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Local Dynamic Path Planning for an Autonomous Forklift in Human Environment

Unclassified Report

Can be made public on the internet

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Promotion 2014

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Internship from 05 Mars 2015 to 28 August 2015

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Acknowledgements

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Résumé

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Abstract

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Work Descritpion

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Part II

Internship Contribution

Chapter 2

Global Near-optimal Solution for Path Planning

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2.1 Description of the Problem

Chapter 3

Algorithmic Approach

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3.1 Representation of the optimization problem

3.1.1 Optimizers

There is a variety of numerical optimization packages implemented in many different programming languages available for solving optimization problems [?]. Each of them may have their own way of defining the optimization problem and may or may not support specific kinds of constraints (equations, inequations or boundaries).

For the initial implementation written in python two packages stood out as good, easy-to-use options for solving the constrained optimization problem that models the planning motion task.

Scipy is a vast open-source scientific package based on python that happens to have a minimization module. Within this module many minimization methods can be found. For this specific optimization problem, only the method SLSPQ was appropriate. It was the only one to handle constrained minimization where the constraints could be equations as well as inequations.

pyOpt is a much smaller ecosystem than Scipy that is specialized in optimization. It gathers many different numerical optimization algorithms some of them free and some licensed. Again, among all of them there were only a few suitable for this problem which were also free: SLSQP (same as the one implemented within Sicpy), PSQP and ALGENCAN.

SLSQP and PSQP are both SQP (for sequential quadratic programming) methods. A SQP method attempts to solve a nonlinearly constrained optimization problem where the object function and the constraints are twice continuously differentiable. It does so by modeling the object function ($\min f(x)$) at the current iterate x_k by a quadratic programming subproblem and using the minimizer of this subproblem to define a new iterate x_{k+1} [?].

The ALGENCAN method

describe algecan

$$\min_{(t_{final}, C_0, \ldots, C_{d+n_{knot}-2})} J = (t_{final} - t_{initial})^2 \tag{3.1.1}$$

under the following constraints $\forall k \in \{0, \ldots, N_s - 1\}$:

$$\begin{cases} \varphi_1(z(t_{initial}), \ldots, z^{(l-1)}(t_{initial})) & = & q_{initial} \\ \varphi_1(z(t_{final}), \ldots, z^{(l-1)}(t_{final})) & = & q_{final} \\ \varphi_2(z(t_{initial}), \ldots, z^{(l)}(t_{initial})) & = & u_{initial} \\ \varphi_2(z(t_{final}), \ldots, z^{(l)}(t_{final})) & = & u_{final} \\ \varphi_2(z(t_k), \ldots, z^{(l)}(t_k)) & \in & \mathcal{U} \\ d_{O_m}(t_k) & \geq & \rho + r_m, \quad \forall O_m \in \mathcal{Q}_{occupied} \end{cases} \tag{3.1.2}$$

- Problem with discretization

  Try adding CONSTRAINTS related to max acceleration (DONE)

  For that we have to increase the maximum derivative order of the flat output needed so we calculate $[\dot{v}\ \dot{\omega}]$ building a φ_3 function

  Also, the constraints to be added:

  $$\varphi_3(z(t_k), \ldots, z^{(l)}(t_k)) \in \mathcal{A}$$

  where $\mathcal{A}$ is the set of admissible acceleration values.

  The function φ_3 is as follows:

  $$\varphi_3(z(t_k), \ldots, z^{(3)}(t_k)) = \\ = \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial t}\|\dot{z}\| \\ \frac{\partial}{\partial t}\frac{(\dot{z}_1\ddot{z}_2 - \dot{z}_2\ddot{z}_1)}{\|\dot{z}\|^2} \end{bmatrix} = \begin{bmatrix} \frac{\dot{z}_1\ddot{z}_1 + \dot{z}_2\ddot{z}_2}{\|\dot{z}\|} \\ \frac{(\ddot{z}_1\ddot{z}_2 + z_2^{(3)}\dot{z}_1 - (\ddot{z}_2\ddot{z}_1 + z_1^{(3)}\dot{z}_2))\|\dot{z}\|^2 - 2(\dot{z}_1\ddot{z}_2 - \dot{z}_2\ddot{z}_1)\|\dot{z}\|\dot{v}}{\|\dot{z}\|^4} \end{bmatrix}$$

- Remake code using good objected oriented structure. It will be good for C++ part (DONE)

ONLINE T_c and T_p (planning horizon) "given" (arbitrary).

$$\tau_k = t_{initial} + kT_c \quad k \in \mathbb{N}$$

Arbitrary detection radius for the robot sensors. Only if the obstacle characteristic position is inside the detection zone the obstacle is considered detected. Using $2m$.

Evaluate for each time interval $[\tau_{k-1}, \tau_k)(k \in \mathbb{N})$ the trajectory beginning at τ_k until $\tau_k + T_p$:

$$\min_{(C_{(0,\tau_k)},\ldots,C_{(d+n_{knot}-2,\tau_k)})} J_{\tau_k} = \|\varphi_1(z(\tau_k + T_p, \tau_k), \ldots, z^{(l-1)}(\tau_k + T_p, \tau_k)) - q_{final}\|^2 \tag{3.1.3}$$

under the following constraints $\forall t \in [\tau_k, \tau_k + T_p]$:

$$\begin{cases} \varphi_1(z(\tau_k, \tau_k), \ldots, z^{(l-1)}(\tau_k, \tau_k)) & = & q_{ref}(\tau_k, \tau_{k-1}) \\ \varphi_2(z(\tau_k, \tau_k), \ldots, z^{(l)}(\tau_k, \tau_k)) & = & u_{ref}(\tau_k, \tau_{k-1}) \\ \varphi_2(z(t, \tau_k), \ldots, z^{(l)}(t, \tau_k)) & \in & \mathcal{U} \\ d_{O_m}(t, \tau_k) & \geq & \rho + r_m, \quad \forall O_m \in \mathcal{O}(\tau_k) \end{cases} \tag{3.1.4}$$

The period $[\tau_{-1}, \tau_0)$ is what is called by Defoort "the initialization phase" which considers:

$$q_{ref}(\tau_0, \tau_{-1}) = q_{initial}$$

$$u_{ref}(\tau_0, \tau_{-1}) = u_{initial}$$

without no more further changes to the expressions above.

Practical stuff for implementation $q \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$. N_s number of time steps used when computing the problem.

Number of equations: $n + m$

Number of inequations (function of τ_k): $N_s(m + \text{card}(\mathcal{O}(\tau_k)))$

dependencies: sudo apt-get install python python-dev libatlas-base-dev gcc gfortran g++

get source: https://pypi.python.org/pypi/scipy

sudo python setup.py install

3.1.2 The mobile robot

For representing the mobile robot geometry in the planning plane a bounding circle was chosen.

Unicycle kinetic model

Flat output formulation

3.1.3 The obstacles

Two different representations of an obstacle are supported. Obstacles can be seen as circles or convex polygons.

Representing an obstacle as a circle is probably the most simple way of doing so and has great advantages when calculating point-to-obstacle distance compared to other representations.

Nevertheless, obstacles such as walls, boxes and shelves cannot be satisfactorily represented by circles. Thus the need of a polygon representation.

Robot-to-obstacle distance for the convex polygon representation

As sad before the robot's geometric form is represented by a circle. When calculating the robot-to-obstacle distance this simplified representation is quite useful. The first approach to calculate the distance between a point and an obstacle represented by a convex polygon was to separate the problem in three cases with a different expression for the distance computation each. We see in the figure 3.1 that the points A, B and C are placed in three different regions with respect to the obstacle. A is "between" the two lines ($r_{0,1}$ and $r_{0,3}$) that pass through the vertex 0 and are orthogonal to the the two adjacent edges. B is "between" the edge s_3, and the orthogonal lines $r_{0,3}$ and $r_{3,2}$. C is in the interior of the obstacle representation, i.e., surrounded by the four edges.

It is easy to see that the computation of the point-to-obstacle distance for A is a simple point-to-point distance using the appropriate vertex. For B a point-to-line distance equation can be used. Finally, since C is in the interior of the polygon the penetration distance is calculated. It is considered as the shortest of the four distances from the point C to the four edges multiplied by -1 (so, once more, point-to-line distance).

Of course that the performance of this approach is "number of edges"-dependent and present fast results only or polygons with few edges (less than 10).

10 was arbitrary, improve this finding a meaningful value or delete it

An important remark though is that for a given planning horizon several (N_s) point-to-obstacle distances have to be calculated. Intuitively we can say that there is a high probability that most of the N_s points are inside the same zone defined by theirs relative positions to the obstacle. Besides, the probability of finding points inside zones that are "far" from the already occupied zones is smaller. This heuristic can be used to speed up the planning process by having a smarter initialization of point-to-obstacle distance computation when using a convex polygon representation.

Finally, when dealing with more complex obstacles representations and/or with a more complex representation of the mobile robot geometry the Enhanced Gilbert-Johnson-Keerthi distance algorithm is a more suitable and efficient approach.

some code is available on the internet, Google code written in D language and/or the other one on stackverflow

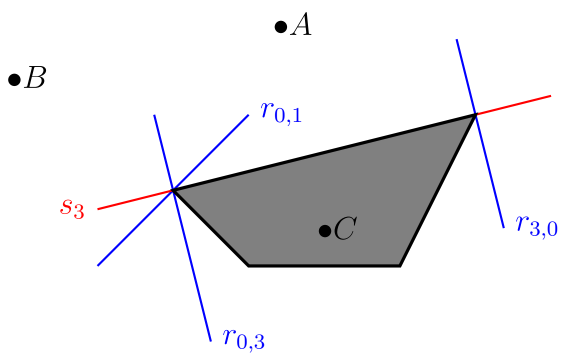


Figure 3.1 – Positioning cases when calculating point-to-obstacle distance in a convex polygon representation.

3.1.4 Analysis of real-time planning feasibility and total time performance

The performance of the motion planning algorithm previously presented depends on several parameters. For starters these parameters can be split into two groups. The **algorithm related** parameters and the **optimization solver related** ones. Among the former group the most important ones are: the number of sample for time discretization (N_s), the number of internal knots for the B-splines curves (n_{knots}), and the planning and computation time horizons (T_p and T_c respectively). The latter kind depends on the optimization solver adopted but since most of them are iterative methods is common to have at least a "maximum number of iterations" and a "stop condition" parameters.

The task of searching for a satisfactory set of parameters' values with regard to a performance metric (e.g. total time to complete the mission) is quite laborious.

We attempt nevertheless to extract some ~~qualitative~~ knowledge about how these parameters impact the generated solution based on several simulations run with different parameters configurations. The main objective here is to be able to support the feasibility of a real-time motion planner based on this algorithm.

Aiming for a scenario invariant understanding of the impact of these parameters three different scenarios were studied. A fist scenario where the robot did not had to avoid any obstacle to complete its mission, a second one where three round obstacles were randomly generated in a region where the robot was probably going to pass through and a third similar to the second only with six instead of three obstacles.

In the other hand, to reduce the problem's size, an unique optimization solver with fixed parameters was used for all simulations. The used parameters can be seen in table 3.1. Different maximum numbers of iterations are used for different stages of the planning process. The subscribed words *first*, *inter* and *last* indicate that the respective maximum numbers of iteration are used for the first optimization problem solving, for all intermediaries ones and for the last one.

talk about accuracy

Real-time feasibility in this context can be considered as having the *maximum computational time* spend for planning the path sections[1] less than or equal to the computation horizon (T_c) . Here though we are only interest in understanding the variation of *maximum computational time*$/T_c$ with changes on T_c, T_p, N_s, N_{knots}.

Another natural performance metric that should be kept in mind is the total time spend to complete the mission (going from the initial configuration to the final).

After this analysis we shall be able to identify sets of parameters' values that minimize

1. All computational times spend for planning all sections are considered for finding the maximum value but the first one

the total time spend to complete the mission, respecting the problem constraints and minimizing the *maximum computational time*$/T_c$ ratio.

Table 3.1 – Optimization solver parameters

Optimization solver type	SLSQP
$MAXIT_{first}$	40
$MAXIT_{inter}$	15
$MAXIT_{last}$	20
accuracy	10^{-3}

3.1.5 Computation time analysis

No obstacles scenario The images in the figure 3.2 try to make prominent the effect of changes in the in the number of samples (N_s) and number of internal knots (N_{knots}). In the ordinate axis we have the *maximum computational time*$/T_c$ ratio and in the abscissa we have the T_c/T_p. For each N_s we took the average of the *maximum computational time*$/T_c$ ratio for a give T_c/T_p among different T_p in order to be T_p invariant.

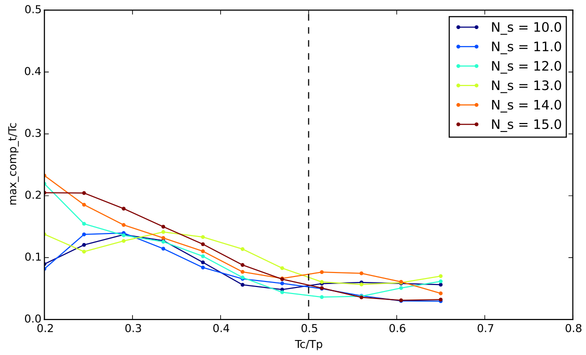
We can see that for a "no obstacles" scenario the overall performance with respect to the *maximum computational time*$/T_c$ ratio is not greatly impacted by variations in the number of samples (N_s) nor in the number of internal knots (N_{knots}). Within a given image the lines are close together showing that variations in N_s have weak impact. In addition, comparing the three images (3.2a, 3.2b, 3.2c) we see that variations in the N_{knots} have also a weak influence.

For the sake of an example we present a simulation result in the figure 3.3 run with the parameters presented in the table 3.2 for a "no obstacles" scenario.

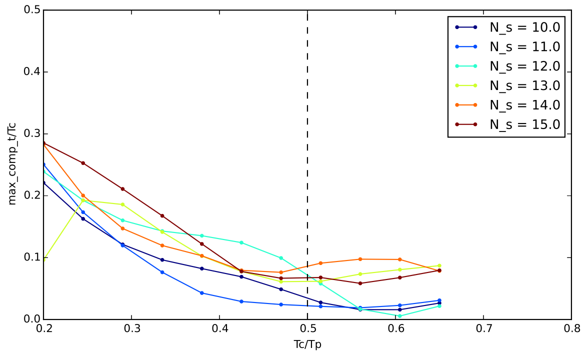
Three obstacles scenario For this new scenario, a greater impact of the number of samples (N_s) and number of non-null internal knots (N_{knots}) is observed. The greater the N_{knots} or the N_s the greater is the *maximum computational time*$/T_c$. This behavior is the one expected since the number of constraints and the number of arguments for the cost function to be minimized depend on these two parameters respectively.

Again, in the figure 3.3 we show a simulation example run with the parameters' values presented in table 3.4.

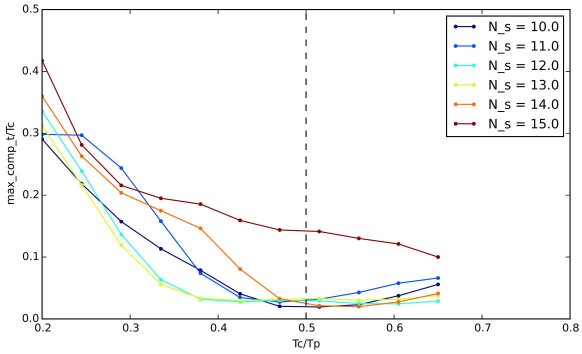
Six obstacles scenario As for the scenario with six obstacles we realize that the observations for the latest scenario are accentuated.



(a) Four internal knots. Average variance between lines is 0.098×10^{-2}



(b) Five internal knots. Average variance between lines is 0.205×10^{-2}



(c) Six internal knots. Average variance between lines is 0.295×10^{-2}

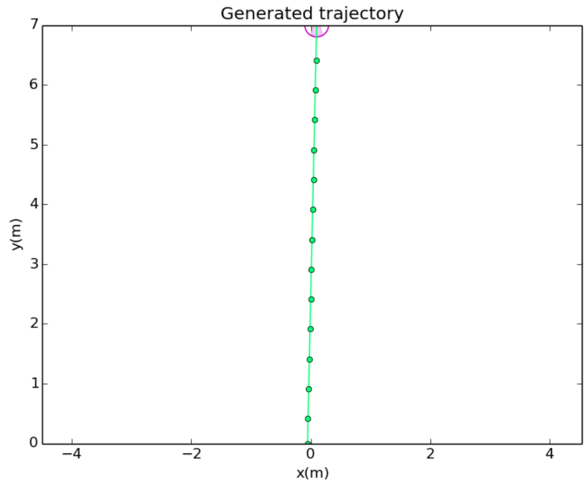
Figure 3.2 – Zero obstacles scenario.

Table 3.2 – Motion planner main parameters

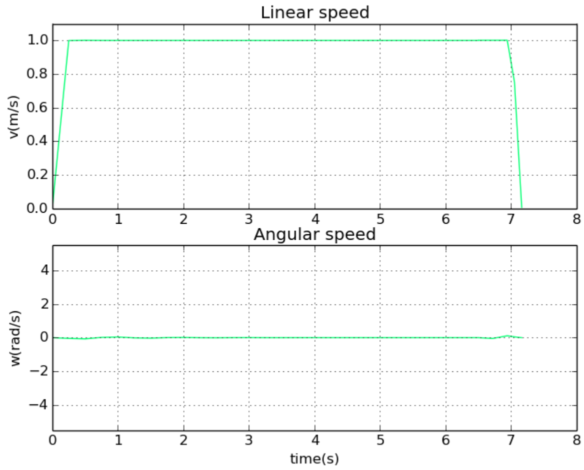
T_p	2.00 s
T_c	0.40 s
N_s	9
N_{knots}	5
v_{max}	1.00 m/s
ω_{max}	5.00 rad/s
q_{inital}	$[-0.05\ 0.00\ \pi/2]^T$
q_{final}	$[0.10\ 7.00\ \pi/2]^T$
u_{final}	$[0.00\ 0.00]^T$
u_{final}	$[0.00\ 0.00]^T$

Total time analysis

The time spend for finding the solution for a given set of parameters values does not impact the solution it self. Let's analyse than how the solution behaves for different scenarios.

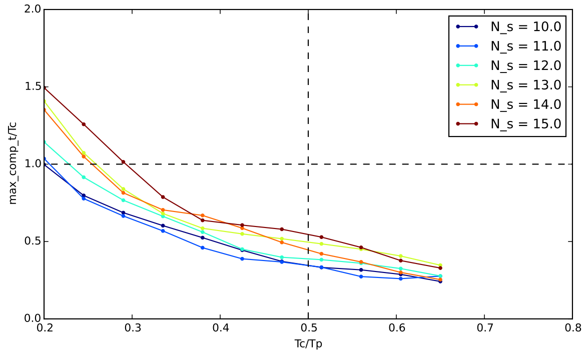


(a) Robot's path.

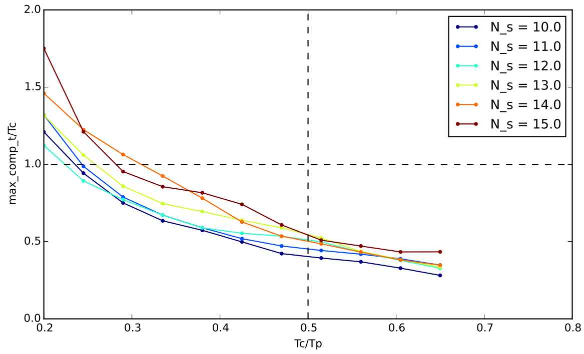


(b) Robot's input.

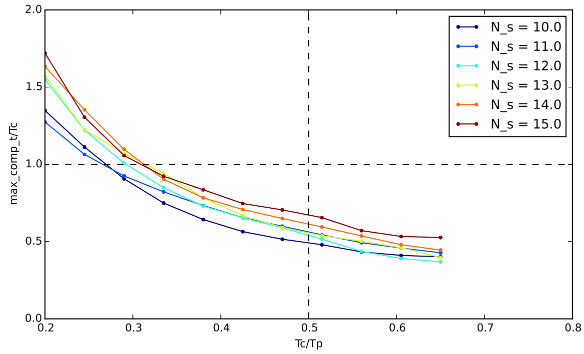
Figure 3.3 – No obstacle scenario simulation example where the *maximum computational time* was about 78% of T_c and the mission total time equals to 7.16 s.



(a) Four internal knots. Average variance between lines is 1.65×10^{-2}



(b) Five internal knots. Average variance between lines is 2.40×10^{-2}



(c) Six internal knots. Average variance between lines is 2.74×10^{-2}

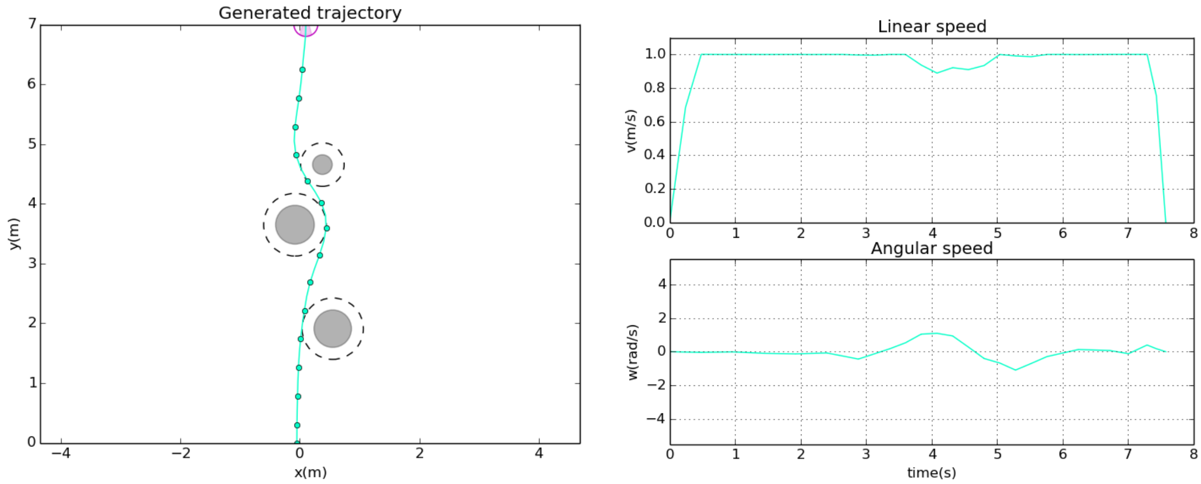
Figure 3.4 – Three obstacles scenario.

Intuitively we can say that the greater the number

- Distance inter robots

Table 3.3 – Motion planner main parameters

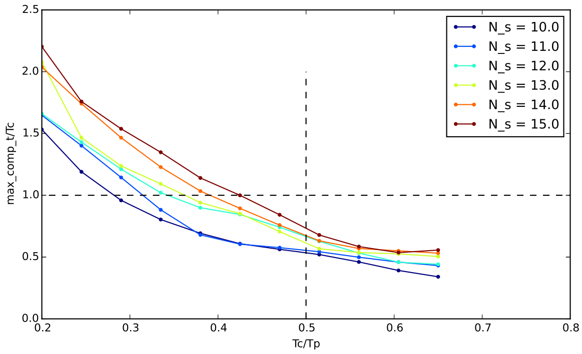
T_p	2.40 s
T_c	0.48 s
N_s	11
N_{knots}	4
v_{max}	1.00 m/s
ω_{max}	5.00 rad/s
q_{inital}	$[-0.05\ 0.00\ \pi/2]^T$
q_{final}	$[0.10\ 7.00\ \pi/2]^T$
u_{final}	$[0.00\ 0.00]^T$
u_{final}	$[0.00\ 0.00]^T$
O_0	[0.55 1.91 0.31]
O_1	[−0.08 3.65 0.32]
O_2	[0.38 4.65 0.16]



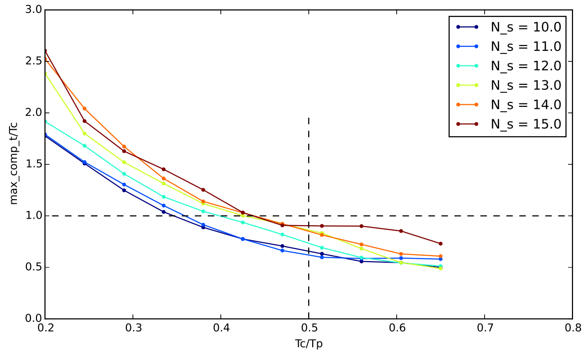
(a) Robot's path. (b) Robot's input.

Figure 3.5 – Three obstacle scenario simulation example where the *maximum computational time* was about 84% of T_c and the mission total time equals to 7.57 s.

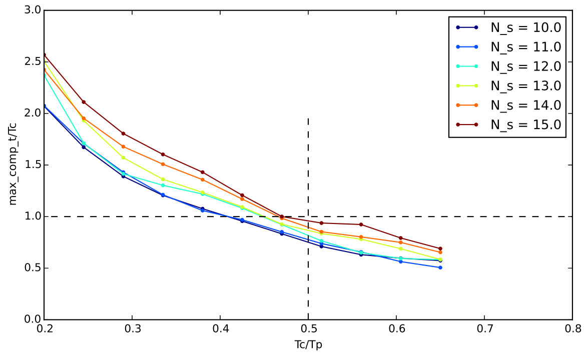
— Min dist to obstacles



(a) Four internal knots. Average variance between lines is 2.92×10^{-2}



(b) Five internal knots. Average variance between lines is 5.78×10^{-2}

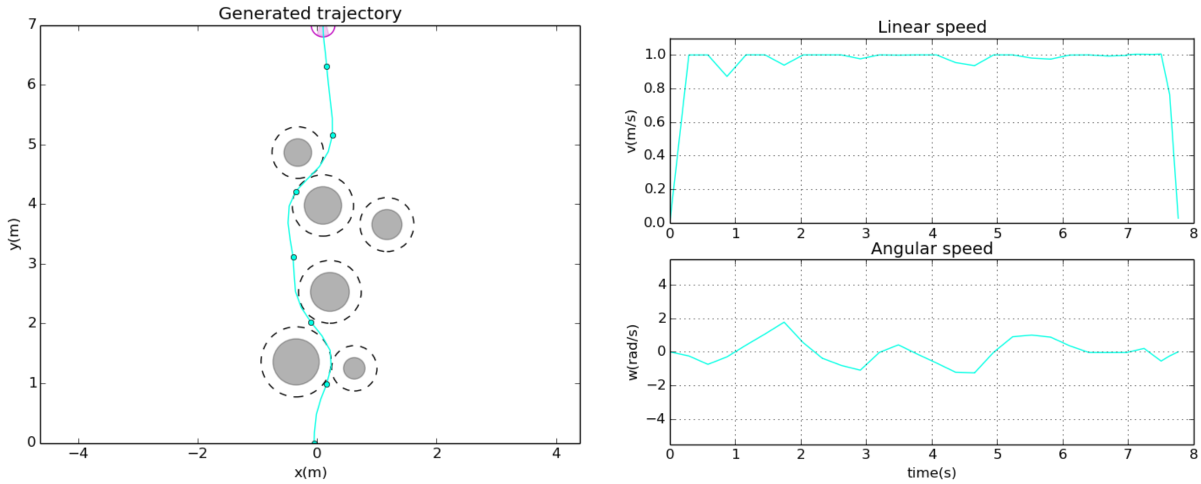


(c) Six internal knots. Average variance between lines is 5.89×10^{-2}

Figure 3.6 – Six obstacles scenario.

Table 3.4 – Motion planner main parameters

T_p	3.20 s
T_c	1.28 s
N_s	12
N_{knots}	6
v_{max}	1.00 m/s
ω_{max}	5.00 rad/s
q_{inital}	$[-0.05\ 0.00\ \pi/2]^T$
q_{final}	$[0.10\ 7.00\ \pi/2]^T$
u_{final}	$[0.00\ 0.00]^T$
u_{final}	$[0.00\ 0.00]^T$
O_0	$[-0.35\ 1.36\ 0.39]$
O_1	$[0.21\ 2.53\ 0.33]$
O_2	$[-0.32\ 4.86\ 0.23]$
O_3	$[0.10\ 3.98\ 0.31]$
O_4	$[0.62\ 1.25\ 0.18]$
O_5	$[1.17\ 3.66\ 0.25]$



(a) Robot's path.

(b) Robot's input.

Figure 3.7 – Three obstacle scenario simulation example where the *maximum computational time* was about 90% of T_c and the mission total time equals to $7.76\ s$.

Appendix A

Random Graphs

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