frac{u_2 L}{I_{yy}} \\ \frac{u_3 L}{I_{zz}} \end{bmatrix}$$

2.3 Control law

Based on the linearised moment equations, a control law can be defined as follows:

$u_1 = -K_{p\phi}\phi - K_{I\phi}\int \phi dt$

$u_2 = -K_{p\theta}\theta - K_{I\theta}\int \theta dt$

$u_3 = -K_{p\psi}\psi - K_{I\psi}\int \psi dt$

2.4 Expressions for $T_1, T_2, \alpha_1, \alpha_2$

Expression for F

A simplified expression for F is derived using Blade Element theory and Conservation of Momentum principle and is given by:

$$F = T\sin\alpha \times \frac{A}{\pi R^2}$$

Here, A is the suface area of one face of the keel, R is the radius of the rotor blade and T and $\alpha$are as defined earlier.

Further, the following assumptions are made to obtain expressions for T and α in terms of the control inputs u.

$\cos\alpha \simeq 1$, $\sin\alpha \simeq \alpha$

Equation for Roll

Assumption:$T_1 = T_2 \simeq \frac{mg}{2}$

$\Rightarrow u_1 \simeq \frac{mg}{2} \times \frac{A}{\pi R^2}(\alpha_1 + \alpha_2)$

Equation for Pitch

Assumption:$\alpha_1 \simeq \alpha_2 \simeq 0$

$\Rightarrow u_2 \simeq (T_2 - T_1)$

Equation for Yaw

Assumption:$T_1 = T_2 \simeq \frac{mg}{2}$

$\Rightarrow u_3 \simeq \frac{mg}{2} \times \frac{A}{\pi R^2}(\alpha_1 - \alpha_2)$

Force equation along vertical axis

Assumption:$\phi = \theta \simeq 0, \ddot{r_3} = 0, \alpha_1 = \alpha_2 \simeq 0$

$\Rightarrow T_1 + T_2 = mg$

Thus,

$$T_1 = \frac{mg - u_2}{2}$$
$$T_2 = \frac{mg + u_2}{2}$$
$$\alpha_1 = \frac{\pi R^2}{A}(u_1 + u_3)$$
$$\alpha_2 = \frac{\pi R^2}{A}(u_1 - u_3)$$