V.6

a) Rearranging to get $\ddot{\tilde{y}}$ and $\ddot{\tilde{\theta}}$ alone gives

$$\ddot{\tilde{y}} = -g\tilde{\theta}$$

$$\ddot{\tilde{\theta}} = \left(\frac{\ell}{\frac{m_2\ell^2}{3} + m_1 y_c^2}\right)\tilde{F} - \left(\frac{m_1 g}{\frac{m_2\ell^2}{3} + m_1 y_c^2}\right)\tilde{y}$$

taking the Laplace transform gives.

$$s^2 Y(s) = -g\tilde{\theta}(s)$$

$$s^2\tilde{\theta}(s) = \left(\frac{\ell}{\frac{m_2\ell^2}{3} + m_1 y_c^2}\right)\tilde{F}(s) - \left(\frac{m_1 g}{\frac{m_2\ell^2}{3} + m_1 y_c^2}\right)\tilde{y}(s)$$

b) Solving for $Y(s)$ and $\tilde{\theta}(s)$ gives

$$Y(s) = \frac{-g}{s^2}\tilde{\theta}(s)$$

$$\tilde{\theta}(s) = \frac{\left(\frac{\ell}{\frac{m_2\ell^2}{3} + m_1 y_c^2}\right)}{s^2}F(s) - \underbrace{\frac{\left(\frac{m_1 g}{\frac{m_2\ell^2}{3} + m_1 y_c^2}\right)}{s^2}\tilde{y}(s)}_{\triangleq\, d(s)}$$

(c)

(d) Block diagram is