



PROJET DE FIN D'ÉTUDES (PFE)  
INGÉNIERIE SYSTÈME : ROBOTIQUE ET SYSTÈMES  
EMBARQUÉS

2014/2015

---

# Local Dynamic Path Planning for an Autonomous Forklift in Human Environment

---

**Unclassified Report**  
**Can be made public on the internet**

*Author:*

José Magno MENDES FILHO

Promotion 2014

*Supervisor - ENSTA:*

David FILLIAT

*Supervisors - CEA:*

Éric LUCET

Internship from 05 Mars 2015 to 28 August 2015

CEA LIST Digiteo Moulon  
Bât. 660 91191 GIF-SUR-YVETTE Cedex, France



# Acknowledgements

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.



## Résumé

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

## Abstract

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.



# Contents

<b>I</b>	<b>Internship Description</b>	<b>3</b>
1	Work Descriptpion	5
<b>II</b>	<b>Internship Contribution</b>	<b>7</b>
<b>2</b>	<b>Global Near-optimal Solution for Path Planning</b>	<b>9</b>
2.1	Description of the Problem . . . . .	9
<b>3</b>	<b>Algorithmic Approach</b>	<b>11</b>
3.1	Representation of the optimization problem . . . . .	12
3.1.1	Optimizers . . . . .	12
3.1.2	The mobile robot . . . . .	14
3.1.3	The obstacles . . . . .	14
3.1.4	Analysis of real-time planning feasibility . . . . .	16
3.2	Class Macther . . . . .	23
3.2.1	Max-Flow Matcher . . . . .	23
<b>4</b>	<b>Incomplete Patches</b>	<b>25</b>
<b>A</b>	<b>Random Graphs</b>	<b>29</b>





# Part I

## Internship Description



# Chapter 1

## Work Descriptpion

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.



## Part II

# Internship Contribution



## Chapter 2

# Global Near-optimal Solution for Path Planning

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

### 2.1 Description of the Problem





# Chapter 3

## Algorithmic Approach

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

## 3.1 Representation of the optimization problem

### 3.1.1 Optimizers

There is a variety of numerical optimization packages implemented in many different programming languages available for solving optimization problems [?]. Each of them may have their own way of defining the optimization problem and may or may not support specific kinds of constraints (equations, inequations or boundaries).

For the initial implementation written in python two packages stood out as good, easy-to-use options for solving the constrained optimization problem that models the planning motion task.

**Scipy** is a vast open-source scientific package based on python that happens to have a minimization module. Within this module many minimization methods can be found. For this specific optimization problem, only the method SLSPQ was appropriate. It was the only one to handle constrained minimization where the constraints could be equations as well as inequations.

**pyOpt** is a much smaller ecosystem than Scipy that is specialized in optimization. It gathers many different numerical optimization algorithms some of them free and some licensed. Again, among all of them there were only a few suitable for this problem which were also free: SLSQP (same as the one implemented within Scipy), PSQP and ALGENCAN.

SLSQP and PSQP are both SQP (for sequential quadratic programming) methods. A SQP method attempts to solve a nonlinearly constrained optimization problem where the object function and the constraints are twice continuously differentiable. It does so by modeling the object function ( $\min f(x)$ ) at the current iterate  $x_k$  by a quadratic programming subproblem and using the minimizer of this subproblem to define a new iterate  $x_{k+1}$  [?].

The ALGENCAN method

describe algecan

$$\min_{(t_{final}, C_0, \dots, C_{d+n_{knot}-2})} J = (t_{final} - t_{initial})^2 \quad (3.1.1)$$

under the following constraints  $\forall k \in \{0, \dots, N_s - 1\}$ :

$$\begin{cases} \varphi_1(z(t_{initial}), \dots, z^{(l-1)}(t_{initial})) &= q_{initial} \\ \varphi_1(z(t_{final}), \dots, z^{(l-1)}(t_{final})) &= q_{final} \\ \varphi_2(z(t_{initial}), \dots, z^{(l)}(t_{initial})) &= u_{initial} \\ \varphi_2(z(t_{final}), \dots, z^{(l)}(t_{final})) &= u_{final} \\ \varphi_2(z(t_k), \dots, z^{(l)}(t_k)) &\in \mathcal{U} \\ d_{O_m}(t_k) &\geq \rho + r_m, \quad \forall O_m \in \mathcal{Q}_{occupied} \end{cases} \quad (3.1.2)$$

— Problem with discretization

Try adding CONSTRAINTS related to max acceleration (**DONE**)

For that we have to increase the maximum derivative order of the flat output needed so we calculate  $[\dot{v} \ \dot{\omega}]$  building a  $\varphi_3$  function

Also, the constraints to be added:

$$\varphi_3(z(t_k), \dots, z^{(l)}(t_k)) \in \mathcal{A}$$

where  $\mathcal{A}$  is the set of admissible acceleration values.

The function  $\varphi_3$  is as follows:

$$\begin{aligned} \varphi_3(z(t_k), \dots, z^{(3)}(t_k)) &= \\ = \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} &= \begin{bmatrix} \frac{\partial}{\partial t} \|\dot{z}\| \\ \frac{\partial}{\partial t} \frac{(\dot{z}_1 \ddot{z}_2 - \dot{z}_2 \ddot{z}_1)}{\|\dot{z}\|^2} \end{bmatrix} = \begin{bmatrix} \frac{\dot{z}_1 \ddot{z}_1 + \dot{z}_2 \ddot{z}_2}{\|\dot{z}\|} \\ \frac{(\ddot{z}_1 \ddot{z}_2 + z_2^{(3)} \dot{z}_1 - (\ddot{z}_2 \ddot{z}_1 + z_1^{(3)} \dot{z}_2)) \|\dot{z}\|^2 - 2(\dot{z}_1 \ddot{z}_2 - \dot{z}_2 \ddot{z}_1) \|\dot{z}\| \dot{v}}{\|\dot{z}\|^4} \end{bmatrix} \end{aligned}$$

— Remake code using good objected oriented structure. It will be good for C++ part (**DONE**)

**ONLINE**  $T_c$  and  $T_p$  (planning horizon) "given" (arbitrary).

$$\tau_k = t_{initial} + kT_c \quad k \in \mathbb{N}$$

Arbitrary detection radius for the robot sensors. Only if the obstacle characteristic position is inside the detection zone the obstacle is considered detected. Using  $2m$ .

Evaluate for each time interval  $[\tau_{k-1}, \tau_k)(k \in \mathbb{N})$  the trajectory beginning at  $\tau_k$  until  $\tau_k + T_p$ :

$$\min_{(C_{(0, \tau_k)}, \dots, C_{(d+n_{knot}-2, \tau_k)})} J_{\tau_k} = \|\varphi_1(z(\tau_k + T_p, \tau_k), \dots, z^{(l-1)}(\tau_k + T_p, \tau_k)) - q_{final}\|^2 \quad (3.1.3)$$

under the following constraints  $\forall t \in [\tau_k, \tau_k + T_p]$ :

$$\begin{cases} \varphi_1(z(\tau_k, \tau_k), \dots, z^{(l-1)}(\tau_k, \tau_k)) &= q_{ref}(\tau_k, \tau_{k-1}) \\ \varphi_2(z(\tau_k, \tau_k), \dots, z^{(l)}(\tau_k, \tau_k)) &= u_{ref}(\tau_k, \tau_{k-1}) \\ \varphi_2(z(t, \tau_k), \dots, z^{(l)}(t, \tau_k)) &\in \mathcal{U} \\ d_{O_m}(t, \tau_k) &\geq \rho + r_m, \quad \forall O_m \in \mathcal{O}(\tau_k) \end{cases} \quad (3.1.4)$$

The period  $[\tau_{-1}, \tau_0)$  is what is called by Defoort "the initialization phase" which con-

siders:

$$q_{ref}(\tau_0, \tau_{-1}) = q_{initial}$$

$$u_{ref}(\tau_0, \tau_{-1}) = u_{initial}$$

without no more further changes to the expressions above.

**Practical stuff for implementation**  $q \in \mathbb{R}^n$  and  $u \in \mathbb{R}^m$ .  $N_s$  number of time steps used when computing the problem.

Number of equations:  $n + m$

Number of inequations (function of  $\tau_k$ ):  $N_s(m + \text{card}(\mathcal{O}(\tau_k)))$

dependencies: `sudo apt-get install python python-dev libatlas-base-dev gcc gfortran g++`

get source: <https://pypi.python.org/pypi/scipy>

`sudo python setup.py install`

### 3.1.2 The mobile robot

For representing the mobile robot geometry in the planning plane a bounding circle was chosen.

#### Unicycle kinetic model

#### Flat output formulation

### 3.1.3 The obstacles

Two different representations of an obstacle are supported. Obstacles can be seen as circles or convex polygons.

Representing an obstacle as a circle is probably the most simple way of doing so and has great advantages when calculating point-to-obstacle distance compared to other representations.

Nevertheless, obstacles such as walls, boxes and shelves cannot be satisfactorily represented by circles. Thus the need of a polygon representation.

#### Robot-to-obstacle distance for the convex polygon representation

As sad before the robot's geometric form is represented by a circle. When calculating the robot-to-obstacle distance this simplified representation is quite useful. The first approach to calculate the distance between a point and an obstacle represented by a convex polygon was to separate the problem in three cases with a different expression for the distance

computation each. We see in the figure 3.1 that the points  $A$ ,  $B$  and  $C$  are placed in three different regions with respect to the obstacle.  $A$  is "between" the two lines ( $r_{0,1}$  and  $r_{0,3}$ ) that pass through the vertex 0 and are orthogonal to the two adjacent edges.  $B$  is "between" the edge  $s_3$ , and the orthogonal lines  $r_{0,3}$  and  $r_{3,2}$ .  $C$  is in the interior of the obstacle representation, i.e., surrounded by the four edges.

It is easy to see that the computation of the point-to-obstacle distance for  $A$  is a simple point-to-point distance using the appropriate vertex. For  $B$  a point-to-line distance equation can be used. Finally, since  $C$  is in the interior of the polygon the penetration distance is calculated. It is considered as the shortest of the four distances from the point  $C$  to the four edges multiplied by  $-1$  (so, once more, point-to-line distance).

Of course that the performance of this approach is "number of edges"-dependent and present fast results only for polygons with few edges (less than 10).

10 was arbitrary, improve this finding a meaningful value or delete it

An important remark though is that for a given planning horizon several ( $N_s$ ) point-to-obstacle distances have to be calculated. Intuitively we can say that there is a high probability that most of the  $N_s$  points are inside the same zone defined by their relative positions to the obstacle. Besides, the probability of finding points inside zones that are "far" from the already occupied zones is smaller. This heuristic can be used to speed up the planning process by having a smarter initialization of point-to-obstacle distance computation when using a convex polygon representation.

Finally, when dealing with more complex obstacles representations and/or with a more complex representation of the mobile robot geometry the Enhanced Gilbert-Johnson-Keerthi distance algorithm is a more suitable and efficient approach.

some code is available on the internet, Google code written in D language and/or the other one on stackoverflow

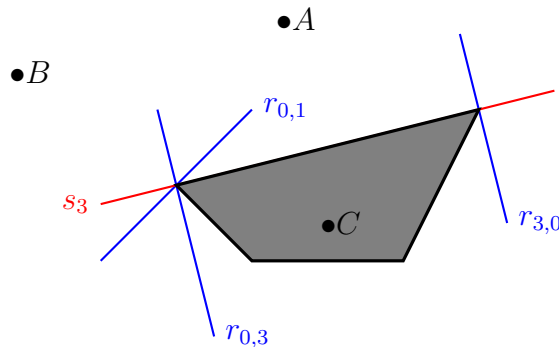


Figure 3.1 – Positioning cases when calculating point-to-obstacle distance in a convex polygon representation.



similar to the second only with six instead of three obstacles.

In the other hand, to reduce the problem size an unique optimization solver with fixed parameters was used for all simulations.

Table 3.1 – Optimization solver parameters

Optimization solver type	SLSQP s
$MAXIT_{first}$	2.40 s
$T_c$	0.48 s
$N_s$	11
$N_{knots}$	4
$v_{max}$	1.00 m/s
$\omega_{max}$	5.00 rad/s
$q_{initial}$	$[-0.05 \ 0.00 \ \pi/2]^T$
$q_{final}$	$[0.10 \ 7.00 \ \pi/2]^T$
$u_{final}$	$[0.00 \ 0.00]^T$
$u_{final}$	$[0.00 \ 0.00]^T$
$O_0$	$[0.55 \ 1.91 \ 0.31]$
$O_1$	$[-0.08 \ 3.65 \ 0.32]$
$O_2$	$[0.38 \ 4.65 \ 0.16]$

### No obstacles scenario

For the sake of an example a simulation done with the following parameters:

Table 3.2 – Motion planner main parameters

$T_p$	2.40 s
$T_c$	0.48 s
$N_s$	11
$N_{knots}$	4
$v_{max}$	1.00 m/s
$\omega_{max}$	5.00 rad/s
$q_{initial}$	$[-0.05 \ 0.00 \ \pi/2]^T$
$q_{final}$	$[0.10 \ 7.00 \ \pi/2]^T$
$u_{final}$	$[0.00 \ 0.00]^T$
$u_{final}$	$[0.00 \ 0.00]^T$
$O_0$	$[0.55 \ 1.91 \ 0.31]$
$O_1$	$[-0.08 \ 3.65 \ 0.32]$
$O_2$	$[0.38 \ 4.65 \ 0.16]$

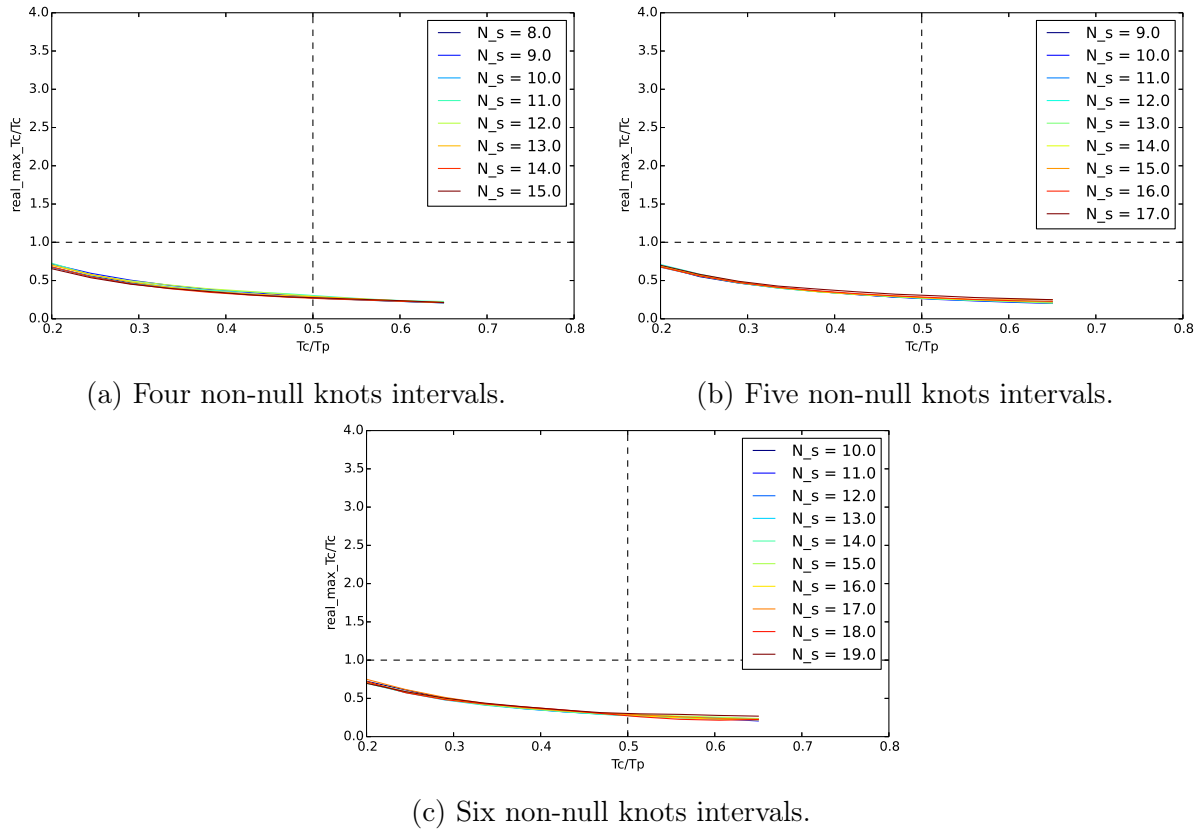


Figure 3.3 – Zero obstacles scenario.



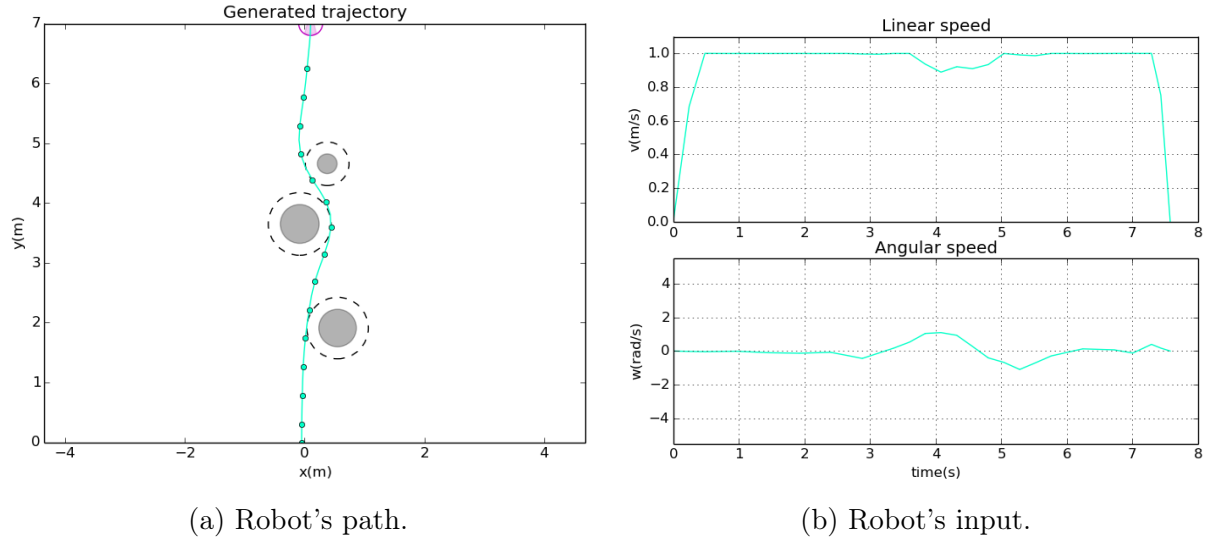


Figure 3.4 – Three obstacles scenario.

gave could be performed in within computing time smaller than the  $T_c$  value for every planning section.

### Three obstacles scenario

For the sake of an exemple a simulation done with the following parameters:

Table 3.3 – Motion planner main parameters

$T_p$	2.40 s
$T_c$	0.48 s
$N_s$	11
$N_{knots}$	4
$v_{max}$	1.00 m/s
$\omega_{max}$	5.00 rad/s
$q_{initial}$	$[-0.05 \ 0.00 \ \pi/2]^T$
$q_{final}$	$[0.10 \ 7.00 \ \pi/2]^T$
$u_{final}$	$[0.00 \ 0.00]^T$
$u_{final}$	$[0.00 \ 0.00]^T$
$O_0$	$[0.55 \ 1.91 \ 0.31]$
$O_1$	$[-0.08 \ 3.65 \ 0.32]$
$O_2$	$[0.38 \ 4.65 \ 0.16]$

gave could be performed in within computing time smaller than the  $T_c$  value for every planning section.

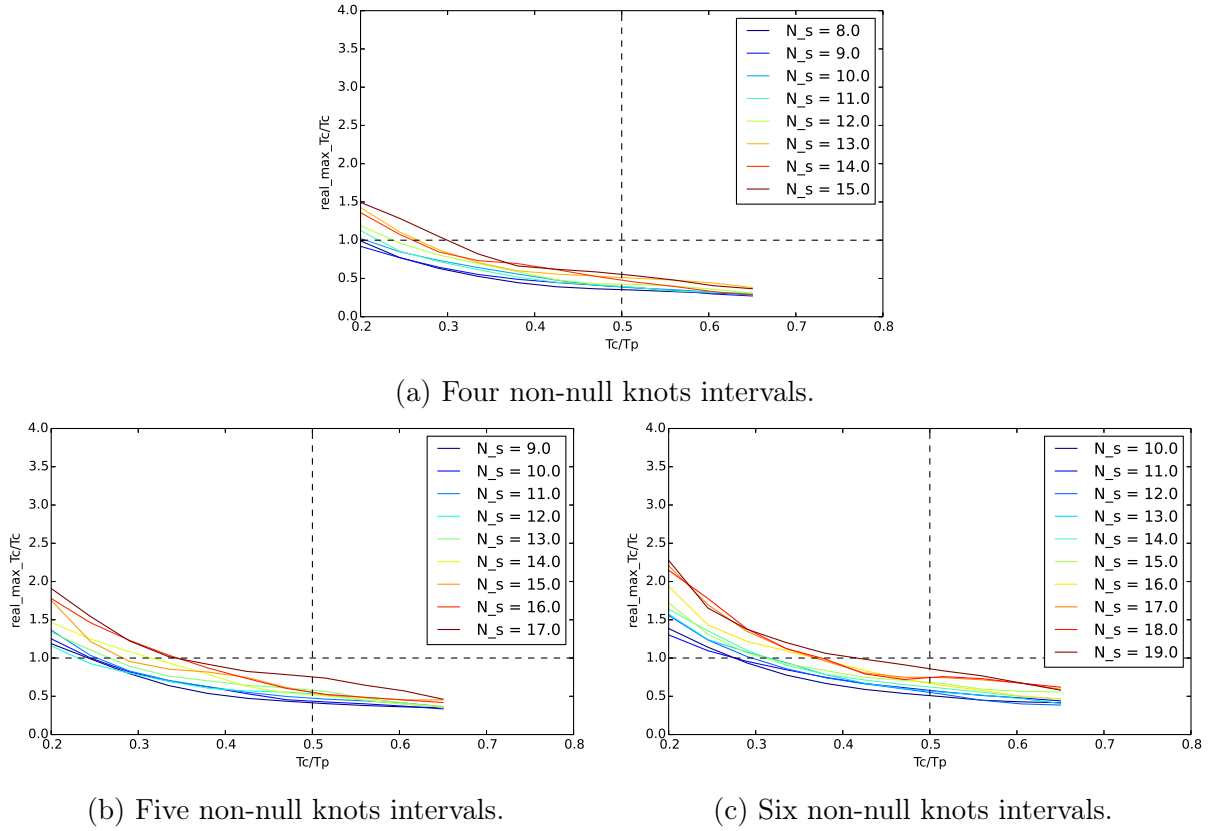


Figure 3.5 – Three obstacles scenario.

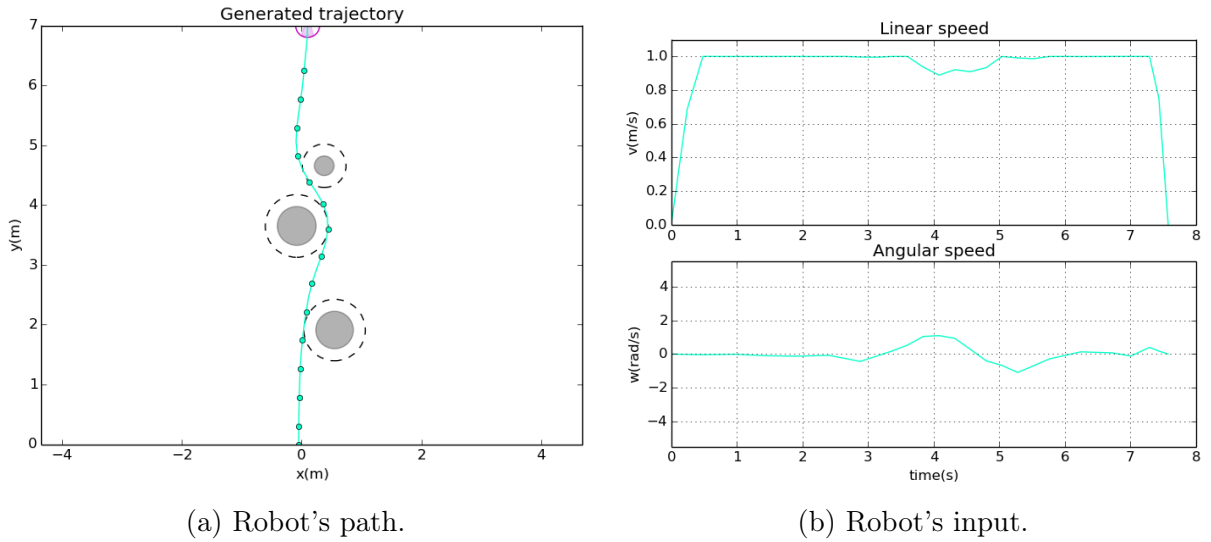


Figure 3.6 – Three obstacles scenario.

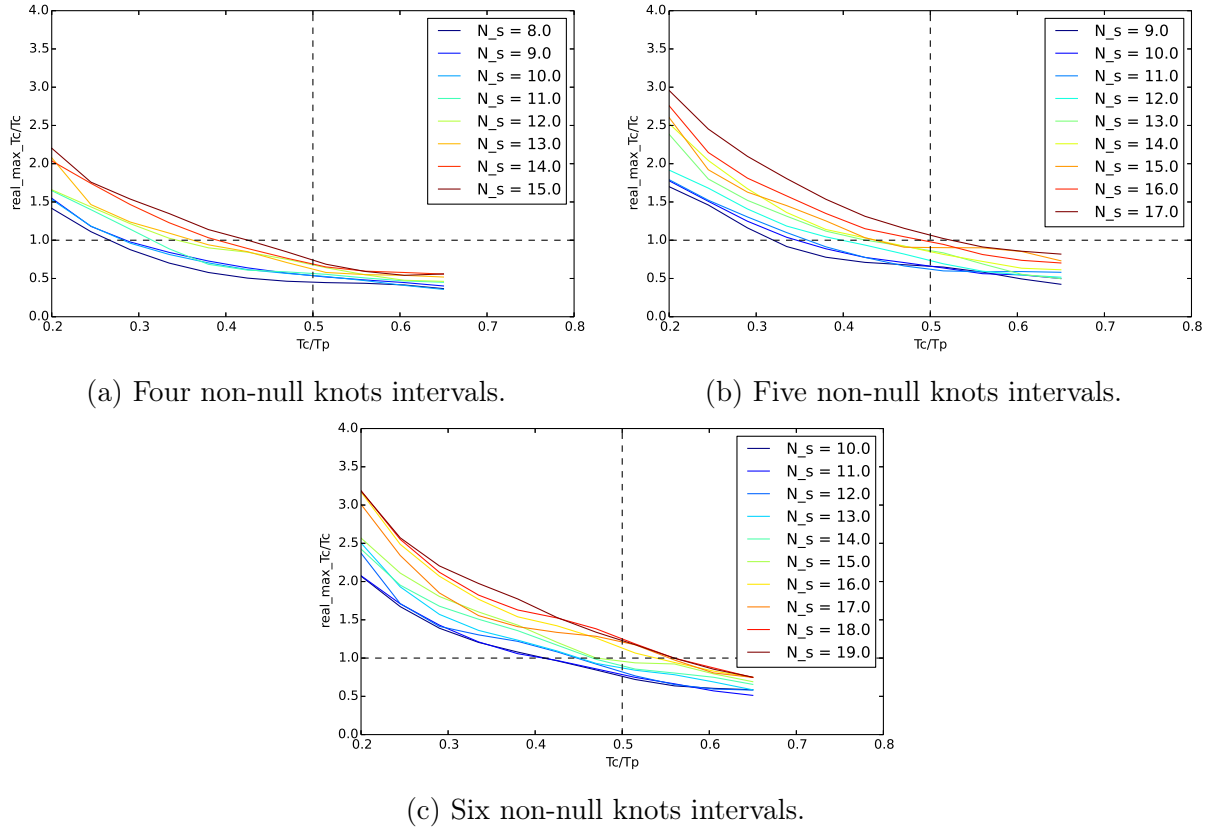


Figure 3.7 – Six obstacles scenario.

### Six obstacles scenario

For the sake of an exemple a simulation done with the following parameters:

Table 3.4 – Motion planner main parameters

$T_p$	2.40 s
$T_c$	0.48 s
$N_s$	11
$N_{knots}$	4
$v_{max}$	1.00 m/s
$\omega_{max}$	5.00 rad/s
$q_{initial}$	$[-0.05 \ 0.00 \ \pi/2]^T$
$q_{final}$	$[0.10 \ 7.00 \ \pi/2]^T$
$u_{final}$	$[0.00 \ 0.00]^T$
$u_{final}$	$[0.00 \ 0.00]^T$
$O_0$	$[0.55 \ 1.91 \ 0.31]$
$O_1$	$[-0.08 \ 3.65 \ 0.32]$
$O_2$	$[0.38 \ 4.65 \ 0.16]$

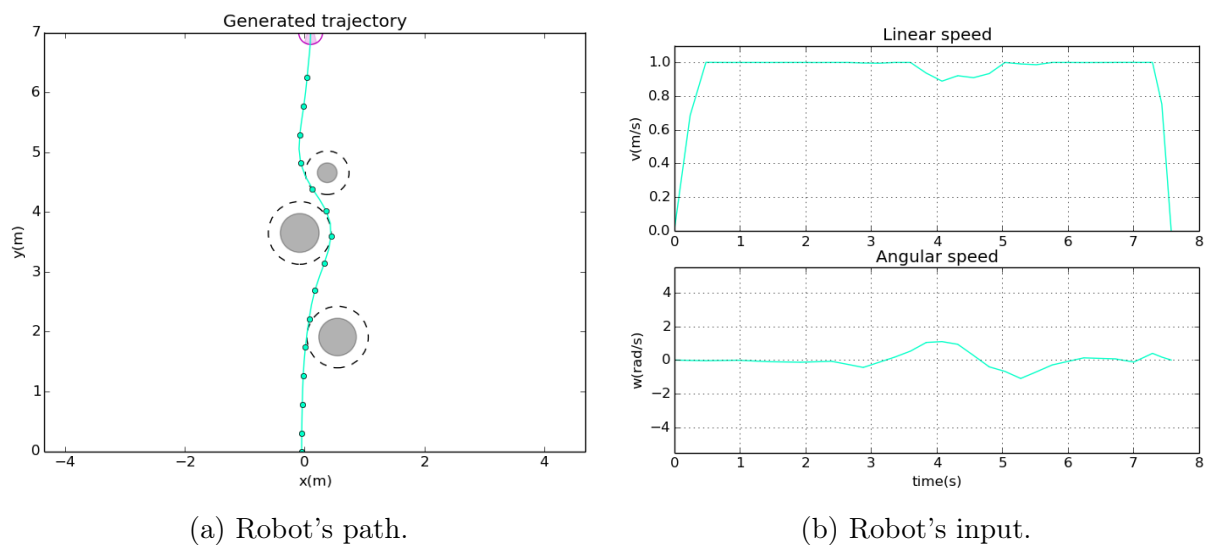


Figure 3.8 – Three obstacles scenario.

gave could be performed in within computing time smaller than the  $T_c$  value for every planning section.

## 3.2 Class Macther

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

### 3.2.1 Max-Flow Matcher

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus

luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

# Chapter 4

## Incomplete Patches

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetur adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetur.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetur at, consectetur sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui.



Morbi luctus, wisi viverra faucibus pretium, nibh est placerat odio, nec commodo wisi enim eget quam. Quisque libero justo, consectetur a, feugiat vitae, porttitor eu, libero. Suspendisse sed mauris vitae elit sollicitudin malesuada. Maecenas ultricies eros sit amet ante. Ut venenatis velit. Maecenas sed mi eget dui varius euismod. Phasellus aliquet volutpat odio. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Pellentesque sit amet pede ac sem eleifend consectetur. Nullam elementum, urna vel imperdiet sodales, elit ipsum pharetra ligula, ac pretium ante justo a nulla. Curabitur tristique arcu eu metus. Vestibulum lectus. Proin mauris. Proin eu nunc eu urna hendrerit faucibus. Aliquam auctor, pede consequat laoreet varius, eros tellus scelerisque quam, pellentesque hendrerit ipsum dolor sed augue. Nulla nec lacus.

Suspendisse vitae elit. Aliquam arcu neque, ornare in, ullamcorper quis, commodo eu, libero. Fusce sagittis erat at erat tristique mollis. Maecenas sapien libero, molestie et, lobortis in, sodales eget, dui. Morbi ultrices rutrum lorem. Nam elementum ullamcorper leo. Morbi dui. Aliquam sagittis. Nunc placerat. Pellentesque tristique sodales est. Maecenas imperdiet lacinia velit. Cras non urna. Morbi eros pede, suscipit ac, varius vel, egestas non, eros. Praesent malesuada, diam id pretium elementum, eros sem dictum tortor, vel consectetur odio sem sed wisi.



# Appendix A

## Random Graphs

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetur adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetur.