TW 244 Huiswerk #1 2004

Probleem 1

(a) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = 2\int \sin u\,du$ $\qquad u = \sqrt{x}$, $\quad du = \frac{1}{2}x^{-1/2}\,dx$

$= -2\cos u + C$

$= -2\cos\sqrt{x} + C$

(b) $\int x \ln x\,dx = \ln x\left(\frac{1}{2}x^2\right) - \int \frac{1}{x}\left(\frac{1}{2}x^2\right)dx + C$

deelwyse integrasie

$= \frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx + C$

$= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$

(c) $\int \frac{x^3+1}{x-1}\,dx = \int\left(x^2+x+1+\frac{2}{x-1}\right)dx$

langdeling

$= \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + 2\ln|x-1| + k$

(d) $$\int \frac{1}{x^2-4}\,dx = \int \frac{1}{(x-2)(x+2)}\,dx$$

parsiële breuke

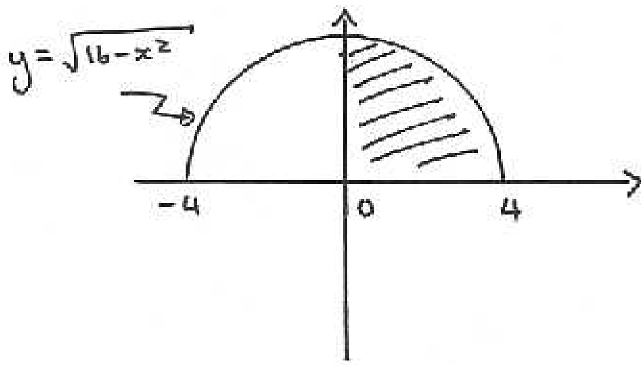
$$= \int \left(\frac{1/4}{x-2} + \frac{-1/4}{x+2} \right) dx$$

$$= \frac{1}{4}\left(\ln|x-2| - \ln|x+2| \right) + C$$

(e) $$\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx$$

$$= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x + k \right]$$

(f) $$\int_0^4 \sqrt{16-x^2}\,dx$$ = area van $\frac{1}{4}$ van sirkel met radius 4



$$= \frac{1}{4}\pi\,(\text{radius})^2$$

$$= \frac{1}{4}\pi\,(4)^2 = 4\pi$$

Probleem 2

(a) $y'(t) = e^{-t} \Rightarrow y(t) = -e^{-t} + k$

beginwaarde $y(0)=1 \Rightarrow 1 = -e^{-0} + k$

$1 = -1 + k$

$k = 2$

$\Rightarrow y(t) = -e^{-t} + 2$

(b) $y''(t) = \cos t \Rightarrow y'(t) = \sin t + k$

beginwaarde $y'(0) = 1 \Rightarrow 1 = 0 + k$

$k = 1$

$y'(t) = \sin t + 1$

$\Rightarrow y(t) = -\cos t + t + c$

beginwaarde $y(0) = 0 \Rightarrow 0 = -1 + 0 + c$

$c = 1$

$\Rightarrow y(t) = -\cos t + t + 1$

Probleem 3

(a) $x(t) = (\alpha + \beta t)e^t$

$x'(t) = (\alpha + \beta t)e^t + \beta e^t = (\alpha + \beta + \beta t)e^t$

$x''(t) = (\alpha + \beta + \beta t)e^t + \beta e^t = (\alpha + 2\beta + \beta t)e^t$

Stel in DV

$x''(t) - 2x'(t) + x(t)$

$= (\alpha + 2\beta + \beta t)e^t - 2(\alpha + \beta + \beta t)e^t + (\alpha + \beta t)e^t$

$= \left[(\alpha - 2\alpha + \alpha) + (2\beta - 2\beta) + (\beta t - 2\beta t + \beta t)\right]e^t$

$= 0 \quad \Rightarrow$ bevredig DV.

(b) $x(0) = 1 \quad \Rightarrow \quad 1 = \alpha + 0 \quad \Rightarrow \quad \alpha = 1$

$x'(0) = 0 \quad \Rightarrow \quad 0 = 1 + \beta + 0 \quad \Rightarrow \quad \beta = -1$

$\Rightarrow$ vir gegewe beginwaardes $x(t) = (1 - t)e^t$

(c) $x(-1) = 0 \quad \Rightarrow \quad 0 = (\alpha - \beta)e^{-1} \quad \Rightarrow \quad \alpha - \beta = 0$

$x(1) = 1 \quad \Rightarrow \quad 1 = (\alpha + \beta)e \quad \Rightarrow \quad \alpha + \beta = e^{-1}$

twee vergelykings in twee onbekendes

Moet vir α en β oplos in

$$\alpha - \beta = 0 \quad \ldots \text{①}$$

$$\alpha + \beta = e^{-1} \quad \ldots \text{②}$$

Uit ① $\alpha = \beta$

$\Rightarrow$ uit ② $2\alpha = e^{-1}$

$$\alpha = \frac{1}{2}e^{-1}$$

dan $\beta = \frac{1}{2}e^{-1}$

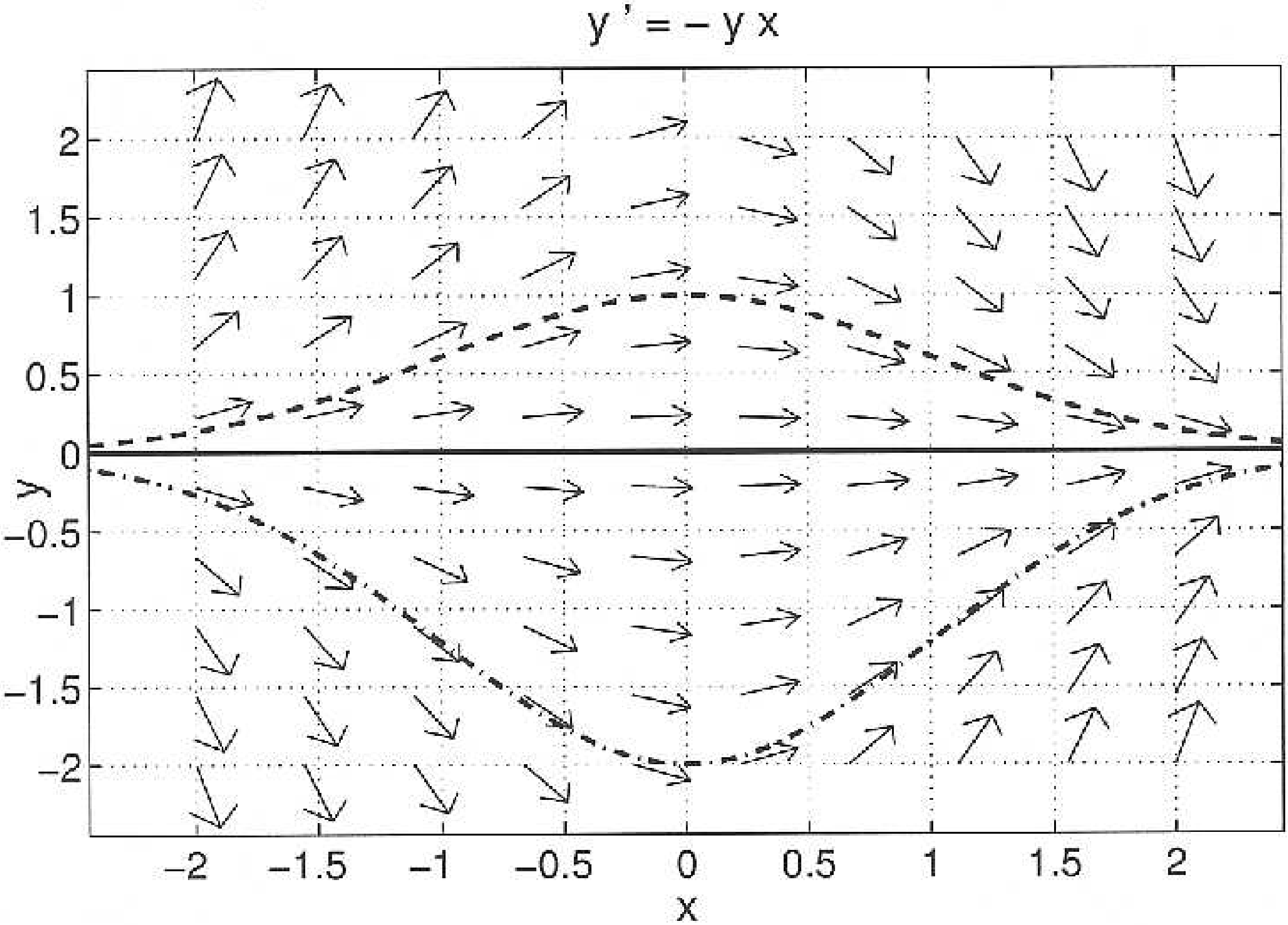
Vir gegewe randvoorwaardes

$$x(t) = \frac{1}{2}e^{-1}(1+t)e^{t}$$

$$= \frac{1}{2}(1+t)e^{t-1}$$

Probleem 4:

(a) & (b)



(c) $y(x) = C e^{-\frac{1}{2}x^2}$

$$y'(x) = C e^{-\frac{1}{2}x^2}(-x) = y(-x) = -xy$$

$$\Rightarrow \quad \frac{dy}{dx} = -xy .$$