G.6 Soln

The linear dynamics are

$$(m_c+2m_r)\ddot{z} = -F_0\theta - \mu\dot{z}$$

$$(m_c+2m_r)\ddot{h} = \tilde{F}$$

$$(J_c+2m_rd^2)\ddot{\theta} = \tilde{\tau}$$

Taking the Laplace transform & setting initial conditions to zero gives

$$(m_c+2m_r)\,s^2 Z(s) = -F_0\,\tilde{\theta}(s) - \mu s Z(s) \qquad (1)$$

$$(m_c+2m_r)\,s^2 H(s) = \tilde{F}(s) \qquad (2)$$

$$(J_c+2m_rd^2)\,s^2\,\tilde{\theta}(s) = \tilde{\tau}(s) \qquad (3)$$

Solving for $H(s)$ in (2) gives

$$\boxed{H(s) = \frac{\left(\frac{1}{m_c+2m_r}\right)}{s^2}\tilde{F}(s)} \qquad (4)$$

these are the ~~tlog~~ linearized longitudinal dynamics.

Solving for $\Theta(s)$ in (3) gives

$$\boxed{\Theta(s) = \frac{\left(\frac{1}{J_c+2m_rd^2}\right)}{s^2}\tilde{\tau}(s)} \qquad (5)$$

Solving for $Z(s)$ in (1) gives

$$\left[(m_c+2m_r)s^2 + \mu s\right] Z(s) = -F_0\, \Theta(s) \qquad (6)$$

$$\Rightarrow \quad Z(s) = \frac{-F_0}{(m_c+2m_r)s^2+\mu s}\, \Theta(s) \qquad (7)$$

$$\Rightarrow \quad \boxed{Z(s) = \frac{-\left(\frac{F_0}{m_c+2m_r}\right)}{s^2+\left(\frac{\mu}{m_c+2m_r}\right)s}\, \Theta(s)} \qquad (8)$$

The Block diagram for the longitudinal motion is therefore

$$\tilde{F}(s) \longrightarrow \boxed{\frac{\left(\frac{1}{m_c+2m_r}\right)}{s^2}} \longrightarrow H(s)$$

and for the lateral motion it is

$$\tau(s) \longrightarrow \boxed{\frac{\left(\frac{1}{J_c+2m_r d^2}\right)}{s^2}} \xrightarrow{\Theta(s)} \boxed{\frac{-\left(\frac{F_0}{m_c+2m_r}\right)}{s^2+\left(\frac{\mu}{m_c+2m_r}\right)s}} \longrightarrow Z(s)$$