

# Advanced Algorithms: Homework 1

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## 1 R-5.3

Skedulering van  $T = (1, 2), (1, 3), (1, 4), (2, 5), (3, 7), (4, 9), (5, 6), (6, 8), (7, 9)$

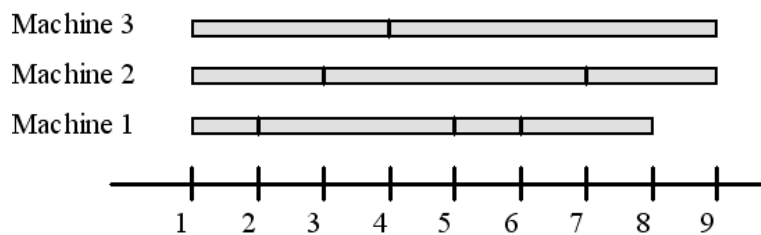


Figure 1: Skedulering van T

## 2 R-5.4

### 2.1 (a)

$$T(n) = 2T(n/2) + \log n$$

$$a = 2, b = 2$$

$$n^{\log_2 2} = n$$

Case 2 met  $k = 1$

$T(n)$  is  $O(n \log^2 n)$

## 2.2 (b)

$$T(n) = 8T(n/2) + n^2$$

$$a = 8, b = 2$$

$$n^{\log_2 8} = n^3$$

Case 1 met  $\epsilon = 1$

$T(n)$  is  $\theta(n^3)$

## 2.3 (c)

$$T(n) = 16T(n/2) + (n \log n)^4$$

$$a = 16, b = 2$$

$$n^{\log_2 16} = n^4$$

Case 2 met  $k = 4$

$T(n)$  is  $\theta(n^4 \log^5 n)$

## 2.4 (d)

$$T(n) = 7T(n/3) + n$$

$$a = 7, b = 3$$

$$n^{\log_3 7} = n^{1.77124}$$

Case 1 met  $\epsilon = 0.77124$

$T(n)$  is  $\theta(n^{1.77124})$

## 2.5 (e)

$$T(n) = 9T(n/3) + n^3 \log n$$

$$a = 9, b = 3$$

$$n^{\log_3 9} = n^3$$

Case 2 met  $k = 1$

$T(n)$  is  $\theta(n^3 \log^2 n)$

## 3 R-5.6

$$Z = XY$$

$$X = \begin{bmatrix} 3 & 2 \\ 4 & 8 \end{bmatrix} Y = \begin{bmatrix} 1 & 5 \\ 9 & 6 \end{bmatrix}$$

$$\begin{aligned}
S_1 &= 3(5 - 6) = -3 \\
S_2 &= (3 + 2)6 = 30 \\
S_3 &= (4 + 8) = 12 \\
S_4 &= 8(9 - 1) = 64 \\
S_5 &= (3 + 8)(1 + 6) = 77 \\
S_6 &= (2 - 8)(9 + 6) = -90 \\
S_7 &= (3 - 4)(1 + 5) = -6
\end{aligned}$$

$$\begin{aligned}
I &= S_5 + S_6 + S_4 - S_2 = 21 \\
J &= S_1 + S_2 = 27 \\
K &= S_3 + S_4 = 76 \\
L &= S_1 - S_7 - S_3 + S_5 = 68
\end{aligned}$$

$$Z = \begin{bmatrix} 21 & 27 \\ 76 & 68 \end{bmatrix}$$

#### 4 R-5.7

$$\begin{aligned}
(a + bi)(c + di) &= e + fi \\
(ac - bd) + (ad + bc)i &= e + fi
\end{aligned}$$

$$\begin{aligned}
ac - bd &= e & ad + bc &= f \\
ac - b(c + d) + bc &= e & ad + b(c + d) - bd &= f \\
c(a + b) - b(c + d) &= e & b(c + d) + d(a - b) &= f
\end{aligned}$$

Dus om  $e + fi$  in slegs drie vermenigvuldig operasies te bereken moet die volgende drie uitgevoer word:

$$\begin{aligned}
c(a + b) \\
d(a - b) \\
b(c + d)
\end{aligned}$$

#### 5 R-5.9

Gegee die matrikse: A is 10x5  
 B is 5x2  
 C is 2x20  
 D is 20x12  
 E is 12x4  
 F is 4x60

As die greedy method (minste vermenigvuldig operasies) gebruik word om  $ABCDEF$  op te los dan word die volgende gekry:

$$\begin{aligned}
& A \cdot B \\
& (A \cdot B) \cdot C \\
& ((A \cdot B) \cdot C) \cdot (D \cdot E) \\
& (((A \cdot B) \cdot C) \cdot (D \cdot E)) \cdot F \\
& 100 + 400 + 960 + 800 + 2400 = 4660
\end{aligned}$$

Dit sal dus 4660 vermenigvuldig operasies vat om te bereken.

## 6 P-5.2

### 6.1 Fractional Knapsack

Om uit te voer tik in *java FraqKnapsack*.

### 6.2 0-1 Knapsack

Om uit te voer tik in *java Knapsack*.

### 6.3 Bruteforce 0-1 Knapsack

Om uit te voer tik in *java bruteKnapSack*.