NAAM: Oplossings US NR. _____

INSTRUKSIES:

(a) 40 minute, 2 probleme, 20 punte, toeboek.

(b) Toon al jou bewerkings en motiveer alle stappe. Korrekte antwoorde verdien nie volpunte sonder die nodige verduideliking nie.

(c) **Let Wel:** Die wenke hieronder kan enige plek in die vraestel **sonder bewys** gebruik word.

(d) Moenie omblaai voordat u aangesê word om dit te doen nie.

INSTRUCTIONS:

(a) 40 minutes, 2 problems, 20 marks, closed book.

(b) Calculations are to be shown and all steps must be justified. Correct answers do not earn full marks without the necessary explanation.

(c) **Note:** *The hints below may be used* **without proof** *anywhere in the paper.*

(d) Do not turn the page until you are told to do so.

Wenke/Hints:

$$\ln(1+x) = x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 - \ldots, \quad |x| < 1, \qquad \sin x = x - \tfrac{1}{3!}x^3 + \tfrac{1}{5!}x^5 - \ldots,$$

$$\arctan x = x - \tfrac{1}{3}x^3 + \tfrac{1}{5}x^5 - \ldots, \quad |x| < 1, \qquad \cos x = 1 - \tfrac{1}{2!}x^2 + \tfrac{1}{4!}x^4 - \ldots.$$

$$\int \frac{dx}{x^2+1} = \arctan x + C, \qquad \int \frac{dx}{x^2-1} = \frac{1}{2}\ln\left(\frac{x-1}{x+1}\right) + C$$

$$\frac{dN}{dt} = bN - sN^2, \quad N(0) = \alpha \qquad \Longrightarrow \qquad N = \frac{b\alpha}{s\alpha + (b - s\alpha)e^{-bt}}$$

$$\frac{dy}{dx} + p(x)\,y = q(x) \qquad \Longrightarrow \qquad \text{Int. Faktor} = e^{\int p(x)\,dx}$$

$$\text{Newton:} \quad f(x) = 0 \qquad \Longrightarrow \qquad x_{n+1} = x_n - f(x_n)/f'(x_n)$$

Vraag 1 (10 punte)

Gebruik Laplace transforms (tabel op laaste bladsy van toets) om die volgende aanvangswaardeprobleem op te los

Question 1 *(10 marks)*

Use Laplace transforms (table on last page of test) to solve the following initial value problem

$$y'' + 4y = 3\cos(t), \qquad y(0) = y'(0) = 0.$$

$$s^2Y(s) - sy(0) - y'(0) + 4Y(s) = 3\,\frac{s}{s^2+1}$$

$$s^2Y(s) + 4Y(s) = \frac{3s}{s^2+1}$$

$$Y(s) = \frac{3s}{(s^2+1)(s^2+4)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+4}$$

$$3s = (As+B)(s^2+4) + (Cs+D)(s^2+1)$$

$s=0 \Rightarrow \quad 0 = 4B + D \quad \ldots\ldots ①$

$s=i \Rightarrow \quad 3i = (Ai+B)(-1+4) + 0 \quad \ldots ②$

$3i = 3Ai + 3B \quad \Rightarrow \quad A=1$ en $B=0$

$\Rightarrow$ vanaf ① is $D=0$

$3s = s(s^2+4) + Cs(s^2+1)$ ← Stel in en maal uit om koëffisiënt te vergelyk en dan vir C op te los.

$= s^3 + 4s + Cs^3 + Cs$

$= (C+1)s^3 + (C+4)s$

$\Rightarrow \quad C+1 = 0 \quad \Rightarrow \quad C = -1$

$$\Rightarrow \quad Y(s) = \frac{s}{s^2+1} - \frac{s}{s^2+4}$$

$$\Rightarrow \quad \boxed{y(t) = \cos t - \cos 2t}$$

Vraag 2 (10 punte)

Gebruik Laplace transforms (tabel op laaste bladsy van toets) om die volgende lineêre stelsel van DV's op te los

Question 2 *(10 marks)*

Use Laplace transforms (table on last page of test) to solve the following system of linear DEs

$$\begin{aligned} x' + x &= y \\ y' - y &= x \end{aligned}$$

met aanvangswaardes

with initial values

$$x(0) = y(0) = 1.$$

$$sX(s) - x(0) + X(s) = Y(s)$$

$$sY(s) - y(0) - Y(s) = X(s)$$

$$\left.\begin{aligned} (s+1)X(s) - Y(s) &= 1 \quad \cdots \textcircled{1} \\ -X(s) + (s-1)Y(s) &= 1 \quad \cdots \textcircled{2} \end{aligned}\right\}$$

Maal ② met $(s+1)$:

$$\left.\begin{aligned} (s+1)X(s) - Y(s) &= 1 \quad \cdots \textcircled{3} \\ -(s+1)X(s) + (s^2-1)Y(s) &= s+1 \quad \cdots \textcircled{4} \end{aligned}\right\}$$

③ + ④

$$(s^2 - 1 - 1)Y(s) = s+2$$

$$Y(s) = \frac{s+2}{s^2-2} = \frac{s}{s^2-2} + \frac{2}{s^2-2}$$

Stel in ② om te kry:

$$X(s) = -1 + (s-1)\frac{s+2}{s^2-2} = \frac{s}{s^2-2}$$

$$y(t) = \cosh\sqrt{2}t + \frac{2}{\sqrt{2}}\sinh\sqrt{2}t$$

$$= \cosh\sqrt{2}t + \sqrt{2}\sinh\sqrt{2}t$$

$$x(t) = \cosh\sqrt{2}t$$

Nommer	$F(s) = \int_0^\infty e^{-st} f(t)\ dt$	$f(t)$
1	$\frac{n!}{s^{n+1}}$	$t^n \quad (n = 0, 1, 2, \ldots)$
2	$\frac{1}{s-a}$	e^{at}
3	$\frac{n!}{(s+a)^{n+1}}$	$t^n e^{-at} \quad (n = 0, 1, 2, \ldots)$
4	$\frac{1}{2}\sqrt{\pi} s^{-\frac{3}{2}}$	$\sqrt{t}$
5	$\frac{\Gamma(k+1)}{s^{k+1}}$	$t^k \quad (k > -1)$
6	$\frac{\Gamma(k+1)}{(s+a)^{k+1}}$	$t^k e^{-at} \quad (k > -1)$
7	$\frac{a}{s^2+a^2}$	$\sin at$
8	$\frac{s}{s^2+a^2}$	$\cos at$
9	$\frac{a}{s^2-a^2}$	$\sinh at$
10	$\frac{s}{s^2-a^2}$	$\cosh at$
11	$\frac{b}{(s-a)^2+b^2}$	$e^{at} \sin bt$
12	$\frac{s-a}{(s-a)^2+b^2}$	$e^{at} \cos bt$
13	$\frac{2a^3}{(s^2+a^2)^2}$	$\sin at - at \cos at$
14	$\frac{2as}{(s^2+a^2)^2}$	$t \sin at$
15	$\frac{s^2-a^2}{(s^2+a^2)^2}$	$t \cos at$
16	$\frac{s^3}{(s^2+a^2)^2}$	$\cos at - \frac{1}{2} at \sin at$