



PROJET DE FIN D'ÉTUDES (PFE)
INGÉNIERIE SYSTÈME : ROBOTIQUE ET SYSTÈMES
EMBARQUÉS

2014/2015

Local Dynamic Path Planning for an Autonomous Forklift in Human Environment

Unclassified Report
Can be made public on the internet

Author:

José Magno MENDES FILHO

Promotion 2014

Supervisor - ENSTA:

David FILLIAT

Supervisors - CEA:

Éric LUCET

Internship from 05 Mars 2015 to 28 August 2015

CEA LIST Digiteo Moulon
Bât. 660 91191 GIF-SUR-YVETTE Cedex, France

Acknowledgements

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Résumé

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Abstract

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Contents

I	Internship Description	3
1	Work Descriptpion	5
II	Internship Contribution	7
2	Global Near-optimal Solution for Path Planning	9
2.1	Description of the Problem	9
3	Algorithmic Approach	11
3.1	Representation of the optimization problem	12
3.1.1	Optimizers	12
3.1.2	The mobile robot	14
3.1.3	The obstacles	15
3.1.4	Analysis of real-time planning feasibility and total time performance	16
3.1.5	Computation time analysis	17
	Bibliography	25
A	Random Graphs	27

Part I

Internship Description

Chapter 1

Work Descriptpion

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Part II

Internship Contribution

Chapter 2

Global Near-optimal Solution for Path Planning

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

2.1 Description of the Problem

Chapter 3

Algorithmic Approach

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

3.1 Representation of the optimization problem

3.1.1 Optimizers

There is a variety of numerical optimization packages implemented in many different programming languages available for solving optimization problems [5]. Each of them may have their own way of defining the optimization problem and may or may not support specific kinds of constraints (equations, inequations or boundaries).

For the initial implementation written in python two packages stood out as good, easy-to-use options for solving the constrained optimization problem that models the planning motion task.

Scipy is a vast open-source scientific package based on python that happens to have a minimization module. Within this module many minimization methods can be found. For this specific optimization problem, only the method SLSPQ was appropriate. It was the only one to handle constrained minimization where the constraints could be equations as well as inequations.

pyOpt is a much smaller ecosystem than Scipy that is specialized in optimization. It gathers many different numerical optimization algorithms some of them free and some licensed. Again, among all of them there were only a few suitable for this problem which were also free: SLSQP (same as the one implemented within Scipy), PSQP and ALGENCAN.

SLSQP and PSQP are both SQP (for sequential quadratic programming) methods. A SQP method attempts to solve a nonlinearly constrained optimization problem where the object function and the constraints are twice continuously differentiable. It does so by modeling the object function ($\min f(x)$) at the current iterate x_k by a quadratic programming subproblem and using the minimizer of this subproblem to define a new iterate x_{k+1} [4].

The ALGENCAN method

describe algecan

$$\min_{(t_{final}, C_0, \dots, C_{d+n_{knot}-2})} J = (t_{final} - t_{initial})^2 \quad (3.1.1)$$

under the following constraints $\forall k \in \{0, \dots, N_s - 1\}$:

$$\begin{cases} \varphi_1(z(t_{initial}), \dots, z^{(l-1)}(t_{initial})) &= q_{initial} \\ \varphi_1(z(t_{final}), \dots, z^{(l-1)}(t_{final})) &= q_{final} \\ \varphi_2(z(t_{initial}), \dots, z^{(l)}(t_{initial})) &= u_{initial} \\ \varphi_2(z(t_{final}), \dots, z^{(l)}(t_{final})) &= u_{final} \\ \varphi_2(z(t_k), \dots, z^{(l)}(t_k)) &\in \mathcal{U} \\ d_{O_m}(t_k) &\geq \rho + r_m, \quad \forall O_m \in \mathcal{Q}_{occupied} \end{cases} \quad (3.1.2)$$

— Problem with discretization

Try adding CONSTRAINTS related to max acceleration (**DONE**)

For that we have to increase the maximum derivative order of the flat output needed so we calculate $[\dot{v} \ \dot{\omega}]$ building a φ_3 function

Also, the constraints to be added:

$$\varphi_3(z(t_k), \dots, z^{(l)}(t_k)) \in \mathcal{A}$$

where \mathcal{A} is the set of admissible acceleration values.

The function φ_3 is as follows:

$$\begin{aligned} \varphi_3(z(t_k), \dots, z^{(3)}(t_k)) &= \\ &= \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial t} \|\dot{z}\| \\ \frac{\partial}{\partial t} \frac{(\dot{z}_1 \ddot{z}_2 - \dot{z}_2 \ddot{z}_1)}{\|\dot{z}\|^2} \end{bmatrix} = \begin{bmatrix} \frac{\dot{z}_1 \ddot{z}_1 + \dot{z}_2 \ddot{z}_2}{\|\dot{z}\|} \\ \frac{(\ddot{z}_1 \ddot{z}_2 + z_2^{(3)} \dot{z}_1 - (\ddot{z}_2 \ddot{z}_1 + z_1^{(3)} \dot{z}_2)) \|\dot{z}\|^2 - 2(\dot{z}_1 \ddot{z}_2 - \dot{z}_2 \ddot{z}_1) \|\dot{z}\| \dot{v}}{\|\dot{z}\|^4} \end{bmatrix} \end{aligned}$$

— Remake code using good objected oriented structure. It will be good for C++ part (**DONE**)

ONLINE T_c and T_p (planning horizon) "given" (arbitrary).

$$\tau_k = t_{initial} + kT_c \quad k \in \mathbb{N}$$

Arbitrary detection radius for the robot sensors. Only if the obstacle characteristic position is inside the detection zone the obstacle is considered detected. Using $2m$.

Evaluate for each time interval $[\tau_{k-1}, \tau_k)(k \in \mathbb{N})$ the trajectory beginning at τ_k until $\tau_k + T_p$:

$$\min_{(C_{(0, \tau_k)}, \dots, C_{(d+n_{knot}-2, \tau_k)})} J_{\tau_k} = \|\varphi_1(z(\tau_k + T_p, \tau_k), \dots, z^{(l-1)}(\tau_k + T_p, \tau_k)) - q_{final}\|^2 \quad (3.1.3)$$

under the following constraints $\forall t \in [\tau_k, \tau_k + T_p]$:

$$\begin{cases} \varphi_1(z(\tau_k, \tau_k), \dots, z^{(l-1)}(\tau_k, \tau_k)) &= q_{ref}(\tau_k, \tau_{k-1}) \\ \varphi_2(z(\tau_k, \tau_k), \dots, z^{(l)}(\tau_k, \tau_k)) &= u_{ref}(\tau_k, \tau_{k-1}) \\ \varphi_2(z(t, \tau_k), \dots, z^{(l)}(t, \tau_k)) &\in \mathcal{U} \\ d_{O_m}(t, \tau_k) &\geq \rho + r_m, \quad \forall O_m \in \mathcal{O}(\tau_k) \end{cases} \quad (3.1.4)$$

The period $[\tau_{-1}, \tau_0)$ is what is called by Defoort "the initialization phase" which con-

siders:

$$q_{ref}(\tau_0, \tau_{-1}) = q_{initial}$$

$$u_{ref}(\tau_0, \tau_{-1}) = u_{initial}$$

without no more further changes to the expressions above.

Practical stuff for implementation $q \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$. N_s number of time steps used when computing the problem.

Number of equations: $n + m$

Number of inequations (function of τ_k): $N_s(m + \text{card}(\mathcal{O}(\tau_k)))$

dependencies: `sudo apt-get install python python-dev libatlas-base-dev gcc gfortran g++`

get source: <https://pypi.python.org/pypi/scipy>

`sudo python setup.py install`

3.1.2 The mobile robot

For representing the mobile robot geometry in the planning plane a bounding circle was chosen.

Unicycle kinetic model

Flat output formulation

3.1.3 The obstacles

Two different representations of an obstacle are supported. Obstacles can be seen as circles or convex polygons.

Representing an obstacle as a circle is probably the most simple way of doing so and has great advantages when calculating point-to-obstacle distance compared to other representations.

Nevertheless, obstacles such as walls, boxes and shelves cannot be satisfactorily represented by circles. Thus the need of a polygon representation.

Robot-to-obstacle distance calculation for the convex polygon representation

As said before the robot's geometric form is represented by a circle. When calculating the robot-to-obstacle distance this simplified representation is quite useful. The first approach to calculate the distance between a point and an obstacle represented by a convex polygon was to separate the problem in three cases with a different expression for the distance computation each. We see in the figure 3.1 that the points A , B and C are placed in three different regions with respect to the obstacle. A is "between" the two lines ($r_{0,1}$ and $r_{0,3}$) that pass through the vertex 0 and are orthogonal to the two adjacent edges. B is "between" the edge s_3 , and the orthogonal lines $r_{0,3}$ and $r_{3,2}$. C is in the interior of the obstacle representation, i.e., surrounded by the four edges.

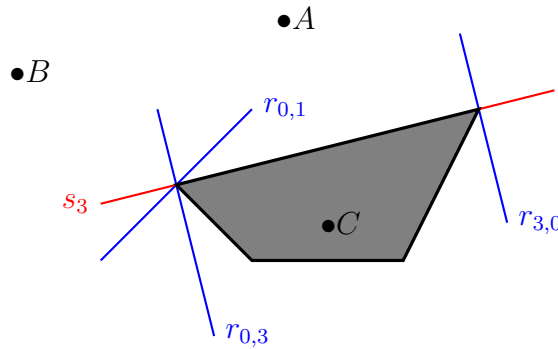


Figure 3.1 – Voronoi regions used for case differentiation.

These cases make use of Voronoi regions [3]. Based on the three types of regions (here we consider the interior of the polygon as a third one) a case differentiation is made and, depending on the case, equations are solved.

It is easy to see that the computation of the point-to-obstacle distance for A is a simple point-to-point distance using the appropriate vertex. For B a point-to-line distance equation can be used. Finally, since C is in the interior of the polygon the penetration distance is calculated. It is considered as the shortest of the four distances from the point C to the four edges multiplied by -1 (so, once more, point-to-line distance).

Of course that the performance of this approach is "number of edges"-dependent and present fast results only for polygons with few edges (less than 10).

10 was arbitrary, improve this finding a meaningful value or delete it

An important remark though is that for a given planning horizon N_s point-to-obstacle distances have to be calculated. Intuitively we can say that there is a high probability that most of the N_s points are inside the same region defined by their relative positions to the obstacle. Besides, the probability of finding points inside regions that are "far" from the already occupied zones is smaller. This heuristic can be used to speed up the planning process by having a smarter initialization of point-to-obstacle distance computation when using a convex polygon representation.

Finally, when dealing with more complex obstacles representations and/or with a more complex representation of the mobile robot geometry the Enhanced Gilbert-Johnson-Keerthi distance algorithm [3] is a more suitable and efficient approach.

some code is available on the internet, Google code written in D language and/or the other one on stackverflow, see bookmarks

3.1.4 Analysis of real-time planning feasibility and total time performance

The performance of the motion planning algorithm previously presented depends on several parameters. For starters these parameters can be split into two groups. The **algorithm related** parameters and the **optimization solver related** ones. Among the former group the most important ones are: the number of sample for time discretization (N_s), the number of internal knots for the B-splines curves (n_{knots}), and the planning and computation time horizons (T_p and T_c respectively). The latter kind depends on the optimization solver adopted but since most of them are iterative methods is common to have at least a "maximum number of iterations" and a "stop condition" parameters.

The task of searching for a satisfactory set of parameters' values with regard to a performance metric (e.g. total time to complete the mission) is quite laborious.

We attempt nevertheless to extract some quantitative knowledge about how these parameters impact the generated solution based on several simulations run with different parameters configurations. The main objective here is to be able to support the feasibility of a real-time motion planner based on this algorithm.

Aiming for a scenario invariant understanding of the impact of these parameters three different scenarios were studied. A first scenario where the robot did not had to avoid any obstacle to complete its mission, a second one where three round obstacles were randomly generated in a region where the robot was probably going to pass through and a third similar to the second only with six instead of three obstacles.

In the other hand, to reduce the problem's size, a unique optimization solver with fixed parameters was used for all simulations. The used parameters can be seen in table 3.1. Different maximum numbers of iterations are used for different stages of the planning process. The subscribed words *first*, *inter* and *last* indicate that the respective maximum numbers of iteration are used for the first optimization problem solving, for all intermediaries ones and for the last one.

talk about accuracy

Table 3.1 – Optimization solver parameters

Optimization solver type	SLSQP
$MAXIT_{first}$	40
$MAXIT_{inter}$	15
$MAXIT_{last}$	20
accuracy	10^{-3}

Real-time feasibility in this context can be considered as having the *maximum computational time* spend for planning the path sections¹ less than or equal to the computation horizon (T_c). Here though we are only interest in understanding the variation of *maximum computational time*/ T_c with changes on T_c , T_p , N_s , N_{knots} .

Another natural performance metric that should be kept in mind is the total time spend to complete the mission (going from the initial configuration to the final).

After this analysis we shall be able to identify sets of parameters' values that minimize the total time spend to complete the mission, respecting the problem constraints and minimizing the *maximum computational time*/ T_c ratio.

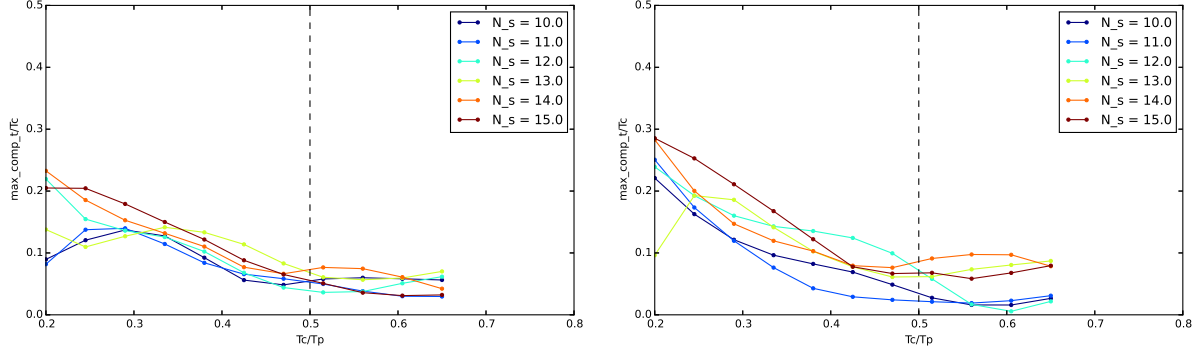
3.1.5 Computation time analysis

No obstacles scenario The images in the figure 3.2 try to make prominent the effect of changes in the in the number of samples (N_s) and number of internal knots (N_{knots}). In the ordinate axis we have the *maximum computational time*/ T_c ratio and in the abscissa we have the T_c/T_p . For each N_s we took the average of the *maximum computational time*/ T_c ratio for a give T_c/T_p among different T_p values in order to be T_p invariant.

We can see that for a "no obstacles" scenario the overall performance with respect to the *maximum computational time*/ T_c ratio is only slightly impacted by variations in the number of samples (N_s) and in the number of internal knots (N_{knots}). Within a given image the lines are close together showing that variations in N_s have weak impact. In addition,

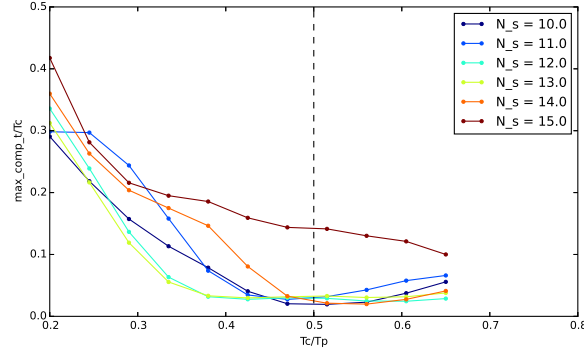
1. All computational times spend for planning all sections are considered for finding the maximum value but the first one

comparing the three images (3.2a, 3.2b, 3.2c) we see that variations in the N_{knots} have also a weak influence.



(a) Four internal knots. Average variance between lines is 0.098×10^{-2}

(b) Five internal knots. Average variance between lines is 0.205×10^{-2}



(c) Six internal knots. Average variance between lines is 0.295×10^{-2}

Figure 3.2 – Zero obstacles scenario.

For the sake of an example we present a simulation result in the figure 3.3 run with the parameters presented in the table 3.2 for a "no obstacles" scenario.

Table 3.2 – Motion planner main parameters

T_p	2.00 s
T_c	0.40 s
N_s	9
N_{knots}	5
v_{max}	1.00 m/s
ω_{max}	5.00 rad/s
$q_{initial}$	$[-0.05 \ 0.00 \ \pi/2]^T$
q_{final}	$[0.10 \ 7.00 \ \pi/2]^T$
u_{final}	$[0.00 \ 0.00]^T$
\dot{u}_{final}	$[0.00 \ 0.00]^T$

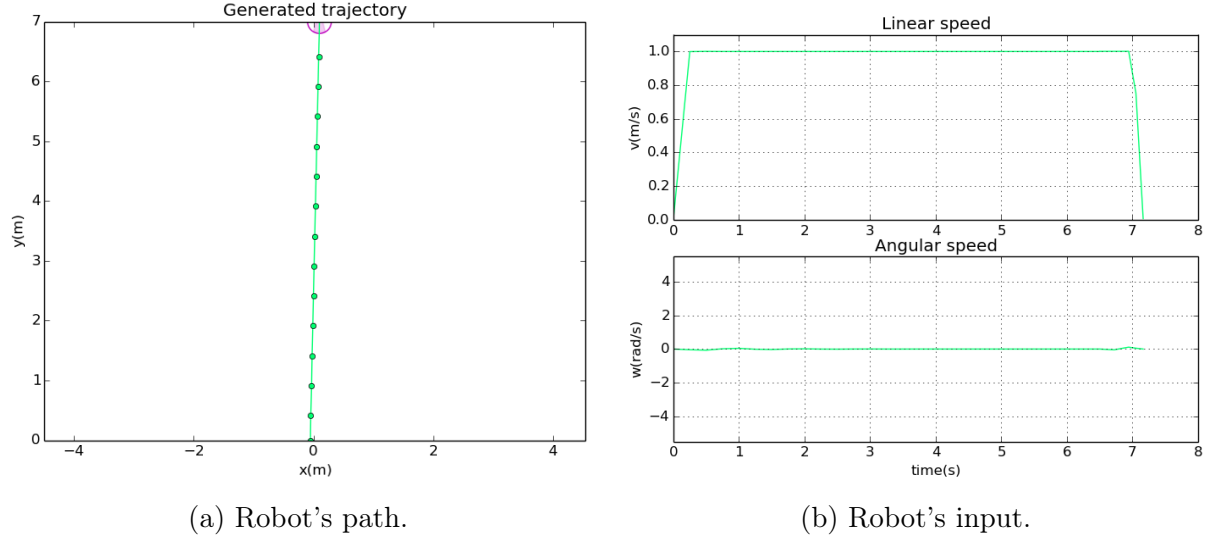


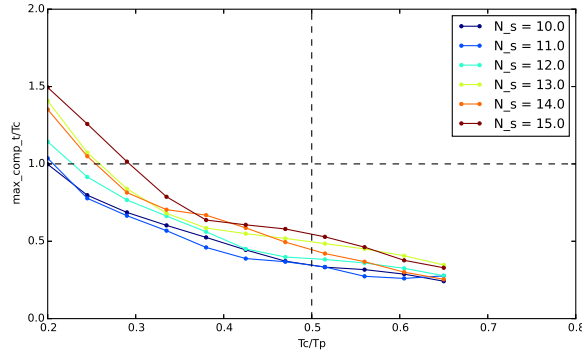
Figure 3.3 – No obstacle scenario simulation example where the *maximum computational time* was about 78% of T_c and the mission total time equals to 7.16 s.

Three obstacles scenario For this new scenario, a greater impact of the number of samples (N_s) and number of non-null internal knots (N_{knots}) is observed. The greater the N_{knots} or the N_s the greater is the *maximum computational time*/ T_c . This behavior is the one expected since the number of constraints and the number of arguments for the cost function to be minimized depend on these two parameters respectively.

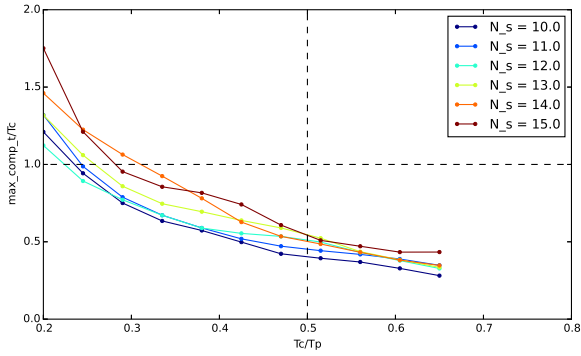
Again, in the figure 3.3 we show a simulation example run with the parameters' values presented in table 3.4.

Table 3.3 – Motion planner main parameters

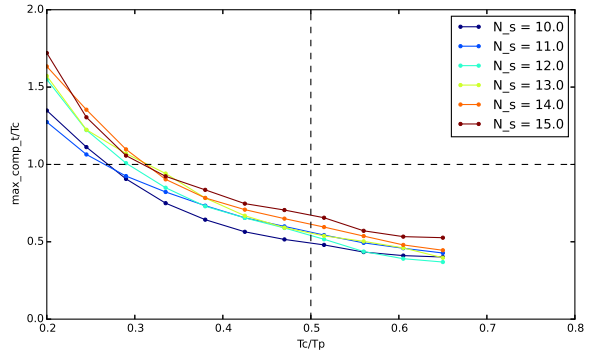
T_p	2.40 s
T_c	0.48 s
N_s	11
N_{knots}	4
v_{max}	1.00 m/s
ω_{max}	5.00 rad/s
$q_{initial}$	$[-0.05 \ 0.00 \ \pi/2]^T$
q_{final}	$[0.10 \ 7.00 \ \pi/2]^T$
u_{final}	$[0.00 \ 0.00]^T$
u_{final}	$[0.00 \ 0.00]^T$
O_0	$[0.55 \ 1.91 \ 0.31]$
O_1	$[-0.08 \ 3.65 \ 0.32]$
O_2	$[0.38 \ 4.65 \ 0.16]$



(a) Four internal knots. Average variance between lines is 1.65×10^{-2}

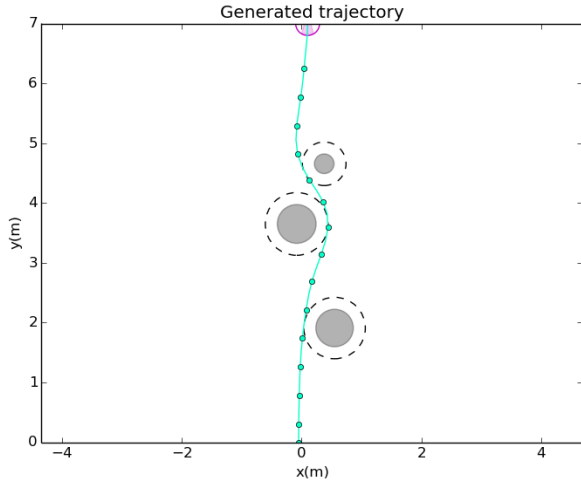


(b) Five internal knots. Average variance between lines is 2.40×10^{-2}

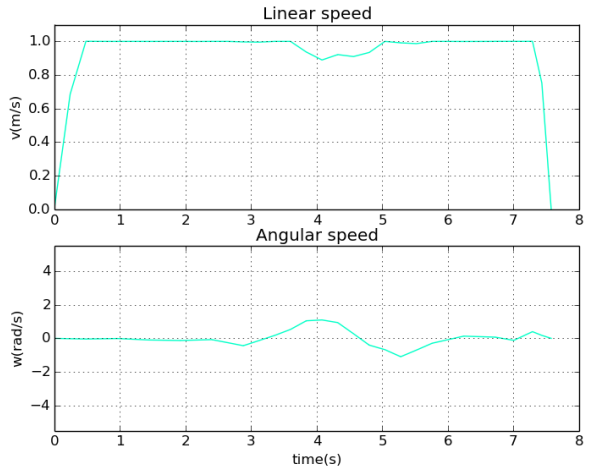


(c) Six internal knots. Average variance between lines is 2.74×10^{-2}

Figure 3.4 – Three obstacles scenario.



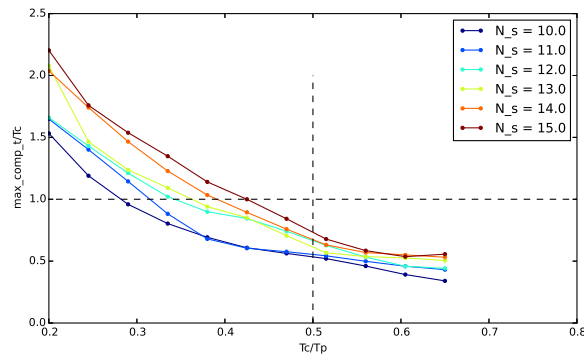
(a) Robot's path.



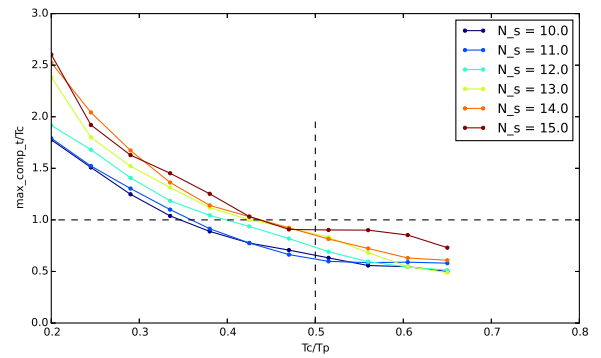
(b) Robot's input.

Figure 3.5 – Three obstacle scenario simulation example where the *maximum computational time* was about 84% of T_c and the mission total time equals to 7.57 s.

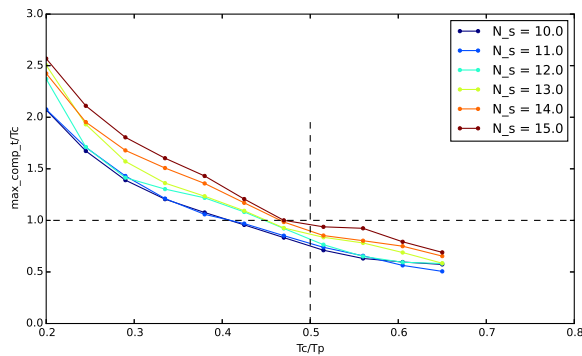
Six obstacles scenario As for the scenario with six obstacles we realize that the observations for the latest scenario are accentuated.



(a) Four internal knots. Average variance between lines is 2.92×10^{-2}



(b) Five internal knots. Average variance between lines is 5.78×10^{-2}

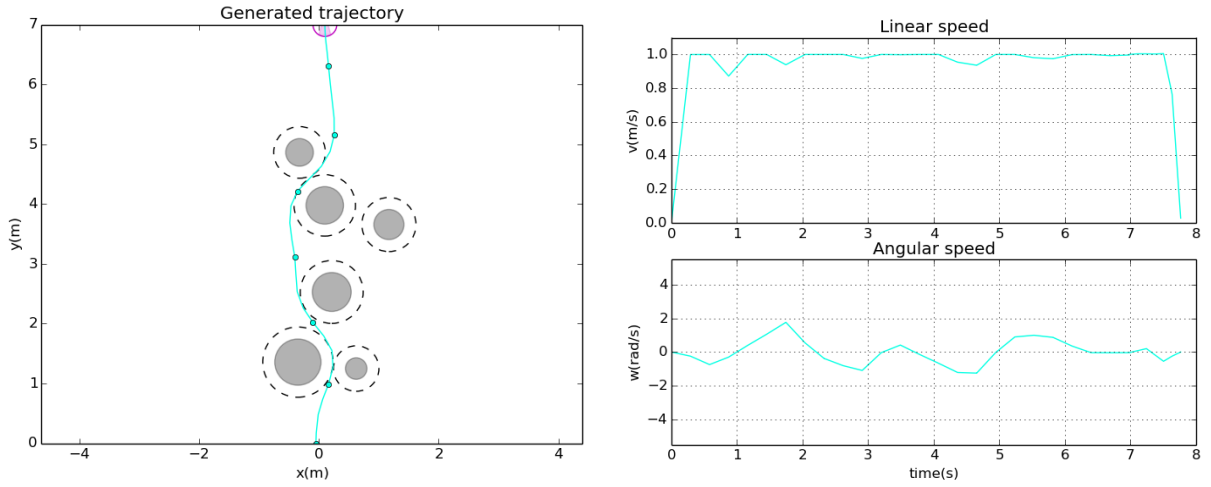


(c) Six internal knots. Average variance between lines is 5.89×10^{-2}

Figure 3.6 – Six obstacles scenario.

Table 3.4 – Motion planner main parameters

T_p	3.20 s
T_c	1.28 s
N_s	12
N_{knots}	6
v_{max}	1.00 m/s
ω_{max}	5.00 rad/s
$q_{initial}$	$[-0.05 \ 0.00 \ \pi/2]^T$
q_{final}	$[0.10 \ 7.00 \ \pi/2]^T$
u_{final}	$[0.00 \ 0.00]^T$
u_{final}	$[0.00 \ 0.00]^T$
O_0	$[-0.35 \ 1.36 \ 0.39]$
O_1	$[0.21 \ 2.53 \ 0.33]$
O_2	$[-0.32 \ 4.86 \ 0.23]$
O_3	$[0.10 \ 3.98 \ 0.31]$
O_4	$[0.62 \ 1.25 \ 0.18]$
O_5	$[1.17 \ 3.66 \ 0.25]$



(a) Robot's path.

(b) Robot's input.

 Figure 3.7 – Three obstacle scenario simulation example where the *maximum computational time* was about 90% of T_c and the mission total time equals to 7.76 s.

Total time analysis

The time spend for finding the solution for a given set of parameters values does not impact the solution it self. Let's analyse than how the solution behaves for different scenarios.

Intuitively we can say that the greater the number

— Distance inter robots

- Min dist to obstacles

Bibliography

- [1] M. Defoort, J. Palos, a. Kokosy, T. Floquet, W. Perruquetti, and D. Boulinguez. Experimental Motion Planning and Control for an Autonomous Nonholonomic Mobile Robot. *Proceedings 2007 IEEE International Conference on Robotics and Automation*, (April):10–14, 2007.
- [2] Michael Defoort. Contributions à la planification et à la commande pour les robots mobiles coopératifs. *Ecole Centrale de Lille*, 2007.
- [3] C. Ericson. *Real-Time Collision Detection*. M038/the Morgan Kaufmann Ser. in Interactive 3D Technology Series. Taylor & Francis, 2004.
- [4] Jorge Nocedal and Steve J. Wright. *Numerical optimization : with 85 illustrations*. Springer series in operations research. Springer, New York, Berlin, Heidelberg, 1999. Tirage corrigé: 2000.
- [5] Ruben E. Perez, Peter W. Jansen, and Joaquim R. R. A. Martins. pyOpt: A Python-based object-oriented framework for nonlinear constrained optimization. *Structures and Multidisciplinary Optimization*, 45(1):101–118, 2012.

Appendix A

Random Graphs

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetur adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetur.