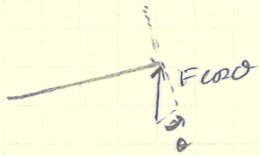
V.3 Ball on Beam

Generalized coordinates : y, θ

Generalized force for y : $\tau_1 = 0$

Generalized force for θ : $\tau_2 = \underbrace{l F \cos\theta}_{\text{torque}}$



Kinetic Energy :

$$K = \tfrac{1}{2} m_1 \dot{y}^2 + \tfrac{1}{2}\left(\frac{m_2 l^2}{3} + m_1 y^2\right)\dot{\theta}^2$$

Potential Energy

$$P = P_0 + m_1 g (y \sin\theta) + m_2 g \tfrac{l}{2} \sin\theta + \tfrac{1}{2} k y^2$$

Lagrangian

$$L = K - P = \tfrac{1}{2} m_1 \dot{y}^2 + \tfrac{1}{2}\left(\frac{m_2 l^2}{3} + m_1 y^2\right)\dot{\theta}^2 - P_0$$
$$- m_1 g y \sin\theta - m_2 g \frac{l}{2} \sin\theta - \tfrac{1}{2} k y^2$$

Euler Lagrange Equations are :

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = \tau_1$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \tau_2$$

where

$$\frac{\partial L}{\partial \dot{y}} = m_1 \dot{y}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) = m_1 \ddot{y}$$

IV.3 cont

$$\frac{\partial L}{\partial y} = m_1 y \dot{\theta}^2 - m_1 g \sin\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = \left(\frac{m_2 l^2}{3} + m_1 y^2\right)\dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \left(\frac{m_2 l^2}{3} + m_1 y^2\right)\ddot{\theta} + 2m_1 y\dot{y}\dot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -m_1 g y \cos\theta - m_2 g \frac{l}{2}\cos\theta$$

$\Rightarrow$ Equations of motion are

$$m_1\ddot{y} - m_1 y\dot{\theta}^2 + m_1 g\sin\theta = 0$$

$$\left(\frac{m_2 l^2}{3} + m_1 y^2\right)\ddot{\theta} + 2m_1 y\dot{y}\dot{\theta} + m_1 g y\cos\theta + m_2 g \tfrac{l}{2}\cos\theta = lF\cos\theta$$