TW324   Tut Toets #1 (Antwoorde)   2004

Probleem 1:

(a) x_+ is nie akkwaat bereken nie want

$$x_+ = \frac{-3001 + \sqrt{3001^2 + 12}}{6} = \frac{-3001 + 3001.00199\ldots}{6}$$

Katastrofiese kansellasie.

x_- is akkwaat, want die berekening van

$$x_- = \frac{-3001 - 3001.00199\ldots}{6}$$

behels geen kansellasie nie.

(b) $x_+ x_- = \frac{c}{a} = -\frac{1}{3}$

$$\Rightarrow x_+ = -\frac{1}{3} x_- .$$

In MATLAB

>> xplus = -1 / (3 * xminus);

Probleem 2

(a)   Aangesien $\sin x \approx x$, $\tan x \approx x$
en $\cos x \approx 1$ as $|x| \ll 1$

volg dat formules in f en g katastrofiese kansellasie behels. Dus $\underline{\underline{A}}$ & $\underline{\underline{B}}$

(b) Aangesien $\cos x \approx -1$ as $x \approx \pi$
behels die formule in 'n katastrofiese
kansellasie.   Dus $\underline{\underline{C}}$.

Probleem 3:

(a) $y_n = \int_1^e (\ln x)^n \, dx$

$$= \int_1^e (\ln x)^n \frac{d}{dx}(x) \, dx$$

$$= (\ln x)^n x \Big|_1^e - n\int_1^e (\ln x)^{n-1} \cdot \frac{1}{x} \cdot x \, dx$$

$$= (\ln e)^n \cdot e - (\ln 1)^n \cdot 1 - n \underbrace{\int_1^e (\ln x)^{n-1} \, dx}_{= \, y_{n-1}}$$

$$\underset{1}{{}^{\shortmid\shortmid}} \qquad \underset{0}{{}^{\shortmid\shortmid}}$$

$$= e - n\, y_{n-1}.$$

$$y_0 = \int_1^e (\ln x)^0 \, dx = \int_1^e dx = e - 1.$$

(b) Gestel y_0 word verteen na $y_0 + \varepsilon = \tilde{y}_0$.
Noem die nuwe ry van getalle $\{\tilde{y}_n\}$.

Nuwe rekursie $\tilde{y}_n = e - n\,\tilde{y}_{n-1}$
Oorspronklike $y_n = e - n\,y_{n-1}$

Trek af: $\tilde{y}_n - y_n = -n(\tilde{y}_{n-1} - y_{n-1})$

Laat
$E_n = \tilde{y}_n - y_n$ $E_n = -n\, E_{n-1}$
fout op stap n
voorstel. $E_0 = \tilde{y}_0 - y_0 = \varepsilon$

$$E_1 = -1 \cdot \hat{E}_0 = -\varepsilon$$

$$E_2 = -2 \cdot E_1 = 2\varepsilon$$

$$E_3 = -3 \cdot E_2 = -3 \cdot 2 \cdot \varepsilon$$

$$E_4 = -4 \cdot E_3 = 4 \cdot 3 \cdot 2 \cdot \varepsilon$$

$$\vdots$$

$$E_n = \underbrace{(-1)^n \, n!} \, \varepsilon$$

$$\longrightarrow \infty \text{ as } n \to \infty$$

Afrondingsfout word dus onbegrens vergroot sodat die algoritme onstabiel sal wees.