Universiteit van Stellenbosch
Departement Rekenaarwetenskap

Rekenaarwetenskap 324: Semestertoets

19 April 2004

Dosent:Brink vd Merwe
Naam: Memo
Studentenr: XXX

Tyd: 2 uur
VolPunte: 50 (55 beskikbaar)
Beantwoord al die vrae.
Answer all the questions.

Question 1 [31 marks]

(a) [3] True or False: (Points will be deducted for an incorrect answer.)
(i) If $L_1 \subseteq L_2$ and L_1 is not regular, then L_2 is not regular.

False, $\mathbf{L_1 = \{0^n1^n | n \geq 0\}, L_2 = \{0,1\}^*}$

(ii) If L_1 and L_2 are nonregular, then $L_1 \cup L_2$ is nonreguler.

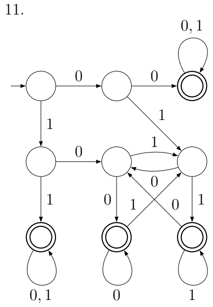
False, $\mathbf{L_1 = \{0^n1^n | n \geq 0\}, L_2 = \{0,1\}^* \setminus L_1}$

(iii) If L is nonregular, then the complement of L is nonregular.

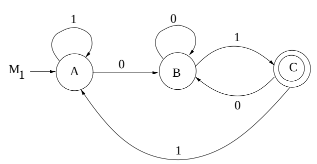
True

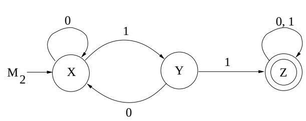
(b) [1] Fill in the blank. Suppose $L \subseteq \Sigma^*$ is a regular language. If every DFA accepting L has at least n states, then every NFA accepting L has at least .... $\lceil \log_2 \mathbf{n} \rceil$........ states.

(c) [3] Draw a DFA recognizing the language of all strings in $\{0,1\}^*$ that begin or end with 00 or 11.

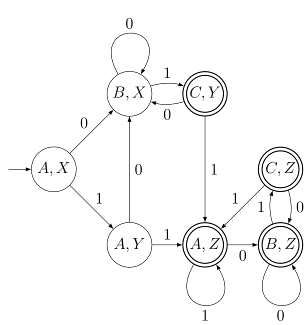


(d) [6] The DFA M_1 below recognizes the language L_1 and M_2 recognizes the language L_2.

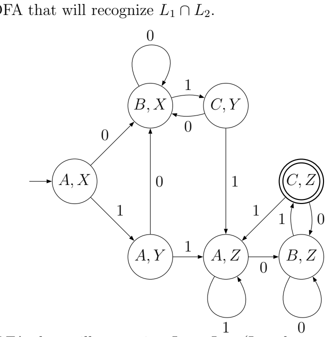




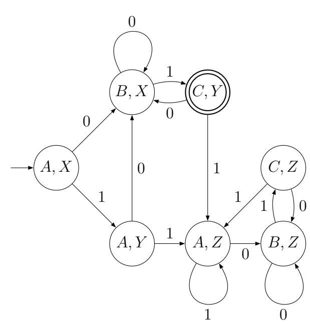
(i) Draw a DFA that will recognize $L_1 \cup L_2$.



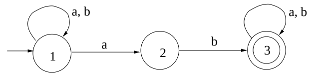
(ii) Draw a DFA that will recognize $L_1 \cap L_2$.

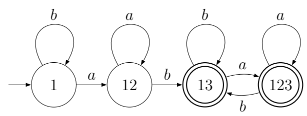


(iii)Draw a DFA that will recognize $L_1 - L_2$. (In other words, this DFA should recognize the strings that are L_1 but not in L_2.)

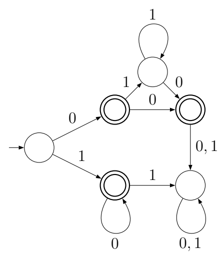


(e) [3] Use the subset construction to draw a DFA that is equivalent to the following NFA.





(f) [3] Draw a DFA equivalent to the regular expression $0 + 10^* + 01^*0$



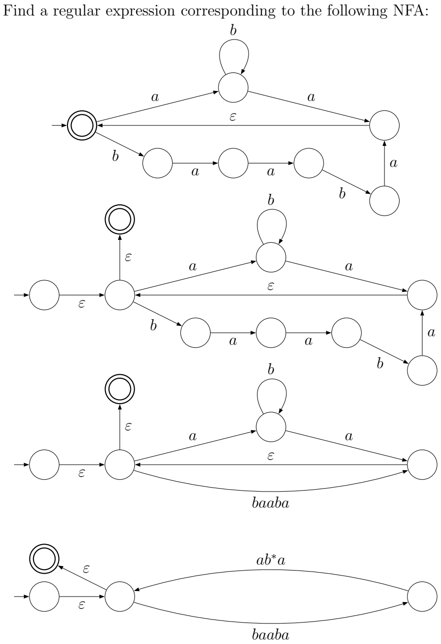
(g) [4] Find regular expressions corresponding to each of the following subsets of $\{0, 1\}^*$.
(i) The language of all strings that do not end with 01.

$$\varepsilon \cup \mathbf{1} \cup (\mathbf{0} \cup \mathbf{1})^*\mathbf{0} \cup (\mathbf{0} \cup \mathbf{1})^*\mathbf{11}$$

(ii) The language of all strings in which the number of 0's is even.

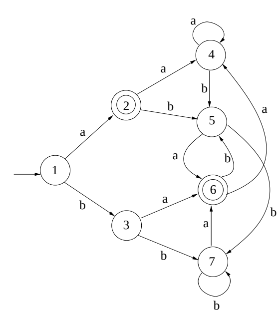
$$\mathbf{1}^*(\mathbf{01}^*\mathbf{01}^*)^*$$

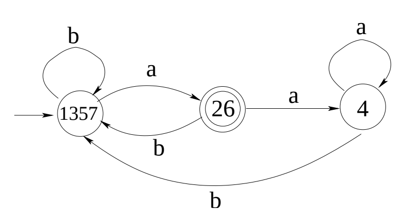
(h) [4] Find a regular expression corresponding to the following NFA:



$(\mathbf{ab}^*\mathbf{a} \cup \mathbf{baaba})^*$

(i) [4] Find the minimal DFA equivalent to the following DFA:





1						
X	2					
	X	3				
X	X	X	4			
	X		X	5		
X		X	X	X	6	
	X		X		X	7

Question 2 [11 marks]

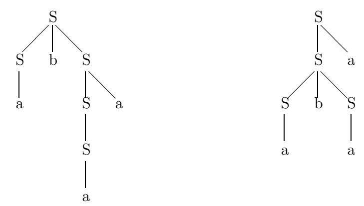
(a) [4] Find CFG's for the following languages:
(i) The language of odd-length strings in $\{a,b\}^*$ with middle symbol a.

S→aSa|aSb|bSa|bSb|a

(ii) $\{a^i b^j c^k | i = j + k; \;\; j, k \geq 0\}$

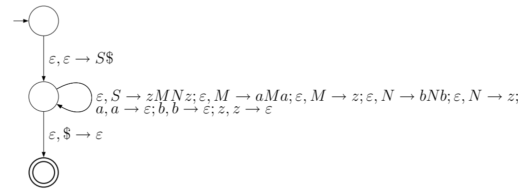
S→aSc|T
T→aTb|ε

(b) [2] Show that the CFG with productions
$S \rightarrow a \mid Sa \mid bSS \mid SSb \mid SbS$
is ambiguous by considering the string $abaa$.

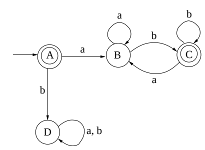


(c) [3] Draw a state diagram diagram for a pushdown automaton that recognizes the language generated by the following CFG. You may use transitions where you push more than one symbol on the stack at a time.

$$S \rightarrow zMNz$$
$$M \rightarrow aMa|z$$
$$N \rightarrow bNb|z$$



(d) [2] Let L be the language recognized by the DFA below. Give a CFG without ε-productions that generates the language $L - \{\varepsilon\}$.



A → aB|bD
D → aD|bD
B → aB|b
C → b|bC|aB

Question 3 [13 marks]

(a) [4] State the pumping lemma for regular languages.

Suppose h is regular, then there exists a value for p, so that if s ∈ h and |s| ≥ p, we can write s as xyz such that

1. **for all i ≥ 0, $xy^iz \in h$**
2. **$|y| > 0$**
3. **$|xy| \leq p$**

(b) [4] State the pumping lemma for context-free languages.

Suppose h is context free, then there exists a value for p, so that if s ∈ h and |s| ≥ p, we can write s as uvwxy such that

1. **for all i ≥ 0, $uv^iwx^iy \in h$**
2. **$|vx| > 0$**
3. **$|vwx| \leq p$**

(c) [5] Decide whether the language $L = \{xcx | x \in \{a,b\}^*\}$ is context-free, and prove your answer.

Not context free
Assume L is context-free and use the pumping lemma to obtain a contradiction by pumping up and obtaining a string not in L.
Let s = $a^pb^pca^pb^p$ = uvwxy

1. **If v or x contains a c, pumping up will give a string with too many c's**
2. **If v and x are both left of c, pumping up will give a string of the form ycx with |y| > |x|**
3. **Similarly if v and x are both right of c, pumping up will give a string of the form xcy with |y| > |x|**
4. **If v is left of c and x right of c, we use |vwx| ≤ p, and pump up to get a sting of the form ycz where y has fewer b's than z**
5. **The cases v = ε and x = ε are included in the 2nd and 3rd case.**