PROJET DE FIN D'ÉTUDES (PFE)

INGÉNIERIE SYSTÈME : ROBOTIQUE ET SYSTÈMES EMBARQUÉS

2014/2015

Local Dynamic Path Planning for an Autonomous Forklift in Human Environment

Unclassified Report

Can be made public on the internet

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Promotion 2014

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Internship from 05 Mars 2015 to 28 August 2015

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Acknowledgements

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Résumé

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Abstract

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Work Descritpion

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Part II

Internship Contribution

Chapter 2

Global Near-optimal Solution for Path Planning

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2.1 Description of the Problem

Chapter 3

Algorithmic Approach

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3.1 Representation of the optimization problem

3.1.1 Optimizers

There is a variety of numerical optimization packages implemented in many different programming languages available for solving optimization problems [?]. Each of them may have their own way of defining the optimization problem and may or may not support specific kinds of constraints (equations, inequations or boundaries).

For the initial implementation written in python two packages stood out as good, easy-to-use options for solving the constrained optimization problem that models the planning motion task.

Scipy is a vast open-source scientific package based on python that happens to have a minimization module. Within this module many minimization methods can be found. For this specific optimization problem, only the method SLSPQ was appropriate. It was the only one to handle constrained minimization where the constraints could be equations as well as inequations.

pyOpt is a much smaller ecosystem than Scipy that is specialized in optimization. It gathers many different numerical optimization algorithms some of them free and some licensed. Again, among all of them there were only a few suitable for this problem which were also free: SLSQP (same as the one implemented within Sicpy), PSQP and ALGENCAN.

SLSQP and PSQP are both SQP (for sequential quadratic programming) methods. A SQP method attempts to solve a nonlinearly constrained optimization problem where the object function and the constraints are twice continuously differentiable. It does so by modeling the object function ($\min f(x)$) at the current iterate x_k by a quadratic programming subproblem and using the minimizer of this subproblem to define a new iterate x_{k+1} [?].

The ALGENCAN method

describe algecan

$$\min_{(t_{final}, C_0, \dots, C_{d+n_{knot}-2})} J = (t_{final} - t_{initial})^2 \tag{3.1.1}$$

under the following constraints $\forall k \in \{0, \dots, N_s - 1\}$:

$$\begin{cases} \varphi_1(z(t_{initial}), \dots, z^{(l-1)}(t_{initial})) & = & q_{initial} \\ \varphi_1(z(t_{final}), \dots, z^{(l-1)}(t_{final})) & = & q_{final} \\ \varphi_2(z(t_{initial}), \dots, z^{(l)}(t_{initial})) & = & u_{initial} \\ \varphi_2(z(t_{final}), \dots, z^{(l)}(t_{final})) & = & u_{final} \\ \varphi_2(z(t_k), \dots, z^{(l)}(t_k)) & \in & \mathcal{U} \\ d_{O_m}(t_k) & \geq & \rho + r_m, \quad \forall O_m \in \mathcal{Q}_{occupied} \end{cases} \tag{3.1.2}$$

— Problem with discretization
  Try adding CONSTRAINTS related to max acceleration (DONE)
  For that we have to increase the maximum derivative order of the flat output needed so we calculate $[\dot{v}\ \dot{\omega}]$ building a φ_3 function
  Also, the constraints to be added:

$$\varphi_3(z(t_k), \dots, z^{(l)}(t_k)) \in \mathcal{A}$$

  where $\mathcal{A}$ is the set of admissible acceleration values.
  The function φ_3 is as follows:

$$\varphi_3(z(t_k), \dots, z^{(3)}(t_k)) =$$
$$= \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial t}\|\dot{z}\| \\ \frac{\partial}{\partial t}\frac{(\dot{z}_1\ddot{z}_2 - \dot{z}_2\ddot{z}_1)}{\|\dot{z}\|^2} \end{bmatrix} = \begin{bmatrix} \frac{\dot{z}_1\ddot{z}_1 + \dot{z}_2\ddot{z}_2}{\|\dot{z}\|} \\ \frac{(\ddot{z}_1\ddot{z}_2 + z_2^{(3)}\dot{z}_1 - (\ddot{z}_2\ddot{z}_1 + z_1^{(3)}\dot{z}_2))\|\dot{z}\|^2 - 2(\dot{z}_1\ddot{z}_2 - \dot{z}_2\ddot{z}_1)\|\dot{z}\|\dot{v}}{\|\dot{z}\|^4} \end{bmatrix}$$

— Remake code using good objected oriented structure. It will be good for C++ part (DONE)

ONLINE T_c and T_p (planning horizon) "given" (arbitrary).

$$\tau_k = t_{initial} + kT_c \quad k \in \mathbb{N}$$

Arbitrary detection radius for the robot sensors. Only if the obstacle characteristic position is inside the detection zone the obstacle is considered detected. Using $2m$.

Evaluate for each time interval $[\tau_{k-1}, \tau_k)(k \in \mathbb{N})$ the trajectory beginning at τ_k until $\tau_k + T_p$:

$$\min_{(C_{(0,\tau_k)},\dots,C_{(d+n_{knot}-2,\tau_k)})} J_{\tau_k} = \|\varphi_1(z(\tau_k + T_p, \tau_k), \dots, z^{(l-1)}(\tau_k + T_p, \tau_k)) - q_{final}\|^2 \quad (3.1.3)$$

under the following constraints $\forall t \in [\tau_k, \tau_k + T_p]$:

$$\begin{cases} \varphi_1(z(\tau_k, \tau_k), \dots, z^{(l-1)}(\tau_k, \tau_k)) & = & q_{ref}(\tau_k, \tau_{k-1}) \\ \varphi_2(z(\tau_k, \tau_k), \dots, z^{(l)}(\tau_k, \tau_k)) & = & u_{ref}(\tau_k, \tau_{k-1}) \\ \varphi_2(z(t, \tau_k), \dots, z^{(l)}(t, \tau_k)) & \in & \mathcal{U} \\ d_{O_m}(t, \tau_k) & \geq & \rho + r_m, \quad \forall O_m \in \mathcal{O}(\tau_k) \end{cases} \quad (3.1.4)$$

The period $[\tau_{-1}, \tau_0)$ is what is called by Defoort "the initialization phase" which considers:

$$q_{ref}(\tau_0, \tau_{-1}) = q_{initial}$$

$$u_{ref}(\tau_0, \tau_{-1}) = u_{initial}$$

without no more further changes to the expressions above.

Practical stuff for implementation $q \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$. N_s number of time steps used when computing the problem.

Number of equations: $n + m$

Number of inequations (function of τ_k): $N_s(m + \text{card}(\mathcal{O}(\tau_k)))$

dependencies: sudo apt-get install python python-dev libatlas-base-dev gcc gfortran g++

get source: https://pypi.python.org/pypi/scipy

sudo python setup.py install

3.1.2 The mobile robot

For representing the mobile robot geometry in the planning plane a bounding circle was chosen.

Unicycle kinetic model

Flat output formulation

3.1.3 The obstacles

Two different representations of an obstacle are supported. Obstacles can be seen as circles or convex polygons.

Representing an obstacle as a circle is probably the most simple way of doing so and has great advantages when calculating point-to-obstacle distance compared to other representations.

Nevertheless, obstacles such as walls, boxes and shelves cannot be satisfactorily represented by circles. Thus the need of a polygon representation.

Robot-to-obstacle distance for the convex polygon representation

As sad before the robot's geometric form is represented by a circle. When calculating the robot-to-obstacle distance this simplified representation is quite useful. The first approach to calculate the distance between a point and an obstacle represented by a convex polygon was to separate the problem in three cases with a different expression for the distance computation each. We see in the figure 3.1 that the points A, B and C are placed in three different regions with respect to the obstacle. A is "between" the two lines ($r_{0,1}$ and $r_{0,3}$) that pass through the vertex 0 and are orthogonal to the the two adjacent edges. B is "between" the edge s_3, and the orthogonal lines $r_{0,3}$ and $r_{3,2}$. C is in the interior of the obstacle representation, i.e., surrounded by the four edges.

It is easy to see that the computation of the point-to-obstacle distance for A is a simple point-to-point distance using the appropriate vertex. For B a point-to-line distance equation can be used. Finally, since C is in the interior of the polygon the penetration distance is calculated. It is considered as the shortest of the four distances from the point C to the four edges multiplied by -1 (so, once more, point-to-line distance).

Of course that the performance of this approach is "number of edges"-dependent and present fast results only or polygons with few edges (less than 10).

10 was arbitrary, improve this finding a meaningful value or delete it

An important remark though is that for a given planning horizon several (N_s) point-to-obstacle distances have to be calculated. Intuitively we can say that there is a high probability that most of the N_s points are inside the same zone defined by theirs relative positions to the obstacle. Besides, the probability of finding points inside zones that are "far" from the already occupied zones is smaller. This heuristic can be used to speed up the planning process by having a smarter initialization of point-to-obstacle distance computation when using a convex polygon representation.

Finally, when dealing with more complex obstacles representations and/or with a more complex representation of the mobile robot geometry the Enhanced Gilbert-Johnson-Keerthi distance algorithm is a more suitable and efficient approach.

some code is available on the internet, Google code written in D language and/or the other one on stackverflow

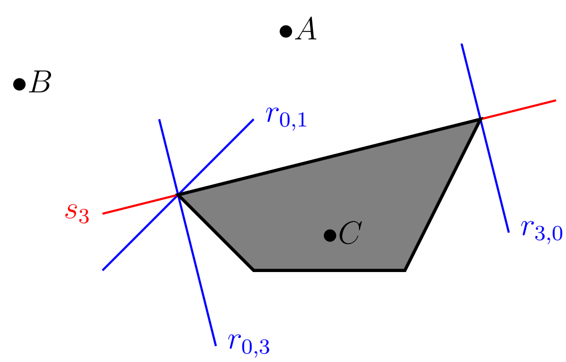


Figure 3.1 – Positioning cases when calculating point-to-obstacle distance in a convex polygon representation.

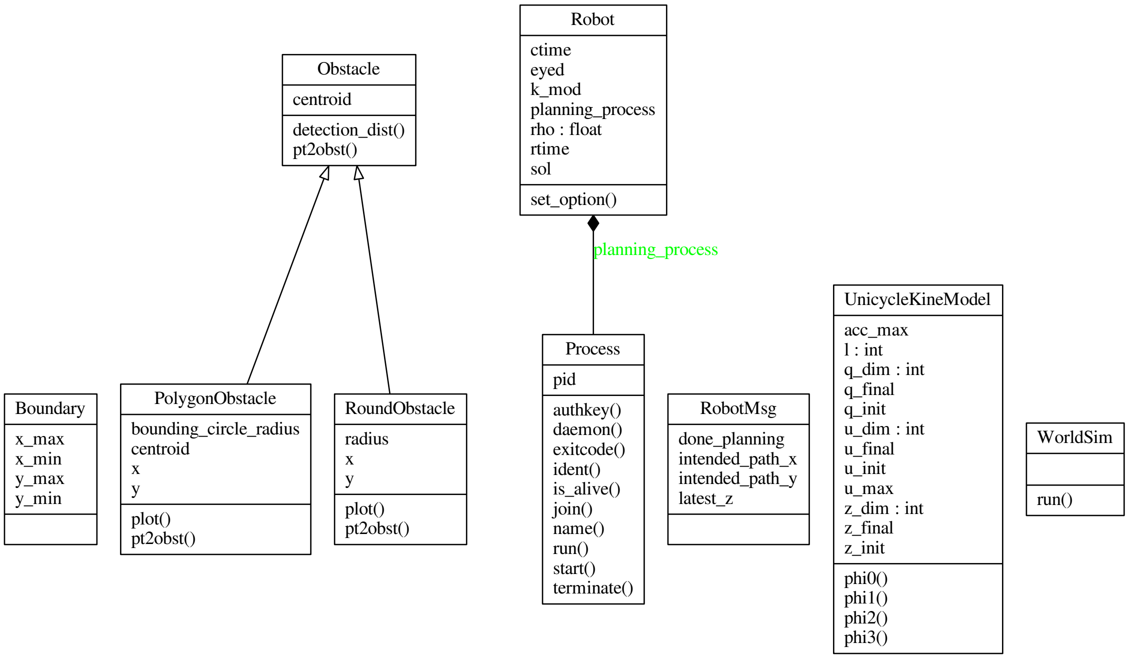


Figure 3.2 – Class diagram.

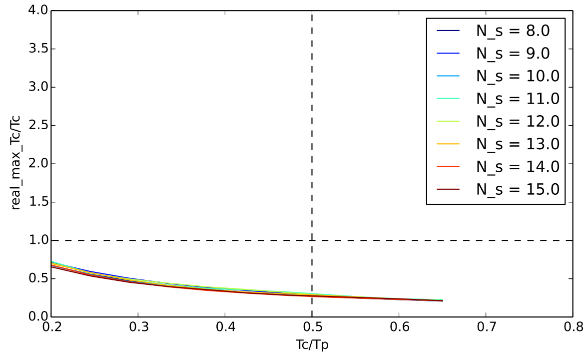
3.1.4 Analysis of real-time planning feasibility

For the sake of an exemple a simulation done with the following parameters: 1 gave

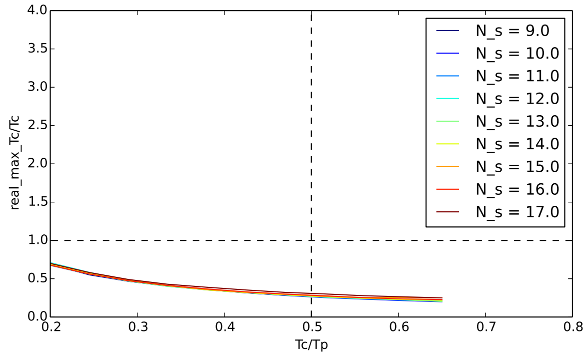
Table 3.1 – Motion planner main parameters

T_p	1.200 s
T_c	0.446 s
N_s	9
N_{knots}	4
v_{max}	1.00 m/s
ω_{max}	5.00 rad/s
q_{inital}	$[-0.05\ 0.00\ \pi/2]^T$
q_{final}	$[0.10\ 7.00\ \pi/2]^T$
u_{final}	$[0.00\ 0.00]^T$
u_{final}	$[0.00\ 0.00]^T$
M	3
O_0	$[0.55\ 1.91\ 0.31]$
O_1	$[-0.08\ 3.65\ 0.32]$
O_2	$[0.38\ 4.65\ 0.16]$

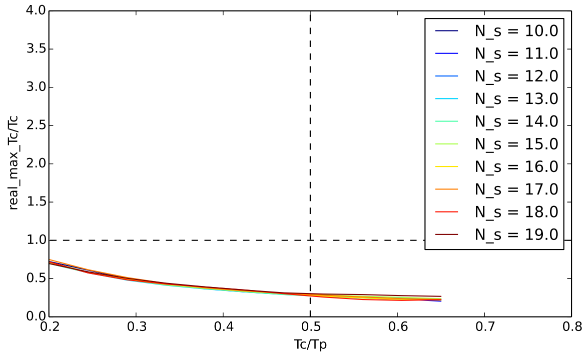
could be perfomed in within computing time smaller than the T_c value for every planning



(a) Four non-null knots intervals.

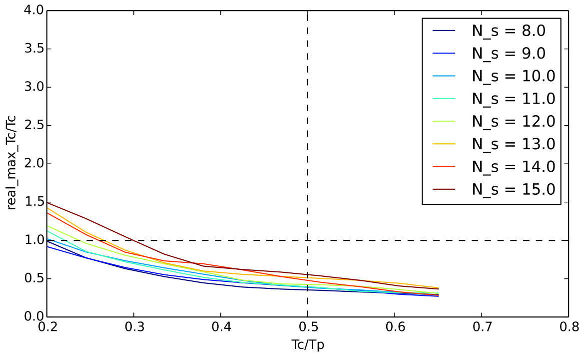


(b) Five non-null knots intervals.

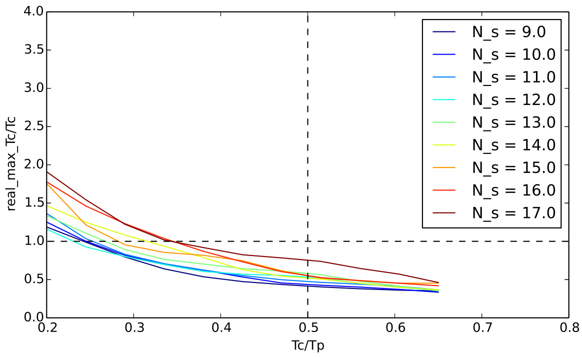


(c) Six non-null knots intervals.

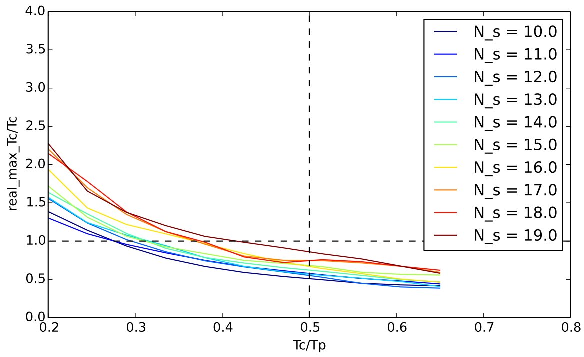
Figure 3.3 – Zero obstacles scenario.



(a) Four non-null knots intervals.

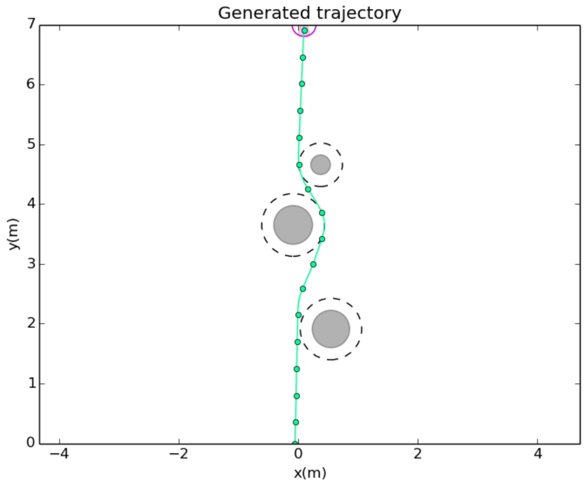


(b) Five non-null knots intervals.

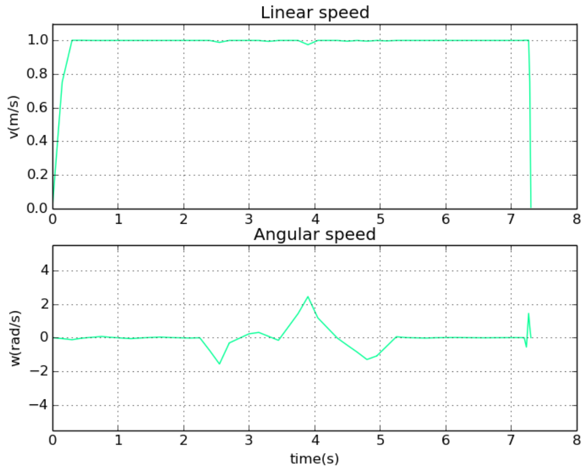


(c) Six non-null knots intervals.

Figure 3.4 – Three obstacles scenario.



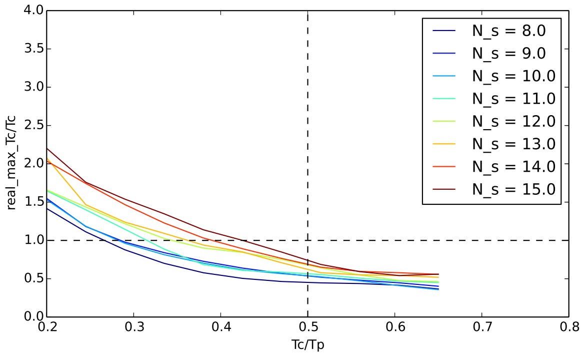
(a) Robot's path.



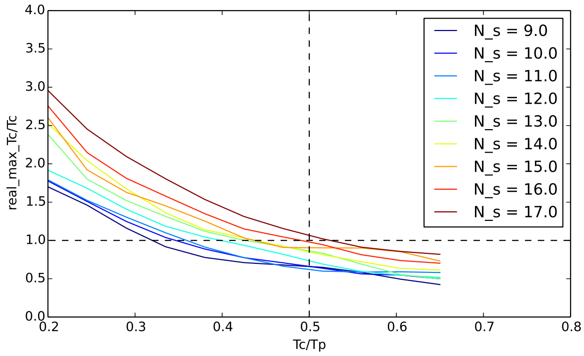
(b) Robot's input.

Figure 3.5 – Three obstacles scenario.

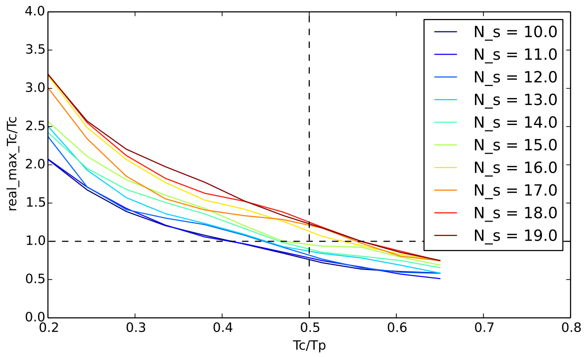
section.



(a) Four non-null knots intervals.



(b) Five non-null knots intervals.



(c) Six non-null knots intervals.

Figure 3.6 – Six obstacles scenario.

3.2 Class Macther

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3.2.1 Max-Flow Matcher

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Chapter 4

Incomplete Patches

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Appendix A

Random Graphs

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