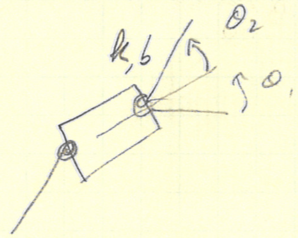
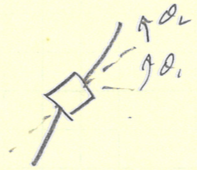
Case Study III — Satellite attitude control



$$P = \tfrac{1}{2} k \theta_2^2 + \tfrac{1}{2} k \theta_2^2 = k \theta_2^2$$

Case Study III Satellite Attitude Control



Previous Lectures

Generalized coordinates : θ_1, θ_2

Generalized forces on θ_1 : $\tau_1 = \tau - b(\dot\theta_1 - \dot\theta_2)$

Generalized forces on θ_2 : $\tau_2 = -b(\dot\theta_2 - \dot\theta_1)$

Kinetic energy :

$$K = \tfrac{1}{2}\left[J_1 + 2m_2 l_1^2 + 2m_2 l_2^2 + 2m_2 l_1 l_2 \cos\theta_2\right]\dot\theta_1^2$$
$$+\tfrac{1}{2}\left[2J_1 + 2m_2 l_2^2\right]\dot\theta_2^2$$
$$+\left[m_2 l_1 l_2 \cos\theta_2 + 2 l_2^2 m_2\right]\dot\theta_1 \dot\theta_2$$

Potential Energy :

$$P = k\,\theta_2^2$$

Lagrangian

$$L = K - P = \tfrac{1}{2}\left[J_1 + 2m_2 l_1^2 + 2m_2 l_2^2 + 2m_2 l_1 l_2 \cos\theta_2\right]\dot\theta_1^2$$
$$+\tfrac{1}{2}\left[2J_1 + 2m_1 l_1^2\right]\dot\theta_2^2$$
$$+\left[m_2 l_1 l_2 \cos\theta_2 + 2 l_2^2 m_2\right]\dot\theta_1\dot\theta_2 - k\,\theta_2^2$$

The Euler Lagrange equations are :

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta_1}\right) - \frac{\partial L}{\partial \theta_1} = \tau_1$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \theta_2}\right) - \frac{\partial L}{\partial \theta_2} = \tau_2$$

where

$$\frac{\partial L}{\partial \dot\theta_1} = [J_1 + 2m_2 l_1^2 + 2m_2 l_2^2 + 2m_2 l_1 l_2 \cos\theta_2]\,\dot\theta_1 + [m_2 l_1 l_2 \cos\theta_2 + 2 l_2^2 m_2]\,\dot\theta_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta_1}\right) = [J_1 + 2m_2 l_1^2 + 2m_2 l_2^2 + 2m_2 l_1 l_2 \cos\theta_2]\,\ddot\theta_1 - 2m_2 l_1 l_2 \sin\theta_2\, \dot\theta_1 \dot\theta_2 + [m_1 l_1 l_2 \cos\theta_2 + 2m_2 l_2^2]\,\ddot\theta_2 - m_1 l_1 l_2 \sin\theta_2\, \dot\theta_2^2$$

$$\frac{\partial L}{\partial \theta_1} = 0$$

$$\frac{\partial L}{\partial \dot\theta_2} = [2J_1 + 2m_2 l_1^2]\,\dot\theta_2 + [m_2 l_1 l_2 \cos\theta_2 + 2 l_2^2 m_2]\,\dot\theta_1$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta_2}\right) = [2J_1 + 2m_1 l_1^2]\,\ddot\theta_2 + [m_2 l_1 l_2 \cos\theta_2 + 2 l_2^2 m_2]\,\ddot\theta_1 - m_2 l_1 l_2 \sin\theta_2\, \dot\theta_1 \dot\theta_2$$

$$\frac{\partial L}{\partial \theta_2} = -m_2 l_1 l_2 \sin\theta_2\, \dot\theta_1^2 - m_2 l_1 l_2 \sin\theta_2\, \dot\theta_1 \dot\theta_2 - 2k\,\theta_2$$

Which gives

$$\begin{bmatrix} J_1 + 2m_2 l_1^2 + 2m_2 l_2^2 + 2m_2 l_1 l_2 \cos\theta_2 & m_2 l_1 l_2 \cos\theta_2 + 2m_2 l_2^2 \\ m_2 l_1 l_2 \cos\theta_2 + 2m_2 l_2^2 & 2J_1 + 2m_2 l_1^2 \end{bmatrix} \begin{pmatrix} \ddot\theta_1 \\ \ddot\theta_2 \end{pmatrix}$$

$$= \begin{bmatrix} +2m_2 l_1 l_2 \sin\theta_2\, \dot\theta_1 \dot\theta_2 + 2m_1 l_1 l_2 \sin\theta_2\, \dot\theta_2^2 + \tau - b(\dot\theta_1 - \dot\theta_2) \\ -m_2 l_1 l_2 \sin\theta_2\, \dot\theta_1^2 - 2k\theta_2 - b(\dot\theta_2 - \dot\theta_1) \end{bmatrix}$$