

NORTHERN ARIZONA UNIVERSITY

MASTER'S THESIS

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# Quadrotor Flight Path Energy Optimization

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*A thesis submitted in fulfilment of the requirements  
for the degree of Master of Science*

*in the*

Department of Electrical Engineering

March 2014

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- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

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# *Abstract*

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Master of Science

## **Quadrotor Flight Path Energy Optimization**

by Edward KEMPER

*thesis*  
In this paper we develop and compare two methods for the flight path energy optimisation of a quadrotor UAV between two known points. First we use classical optimal control techniques and find an approximate solution to the resulting boundary value problem. (This method is shown to be too computationally intensive to provide a solution in a reasonable amount of time.) The second method that is developed is a heuristic technique which minimizes the energy of the flight path through optimal PID controller tuning. (Simulation results of the heuristic show that both reliable control of the system and energy minimization are achieved.)

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# *Acknowledgements*

The acknowledgements and the people to thank go here, don't forget to include your project advisor. . .

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# Symbols

$a$  distance m

$P$  power W ( $\text{Js}^{-1}$ )

$\omega$  angular frequency  $\text{rads}^{-1}$

more?

*For my Parents Jack and Carol...*