<u>Example</u> Quadrotor roll control:

Differential equation (after linearization) is

$$J\ddot{\phi} = \tau + w$$

neglected dynamics/wind/etc

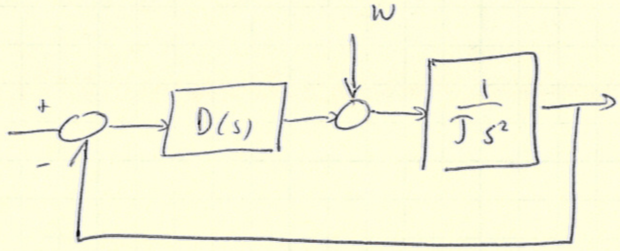
applied rolling torque

roll angle

rolling inertia

The transfer function is

$$\phi(s) = \frac{1}{Js^2}\left(\tau(s) + w\right)$$



Assuming stability,
what kind of disturbances can this system reject if

a) $D(s) = k_p + k_d s$

b) $D(s) = k_p + \frac{k_I}{s} + k_d s$?

$$e_{ss} = \lim_{s\to 0} \frac{-G}{1+GD}\frac{1}{s^q}$$

$$= \lim_{s\to 0} \frac{1/Js^2}{1+\frac{1}{Js^2}\left(k_p + k_d s + \frac{k_I}{s}\right)}\frac{1}{s^q}$$

$$= \lim_{s\to 0} \frac{1}{Js^2 + k_p + k_d s + \frac{k_I}{s}}\frac{1}{s^q}$$

$$= \lim_{s\to 0} \frac{s}{Js^3 + k_d s^2 + k_p s + k_I}\frac{1}{s^q}$$

a) If ~~the~~ $k_I = 0$ then

$$e_{ss} = \lim_{s \to 0} \frac{1}{Js^2 + k_d s + k_p} \frac{1}{s^q}$$

which is finite if $q = 0$. So the system is type 0 and the steady state error induced by a step disturbance is $\cancel{\frac{1}{k_p}}$ $e_{ss} = \frac{1}{k_p}$

b) If $k_I \neq 0$ then

$$e_{ss} = \lim_{s \to 0} \frac{s}{Js^3 + k_d s^2 + k_p s + k_I} \frac{1}{s^q}$$

which is zero if $q = 0$, and finite if $q = 1$.
$\therefore$ the system is type 1 and the steady state error induced by a ramp disturbance is $e_{ss} = \frac{1}{k_I}$.