Probleem 1:

$$y'' + 25y = \cos(5.6t)$$

$$y(0) = 0, \quad y'(0) = 0.$$

Die Algemene oplossing bestaan uit die oplossing tot die homogene DV, y_h, en 'n partikuliere integraal:

$$y = y_h + y_p.$$

Bepaal eers die homogene oplossing, dus die oplossing tot

$$y'' + 25y = 0.$$

Hierdie is EHB, dus ken ons die oplossing as

$$y_h = y = c_1 \cos 5t + c_2 \sin 5t.$$

Vir die partikuliere oplossing raai

$$y = A\cos(5.6t)$$

Dan is

$$y' = -5.6 A \sin(5.6t)$$

en $$y'' = -(5.6)^2 A \cos(5.6t).$$

Stel in DV om vir A op te los

$$-(5.6)^2 A \cos(5.6t) + 25 A\cos(5.6t) = \cos(5.6t)$$

$$A\left(25 - (5.6)^2\right) = 1$$

$$\Rightarrow \quad A = \frac{1}{(5-5.6)(5+5.6)} = -0.157$$

$$\Rightarrow \quad y_p = y = -0.157 \cos(5.6t)$$

Dus is die algemene oplossing van die DV

$$y = y_h + y_p$$

$$= c_1 \cos 5t + c_2 \sin 5t - 0.157 \cos(5.6t)$$

Gebruik nou die aanvangswaardes

$y(0) = y'(0) = 0$ om vir die konstantes

c_1 en c_2 op te los:

$$y(0) = 0 = c_1 + c_2 \times 0 - 0.157 \quad \Rightarrow c_1 = 0.157$$

$$y'(t) = -c_1(\sin 5t)5 + c_2(\cos 5t)5 + 0.157(5.6)\sin(5.6t)$$

$$y'(0) = 0 = 0 + 5c_2 \quad \Rightarrow \quad c_2 = 0$$

$$\Rightarrow \quad y(t) = 0.157\cos 5t - 0.157\cos 5.6t$$

$$= 0.157\left(\cos 5t - \cos 5.6t\right)$$

Gebruik

$$\cos(a+b) = \cos a \cos b - \sin a \sin b \quad \ldots (1)$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b \quad \ldots (2)$$

(2) − (1)

$$\cos(a-b) - \cos(a+b) = 2\sin a \sin b$$

dus met $a-b = 5t$ $\Rightarrow 2a = (5.+5.6)t$

en $a+b = 5.6t$ $\quad a = 5.3t$

$\Rightarrow b = 0.3t$

is $\cos 5t - \cos 5.6t = 2(\sin 5.3t)(\sin 0.3t)$

en ons kan die oplossing tot die gegewee aanvangsprobleem skryf as

$$y(t) = 0.157(2)(\sin 5.3t)(\sin 0.3t)$$

$$= 0.314 \sin(5.3t)\sin(0.3t)$$

The solution curve is plotted in Figure 4.2.5 (solid curve). The beat is the function

$$\frac{1}{3.18}\sin(0.3t)$$

with circular frequency 0.3, period $2\pi/0.3 \approx 20.94$, and amplitude $1/3.18$.

Graphs of $\pm(\sin 0.3t)/3.18$ (dashed curves in Figure 4.2.5) envelop the solution curve of IVP (13). Each hump in the solution curve has time width $\pi/0.3 \approx 10.47$.

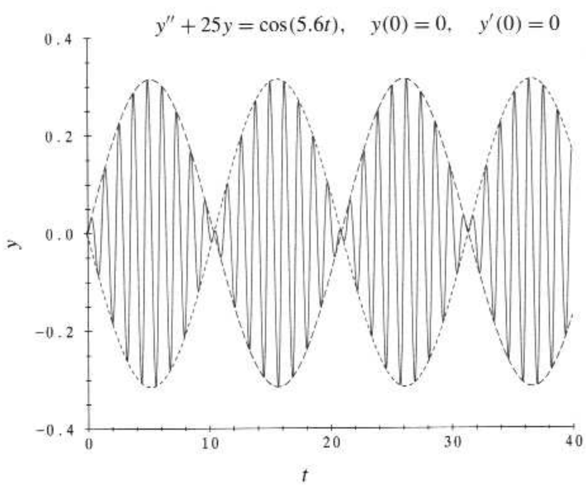
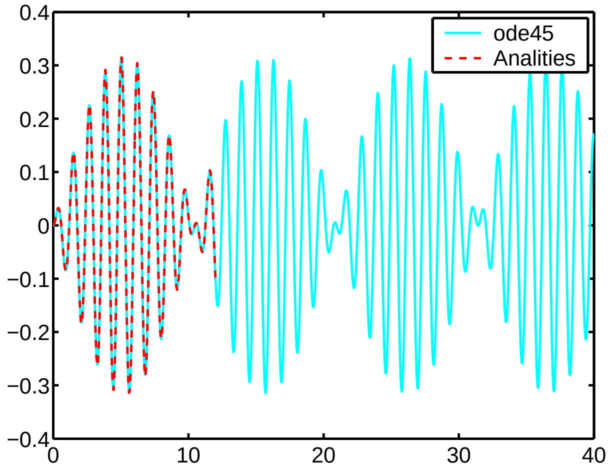


FIGURE 4.2.5 Beats with low circular beat frequency 0.3, long period $2\pi/0.3 \approx 20.94$, and zero initial data

```
>> tydsduur = [0 40];
>> aanvangs = [0 0];
>> [t,z] = ode45(inline('[z(2); cos(5.6*t)-25*z(1)]','t','z'),tydsduur,aanvangs);
```



Probleem 2:

Trial solution try a particular solution of the form $x_p(t) = At\sin 10t + Bt\cos 10t$. The t is present in this particular solution because the normal trial solution, $A\sin 10t + B\cos 10t$, is a solution of the associated homogeneous differential equation.

Substituting $x_p(t)$ into (7.26) gives $20A\cos 10t - 20B\sin 10t = 19\cos 10t$, so we need $B = 0$ and $A = 19/20$. This gives our particular solution as $(19/20)t\sin 10t$, and

General solution the general solution of (7.26) as

$$x(t) = C_1\sin 10t + C_2\cos 10t + \frac{19}{20}t\sin 10t.$$

Imposing our previous initial conditions on this solution gives $C_1 = C_2 = 0$, so the final form of our solution is

$$x(t) = \frac{19}{20}t\sin 10t. \qquad (7.27)$$

Here the solution will oscillate between t and $-t$ and will be unbounded as $t \to \infty$. Its graph is shown in Figure 7.13.

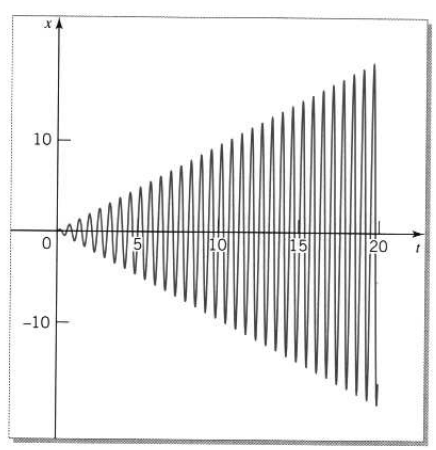


FIGURE 7.13 Graph of $(19t/20)\sin 10t$

Die 'amplitude' van die oplossing neem toe soos t toe neem. Hierdie verskynsel word resonansie genoem.

Probleem 3:

Interpret and solve the initial-value problem

$$\frac{1}{5}\frac{d^2x}{dt^2} + 1.2\frac{dx}{dt} + 2x = 5\cos 4t, \qquad x(0) = \frac{1}{2}, \quad x'(0) = 0. \tag{26}$$

Solution We can interpret the problem to represent a vibrational system consisting of a mass ($m = 1/5$ slug or kilogram) attached to a spring ($k = 2$ lb/ft or N/m). The mass is released from rest $\frac{1}{2}$ unit (foot or meter) below the equilibrium position. The motion is damped ($\beta = 1.2$) and is being driven by an external periodic ($T = \pi/2$ seconds) force beginning at $t = 0$. Intuitively we would expect that even with damping the system would remain in motion until such time as the forcing function is "turned off," in which case the amplitudes diminish. However, as the problem is given, $f(t) = 5\cos 4t$ will remain "on" forever.

We first multiply the differential equation in (26) by 5 and solve

$$\frac{dx^2}{dt^2} + 6\frac{dx}{dt} + 10x = 0$$

by the usual methods. Since $m_1 = -3 + i$, $m_2 = -3 - i$, it follows that

$$x_c(t) = e^{-3t}(c_1\cos t + c_2\sin t).$$

Using the method of undetermined coefficients, we assume a particular solution of the form $x_p(t) = A\cos 4t + B\sin 4t$. Now

$$x_p' = -4A\sin 4t + 4B\cos 4t, \quad x_p'' = -16A\cos 4t - 16B\sin 4t$$

so that

$$x_p'' + 6x_p' + 10x_p = (-6A + 24B)\cos 4t + (-24A - 6B)\sin 4t = 25\cos 4t.$$

The resulting system of equations

$$-6A + 24B = 25, \qquad 24A - 6B = 0$$

yields $A = -25/102$ and $B = 50/51$. It follows that

$$x(t) = e^{-3t}(c_1\cos t + c_2\sin t) - \frac{25}{102}\cos 4t + \frac{50}{51}\sin 4t. \tag{27}$$

When we set $t = 0$ in the above equation, we obtain $c_1 = 38/51$. By differentiating the expression and then setting $t = 0$, we also find that $c_2 = -86/51$. Therefore the equation of motion is

$$x(t) = e^{-3t}\left(\frac{38}{51}\cos t - \frac{86}{51}\sin t\right) - \frac{25}{102}\cos 4t + \frac{50}{51}\sin 4t. \qquad \textbf{28}$$

Transient and Steady-State Terms

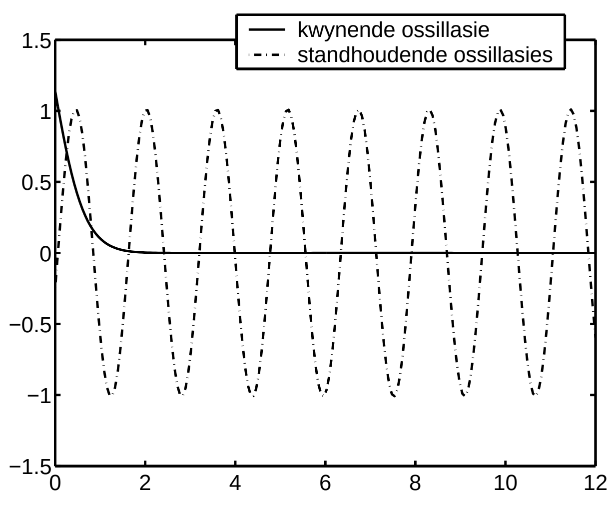
Notice that the complementary function

$$x_c(t) = e^{-3t}\left(\frac{38}{51}\cos t - \frac{86}{51}\sin t\right)$$

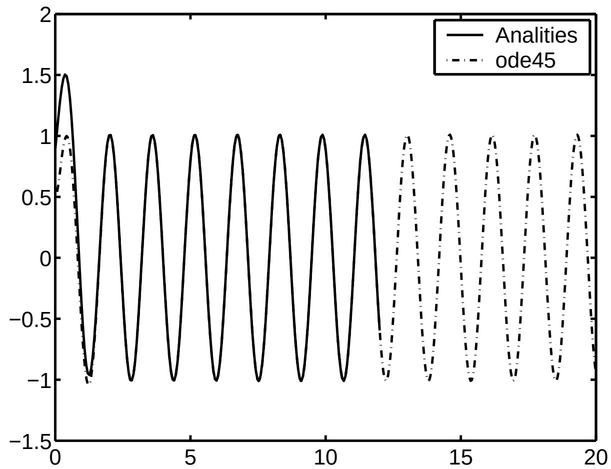
in (28) possesses the property that $\lim_{t\to\infty} x_c(t) = 0$. Since $x_c(t)$ becomes negligible (namely, $\to 0$) as $t \to \infty$, it is said to be a **transient term,** or **transient solution.** Thus for large time the displacements of the weight in the preceding problem are closely approximated by the particular solution $x_p(t)$. This latter function is also called the **steady-state solution**. When F is a periodic function, such as $F(t) = F_0 \sin \gamma t$ or $F(t) = F_0 \cos \gamma t$, the general solution of (25) consists of

$$x(t) = \textit{transient} + \textit{steady-state}.$$

$$\begin{aligned}
x(t) &= x_{kwynende}(t) + x_{standhoudend}(t) \\
\text{waar} & \\
x_{kwynende}(t) &= e^{-3t}\Big[\frac{38}{51}\cos(t) - \frac{86}{51}\sin(t)\Big] \\
x_{standhoudend}(t) &= -\frac{25}{102}\cos(4t) + \frac{50}{51}\sin(4t)
\end{aligned}$$



(d)



(d)