RW778 Concurrent Programming

Homework Assignment H1

Due: Thu 2006.08.10

1. A coffee ccan contains black and white beans. The following process is repeated as long as possible:

   > Randomly select two beans from the can. If they are the same colour, throw them away but place one black bean into the can. (Assume an adequate supply of black beans.) If they are different colours, throw the black one away and place the white bean back into the can.

   Determine the relation between the initial contents of the coffee can and the colour of the final bean that remains. [2]

2. Determine $wp(S, R)$ for the following S and R.

	S	R
(a)	$i := i + 1$	$i > 0$
(b)	$i := i + 2 \; ; \; j := j - 2$	$i + j = 0$
(c)	$i := i + 1 \; ; \; j := j - 1$	$i * j = 0$
(d)	$z := z * j \; ; \; i := i - 1$	$z * j^i = c$

   [4]

3. Calculate and simplify $wp(I, qw + r = x \wedge r \geq 0)$ where

   $$I : \textbf{if } w \leq r \to r := r - w \; ; \; q := q + 1$$
   $$[] \; w > r \to \textbf{skip}$$
   $$\textbf{fi} \; .$$

   [2]

4. Calculate and simplify $wp(J, a > 0 \wedge b > 0)$ where

   $$J : \textbf{if } a > b \to a := a - b$$
   $$[] \; b > a \to b := b - a$$
   $$\textbf{fi} \; .$$

   [2]

5. Prove the following theorem:

$$\{b \geq 0\}$$
$$x := a \; ; \; y := b \; ; \; z := 0 \; ;$$
$$\textbf{do } (y > 0) \wedge even(y) \rightarrow y := y \div 2 \; ; \; x := x + x$$
$$[\,] \; odd(y) \qquad\qquad\quad \rightarrow y := y - 1 \; ; \; z := z + x$$
$$\textbf{od}$$
$$\{z = ab\}$$

using the invariant $y \geq 0 \wedge z + xy = ab$. [6]

6. Prove the following theorem:

$$\{0 < n\}$$
$$i := 1 \; ; \; k := 0 \; ;$$
$$\textbf{do } i < n \rightarrow \textbf{if } b[i] \leq b[k] \rightarrow \textbf{skip}$$
$$[\,] \; b[i] \geq b[k] \rightarrow k := i$$
$$\textbf{fi} \; ;$$
$$i := i + 1$$
$$\textbf{od}$$
$$\{\forall j : 0 \leq j < n : b[k] \geq b[j]\}$$

using the invariant $0 < i \leq n \wedge (\forall j : 0 \leq j < i : b[k] \geq b[j])$. [6]

Total marks: 22