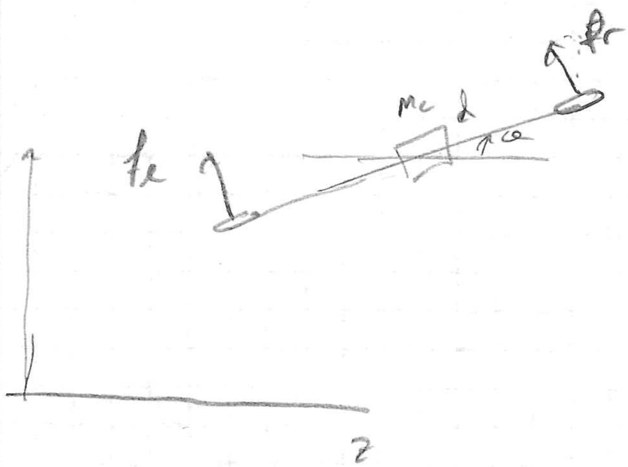
Q.3



Generalized coordinates : $z, h, \theta \Rightarrow q = \begin{pmatrix} z \\ h \\ \theta \end{pmatrix}$

Generalized forces on z : $-(f_r + f_e)\sin\theta$

on h : $(f_r + f_e)\cos\theta$

on θ : $d(f_r - f_e)$

$$\Rightarrow \tau = \begin{pmatrix} -(f_r + f_e)\sin\theta \\ (f_r + f_e)\cos\theta \\ d(f_r - f_e) \end{pmatrix}$$

Kinetic Energy :

$$K = \tfrac{1}{2}(m_c + 2m_r)\dot{z}^2 + \tfrac{1}{2}(m_c + 2m_r)\dot{h}^2 + \tfrac{1}{2}(J_c + 2m_r d^2)\dot{\theta}^2$$

Potential Energy :

$$P = m_c g h + m_r g(h + d\sin\theta) + m_e g(h - d\sin\theta) + P_0$$

$$= (m_c + 2m_r) g h + P_0$$

Lagrangian :

$$L = K - P = \tfrac{1}{2}(m_c + 2m_r)\dot{z}^2 + \tfrac{1}{2}(m_c + 2m_r)\dot{h}^2 + \tfrac{1}{2}(J_c + 2m_r d^2)\dot{\theta}^2$$

$$- (m_c + 2m_r) g h - P_0$$

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The Euler-Lagrange equations are

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) - \frac{\partial L}{\partial z} = \tau_1$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{h}}\right) - \frac{\partial L}{\partial h} = \tau_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \tau_3$$

where

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) = \frac{d}{dt}\left((m_c+2m_r)\dot{z}\right) = (m_c+2m_r)\ddot{z}$$

$$\frac{\partial L}{\partial z} = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{h}}\right) = \frac{d}{dt}\left((m_c+2m_r)\dot{h}\right) = (m_c+2m_r)\ddot{h}$$

$$\frac{\partial L}{\partial h} = -(m_c+2m_r)g$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{d}{dt}\left((J_c+2m_r d^2)\dot{\theta}\right) = (J_c+2m_r d^2)\ddot{\theta}$$

$\leftarrow \frac{\partial L}{\partial \theta} = 0$

$\therefore$ The Euler Lagrange equations give

$$(m_c+2m_r)\ddot{z} = -(f_r+f_e)\sin\theta$$

$$(m_c+2m_r)\ddot{h} + (m_c+2m_r)g = (f_r+f_e)\cos\theta$$

$$(J_c+2m_r d^2)\ddot{\theta} = d(f_r-f_e)$$

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or in Matrix form

$$\begin{pmatrix} (m_c+2m_r) & 0 & 0 \\ 0 & (m_c+2m_r) & 0 \\ 0 & 0 & (J_c+2m_r d^2) \end{pmatrix} \begin{pmatrix} \ddot{z} \\ \ddot{h} \\ \ddot{\theta} \end{pmatrix} = \begin{pmatrix} -(f_r+f_e)\sin\theta \\ -(m_c+2m_r)g + (f_r+f_e)\cos\theta \\ d(f_r-f_e \end{pmatrix}$$