VI.8

(a) From VI.6 (b), neglecting m_2, l_2, b, and θ_2 dynamics,

$$\underbrace{\begin{bmatrix}(m_c+m)s^2 & mls^2 \\ s^2 & (ls^2+g)\end{bmatrix}}_{A}\begin{bmatrix}X \\ \Theta\end{bmatrix} = \begin{bmatrix}1 \\ 0\end{bmatrix}F$$

$$\begin{aligned}\det A &= (m_c+m)s^2(ls^2+g) - mls^4 \\ &= (m_c+\cancel{m})ls^4 + (m_c+m)gs^2 - \cancel{mls^4} \\ &= m_c l s^4 + (m_c+m)gs^2 \\ &= s^2\left[m_c l s^2 + (m_c+m)g\right]\end{aligned}$$

$$\frac{\Theta}{F} = \frac{1}{\det A}\cdot\det\begin{bmatrix}(m_c+m)s^2 & 1 \\ s^2 & 0\end{bmatrix}$$

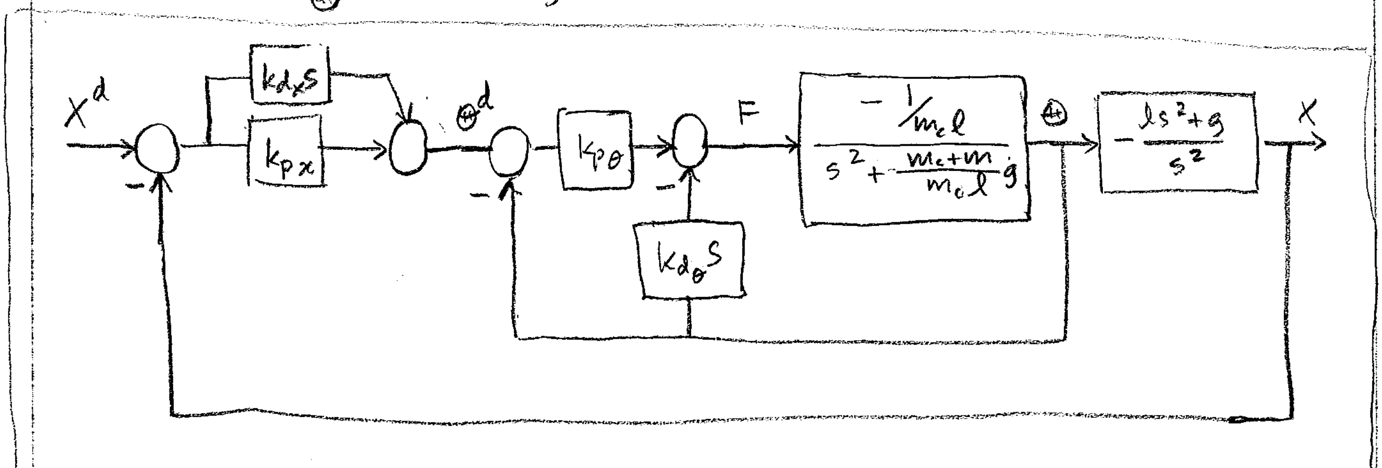
$$= \frac{-s^2}{s^2\left[m_c l s^2 + (m_c+m)g\right]}$$

$$\frac{\Theta}{F} = \frac{-1/m_c l}{s^2 + \frac{m_c+m}{m_c l}g}$$

From 2nd row of matrix eqn,

$$s^2 X = -(ls^2+g)\Theta$$

$$\frac{X}{\Theta} = -\frac{ls^2+g}{s^2}$$



(b) Inner loop TF :

$$\frac{\Theta}{\Theta^d} = \frac{k_{p\theta}\left(\frac{\Theta}{F}\right)}{1 + (k_{d\theta}s + k_{p\theta})\left(\frac{\Theta}{F}\right)}$$

$$= \frac{-k_{p\theta}/m_c l}{s^2 + \frac{m_c+m}{m_c l}g - \frac{1}{m_c l}(k_{d\theta}s + k_{p\theta})}$$

$$\boxed{\frac{\Theta}{\Theta^d} = \frac{-k_{p\theta}/m_c l}{s^2 - \frac{k_{d\theta}}{m_c l}s + \frac{(m_c+m)g - k_{p\theta}}{m_c l}}}$$

(c) $t_r = 1\,sec$, $M_p = 50\%$

$$\omega_{n_\theta} = \frac{1.8}{t_r} \Rightarrow \underline{\underline{\omega_{n\theta} = 1.8 \text{ rad/s}}}$$

$$M_p = 50\% \Rightarrow \underline{\underline{\zeta_\theta \approx 0.2}}$$

CLCE desired : $s^2 + 2\zeta_\theta\omega_{n_\theta} + \omega_{n_\theta}^2 = 0$

CLCE actual : $s^2 - \frac{k_{d\theta}}{m_c l}s + \frac{(m_c+m)g - k_{p\theta}}{m_c l} = 0$

Matching coeff's : $(m_c+m)g - k_{p\theta} = m_c l\,\omega_{n\theta}^2$

$$k_{p\theta} = (m_c+m)g - m_c l\,\omega_{n\theta}^2$$

$$\boxed{k_{p\theta} = -28{,}980}$$

$$k_{d\theta} = -2\zeta_\theta\omega_{n\theta} m_c l$$

$$\boxed{k_{d\theta} = -10{,}800}$$

(d) CL poles are roots of

$$s^2 + 2\zeta_\theta \omega_{n\theta} + \omega_{n\theta}^2 = 0$$

$$\Rightarrow \quad s^2 + 0.72s + 3.24 = 0$$

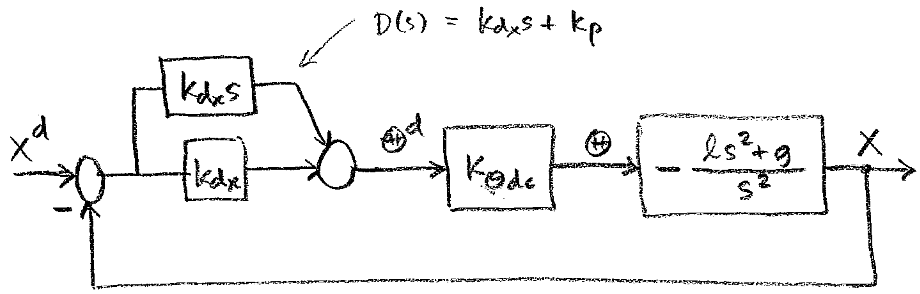
$$\boxed{s_{1,2} = -0.36 \pm 1.736j}$$

(e)

$$K_{\theta dc} = \frac{-k_{p\theta}}{(m_c + m)g - k_{p\theta}}$$

$$\boxed{K_{\theta dc} = 0.5963}$$

(f)



$$\frac{X}{X^d} = \frac{N_{DG}}{D_{DG} + N_{DG}}$$

$$= \frac{-K_{\theta dc}(k_{dx}s + k_{px})(ls^2 + g)}{s^2 - K_{\theta dc}(k_{dx}s + k_{px})(ls^2+g)}$$

$$\boxed{\frac{X}{X_d} = \frac{k_{dx}\left(s + \frac{k_{px}}{k_{dx}}\right)(ls^2+g)}{k_{dx}ls^3 + \left(k_{px}l - \frac{1}{K_{\theta dc}}\right)s^2 + k_{dx}gs + k_{px}g}}$$

(g) See Matlab,

$$D(s) = K(s+a) = k_{dx}\left(s + \frac{k_{px}}{k_{dx}}\right)$$

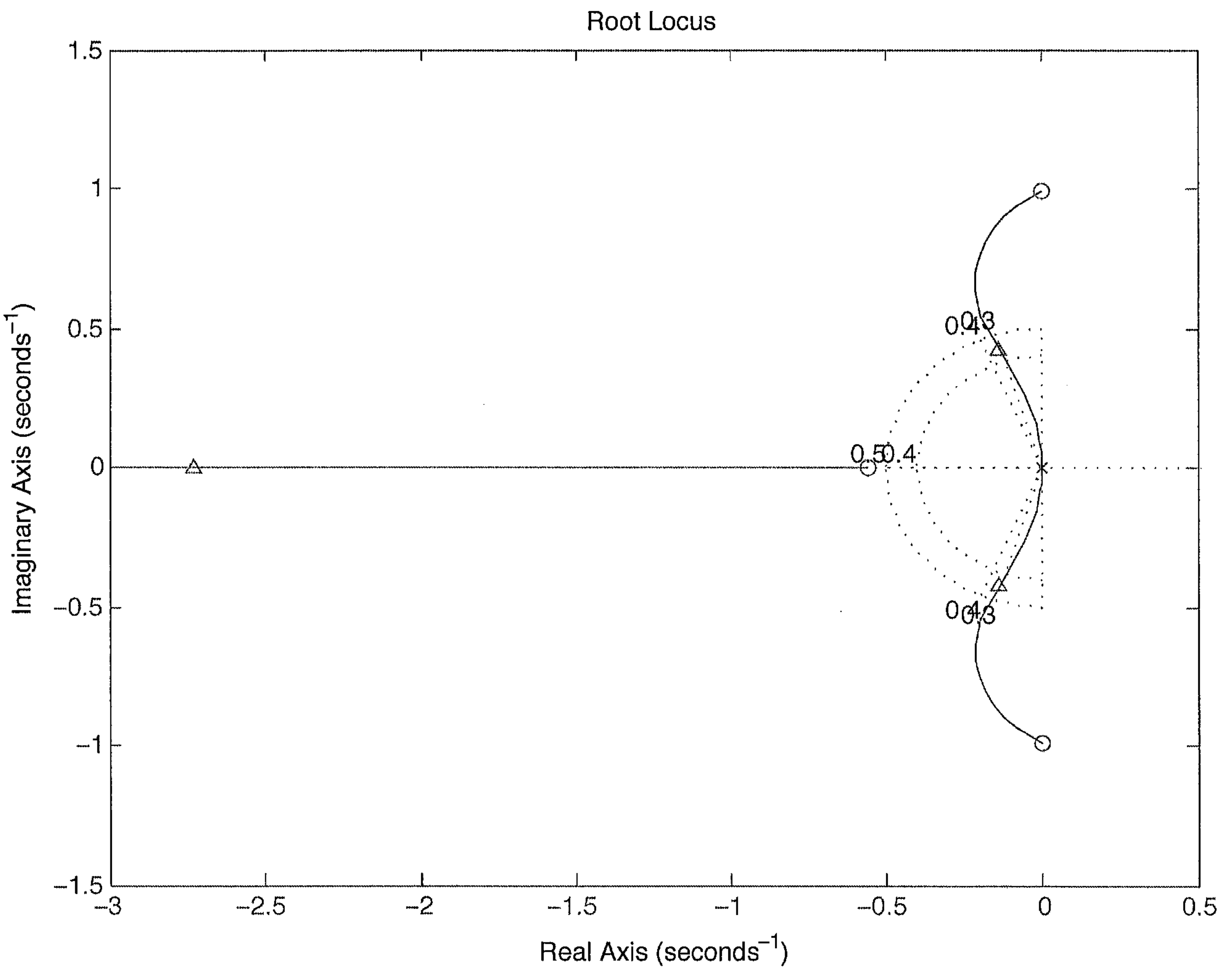
$$\Rightarrow \boxed{\begin{aligned} k_{dx} &= -0.0685 \\ k_{px} &= -0.0385 \end{aligned}}$$

(h) From Matlab,

$$\boxed{\begin{aligned} s_1 &= -2.7929 \\ s_{2,3} &= -0.1367 \pm j0.4455 \end{aligned}}$$

$$\omega_{n_x} = \sqrt{.1367^2 + .4455^2}$$

$$\boxed{\omega_{n_x} = 0.4660 \text{ rad/s}}$$


Root Locus
1.5
1
0.5
0
−0.5
−1
−1.5
−3
−2.5
−2
−1.5
−1
−0.5
0
0.5
Imaginary Axis (seconds⁻¹)
Real Axis (seconds⁻¹)
0.5
0.4
0.4
0.3
0.4
0.3