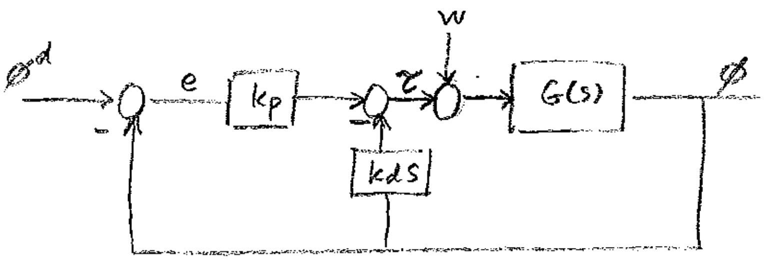
IV.10



$$G(s) = \frac{1/J}{s^2 + \frac{b}{J}s + \frac{mgL}{J}}$$

(a)
$$E(s) = \phi^d - \phi = \phi^d \left[1 - \frac{\phi}{\phi^d}\right]$$

$$= \phi^d \left[\frac{s^2 + \frac{b+k_d}{J}s + \frac{mgL+k_p}{J} - \frac{k_p}{J}}{s^2 + \frac{b+k_d}{J}s + \frac{mgL+k_p}{J}} \right]$$

$$= \phi^d \left[\frac{s^2 + \frac{b+k_d}{J}s + \frac{mgL}{J}}{s^2 + \frac{b+k_d}{J}s + \frac{mgL+k_p}{J}} \right]$$

$$e_{ss} = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} s \left[\frac{s^2 + \frac{b+k_d}{J}s + \frac{mgL}{J}}{s^2 + \frac{b+k_d}{J}s + \frac{mgL+k_p}{J}} \right] \phi^d(s)$$

Let $\phi^d(s) = \frac{1}{s^{k+1}}$.

For $k+1 = 1$, $e_{ss} = \frac{mgL}{mgL+k_p}$ = finite constant

$k = 0 \Rightarrow$ system is Type 0 with PD control

For a step, $e_{ss} = \frac{mgL}{mgL+k_p}$

For a ramp, $e_{ss} = \infty$

For integral control, replace k_p with $\frac{k_p s + k_I}{s}$:

$$\frac{\phi}{\phi^d} = \frac{\frac{1}{J}\left(\frac{k_p s + k_I}{s}\right)}{s^2 + \frac{b+k_d}{J}s + \frac{mgL}{J} + \frac{1}{J}\left(\frac{k_p s + k_I}{s}\right)}$$

$$\frac{\phi}{\phi^d} = \frac{\frac{1}{J}(k_p s + k_I)}{s^3 + \frac{b+k_d}{J}s^2 + \frac{mgL+k_p}{J}s + \frac{k_I}{J}}$$

$$E(s) = \phi^d(s)\left[1 - \frac{\phi}{\phi^d}\right] = \frac{s\left[s^2 + \frac{b+k_d}{J}s + \frac{mgL}{J}\right]}{s^3 + \frac{b+k_d}{J}s^2 + \frac{mgL+k_p}{J}s + \frac{k_I}{J}}\,\phi^d(s)$$

(cont.)

IV.10 (a) cont.

By FVT, $e_{ss} = \lim_{s \to 0} s\,E(s)$

$$= \lim_{s\to 0} s \frac{s\left[s^2 + \frac{b+k_d}{J}s + \frac{mgL}{J}\right]}{s^3 + \frac{b+k_d}{J}s^2 + \frac{mgL+k_p}{J}s + \frac{k_I}{J}} \cdot \phi^d(s)$$

Let $\phi^d(s) = \frac{1}{s^{k+1}}$

For $k+1 = 2$, $e_{ss} = \frac{mgL}{k_I}$ = finite constant

With PID control, system is Type 1 with respect to reference inputs

PID: For a step, $e_{ss} = 0$

PID: For a ramp, $e_{ss} = \frac{mgL}{k_I}$

(b) Want $\frac{E}{W}$: $(\phi^d = 0)$ PD control

$$e = -\phi = -G(s)[\tau + w]$$

$$\tau = k_p e - k_d s\phi = (k_p + k_d s)e$$

$$e = -G(k_p + k_d s)e - Gw$$

$$e[1 + (k_d s + k_p)G] = -Gw$$

$$\frac{E}{W} = \frac{-G}{1 + (k_d s + k_p)G}$$

$$\frac{E}{W} = \frac{-1/J}{s^2 + \frac{b+k_d}{J}s + \frac{mgL+k_p}{J}}$$

For w = constant $\Rightarrow W(s) = \frac{w_0}{s}$,

$$e_{ss} = \lim_{s\to 0} s\left(\frac{E}{W}\right)\frac{w_0}{s}$$

$$\boxed{e_{ss} = -\frac{1}{mgL + k_p}}$$

(cont.)

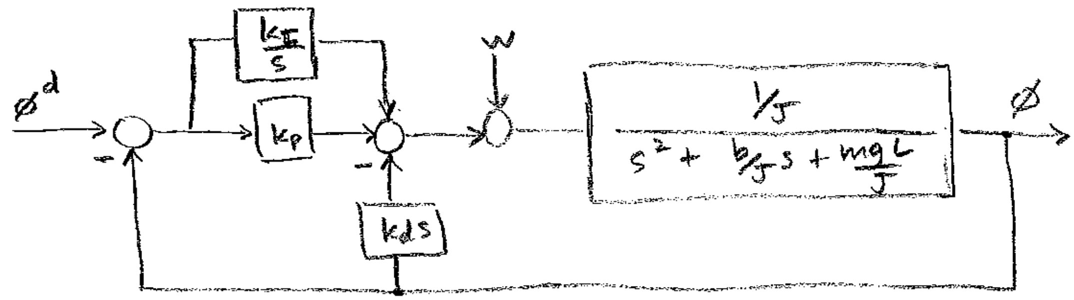
IV.10 (b) cont.

For PID control,
$$\frac{E}{W} = \frac{-G}{1 + \left(k_d s + k_p + \frac{k_I}{s}\right)G}$$
$$= \frac{-Gs}{s + (k_d s^2 + k_p s + k_I)G}$$
$$= \frac{-\frac{1}{J}s}{s^3 + \frac{b}{J}s^2 + \frac{mgL}{J}s + (k_d s^2 + k_p s + k_I)\cdot\frac{1}{J}}$$
$$= \frac{-\frac{1}{J}s}{s^3 + \frac{b+k_d}{J}s^2 + \frac{mgL+k_p}{J}s + \frac{k_I}{J}}$$

For a constant disturbance,
$$e_{ss} = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} s\left(\frac{E}{W}\right)\frac{W_o}{s}$$

$\boxed{e_{ss} = 0}$ for PID control

(c)



From part (a),
$$\boxed{\frac{\phi}{\phi^d} = \frac{\frac{1}{J}(k_p s + k_I)}{s^3 + \frac{b+k_d}{J}s^2 + \frac{mgL+k_p}{J}s + \frac{k_I}{J}}}$$

(d)

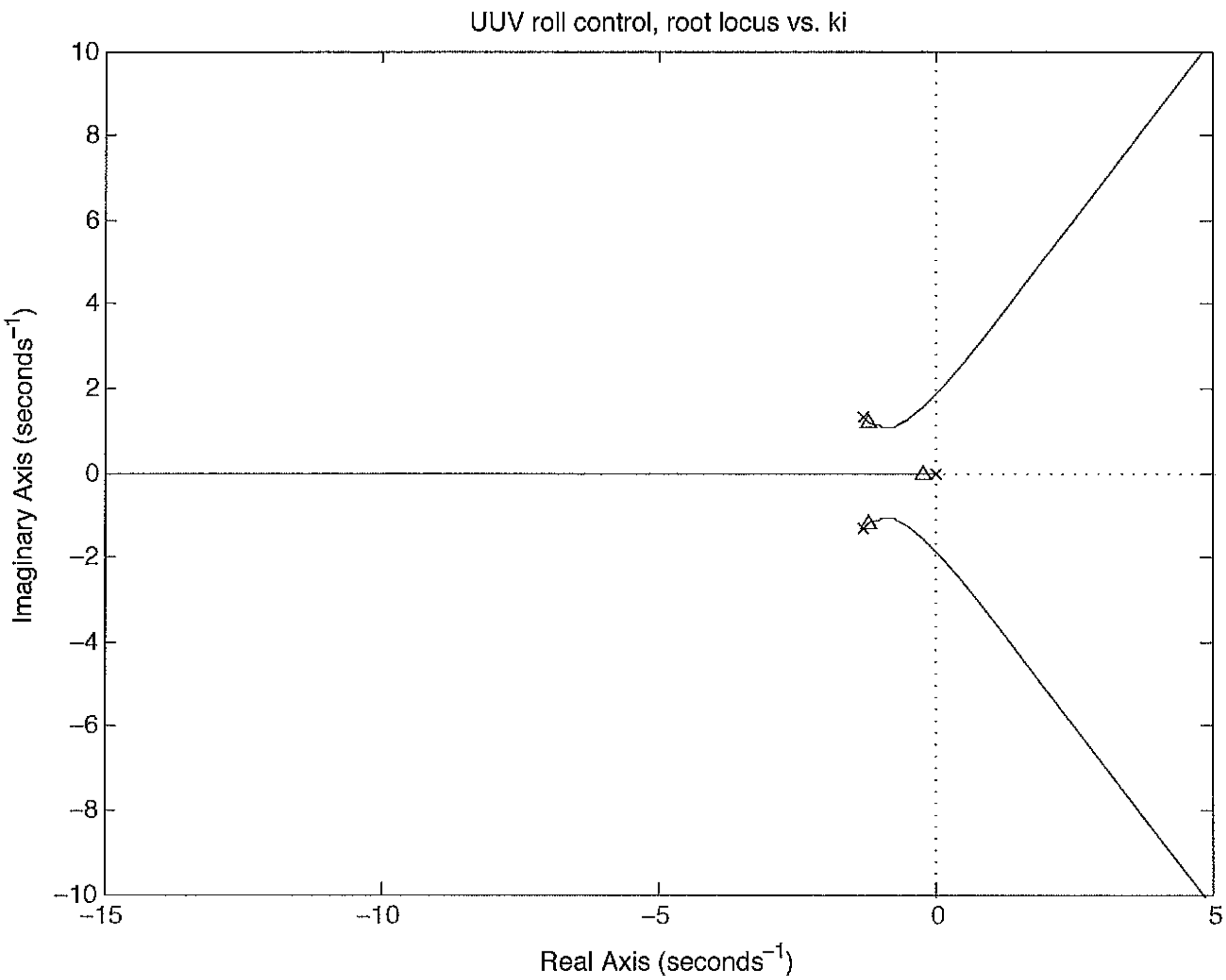
CLCE: $s^3 + \frac{b+k_d}{J}s^2 + \frac{mgL+k_p}{J}s + \frac{k_i}{J} = 0$

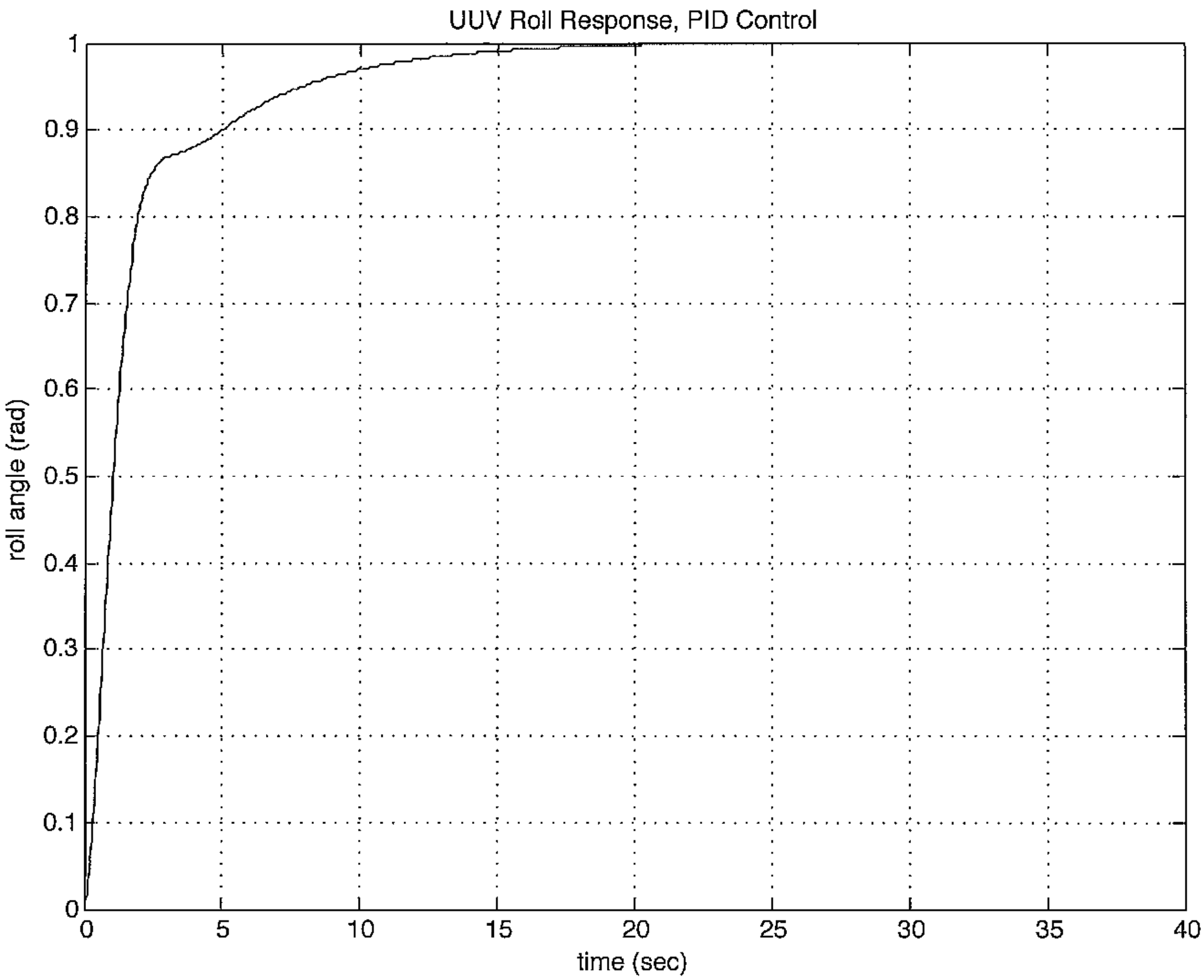
Evan's form
$$\boxed{1 + k_I \frac{1/J}{s\left[s^2 + \frac{b+k_d}{J}s + \frac{mgL+k_p}{J}\right]} = 0}$$

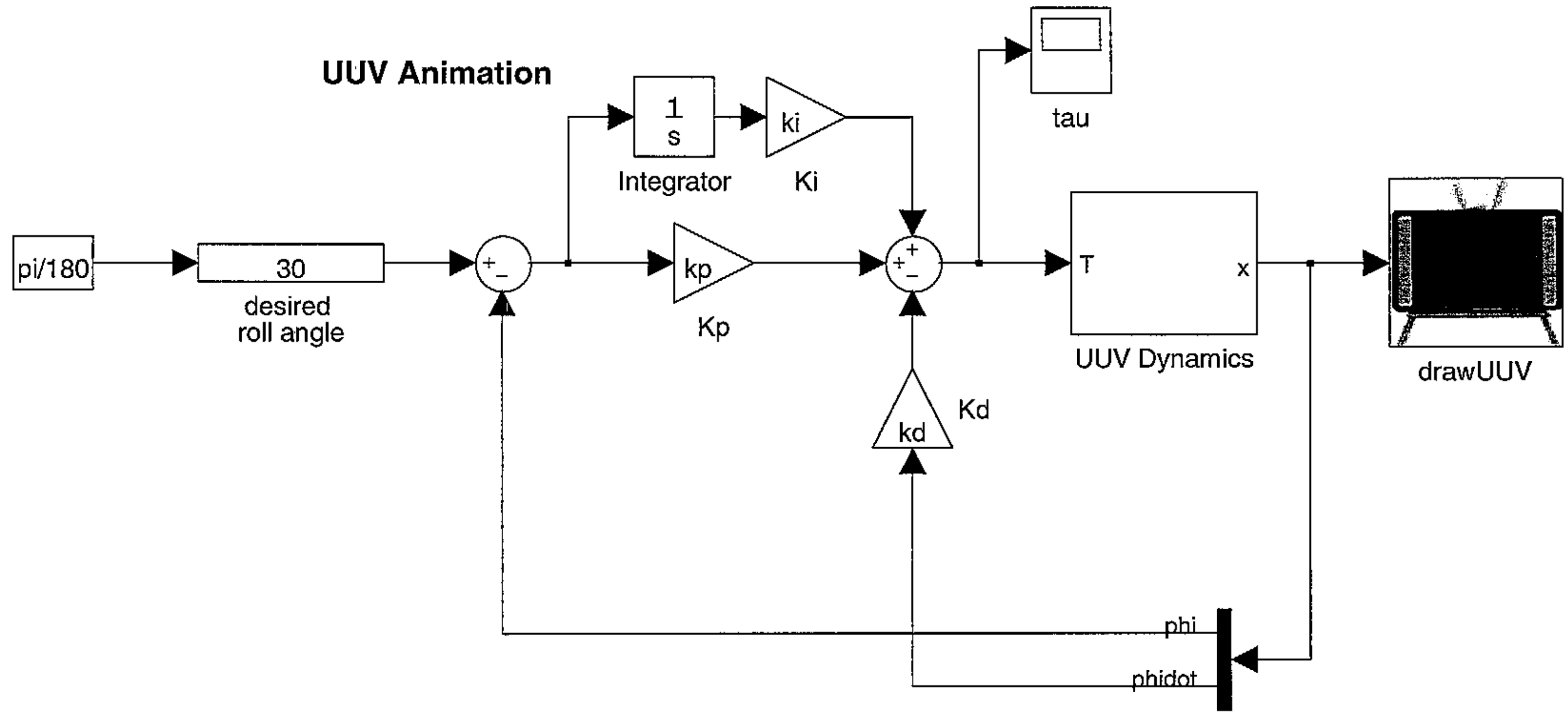
See Matlab results for root locus vs. k_I.

System transient response is not degraded by fairly large values of k_I. $\underline{k_I = 30}$ works well.

```
% simulation of UUV with lead control

J = 45;
m = 200;
g = 9.81;
L = 0.03;
b = 5;

kp = 95;
kd = 112;
ki = 30;

tfinal = 40;

% H is the TF taken from Evan's form for ki
numH = 1/J;
denH = [1 (b+kd)/J (m*g*L+kp)/J 0];
H = tf(numH,denH);

figure(1);
rlocus(H); hold on;
rlocus(H,ki,'b^'); hold off;
title('UUV roll control, root locus vs. ki');
axis([-15 5 -10 10]);

nCL = [kp/J ki/J];
dCL = [1 (b+kd)/J (m*g*L+kp)/J ki/J];
sysCL = tf(nCL,dCL);

figure(2);
[yCL,t,xCL] = step(sysCL,tfinal);
plot(t,yCL); grid;
title('UUV Roll Response, PID Control');
xlabel('time (sec)');
ylabel('roll angle (rad)');
```


UUV roll control, root locus vs. ki
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
10
8
6
4
2
0
−2
−4
−6
−8
−10
−15
−10
−5
0
5


UUV Roll Response, PID Control
roll angle (rad)
time (sec)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
5
10
15
20
25
30
35
40


UUV Animation
pi/180
30
desired
roll angle
1
s
Integrator
ki
Ki
kp
Kp
kd
Kd
tau
T
x
UUV Dynamics
drawUUV
phi
phidot

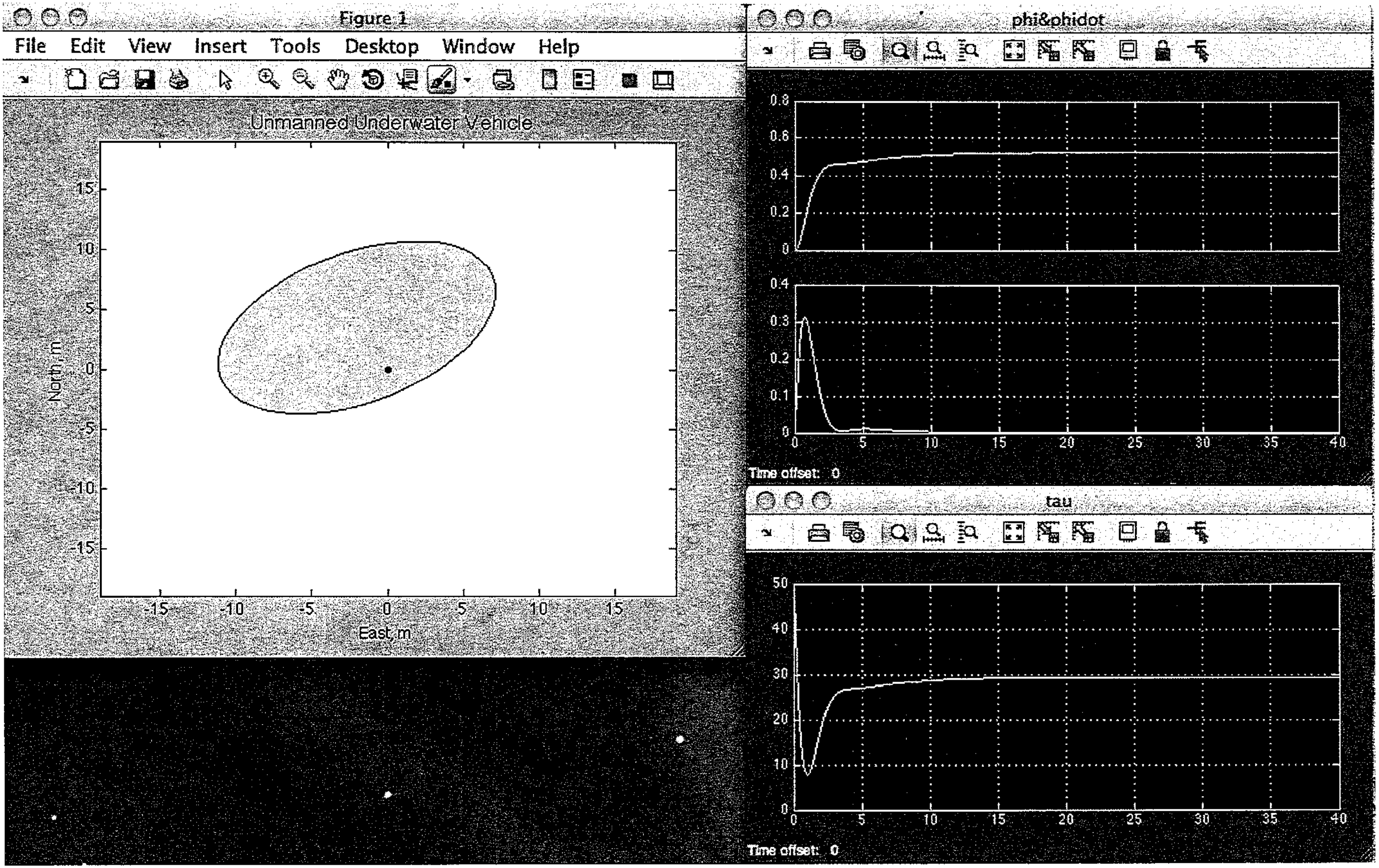


Figure 1
File Edit View Insert Tools Desktop Window Help
Unmanned Underwater Vehicle
North, m
East, m
-15 -10 -5 0 5 10 15
phi&phidot
0.8 0.6 0.4 0.2 0
0.4 0.3 0.2 0.1 0
0 5 10 15 20 25 30 35 40
Time offset: 0
tau
50 40 30 20 10 0
0 5 10 15 20 25 30 35 40
Time offset: 0