NAAM: ______________________________ US NR: ______________

INSTRUKSIES: Drie probleme, 12 punte, 40 minute. Toon alle berekenings by Probleem 3.

INSTRUCTIONS: *Three problems,* 12 *marks,* 40 *minutes. Show all work on Problem 3.*

Probleem 1 (*3 punte*)

Beskou die vergelyking / *Consider the equation*

$$3\,x^2 + 3001\,x - 1 = 0.$$

Die kwadratiese formule, / *The quadratic formula,*

$$x_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

is soos volg in MATLAB ge-implementeer / *was implemented as follows in* MATLAB

```
>> a = 3; b = 3001; c = -1;
>> xplus = (-b+sqrt(b^2-4*a*c))/(2*a)

xplus =   3.332221482802803e-04

>> xminus = (-b-sqrt(b^2-4*a*c))/(2*a)

xminus = -1.000333666555482e+03
```

(a) Watter van die twee wortels, x_+ of x_-, is **nie** tot volle akkuraatheid bereken **nie**? Omsirkel een, geen, of beide.

Which of the two roots, x_+ or x_-, was **not** *computed to full accuracy? Circle one, none, or both.*

(b) In die ruimte hier onder, skryf MATLAB instruksies wat die onakkurate wortel(s) van deel (a) tot volle akkuraatheid sal bereken.

In the space below, write MATLAB *instructions that will compute the inaccurate root(s) of part (a) to full accuracy.*

Probleem 2 (*3 punte*)

Beskou die funksie

Consider the function

$$f(x) = \sin(x) - \tan(x).$$

Met behulp van trigonometriese identiteite kan aangetoon word dat die volgende drie formules identiese voorstellings van $f(x)$ is (aanvaar sonder bewys)

With the aid of trigonometric identities it may be shown that the following three formulas are identical representations of $f(x)$ (assume without proof)

$$g(x) = \tan(x)(\cos(x) - 1), \qquad h(x) = -\frac{\sin^3(x)}{\cos(x)(\cos(x)+1)}, \qquad k(x) = -2\tan(x)\sin^2(\frac{x}{2}).$$

(a) Gestel $f(x)$ moet in MATLAB ge-evalueer word naby die oorsprong, sê by $x = 10^{-7}$ of so. As 'n akkurate antwoord die doel is, watter van die vier formules hierbo behoort **nie** gebruik te word **nie**? (Daar mag dalk meer as een onakkurate formule wees, omsirkel almal).

Suppose $f(x)$ has to be evaluated in MATLAB *near the origin, say at $x = 10^{-7}$ or so. If an accurate answer is the goal, which of the four formulas above should* **not** *be used? (There may be more than one inaccurate formula, circle all.)*

(A) $f(x)$ (B) $g(x)$ (C) $h(x)$ (D) $k(x)$

(E) alle formules sal identiese resultate lewer / *all formulas will yield identical results*

(b) Herhaal deel (a) vir 'n waarde van x naby π.

Repeat part (a) for a value of x near π.

(A) $f(x)$ (B) $g(x)$ (C) $h(x)$ (D) $k(x)$

(E) alle formules sal identiese resultate lewer / *all formulas will yield identical results*

Probleem 3 (*6 punte*)

Beskou die integrale

Consider the integrals

$$y_n = \int_1^e \left(\ln x\right)^n dx, \qquad n = 0, 1, 2, \ldots$$

.

(a) Gebruik deelwyse integrasie om aan te toon dat die integrale rekursief bereken kan word volgens

Use integration by parts to show that the integrals can be computed recursively via

$$y_n = e - n\, y_{n-1}, \qquad n = 1, 2, \ldots,$$

en gee ook die aanvangswaarde y_0.

and also give the initial value y_0.

(b) Modelleer die effek van 'n klein afrondingsfout, ϵ, in die aanvangswaarde. Op grond hiervan, sou u sê dat die rekursie van deel (a) 'n stabiele algoritme is vir die berekening van y_n? (U hoef nie 'n verbeterde algoritme voor te stel nie.)

Model the effect of a small roundoff error, ϵ, in the initial value. On these grounds, would you say the recursion of part (a) is a stable algorithm for computing the y_n? (You need not suggest an improved algorithm.)