Probleem 1:

(a) Singuliere punte $\Rightarrow \frac{dx}{dt} = 0 = \frac{dy}{dt}$

$$\begin{aligned}(y-x)(y-1) &= 0\\(x-y)(x-1) &= 0\end{aligned}$$

Dus $y = x$ in altyd 'n oplossing $\Rightarrow$ singuliere lyn.

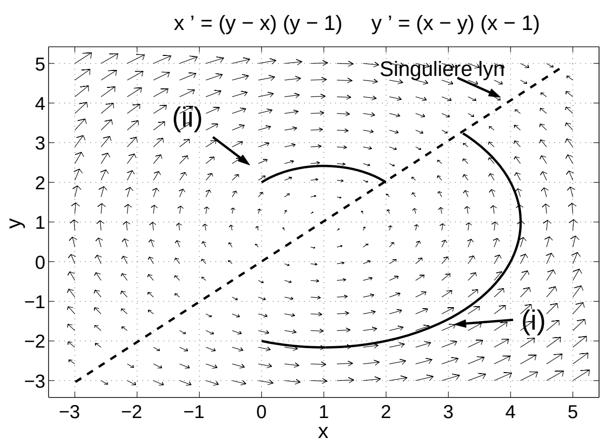
As $y \neq x$ volg $(x, y) = (1, 1)$ in 'n singuliere punt.

(b)

$$\begin{aligned}\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} &= \frac{(y-x)(y-1)}{(x-y)(x-1)}\\ &= -\frac{(y-1)}{(x-1)} \qquad (y \neq x)\\ \Rightarrow \qquad (y-1)\,dy &= -(x-1)\,dx\\ \tfrac{1}{2}(y-1)^2 &= -\tfrac{1}{2}(x-1)^2 + \text{integrasiekonstante}\\ \Rightarrow \qquad (y-1)^2 + (x-1)^2 &= C^2 \qquad C \text{ is 'n konstante}\end{aligned}$$

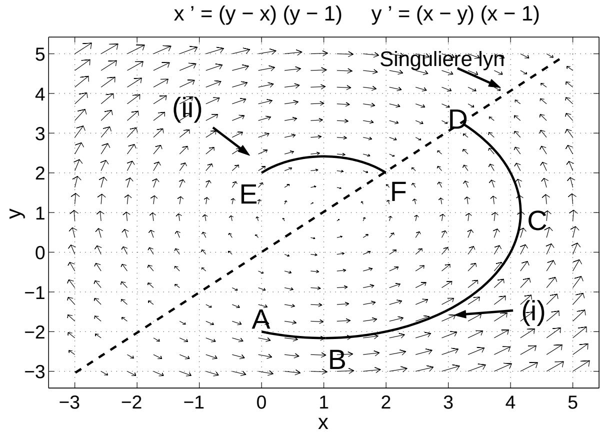
Oplossingskrommes dus 'n familie van sirkels met middelpunt $(x, y) = (1, 1)$ en radius C.

(c)

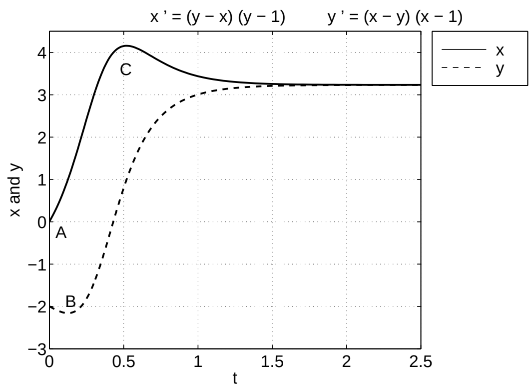


(d) (i) Aanvankilik neem x toe en y af (A $\rightarrow$ B op Figuur 1). Dan neem beide x en y toe tot by C, waarna x weer afneem. Uiteindelik streef beide x en y na 'n waarde net oor 3 (D). [Werklike gedrag in Figuur 2.]

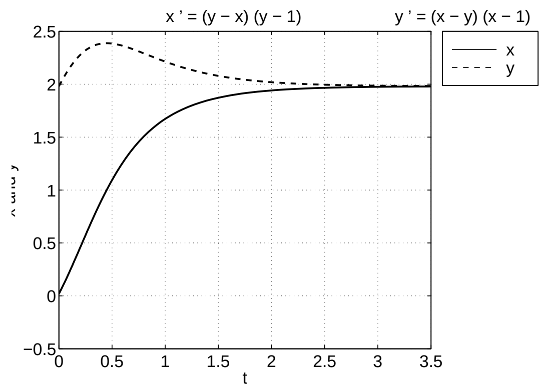
(ii) x neem uniform toe van E na F, terwyl y aanvanklik toeneem en later weer afneem. Uiteindelik streek beide x en y na die waarde 2 (F). [Werklike gedrag in Figuur 3.]



Figuur 1



Figuur 2: Werklike gedrag van (i)



Figuur 3: Werklike gedrag van (ii)

Probleem 2:

(a) $\tau = -5 + 7 = 2$ en $\Delta = (-5)(7) - (2)(3) = -41$
$\tau^2 - 4\Delta = 4(40) > 0 \Rightarrow (0,0)$ is a saalpunt.

(b) $\tau = -5 - 7 = -12$ en $\Delta = 35 - 6 = 29$
$\tau^2 - 4\Delta = 28 > 0 \Rightarrow (0,0)$ is a stabiele node.

(c) $\tau = -5 + 4 = -1$ en $\Delta = -20 + 21 = 1$
$\tau^2 - 4\Delta = -3 < 0 \Rightarrow (0,0)$ is a stabiele spiraal.

(d) $\tau = -1 + 1 = 0$ en $\Delta = -1 + 2 = 1$
$\tau^2 - 4\Delta = -4 < 0 \Rightarrow (0,0)$ is a senter.