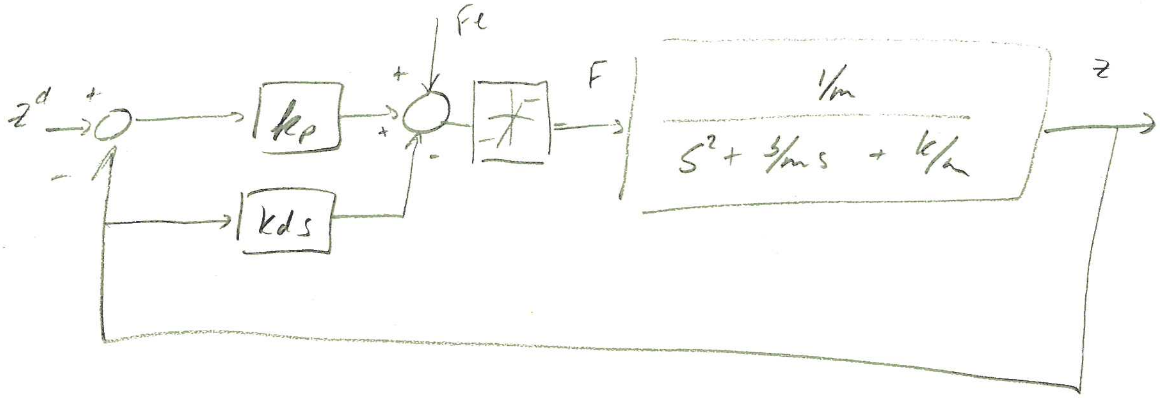
a)



$$Z = \left(\frac{1/m}{s^2 + \frac{b}{m}s + k/m}\right)\left[k_p(z^d - z) - k_d s z\right]$$

$$\Rightarrow \left(s^2 + \frac{b}{m}s + \frac{k}{m}\right)z = \frac{k}{m}z^d - \frac{k_p}{m}z - \frac{k_d}{m}sz$$

$$\Rightarrow \left(s^2 + \frac{b+k_d}{m}s + \frac{k+k_p}{m}\right)z = \frac{k}{m}z^d$$

$$\Rightarrow Z(s) = \boxed{\left[\frac{k/m}{s^2 + \left(\frac{b+k_d}{m}\right)s + \frac{k+k_p}{m}}\right]} z^d(s)$$

actual transfer function from Z^d to Z

Note that we can place the closed-loop poles anywhere using the gains k_p and k_d

b) From problem 5 the equilibrium force is given by

$F_e = k y_e$ where k is the spring constant.

Therefore the max equilibrium force is

$$F_{e,max} = k\, y_{e,max} = \left(3\,\frac{kg}{s^2}\right)(0.5\,m) = 1.5\,\frac{kg\,m}{s^2}$$

Therefore the maximum excess force is bounded by

$$|\tilde{F}| \leq F_{max} = 2N - F_{smax} = 0.5\,N$$

c) From the block diagram, when a step size of A is placed on z^d, the force right after the step is given by

$$\hat{F} = k_p A = F_{max}$$

$$\Rightarrow k_p = \frac{F_{max}}{A} = \frac{0.5}{1} = 0.5$$

d) If the desired transfer function is

$$Z(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} Z^d(s)$$

Then
$$2\xi\omega_n = \frac{b + k_d}{m}$$

$$\omega_n^2 = \frac{k + k_p}{m}$$

$$\therefore \omega_n = \sqrt{\frac{k + k_p}{m}} = \sqrt{\frac{3 + 0.5}{5}} = 0.8367$$

and
$$k_d = 2\xi\omega_n m - b = 2(0.707)(0.8367)(5) - 0.5$$

$$= 5.4161$$

e) The closed loop poles are $-0.5916 \pm j\,0.5916$