Universiteit van Stellenbosch

Toegepaste Wiskunde 314

Tutoriaal 11: Oplossings

(1) (a) Yes

    (b) No

    (c) No

(2) (a) 3

    (b) Parity–check gives $6x_6 + 7x_7 = 7$ or $x_7 = 7x_6 + 1$; solutions for $x_6 x_7$ are...
01, 18, 24, 30, 47, 53, 76, 82, 99.

(3) 0000000000 and 0505000000 are two codewords that are Hamming distance two apart, so the minimum distance of the code is two; hence single errors cannot be corrected.

(4) (a) $S(\mathbf{y}) = (\sum_{i=1}^{10} y_i , \sum_{i=1}^{10} iy_i)$.

    (b) $S(\mathbf{y}) = ((\sum_{i=1}^{10} x_i) + k , (\sum_{i=1}^{10} ix_i) + jk) = (k, jk)$.

    (c) $S(0617960587) = (5, 9)$, so $k = 5$ and $j = 4$ and therefore
codeword is $0617960587 - 0005000000 = 0612960587$.
$S(3617960587) = (8, 1)$, so $k = 8$ and $j = 7$ and therefore
codeword is $3617960587 - 0000008000 = 3617963587$.

(5) A standard form generator matrix for E_n is

$$
G = \begin{bmatrix}
1 & 0 & . & . & 0 & 1 \\
0 & 1 & . & . & 0 & 1 \\
. & . & . & . & . & . \\
. & . & . & . & . & . \\
0 & 0 & . & . & 1 & 1
\end{bmatrix}
$$

and so a generator matrix for $E_n^{\perp}$ is

$$
G = \begin{bmatrix} 1 & 1 & . & . & . & 1 \end{bmatrix}
$$

hence $E_n^{\perp}$ is the binary repetition code of length n.