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Eksamen: RW242

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Voorletters en van:
Studentenommer:

Instruksies *Instructions*

- Beantwoord al die vrae.

  Answer all questions.

- Skryf asseblief met ink.

  Please write in ink.

- Gebruik die agterkant van elke bladsy vir rofwerk.

  Use the reverse side of each page for scribbling.

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1. Dui aan of die volgende uitdrukkings waar ("W") of vals ("V") is.

   Indicate whether the following expressions are true ("T") or false("F").

   (a) $\{n : \mathbb{N} \mid n < 4\} \subseteq \mathbb{P}\,\mathbb{N}$ ☐

   (b) $\#\,\mathbb{P}\{a, b, c, d, e, f, g\} = 127$ ☐

   (c) $\{n : \mathbb{N} \mid n < 3\} \setminus \varnothing = \{1, 2\}$ ☐

   (d) $\{(1,1),(2,2),(3,3)\} \fatsemi \{(1,1),(2,2)\} = \{(1,1),(2,2)\}$ ☐

   (e) $\varnothing \in \mathbb{P}\,\varnothing$ ☐ 10

2. R_1 en R_2 is relasies oor $T_1 \times T_2$ en $T_2 \times T_3$ respektiewelik. Dui aan of die volgende uitdrukkings waar ("W") of vals ("V") is.

   R_1 and R_2 are relations over $T_1 \times T_2$ and $T_2 \times T_3$ respectively. Indicate whether the following expressions are true ("T") or false ("F").

   (a) $(R_1 \fatsemi R_2)^{-1} = R_2^{-1} \fatsemi R_1^{-1}$ ☐

   (b) $\varnothing \oplus R_1 = R_1 \oplus \varnothing$ ☐

   (c) $R_1 \oplus R_2 = R_2 \oplus R_1$ ☐

   (d) $S \lhd\!\!\cdot\, R_1 = (T_1 \cap S) \lhd R_1$ for $S \subseteq T_1$ ☐

   (e) $R_2 \rhd S = R_2 \fatsemi (\mathrm{id}\, S)$ for $S \subseteq T_3$ ☐ 10

3. Vereenvoudig die onderstaande uitdrukking om om vas te stel of dit 'n toutologie is. Dui aan watter afleidingsreëls (De Morgan, distributiewe reël, ens.) gebruik word tydens elke stap.
Simplify the following expression to determine whether it is a tautology. Indicate which derivation rules (De Morgan, distributive law, etc.) are used during each step.
$(p \Rightarrow q) \Rightarrow ((q \Rightarrow r)) \Rightarrow (p \Rightarrow r))$

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4. (a) Herskryf die onderstaande uitdrukking deur slegs die universele kwantor ($\forall$) en die operators $\neg$ en $\vee$ te gebruik.
Rewrite the following expression by using only the universal quantifier ($\forall$) and the operators $\neg$ and $\vee$.
$(\exists\, ch : Channel \bullet Active(ch)) \Rightarrow (\exists\, p : Processor \bullet Busy(p))$

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(b) Skryf 'n uitdrukking in predikaat logika om die volgende vereiste voor te stel: "die skikking van heeltalle x met n elemente (met indeks i vanaf 0 tot $n-1$) is gesorteer in nie-dalende volgorde".
Write an expression in predicate logic to represent the following requirement: the array of integers x with n elements (with index i from 0 to $n-1$) is sorted in non-decreasing order".

. 2

(c) Skryf 'n lambda-uitdrukking om 'n funksie te definieer wat 'n nie-negatiewe heeltal kleiner as 6 afbeeld op 'n paar waarvan die eerste element die gegewe getal is en die tweede element 'n getal wat een groter is as die gegewe getal.
Write a lambda expression to define a function that maps a non-negative integer smaller than 6 to a pair of which the first element is the given number and the second element is a number that is one larger than the given number.

. 2

5. Die spesifikasie van 'n motor se sentrale sluitstelsel bevat die onderstaande formules. Gebruik die metode van indirekte bewysvoering om aan te toon dat as die enjin loop die deure nie gesluit sal wees nie.
   The specification of a car's central locking system contains the formulas shown below. Use the method of indirect proof to show that if the engine is running the doors will not be locked.

   - Indien die alarm in werking is, is die immobiliseerder ook in werking.
     If the alarm is enabled, the immobiliser is also enabled.
     pr1: $alarm \Rightarrow immob$
   - Die enjin kan nie loop terwyl die immobiliseerder in werking is nie.
     The engine cannot be running while the immobiliser is enabled.
     pr2: $\neg(run \land immob)$
   - As die deure gesluit is, sal die alarm in werking wees.
     If the doors are locked the alarm must be enabled.
     pr3: $locked \Rightarrow alarm$

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6. Die volgende is 'n gedeelte van 'n spesifikasie vir 'n lêerstelsel. Voltooi die spesifikasie deur die ontbrekende dele in te vul.
   The following is part of a specification for a file system. Complete the specification by filling in the missing parts.

   A file contains data stored as zero or more sectors (blocks of 512 bytes). Each sector has a unique number. The basic types SEC and $DATA$ are used to represent sectors and data respectively. A file is modelled as a partial function that maps sectors to data as shown by the following schema:

   File
   $contents : SEC \pfun DATA$

   Initially, a file contains no data as shown by schema *Init*.

   Init
   $File'$

   $contents' = \varnothing$

The operation *Read* returns the data associated with sector $s?$ if it exists.

Read

$\Xi File$

$s? : SEC$

$d! : DATA$

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A given sector $s?$ of a file can be overwritten by new data $d?$ by executing the operation *Write*. The command can only be executed if sector $s?$ exists.

Write

$\Delta File$

$s? : SEC$

$d? : DATA$

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4

A new sector $s?$ that stores data $d?$ can be added to a file by executing the operation *Add*. The command can only be executed if the file does not already contain a sector $s?$.

Add

$\Delta File$

$s? : SEC$

$d? : DATA$

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4

The operation *Delete* is used to delete a given sector $s?$ without knowing what data it contains.

Delete

$\Delta File$

$s? : SEC$

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7. Baie stelsels kan gemodelleer word as eindige outomate wat op gespesifiseerde maniere reageer op verskillende toevoerstringe. Teken 'n oorgangsdiagram om 'n outomaat voor te stel wat enige string van 0'e en 1'e aanvaar wat bestaan uit 'n onewe aantal 0'e en 'n onewe aantal 1'e.
   Many systems can be modelled as finite automata that react to different input strings in specified ways. Draw a transition diagram to represent an automaton that accepts any string composed of an odd number of 0's and an odd number of 1's.

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