Homework G.12 - Solution

G.12

a) Given $0 \le f_\ell \le f_{max}$, $0 \le f_r \le f_{max}$

Since

$$F = f_r + f_\ell$$

and $\tilde{F} = F - F_e$ we have

$$|\tilde{F}| \le |f_r + f_\ell| - |F_e|$$

$$\le 2f_{max} - (m_c + 2m_r)g$$

$$\Rightarrow \boxed{\tilde{F}_{max} = 2f_{max} - (m_c + 2m_r)g}$$

Since $\tau = \frac{f_\ell}{d} - \frac{f_r}{d}$

$$\tau_{max} = \frac{f_{max}}{d}$$

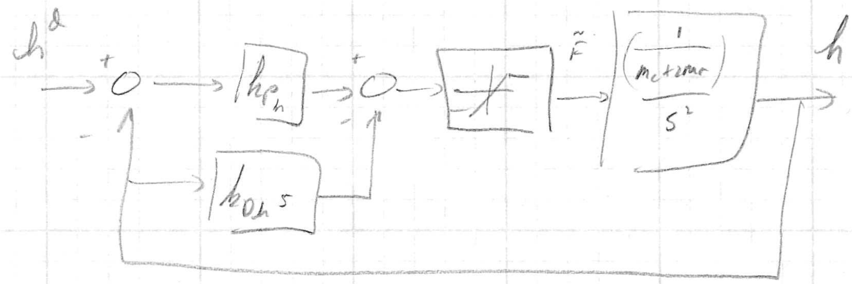
However since each motor must apply $F_e/2$ just to stay in the air the maximum force available to each motor for applied torque is

$$f_{max} - \frac{(m_c + 2m_r)g}{2}$$

$\therefore$ the maximum possible torque is

$$\boxed{\tau_{max} = \frac{f_{max} - \left(\frac{(m_c + 2m_r)g}{2}\right)}{d}}$$

6.12 b) A block diagram of the altitude loop w/ PD control is



When a step of A_h is applied to h^d, the force is $\tilde F = k_{P_h} A_h$

$\therefore$ let $\boxed{k_{P_h} = \frac{\tilde F_{max}}{A_h}}$

The closed-loop char polynomial is

$$\Delta_{cl}(s) = s^2 + \left(\frac{1}{m_c+2m_r}\right) k_{D_h} s + \left(\frac{1}{m_c+2m_r}\right) k_{P_h}$$

comparing to $\Delta_{cl}^d(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$

we have $\omega_{n_h} = \sqrt{\left(\frac{1}{m_c+2m_r}\right) k_{P_h}}$

and $\boxed{k_{D_h} = 2\zeta_h \omega_{n_h} (m_c + 2m_r)}$

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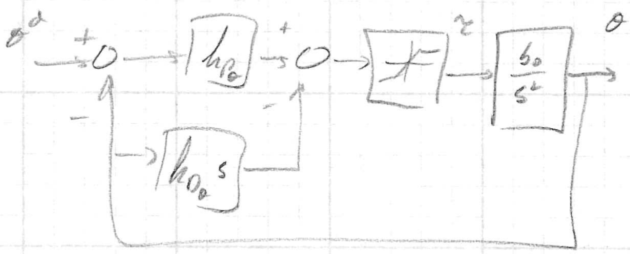
c) The block diagram of the lateral inner loop w/ PD control is



where $b_0 = \frac{1}{J_c + 2 m_r d^2}$

Just after a step of A_θ on θ^d the torque is

$$\tau = A_\theta k_{P_\theta}$$

$$\therefore \text{let } \boxed{k_{P_\theta} = \frac{\tau_{max}}{A_\theta}}$$

The closed loop char polynomial is

$$\Delta_{cl} = s^2 + b_0 k_{D_\theta} s + b_0 k_{P_\theta}$$

Comparing to

$$\Delta_{cl}^d = s^2 + 2\zeta_\theta \omega_{n_\theta} s + \omega_{n_\theta}^2$$

gives $\omega_{n_\theta} = \sqrt{b_0 k_{P_\theta}}$

$$\boxed{k_{D_\theta} = 2\zeta_\theta \omega_{n_\theta} / b_0}$$

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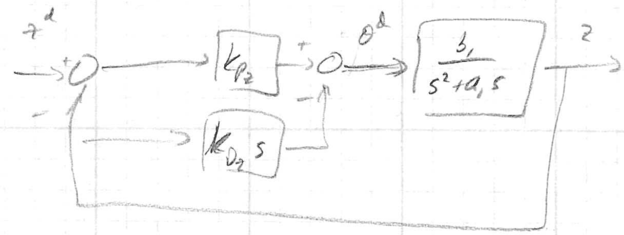
d) The DC gain of the inner loop is one

e) The block diagram of the outer loop w/ PD control is



where $a_1 = \frac{\mu}{m_c + 2m_r}$

$b_1 = \frac{-F_0}{m_c + 2m_r}$

Just after a step of A_z on z^d, the desired angle is

$$\theta^d = k_{P_z} A_z$$

$\therefore$ let $\boxed{k_{P_z} = \frac{A_\theta}{A_z}}$

The closed-loop char polynomial is

$$\Delta_{cl} = s^2 + (a_1 + b_1 k_{D_z}) s + b_1 k_{P_z}$$

Comparing to

$$\Delta_{cl}^d = s^2 + 2\zeta_z \omega_{n_z} s + \omega_{n_z}^2$$

gives $\omega_{n_z} = \sqrt{b_1 k_{P_z}}$

$$\boxed{k_{D_z} = \frac{2\zeta_z \omega_{n_z} - a_1}{b_1}}$$