Flight Schedule Planning with Maintenance Considerations

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Abstract

Airline planning operations typically begin with the specification of the master flight schedule. While the master schedule is typically determined based on strategic business objectives, such as market share and competitive considerations, efficient operation of the schedule leads to a series of challenging problems that are constrained by FAA regulations, labor agreements, market forces, etc. In this paper, we propose a model that integrates a surrogate representation of FAA regulations regarding aircraft maintenance within a master flight schedule planning problem, and examine its impact on the resulting schedules.

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1 Introduction

Airline operations begin with a set of scheduled flights, from which costs and revenues are generated. The development of the flight schedule involves forecasting demand for flights between different airports, including considerations of competition, market share, and airplane and staffing availability. A "good" schedule maximizes profit within the constraints of an airline's human resources and infrastructure. It is fair to say that airline planning problems are characterized by their large and complex nature, and give rise to some of the most challenging of optimization problems. For example, once a flight schedule has been settled upon, a variety of problems related to the operation of the schedule result. Aircraft and crew must be assigned to specific flights in accordance with complicated labor regulations. Aircraft must be maintained in accordance with Federal maintenance guidelines, and ticket pricing issues must be resolved. In general, each of these problems leads to a rich and complex area of research, such as "fleet assignment problems", "crew scheduling", "maintenance scheduling", and "revenue management." Illustrative works in these areas include Hane et al. [1995], Gopalan and Talluri [1998], and McGill and Van Ryzin [1999] respectively. Due to the complexity of these problems, they are typically considered in isolation, and only after the flight schedule has been determined. Exceptions to this tendency include Cordeau et al. [2001], who integrate the aircraft routing and crew scheduling problems for a single fleet type. Other examples include Cohn and Barnhart [2003], Stojković and Soumis [2001], and Freling et al. [2000]. Similarly, Barnhart et al. [1998] investigate a model that integrates the Fleet Assignment Model (FAM), found in Hane et al. [1995], and a duty pairing problem (DPP) model, which is an approximation of the crew pairing problem. Note however, that each of these studies require the master flight schedule as input to their models.

In this paper, we consider a modelling paradigm that permits a movement toward planning methods that permit a representation of these complicated operational problems within the schedule planning phase. Our method involves the development of a surrogate representation of an operational problem — a model that is relatively easy to solve, yet sufficiently representative of the problem to reflect its impact on schedule choices. To illustrate the concept, we develop a surrogate model of maintenance scheduling. In particular, we use the concept of "maintenance opportunities," which stand as surrogates for the more complicated notion of "maintenance feasibility." While the former concept can be determined via the solution of a linear program (which we propose as a subproblem of a schedule planning problem), the latter requires the solution of a complex combinatorial optimization problem designed to ensure strict compliance with FAA maintenance regulations.

2 Maintenance Scheduling

Just as crews require periodic training, aircraft require periodic maintenance, although maintenance requirements are more frequent and varied than crew training. Each US-operated aircraft must meet FAA and airline requirements for scheduled maintenance. Aircraft must undergo four types of maintenance, as explained by Talluri [1998], from infrequent major overhauls to minor visual inspections every few days. Major overhauls are generally accommodated on a rotating basis, and have the impact of an effective reduction in the number of aircraft available at any given time. Visual inspections require one or two hours every 65 flying hours by FAA rules, although most major airlines self-impose a stricter 45-hour maximum interval. Other types of minor maintenance include engine oil changes and other minor tasks needed to keep an airplane in service. Our model applies to consideration of the impact of minor maintenance on the flight schedule.

Current methods tend to consider maintenance schedules after the flight schedule has been determined. The reasons for this are fairly clear from a computational point of view. Scheduled maintenance requires knowledge of the flights assigned to each specific aircraft (called the "tail assignment"). The immense size of an airline's operations, involving up to 2500 flights, 200 airports and 10 distinct fleets, makes it unrealistic to expect to consider the tail assignment and maintenance schedule while initially planning the master schedule. From a decision modelling point of view, the reasons are equally clear. The master schedule, which forms the nucleus of the airline's business operations and revenue generating capacity, is the product of a strategic planning process in which big-picture issues such as competition and market shares are of primary importance. Maintenance, on the other hand, takes place on an operational level. As such, it requires a closer level of attention to detail. Nonetheless, maintenance requirements impose constraints on the operation of the master schedule, which must be "maintenance feasible."

Talluri [1998] and Gopalan and Talluri [1998] developed procedures for routing aircraft to meet basic maintenance requirements. Their method heuristically finds maintenance-feasible four-day aircraft routings in polynomial time, and can identify flights which must be shifted to another fleet to achieve maintenance feasibility. This algorithm can be particularly useful when airlines route specific airplanes late in the planning process. Alternatively, Cohn and Barnhart [2003] show that by planning maintenance routings before crew pairings, the solution quality of both problems improves. They state that crew pairings often produce infeasible maintenance routings, and planning the maintenance routings first limits selected crew pairings to those which allow for required maintenance opportunities. Throughout our development, we assume that all minor maintenance can be performed at a set of properly equipped and staffed *maintenance stations*. A *maintenance opportunity* occurs when an airplane spends a sufficiently long period of time at a maintenance station, whether or not maintenance is actually performed. As a stand-in for the more complex notion of maintenance feasibility, our basic premise is that if a schedule has sufficient maintenance opportunities built into it, the schedule is likely to be maintainable with out excessive delay.

3 Models

We begin with a model for the identification of the master schedule. Our model involves the determination of flights, which are modelled as flows in a dynamic network. Flights require a given amount of time to travel from one airport to another, and airplanes must spend a minimum amount of time at the new location upon arrival. Each airport has a finite gate capacity, which restricts the number of aircraft that can be on the ground at any given time. In addition, each airport has a limited number of takeoff and landing time "slots" in any period of time. We model this as a dynamic network, in a manner that is similar to the network description that appears in Hane et al. [1995].

Airline schedules are cyclic in nature, and are designed to repeat every c units of time. In formulating the model, simultaneous considerations of time and space are of paramount importance. We adopt the following notation throughout this paper.

Sets:

I denotes the set of distinct locations (airports) under consideration

$T = \{0, ..., t^{max}\}$, is the set of distinct times associated with an appropriate discretization of time.

$\mathcal{N} \subset I \times T$, is the set of nodes in the dynamic network.

$\mathcal{A} \subset \mathcal{N} \times \mathcal{N}$ is the set of arcs in the dynamic network.

Thus, each node in the dynamic network corresponds to a specific location in I and a specific time in T. For each $n \in \mathcal{N}$, let

$i(n)$ denote the location to which n corresponds,

$\tau(n)$ denote the time to which n corresponds.

Let

t_{ij} denote the standard time required to travel between locations i and j;

$\gamma(i)$ denote the minimum "turn" time required at location i, the time required for unloading passengers and cargo, cleaning, fueling, inspecting, and reloading the aircraft;

c_{ij} denotes the estimated profit generated by a flight between locations i and j.

An arc $(n, m) \in \mathcal{A}$ is either a flight or time spent on the ground. Let

$\mathcal{A}^{\mathcal{F}}$ denote the set of flight arcs, where $(n,m) \in \mathcal{A}^{\mathcal{F}}$ implies that $i(n) \neq i(m)$ and $\tau(m) - \tau(n) = t_{i(n)i(m)}$,

$\mathcal{A}^{\mathcal{G}}$ denote the set of ground arcs, where $(n,m) \in \mathcal{A}^{\mathcal{G}}$ implies that $i(n) = i(m)$ and $\tau(m) - \tau(n) = 1$.

Finally, for each t, let

$\mathcal{N}(t)$ $=\{n \in \mathcal{N} \mid \tau(n) = t\}$ the set of nodes that correspond to time t

$\mathcal{A}(t)$ $=\{(n,m) \in \mathcal{A} \mid \tau(n) \leq t \text{ and } \tau(m) > t\}$, the set of arcs that are active at time t

$\mathcal{N}_i(t)$ $=\{n \in \mathcal{N}(t) \mid i(n) = i\}$, the set of nodes that correspond to location i and time t.

Typically, the cycle c is one day, and t^{max} may be any time up to one month. In addition, we have the following data:

$r_i^-, (r_i^+)$ is the maximum number of aircraft that can takeoff (land) at location i.

g_i is the maximum number of aircraft that can be on the ground at location i.

V is the number of airplanes available in the system.

Our model is designed to identify values of the following decision variables

x_{nm} is the "flow" (i.e., the number of aircraft) on arc $(n,m) \in \mathcal{A}$.

that maximize "profit" while maintaining an appropriate number of "maintenance opportunities", where the latter is denoted by the function $f_{mx}(x)$ and is defined in §4.

MSP:

$$\text{Max} \sum_{(nm)\in\mathcal{A}^{\mathcal{F}}} c_{i(n)i(m)} x_{nm} \tag{1}$$

$$\begin{aligned}
\text{s.t.} \qquad \sum_m x_{nm} &\leq r^-_{i(n)} \quad \forall\, n \in \mathcal{N}, (nm) \in \mathcal{A}^{\mathcal{F}} && (2)\\
\sum_n x_{nm} &\leq r^+_{i(m)} \quad \forall\, m \in \mathcal{N}, (nm) \in \mathcal{A}^{\mathcal{F}} && (3)\\
\sum_n x_{nm} &\leq g_{i(m)} \quad \forall\, t, m \in \mathcal{N}(t), (nm) \in \mathcal{A}^{\mathcal{G}} && (4)\\
\sum_{(nm)\in\mathcal{A}(0)} x_{nm} &\leq V && (5)\\
f_{mx}(x) &\geq \sum_{(nm)\in\mathcal{A}(0)} x_{nm} && (6)\\
\sum_{(mn)\in\mathcal{A}^{\mathcal{F}},\ni\ n\in\{\mathcal{N}_{i(p)}(\tau(p)-t)\}_{t=0}^{\gamma(i(p))}} x_{mn} &\leq x_{pq} \quad \forall\, (pq) \in \mathcal{A}^{\mathcal{G}} && (7)\\
\sum_m x_{mn} - \sum_m x_{nm} &= 0 \quad \forall\, n \in \mathcal{N} && (8)\\
x_{nm} - x_{n'm'} &= 0 \quad \forall\, (nm) \in \mathcal{A}, i(n) = i(n'), n \in \mathcal{N}(t), \\
&\qquad \tau(n') = \tau(n) + c, t \in \{0, \ldots, \max(t_{ij})\} && (9)\\
x_{nm} &\in Z^+ \quad \forall\, (nm) \in \mathcal{A} && (10)
\end{aligned}$$

The objective function maximizes the net profit associated with the master schedule. Constraints (2) limit the number of flights departing a location to the takeoff capacity, while (3) similarly limit the number of landings at each location in each period. Constraints (4) ensure that holding capacity is not exceeded at any location, while constraint (5) prescribes that the number of assigned airplanes does not exceed the number available, and is called a *plane count* constraint. With (6), we require at least as many maintenance opportunities as the plane count. Equations (7) require that flights stay on the ground for at least the minimum turn times. In other words, given x_{pq}, where $(pq) \in \mathcal{A}^{\mathcal{G}}$ the number of aircraft assigned on the ground arc must include at least all of the flights that have arrived $\gamma(i(p))$ or fewer time units before $\tau(p)$ (and are therefore in their mandatory ground hold time window). Resource flow is maintained by (8). Constraints (9), known as *wraparound constraints*, ensure identical schedules for each cycle.

4 A Model of Maintenance Opportunities

Minor frequent maintenance requires each aircraft to pass through a maintenance station every few days. As previously discussed, scheduling this maintenance results in a large integer program, which is prohibitive as a component of MSP. We incorporate this maintenance requirement by counting *maintenance opportunities*, layovers meeting or exceeding a minimum time interval at a designated maintenance station, within the MSP. Since minor maintenance must occur every $\mathcal{T}$ days (typically, $\mathcal{T}$ is 3-4 days), we can count the number of maintenance opportunities in this time window, and require that the number of maintenance opportunities is at least as large as the number of aircraft (as in (6)). Note that this does not guarantee that each individual aircraft will be serviced, nor does it minimize maintenance cost; it simply offers a measure of whether a schedule is "maintainable." The maintenance opportunity count also acts as a measure for comparison between alternate schedules to estimate which is the most maintainable.

In the following, we assume that $\mathcal{T} > 0$, and present a model from which $f_{mx}(x)$, the number of maintenance opportunities associated with master schedule x, can be derived. We restrict our attention to the case in which there is only a single type of maintenance that must be scheduled, and comment on extensions to multiple types of maintenance at the end of the section. We begin by constructing a "maintenance network" based on the dynamic network used for MSP. To do this, we overlay "maintenance arcs" on the network, which correspond to sequences of enough ground arcs to permit maintenance to take place. Figure 1 illustrates the flight network and the resulting maintenance opportunities. Figure 1a displays the basic MSP structure, where horizontal lines represent potential ground arcs, and diagonal lines represent potential flights. Phoenix (PHX) appears as a maintenance station, while Salt Lake City (SLC) and Los Angeles (LAX) do not have maintenance capability. Figure 1b depicts a collection of flights (and ground holds) between these cities. Assuming that maintenance requires two time periods and that at most two aircraft can undergo maintenance at any time, Figure 1c, illustrates a reorganization of the flights that provides four maintenance opportunities.

In addition to the notation established for MSP,

t^{mx} is the time required to perform scheduled maintenance

$\mathcal{I}^{mx}$ is the set of locations where scheduled maintenance can be performed;

$\mathcal{A}^{mx}$ is the set of maintenance arcs, $(nm) \in \mathcal{A}^{mx}$ if $i(m) = i(n), i(n) \in \mathcal{I}^{mx}$, and $\tau(m) - \tau(n) \geq t_{mx}$;

$\mathcal{A}^{mx}(t)$ $=\{(n,m) \in \mathcal{A}^{mx} \mid \tau(n) \leq t \text{ and } \tau(m) > t\}$, the set of maintenance arcs active at time t;

h_i is the number of crews at maintenance station i;

$\mathcal{T}$ is the maintenance cycle time, typically three to four days.

With x_{nm} as the flight schedule proposed by MSP, we have the following decision variables:

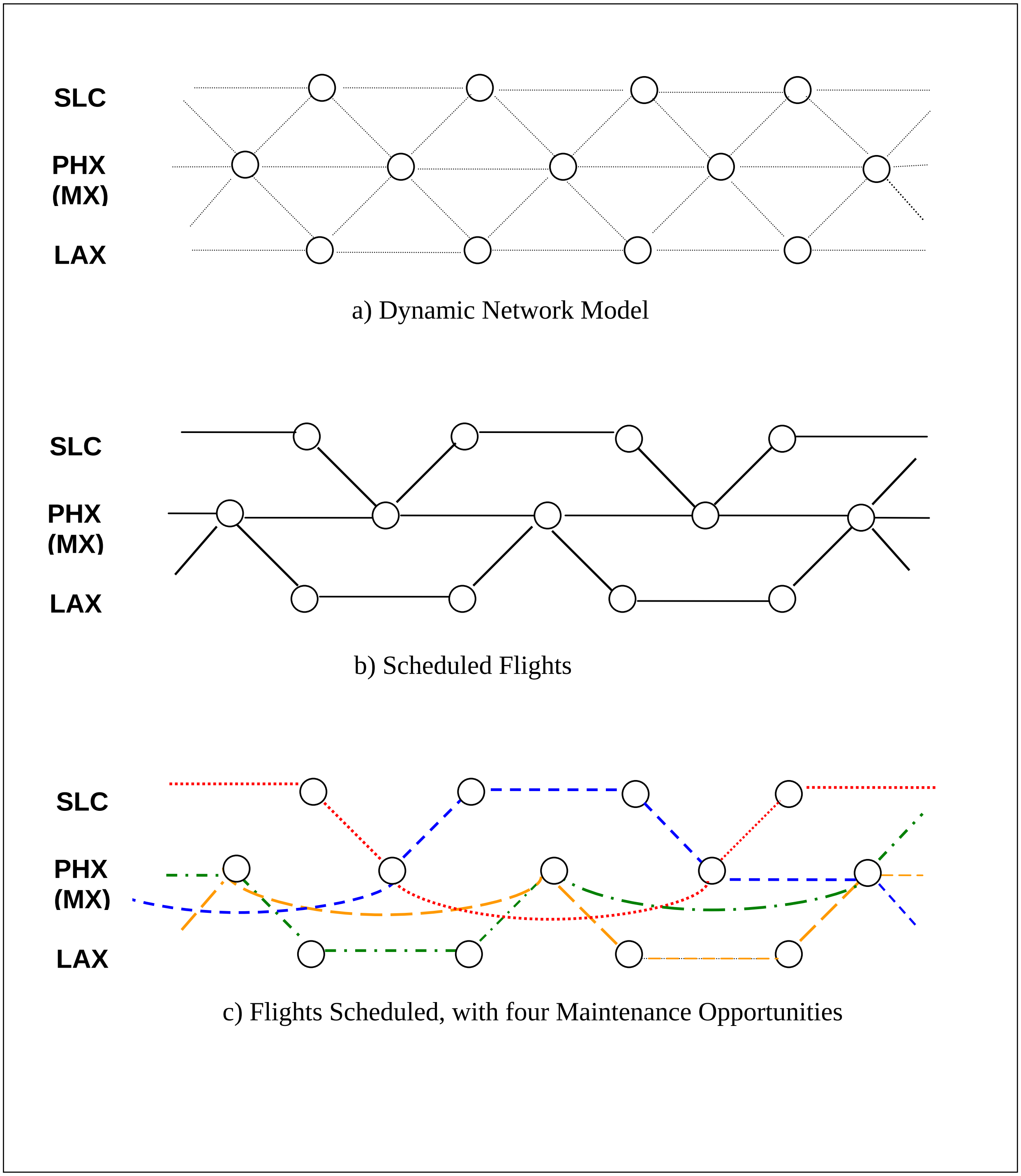

Figure 1. Network Structure. Time moves from left to right.

y_{nm} is the "flow" on maintenance arc $(nm) \in \mathcal{A}^{mx}$

z_{nm} is the "flow" on non-maintenance arc $(nm) \in \mathcal{A}^{\mathcal{G}}, i(n) \in \mathcal{I}^{mx}$

We define the number of maintenance opportunities for a given master schedule, x, as

$$\textbf{MX: } f_{mx}(x) = \text{Max} \sum_{(nm)\in\mathcal{A}^{mx}} y_{nm} \tag{11}$$

$$\text{s.t.} \sum_{(\ell m)\in\mathcal{A}^{mx}(\tau(n))} y_{\ell m} \leq h_{i(n)} \quad \forall\, n \in \mathcal{N}^{mx} \tag{12}$$

$$\sum_{(\ell m)\in\mathcal{A}^{mx}(\tau(n))} y_{\ell m} + z_{np} = x_{np} \quad \forall\, n, (np) \in \mathcal{A}^{\mathcal{G}}, i(n) \in \mathcal{I}^{mx} \tag{13}$$

$$y_{nm} \geq 0 \quad \forall\, (nm) \in \mathcal{A}^{mx} \tag{14}$$

$$z_{nm} \geq 0 \quad \forall\, (nm) \in \mathcal{A}^{\mathcal{G}} \tag{15}$$

Constraints (12) ensure that the number of maintenance opportunities does not exceed the number of available maintenance crews at any place and time. Equations (13) assign the flow on ground arcs at maintenance stations in the master schedule to maintenance or non-maintenance arcs.

Our presentation of the MSP-MX model highlights the potential to solve this problem using a variation of Benders' Decomposition. Indeed, this is readily accomplished. In cases where the flight network is only moderately sized, the maintenance constraints can simply be directly incorporated into the MSP without resorting to a decomposed presentation. In §7 we discuss extensions of the model, many of which favor solution by decomposition. We note that the use of the surrogate measure "maintenance opportunities" in place of the requirement of maintenance-feasibility eases the computational requirements of such integrated decision making. Nonetheless, it simultaneously raises questions regarding the suitability of the surrogate. Ideally, a large number of maintenance opportunities would indicate a readily maintainable schedule, while a small number would indicate a schedule with the potential for maintenance difficulties. In other words, additional effort must be expended in order to determine the validity of the approximation.

5 Validation of the Surrogate Model

To test and demonstrate the validity of the surrogate model, we undertake the more involved task of actually testing the maintenance feasibility of the master schedule identified via the solution of the integrated model (MSP). This is accomplished via a "string model" similar to those described in Barnhart et al. [1998]. A string is a series of flights that are assigned to a single aircraft. Although string models are not the most efficient method for solving the maintenance scheduling model, they are relatively simple and provide a useful basis for comparison with the approximation results. The heuristic method proposed by Talluri [1998] is more efficient computationally, but is more difficult to implement. Our goal in this section is to explore the validity of the output of our surrogate model rather than to efficiently identify an operationally viable maintenance schedule. For this reason, we validate the surrogate representation with the more easily implemented, but time consuming, string model.

Our validation model begins with a collection of binary variables,

f_{nms} is 1 if flow on arc $(nm) \in \mathcal{A}$ is assigned to string s (i.e., aircraft s); 0 otherwise

ϕ_{ns} is 1 if a maintenance arc begins at node n on string s; 0 otherwise

σ_s is 1 if string s contains at least one maintenance arc; 0 otherwise

In this manner, $\sum_s \sigma_s$ denotes the number of aircraft that utilize at least one maintenance arc. If this number is equal to the number of aircraft needed to fly the schedule, then the schedule is "maintenance feasible." If the maximum number of strings that contain maintenance arcs is less than the number of aircraft, then the schedule is not maintenance feasible.

To represent potential maintenance arcs and minimum ground holds, we introduce two sets. These sets are primarily notational aids for expressing the model. For each n such that $i(n) \in \mathcal{I}^{mx}$, let

$\mathcal{A}_n^{mx}$ $=\{(\ell, m) \in \mathcal{A}^{mx} \mid i(\ell) = i(n), \text{ and} \tau(\ell) = \tau(n)\}$, the set of maintenance arcs that originate with node n.

$\mathcal{A}^{\mathcal{G}}{}_n$ $=\{(\ell, m) \in \mathcal{A}^{\mathcal{G}} \mid i(\ell) = i(n), \tau(n) \leq \tau(\ell) < \tau(n) + \gamma_i(n)\}.$, the set of ground arcs necessary to satisfy the minimum ground hold constraint for flights landing at n.

The set $\mathcal{A}_n^{mx}$ represents the arcs that may be used to schedule maintenance at location $i(n) \in \mathcal{I}^{mx}$ beginning at time $t(n)$. Similarly, the set $\mathcal{A}^{\mathcal{G}}{}_n$ represents the set of ground arcs that are associated with the minimum ground hold constraint for flights landing at location $i(n)$ at time $t(n)$.

Given a master schedule x, a more precise representation of a maintenance schedule may be obtained via the solution to

$$\max \sum_{s} \sigma_s \tag{16}$$

$$\text{s.t.} \quad \sum_{n \in \mathcal{I}^{mx}} \phi_{ns} \geq \sigma_s \qquad \forall\, s \tag{17}$$

$$\sum_{(\ell m) \in \mathcal{A}_n^{mx}} f_{\ell ms} \geq \phi_{ns} \qquad \forall\, n \in \mathcal{N}^{mx}, s \tag{18}$$

$$\sum_{s} \sum_{n:i(n)=i,\ \tau(n) \leq t < \tau(n)+t^{mx}} \phi_{ns} \leq h_i \qquad \forall\, t,\ \forall\, i \in \mathcal{I}^{mx} \tag{19}$$

$$\sum_{m} f_{mns} = \sum_{m} f_{nms} \qquad \forall\, n \in \mathcal{N}, s \tag{20}$$

$$\sum_{s} f_{nms} = x_{nm} \qquad \forall\, (nm) \in \mathcal{A} \tag{21}$$

$$\sum_{m:(mn) \in \mathcal{A}^{\mathcal{F}}} f_{mns} \geq \sum_{(\ell p) \in \mathcal{A}^{\mathcal{G}}{}_n} f_{\ell s} \qquad \forall\, n \in \mathcal{N}, s \tag{22}$$

$$\sigma_s, \phi_{ns}, f_{nms} \in [0, 1] \qquad \forall\ n, m, s \tag{23}$$

Constraints (17) and (18) drive variables σ_s to indicate whether string s contains a maintenance arc or not. Since σ_s is binary, the objective is increased by 1 if one or more maintenance arcs is assigned to a string, so that assigning more than one maintenance arc to a string does not result in any additional benefit. In (19), flow on maintenance arcs at any one place and time is limited to the number of available crews, as in (12). Flow is conserved by (20). These constraints also associate individual flight legs with strings. Equations (21) require that each unit of flow on an arc is assigned to a string, and (22) ensure that landing flights enter a ground hold or maintenance arc. In a maintainable flight schedule, every string will include at least one maintenance opportunity, so that $\sum_s \sigma_s$ equals the total number of aircraft.

The string model follows the same maintenance time requirement as the approximation model. The solution obtained from the approximation model indicates whether the number of maintenance opportunities in the schedule is at least as great as the number of aircraft. Consequently, the string model cannot find more maintenance arcs than the surrogate model. If the surrogate model indicates that not enough maintenance arcs exist for the schedule to be maintainable, it is impossible for the string model to show a contradictory result. When the surrogate model indicates maintenance feasibility, it is not immediately clear that the schedule is necessarily maintainable. We will test this empirically.

6 Experiment

Using the string model, we now have a basis from which to test the validity of our surrogate representation of maintenance. Within our experimental setting, we used “small” networks containing five airports. Each of the problems that we used was constructed using randomly generated data, as described below.

To generate problem data, we begin with networks of 5 airports over 144 thirty-minute time periods, or three days, with a maximum of 15 aircraft in each schedule. For each network, we used a pseudo-random number generator to produce grid coordinate pairs for each city. The coordinates were uniformly distributed on the interval [100,3000] in the east-west direction and [100,1500] in the north-south direction. From the coordinates, we calculated Euclidean distances between the cities, and linearly transformed these distances into matrices of travel times. We then randomly generated profit per flight and demand per flight ($\{c_{ij}\}$ and $\{d_{ij}\}$, respectively) using uniform distributions, and selected airport capacities and number of maintenance crews at each airport. Initially, each airport was identified as a maintenance station with probability 0.2. Maintenance stations are then designated sequentially, although this designation is adjusted to ensure that at least one, and no more than two, airports are designated as maintenance stations. Within each maintenance station, there are 2, 3, or 4 maintenance crews with probabilities 0.1, 0.2, and 0.7, respectively.

Flight schedules were generated for each network. Maintenance feasibility of these schedules was evaluated using both the surrogate model and the string model. The results were then compared for consistency. Our goal is to test the predictive capability of the surrogate model for a variety of master schedules on these various networks. If the string model finds a problem to have fewer maintainable strings than the number of aircraft needed to fly the schedule, then we say that the schedule is not maintainable. Similarly, if the surrogate model finds fewer maintenance opportunities than aircraft, the schedule is identified as unmaintainable. For each of the networks generated, we compared the indications of maintenance feasibility obtained from the surrogate model to those obtained from the string model.

Table 1 contains a summary of these comparisons for five different sets of problem data. MSP was used to generate flight schedules, with and without the constraint on maintenance opportunities, (6). We use “a” to indicate an MSP solution based solely on profit (i.e., with (6) omitted), and “b” to indicate the profit maximizing solution with (6) included. Consequently, in the first data set, profit-only considerations yield six maintenance opportunities, and results in six maintainable flight strings for 15 airplanes. Adding the constraint on maintenance opportunities results in an abundance of maintenance opportunities (120 opportunities for 15 airplanes), which the string model confirms as maintainable. In all cases, the surrogate and the string model reached the same conclusion regarding maintenance feasibility. The results of this experiment indicate that the surrogate model is useful for determining whether a schedule is maintainable.

It is interesting to note that in each case, the MSP objective value was not affected by the inclusion of the constraint on maintenance opportunities. This indicates that the algorithm explored alternate optimal solutions as the Benders’ cuts shaped the master schedule towards a maintenance-feasible one.

As expected, the solution times of the two maintenance models differed sharply. The time

Problem Number	# Planes	Surrogate Model MX opportunities	String Model # maintained
1a	15	6	6
1b	15	120	15
2a	15	12	9-12
2b	14	36	14
3a	15	6	6
3b	15	84	15
4a	15	12	0
4b	15	30	15
5a	15	6	6
5b	15	30	15

Table 1: Results of Model Validation Experiment.

required to solve the surrogate model is essentially negligible, while the string model requires much more time to solve, even for these small networks. In some cases (e.g., 2a), the string model failed to identify an optimal solution after several hours (although it indicated a lower bound of 9 maintainable strings and an upper bound of 12). When employing a solution technique that requires the subproblems to be solved many times, such as Benders' decomposition, these time savings are extremely helpful in finding an overall optimal schedule within a "reasonable" amount of time.

To demonstrate scalability, we applied the master schedule planning process to much larger networks, with 20, 25, and 30 airports and 144 time periods. The resulting problems are too large to subject the schedule to an investigation of maintenance feasibility via our string model. However, in each of the problems that we generated and solved, we found that the solutions obtained using the MSP with the constraint on maintenance included sufficient opportunities for the aircraft to be maintained, whereas the initial solutions for the same problems were generally not maintainable schedules. As in the smaller problems, the objective values were not affected by the inclusion of the maintenance constraint. Table 2 shows the results of solving a few large problems with and without maintenance considerations in the model.

Problem Number	# Airports	# Planes	Initial # MX opportunities	Final # MX opportunities
6	20	60	0	66
7	25	75	6	92
8	30	90	60	112
9	30	90	0	112
10	30	90	18	108
11	30	90	24	109

Table 2: Demonstration of Scalability.

7 Extensions

The discussion above explores the validity of a surrogate model in approximating maintenance scheduling, and thus to rapidly judge a schedule for maintenance feasibility. For simplicity, the MX model did not consider several factors that impact real aircraft maintenance scheduling. Some of these considerations may be explored as extensions of our model.

"Safety factor." In our model, we required that the number of maintenance opportunities be at least as great as the number of aircraft needed to fly the schedule. Since the number of aircraft available may vary, especially for larger networks, and since delays and other schedule disruptions may cause deviations from the planned schedule, we may want to include a multiplier on V in the model. For example, we may want the schedule to contain at least $\alpha > 1$ maintenance opportunities per plane, so the right-hand side of constraint (6) becomes $\alpha \sum_{(nm)\in\mathcal{A}(0)} x_{nm}$. Inclusion of this safety factor may decrease the profit of the schedule, so the tradeoffs between profit and reduced risk of loss of maintenance feasibility should be analyzed.

Multiple forms of maintenance. Our investigation assumed a single type of aircraft maintenance. In reality, different types of maintenance must be performed at different intervals, and possibly in different locations. Major maintenance, such as annual inspections, may be modeled by reducing V, reflecting the fact that a given number of aircraft are routinely removed from service for an extended period of time. With varying forms of frequent maintenance required, we may use multiple sets of maintenance arcs, ie., $\{\mathcal{A}_i^{mx}\}$, and constrain each type separately.

Random maintenance times. When mechanics discover problems on an aircraft during a short inspection, repair times can easily exceed t^{mx}. To capture scheduled maintenance events that exceed the predicted time, we may reformulate MX as a stochastic linear program with randomness in the number of time periods required for membership of nm in set A^{mx}. Since the number of potential scenarios for extended maintenance can be large, our solution technique would need to incorporate stochastic programming solution techniques, which typically favor the decomposed presentation of the integrated model.

Unscheduled maintenance. Unscheduled maintenance occurs when an airplane "breaks" during a trip and requires immediate attention to remain in service after landing. These problems range from burned-out lights to engines damaged by birds, and generally must be fixed at the airport where the plane landed. Sometimes technicians or parts must be flown to non-maintenance stations to repair the plane, which takes time and exacerbates schedule disruption. While scheduled maintenance is largely predictable, unscheduled maintenance occurs randomly. While our focus in this paper is on scheduled maintenance, the impact of schedule disruptions on the operation of the flight schedule is examined in a forthcoming paper.

8 Conclusions

As discussed, airline schedules are very large, complex problems which currently are not typically developed in a manner that explicitly recognizes the impact of operational problems on the profitability of the schedule. Approximations of these related operational problems can allow the development of a schedule based on a more realistic representation of its ultimate profitability. Our validation experiment demonstrates the viability of one such approximation model. In particular, it appears that the approximation model is a useful surrogate for the exact model, and can be used for indicating maintenance feasibility of flight schedules.

The data and models used in this demonstration are simplified. For example, the models do not account for common requirements such as overnight layovers. Similarly, the profit-per-flight data depend only on the origin and destination of the flight, ignoring the influence of time of day, crew costs, winds, and myriad other factors. Many of these items can be incorporated in a more comprehensive version of MSP. Nevertheless, the concept of the surrogate model as demonstrated is still effective.

The surrogate model for maintenance is clearly useful in identifying feasible schedules. The model may also be used for long-term planning decisions, such as how many maintenance stations to operate and where to locate them. These types of problems can be analyzed by adding relative cost coefficients for the maintenance stations to the objective function, and by changing the maintenance crew availability data to reflect each option.

Future research in this investigation will explore surrogate models for other subproblems in airline planning, such as crew scheduling and revenue management. Our goal is to develop a set of surrogate models that can be used in concert to find an optimal schedule in an integrated manner.

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