ECEn 483 / ME 431

Practice Final - 2014

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Name: ______________________________

Closed book, three page of notes, Matlab/Similink/MS Word.
Work all problems. Unless directed otherwise, write solutions on this document.
Draw a box around your final answer.
Note that Alt-Printscreen copies the contents of the selected window to the clipboard.

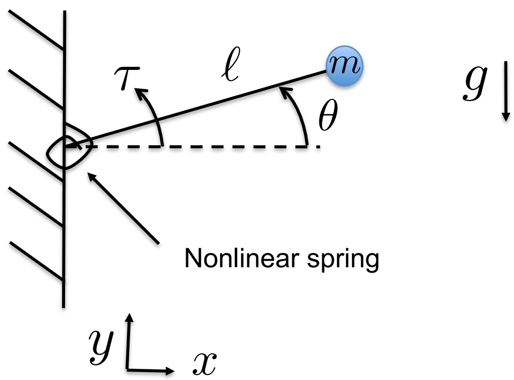
Note: In preparation for the final exam, you may use your own personal materials, materials from Learning Suite, and the wiki. You may collaborate with your classmates, but copying or using others' materials is strictly prohibited. You should not seek out solutions to this practice exam or other final exams on the Internet or from others who took this course in the past. *Violations of these terms will leave me with no other choice than to create an alternative final exam.*

Part 1. Equations of Motion - Simulation Model

The figure below shows a point mass connected to a massless rod which is connected to the wall with a nonlinear spring and damper. Applying Newton's second law yields the following equation of motion

$$m\ell^2\ddot{\theta} + b\dot{\theta} + k_1\theta + k_2\theta^3 + mg\ell\cos\theta = \tau(t).$$

The joint has friction which we will model as viscous friction with damping coefficient of $b = 0.1$ N-m-s. The physical parameters of the system are $g = 9.81$ m/s, $\ell = 0.25$ m, $m = 0.1$ kg, $k_1 = 0.02$ N-m, and $k_2 = 0.01$ N-m. The input torque τ is limited to ± 3.0 N-m.



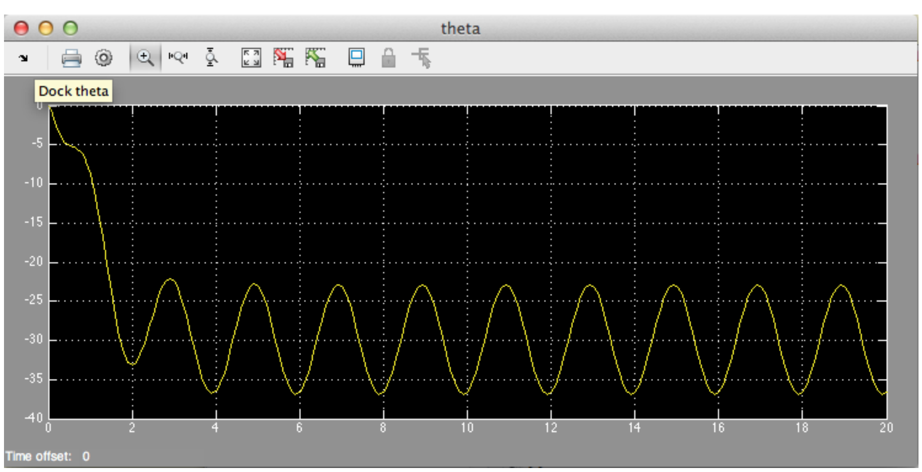
For this section, use the Simulink model `practice_final_part_1.mdl`.

1.1 Express the equation of motion above in state-variable form where the state vector is defined as $x = [\theta, \dot{\theta}]^\top$, the input is $u = \tau(t)$, and the output is defined as $y = \theta$.

1.2 Implement the nonlinear equations of motion in the `mdlDerivatives` function of the Simulink block `rodmass_dynamics.m`. (The functions provided are configured to accept a parameter variable `P` that you may define if you choose.)

1.3 Test your system by providing the system with the torque input $\tau(t) = 0.2 + 0.04\sin(\pi t)$ N-m. Give the states zero initial conditions and plot the response for $t = 0$ to $t = 20$ s. **Insert a plot of the output of the system in the associated Word document.**

For purposes of comparison, here is a plot of my results:



Part 2. Design Models

For this section, use the Simulink diagram `practice_final_part_2.mdl`.

The objective of this part is to use the equations of motion to find the appropriate design models that will be used to design the feedback control strategies.

2.1 Linearize the system around the equilibrium angle θ_e, which may or may not be zero. Find the associated equilibrium torque τ_e so that θ_e is an equilibrium of the system.

2.2 Open the Simulink file `practice_final_part_2.mdl` and insert a constant torque of τ_e on the physical system to verify that the equilibrium torque is correct, assuming that $\theta_e = 0$ degrees. **Insert a plot of the output of the system with initial condition $\theta(0) = \theta_e = 0$ (automatically set) and an input of τ_e in the associated Word document.**

2.3 Linearize the model found in Part 1 around the equilibrium (θ_e, τ_e).

2.4 Find the transfer function from the input τ to the output θ of the linearized model when $\theta_e = 0$.

2.5 Find a state-space model for the system linearized around θ_e, τ_e when $\theta_e = 0$.

Part 3. PID Control

For this section, use the Simulink diagram `practice_final_part_3.mdl` and the controller `ctrl_pid.m`, with sampling rate for the controller of $T_s = 0.01$.

3.1 Using the transfer function derived in Problem 2.4, draw the block diagram for the system using PD control, where the derivative gain multiples the angular rate and not the derivative of the error.

3.2 Derive the closed-loop transfer function from the reference input θ^c to the output angle θ.

3.3 Find the proportional gain k_p such that the control input τ saturates at $\tau_{\max} = 3$ N-m when a step of 20 degrees is placed on the system. *Watch your units – the equations of motion are based on angular units of radians.*

3.4 If the desired transfer function for the inner loop is given by

$$\Theta(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\Theta^c(s),$$

find the natural frequency ω_n and the derivative gain k_d so that the actual transfer function equals the desired transfer function, where $\zeta = 0.707$.

3.5 What are the closed-loop poles of system?

3.6 Using a dirty derivative with time constant $\tau = 0.005$ s, implement PD control in the control file `ctrl_pid.m` where the input θ^c is a square wave with an amplitude of ± 10 deg and a frequency of 0.1 Hz. **Insert a plot in the Word file that shows both θ^c and θ for 20 seconds of simulation.**

3.7 Give your system a disturbance torque input of 0.1 N-m and note the resulting steady-state error. Add an integrator to remove the steady-state error. **Insert a plot in the Word file that shows both θ^c and θ for 20 seconds of simulation.**

3.8 **Insert a copy of the Matlab code** for `ctrl_pid.m` in the Word document.

Part 4. Loopshaping

For this section, use the Simulink diagram `practice_final_part_4.mdl`.

4.1 Graph the Bode plot of the original open-loop system transfer function derived in Problem 2.4. For a cross-over frequency of $\omega_c = 30$ rad/s, what is the phase margin of the open-loop system?

4.2 With $D_{\text{lag}}(s) = 1$, design a lead compensator $D_{\text{lead}}(s)$ to give a phase margin corresponding to a damping ratio of 0.7. List the lead gain, the lead zero, and the lead pole below.

4.3 Implement the lead controller in Simulink and **insert a plot in the Word file that shows both θ^c and θ for 20 seconds of simulation** using the square-wave input of Problem 3.6. Don't forget to calculate and implement τ_e as a feedforward command.

4.4 Give your system a disturbance torque input of 0.1 N-m. Note the magnitude of the steady-state error. Design a lag compensator to complement your lead design to reduce the steady-state error to less than 0.3 deg. List the lag gain, the lag zero, and the lag pole below.

4.5 Implement the lead-lag controller in Simulink with the disturbance torque input acting and **insert a plot in the Word file that shows both θ^c and θ for 20 seconds of simulation.**

4.6 **Insert a graph in the Word file** that simultaneously show open-loop Bode plots for the original plant, the lead controlled plant, and the lead-lag controlled plant.

K_{lead} :

z_{lead} :

p_{lead} :

α_{lag} :

z_{lag} :

p_{lag} :

Part 5. Observer-based Control

For this section, use the Simulink diagram `practice_final_part_5.mdl` and the controller `ctrl_est.m`, with sampling rate for the controller of $T_s = 0.01$.

The objective of this part is to design a state feedback controller of the form

$$u = u_e - K(\hat{x} - N_x \theta^c)$$

to regulate the angle to a commanded input, and where $\hat{x}$ is produced by the observer

$$\dot{\hat{x}} = F(\hat{x} - x_e) + G(u - u_e) + L(y - H\hat{x}).$$

5.1 Find the feedback gain K that places the closed-loop poles at the locations $s = -25 \pm 15j$.

5.2 Find the reference state vector N_x.

5.3 Find the observer gains so that the poles of the observation error, i.e., the eigenvalues of $F - LH$, are five times faster than the eigenvalues of $F - GK$.

5.4 Implement the observer based control in `ctrl_est.m` in such a way that you can easily switch between using the real state (only possible in simulation) and the estimated state.

5.5 Insert a plot in the Word document of the square-wave input response of the system (θ and θ^c) when the true state is being used.

5.6 Insert a plot in the Word document of the state estimation error when the true state is being used verifying that the estimator errors go to zero as expected.

5.7 Insert a plot in the Word document of the step response of the system when the estimated state is being used in the controller.

5.8 Insert in the Word document a copy of `ctrl_est.m`.

Part 6. Root-Locus

For this section, use the Simulink diagram `practice_final_part_6.mdl`.

6.1 In response to a step, we want the closed-loop system to have a rise time less than 0.05 s and percent overshoot less than 10 percent. Plot the root locus of the original open-loop system transfer function derived in Problem 2.4 (no compensation) with acceptable regions of the s-plane for the system closed-loop poles defined.

6.2 With $D_{\text{lag}}(s) = 1$, design a lead compensator $D_{\text{lead}}(s)$ that satisfies the transient response specifications given. List the lead gain, the lead zero, and the lead pole below.

6.3 Implement the lead controller in Simulink and **insert a plot in the Word file that shows both θ^c and θ for 20 seconds of simulation** using the square-wave input of Problem 3.6. Don't forget to calculate and implement τ_e as a feedforward command.

6.4 For a disturbance torque input of 0.1 N-m, calculate the steady-state error in degrees. Show your work and list the steady-state error (in degrees) below. Design a lag compensator to complement your lead design to reduce the steady-state error to less than 0.5 deg. List the lag ratio (gain), the lag zero, and the lag pole below.

6.5 Implement the lead-lag controller in Simulink with the disturbance torque input acting and **insert a plot in the Word file that shows both θ^c and θ for 20 seconds of simulation.** From your plot, it should be clear that the steady-state error specification is met.

6.6 **Insert a graph in the Word file** that simultaneously shows your root locus design (lead and lag) with the closed-loop poles of the system clearly shown.

K_{lead} :

z_{lead} :

p_{lead} :

e_{ss} :

α_{lag} :

z_{lag} :

p_{lag} :