NAAM: Antwoorde US NR: ______________

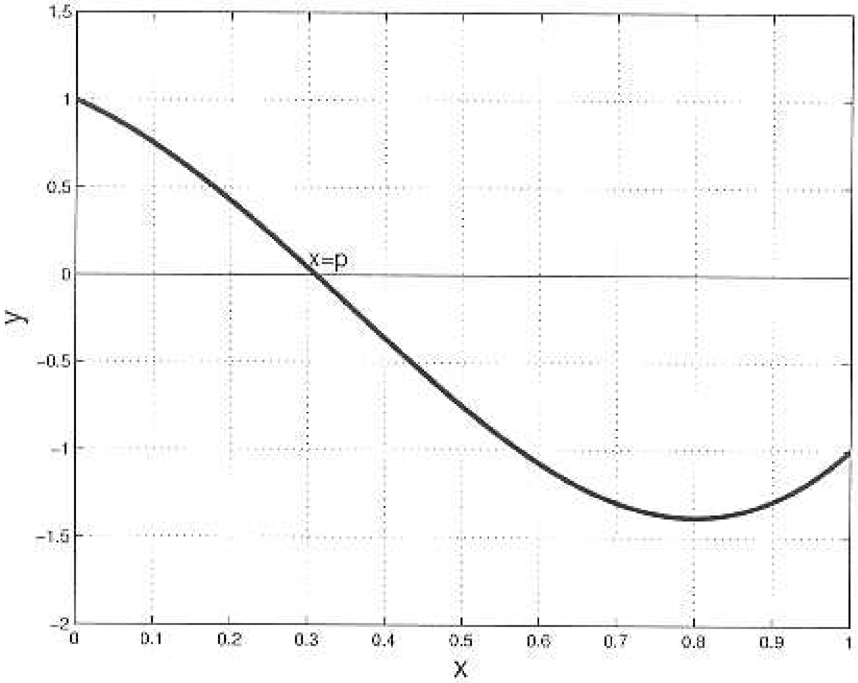
INSTRUKSIES: Drie probleme, 20 punte, 45 minute. Toon alle berekenings by Probleem 3.

INSTRUCTIONS: Three problems, 20 marks, 45 minutes. Show all work on Problem 3.

Probleem 1 (3 + 2 = 5 punte)

Die figuur toon die grafiek van 'n funksie $y = f(x)$, met wortel $x = p$ soos aangetoon.

The figure shows the graph of a function $y = f(x)$, with root $x = p$ as shown.



(a) Gestel die halveringsmetode word gebruik om die wortel p te benader, met aanvanklike interval $[a_0, b_0] = [0, 1]$. Toon aan hoe die metode sal vorder, deur die tabel hier onder te voltooi:

Suppose the bisection method is used to approximate the root p, with initial interval $[a_0, b_0] = [0, 1]$. Indicate how the method will proceed, by completing the table below:

(b) Herhaal deel (a) met die Regula-Falsi metode (slegs benaderde waardes verlang):

Repeat part (a) with the Regula-Falsi method (only approximate values required):

Halvering/*Bisection*

n	a_n	b_n
0	0	1
1	0	1/2
2	1/4	1/2
3	1/4	3/8

Regula-Falsi

n	a_n	b_n
0	0	1
1	0	0.5
2	0.29	0.5

Probleem 2 (6 *punte*)

'n Sekere vergelyking $f(x) = 0$ het eksakte wortel $p = \pi/3$. Wanneer 'n spesifieke numeriese metode op hierdie vergelyking toegepas word, word die volgende drie benaderings, x_n, verkry. Die absolute foute word ook vir u gerief getoon.

A certain equation $f(x) = 0$ has the exact root $p = \pi/3$. When a particular numerical method is applied to this equation, the following three approximations, x_n, are obtained. The absolute errors are also shown for your convenience.

① $1.4028e{-7} \approx C \cdot (6.9751e{-4})^{\alpha}$

② $5.5732e{-15} \approx C \cdot (1.4028e{-7})^{\alpha}$

②÷① $\frac{5.5732e{-15}}{1.4028e{-7}} \approx \left(\frac{1.4028e{-7}}{6.9751e{-4}}\right)^{\alpha}$

	x_n	$\lvert x_n - p\rvert$
1	1.04789506304527	6.9751e-04
2	1.04719769147403	1.4028e-07
3	1.04719755119660	5.7732e-15

$3.9758e{-8} \approx (2.0112e{-4})^{\alpha}$

$\alpha \approx \frac{\log(3.9758e{-8})}{\log(2.0112e{-4})} \approx 2$

(a) Skat die orde van konvergensie, α, deur gebruik te maak van die definisie

Estimate the order of convergence, α, by using the definition

$$\lim_{n\to\infty} \frac{|e_{n+1}|}{|e_n|^{\alpha}} = C.$$

(Omsirkel die beste skatting.) *(Circle the best estimate.)*

(A) $\alpha = 1$ (B) $\alpha = 1.5$ (C) $\alpha = 2$ (D) $\alpha = 2.5$ (E) $\alpha = 3$

(b) Skat ook die foutkonstante, C. (Omsirkel die beste skatting.)

stel $\alpha = 2$ in ① of ② hierbo: $C \approx 0.28\ldots$

Also estimate the error constant, C. (Circle the best estimate.)

(A) $C = 0.03$ (B) $C = 0.06$ (C) $C = 0.1$ (D) $C = 0.3$ (E) $C = 1.1$

(c) Wanneer 'n ander numeriese metode op hierdie vergelyking toegepas word, word 'n waarde $\alpha = 1.6$ verkry. Watter tipe konvergensie is hierdie?

When a different numerical method is applied to this equation a value of $\alpha = 1.6$ is obtained. What type of convergence is this?

(A) lineêr (*linear*) (B) super-lineêr (*super-linear*) (C) kwadraties (*quadratic*)

(D) super-kwadraties (*super-quadratic*) (E) kubies (*cubic*)

(d) Die metode in deel **(c)** is heel waarskynlik

The method in part **(c)** *is most likely*

(A) Secant (B) Newton (C) Halley (D) Heron (E) Horner

(e) Die metode in deel **(a)** is heel waarskynlik

The method in part **(a)** *is most likely*

(A) Secant (B) Newton (C) Halley (D) Brent (E) Horner

Probleem 3 (3 + 3 + 3 = 9 *punte*)

Beskou die vergelyking (x in radiale)

Consider the equation (x in radians)

$$x^2 = 1 - \sin x.$$

(a) Gebruik 'n grafiese metode om vas te stel hoeveel reële wortels daar is.

Use a graphical method to determine how many real roots there are.

(b) Gebruik Newton se metode met aanvanklike skatting $x_0 = 0$ om 'n wortel te benader. Voer twee stappe van die metode uit, d.w.s., bereken x_1 en x_2. *Wenk:*

Use Newton's method with an intial guess $x_0 = 0$ to approximate a root. Execute two steps of the method, i.e., compute x_1 and x_2. Hint:

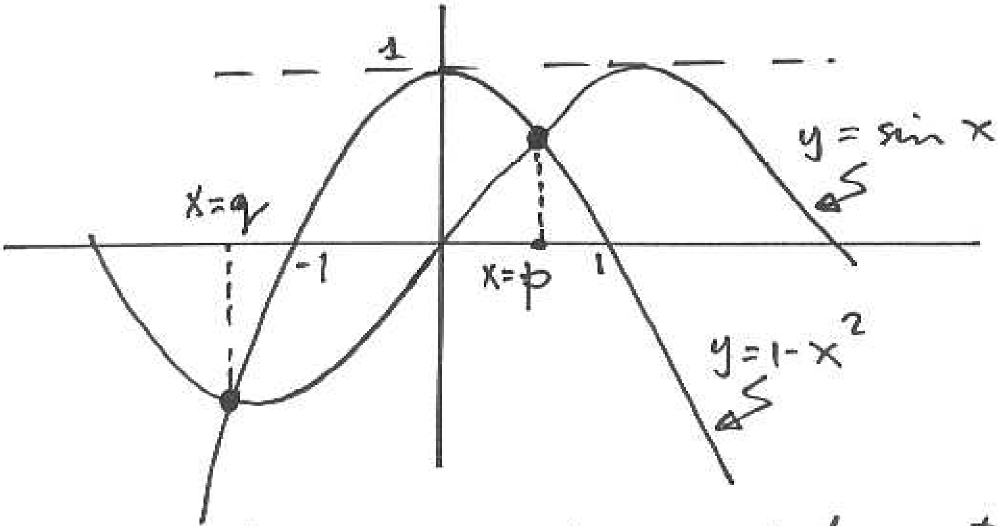
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

(c) Indien mens die iterasie van deel (b) verder sou voer, watter orde van konvergensie verwag jy om waar te neem? Lineêr, kwadraties, kubies? Dalk iets anders? *Wenk:*

Suppose one continues the iteration of part (b). What order of convergence do you expect to observe? Linear, quadratic, cubic? Perhaps something else? Hint:

$$x_{n+1} - p = \frac{f''(\xi_n)}{2f'(x_n)}(x_n - p)^2$$

(a) Skryf die vergelyking as $\sin x = 1 - x^2$ en skets die linker- en regterkante op dieselfde assestelsel:



Die grafiek suggereer twee reële wortels, by $x = q < 0$ en $x = p > 0$.

(b) $f(x) = x^2 + \sin x - 1 \quad , \quad f'(x) = 2x + \cos x$

$$x_{n+1} = x_n - \frac{x_n^2 + \sin x_n - 1}{2x_n + \cos x_n}$$

$$x_0 = 0$$

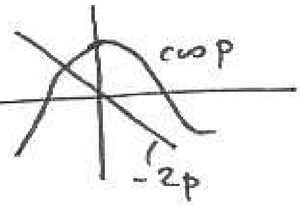
$$x_1 = 1 - \frac{0 + \sin 0 - 1}{2.0 + \cos 0} = 1$$

$$x_2 = 1 - \frac{1 + \sin 1 - 1}{2 \cdot 1 + \cos 1} \doteq 0.66875 \xrightarrow[\text{(konvergeer na } p\text{.)}]{}$$

(c) Newton se metode konvergeer kwadraties, tensy $f'(p) = 0$ of $f''(p) = 0$.

Nou $f'(p) = 0 \Rightarrow 2p + \cos p = 0$
$\Rightarrow \cos p = -2p$

Kan slegs 'n oplossing hê as $p < 0$, wat nie hier die geval is nie.



Ook $f''(x) = 2 + \cos x$
$f''(p) = 2 + \cos p \neq 0$
want $|\cos p| \leq 1$ vir alle p.

Dus $f'(p) = 0$ en $f''(p) = 0$ is beide onmoontlik, sodat die konvergensie kwadraties sal wees.