9

Answers

Section 1.5

1.(a) $X(t = X(0) + kt$

(b) $X(t) \to \infty$ as $t \to \infty$, but the limited amount of space and food in the bottle contradicts this. Hence, the model is not valid for large values of t.

(c To describe the best fit the vertical distance between the line and each data point must all be positive, otherwise errors cancel out. Use absolute values or squares of the differences. The best fit (for squares) is

$$X(t) = \frac{530}{3} + 102t$$

(and once you have read §2.2 you will know how I know this!) At any rate, the fit is not satisfactory, since the data points clearly show that the graph of $X(t)$ should bend downwards.

(d) $X(t) = \ 1 + a)^t X(0$

Interpretation: Again $X(t) \to \infty$ as $t \to \infty$, since $a > 0$, so that the model is not valid for large values of t.
Validation: The best fit (for squares) is

$$209(1.288)^t$$

which is unsatisfactory, since $X(5) = 740$

(e) Go, man, go

3.(a) (i) 4th order (ii) $-\infty < x < \infty$ (iii) linear

(b (i) first order (ii) $0 < x < \infty$ (iii) nonlinear

4.(a) No, $y(\pi) = 1$ is not satisfied

(b) No, $y = |x|$ is not differentiable at $x = 0$

(c) Yes

(d) Yes

7.(a) 0.603

(b) 0.357

(c) 0.655

Section 2.12

1.(a) 146,410

(b) 18.2 hours

2 50,238

3 104

4 (a) 0.25 and 3.95

(b) 22.7

5.(a) 0.20 and 4.98

(b) 20.2

6.(a) Yes

(b) No

(c) Yes

(d) Yes

9.(a) 371

(b) 376

10.(a) $\alpha(b - N)^{1+} \quad N(b - \alpha)^{1+c} e^{-at}$

(b) $N \to b$

(c) Inflection point at

$$\tau \quad \frac{1+c}{a} \ln \frac{c(b-\alpha)}{b(c-1+\sqrt{1+c})} + \frac{1}{a} \ln \frac{b(\sqrt{1+c}-1)}{\alpha c}$$

When $\alpha \quad b$, then $N(t) \quad b$ for $t \geq 0$

11 $m(t) = (\frac{\alpha}{\beta})^3(1 - e^{-\frac{\beta t}{3}})^3$

12. 29.8 years

13.(a) 0.0494 and 1.44×10^{-8}

(b) $1,459,000$ metric tons

(c) $3,435,000$ metric tons

14.(a) 323

(c 163 years

15.(a) 311

(b) 141

16. $w(t) = \frac{1-k}{k}(\alpha - \frac{b}{a})(e^{ka t} - 1)$ for $0 < k \leq 1$;
$w(t) = (a\alpha - b)t$ for $k = 0$;
Assumption: $b \leq a\alpha$

17. $k = 0$, $630,000

18.(a) $(a + 1)^{-1}$

(b) $a >$

19. 16 days

20. $x(t) = -(1-\alpha)e^{-kq\alpha - k(1-\alpha)(t-2p)+(1-\alpha^{-1})(e^{-k\alpha(t-2p)}-1)}$

22 675

23 β increases

24. 403 meters, 209 km/h

25. 92.45 seconds

27 $mpc^{-2}(\ln 3 - \frac{2}{3})$

65.6 km/h, 51.3 seconds

62.9 km/h

30.(b) $v(t) \quad 20000 + 10(250 - t) - 20147.62(250 \quad t)^{0.02}$

(c) $v(200) \quad -1286.75$ No lift-off!

31.(a) $10,574$ meters

(b) It is valid

32 $y \quad x^2 + 1$

33.(a) $y \quad \pm x\sqrt{x^2 + 2}$.

(c $y = \pm\sqrt{(x^2 \quad 1)(x^2 + 3)}$; graph not defined for $|x| <$

34.(a) (i) $y = \sqrt{2(\cos x + 1)}$
(ii) y : $\sqrt{2\cos x - 1}$ for $0 \leq x \leq \frac{\pi}{3}$
(iii) $y = \sqrt{2\cos x + 7}$ for $0 \leq x \leq \frac{2\pi}{3}$

35.(a) $y = \frac{1}{2}(1 + e^{-x^2})$

(b) $y = 1 + x^2 + \sqrt{1 + x^2}$

36.(a) $x^3 + y^3 = 2xy$

(b) $y^2 + 2xy \quad 3x^2 = 5$

37 $\ln(v^2 + cvx + kx^2 \quad \frac{2c}{\sqrt{4k - c^2}} \arctan \frac{2v + cx}{x\sqrt{4k - c^2}}$ constant

Section 2.13

Project F:

a) $p(t) \quad \beta + (\alpha - \beta)e^{-k(b+d)t}$

(b $p(t) = \alpha e^{-Bt} + \frac{A}{B}(1 - e^{-Bt}) + \frac{E}{w^2 + B^2}(B \sin wt - w \cos wt + we^{-Bt}$
with $A = k(a - c)$, $B = k(b + d)$ and $E = kf$.

(c) $p(t) \quad \frac{c-a}{b+d}(e^{\frac{b+d}{m-n}t} - 1) + \alpha e^{\frac{b+d}{m-n}t}$

Section 3.7

$y = \frac{t^2}{4}$ and $y = 0$ for $t \geq 0$. The function $f(t, y) = \sqrt{y}$
is not Lipschitz on any interval containing the point $y \quad 0$.

2. $y = \frac{2}{2 - t^2}$, $|t| < \sqrt{2}$

3. $\delta = 0.5$, $0 \leq y \leq 2$

4 The five values for the approximation are
1, 1, 1.01, 1.0304, 1.0623.

5. The nine values for the approximation are:
1, 1, 1.0025, 1.0075, 1.0151, 1.0254, 1.0386, 1.0548, 1.0742

6. $y \quad t + e$

10. $a(b - c)$

12. $6.129, \quad y = \frac{1}{1-t}$

13.(a) $3.94, 3.27, 2.81$

(c) $|error| \leq 4.06h, \quad h \leq 0.00025$

14.(a) $0.48, 1.16, 1.71, 2.18, 2.61$

(c) $|error| \leq 1.13$

19. $|e_i| \leq 1{,}987{,}975h^4$

Section 4.5

2 $(s - k)^{-1} \qquad (s > k)$

3 $2s^{-3} \qquad (s > 0)$

4. $k(s^2 - k^2)^{-1} \qquad (s > |k|)$

5. $\sqrt{\pi}s^{-\frac{1}{2}} \qquad (s > 0)$

6.(a) Yes

(b) No; undefined at $t \quad 0$.

(c) Yes

(d) No; undefined at $t \quad 2$

(e) Yes

10.(a) Yes

(b) No; not defined at $t = 1$

(c) Yes

(d) Yes

(e) No; undefined at $t = 0$.

14 $2(s \quad k)^3$

16.(a) t

(b) $2t - 2\sin t$

17.(a) $a^{-2}(e^{at} \quad 1) - \frac{t}{a}$

(b) $e^{4t} \quad (3t+1)e^{t}$

18.(a) $2t + 4e^{-t}$

(b) $0.011e^{7t} + 1.889e^{-2t} - 1.900e^{-3t}$

(c) $4\cos t + 3\sin t \quad e^{t}(\cos 2t + 0.5\sin 2t)$

(d) $2e^{-t} + e^{-2t}(\sin 3t - \cos 3t)$

19.(a) te^{1-t}

(b) $t + \pi\cos t + k\sin t$ (k an arbitrary constant)

(c) $t + \pi\cos t$ (d) no solution

20. $0.045(e^{t}\cos 2t \quad 1) + 0.477e^{t}\sin 2t$

21.(a) $0.5t^2$

(b) kt^2e^{-t} (k an arbitrary constant)

22 $\cos t$

23.(a) $2te^{-t}$

(b) $\sin 2t$

(c) $t \quad 0.5t^2$

25. 3.323

Section 5.8

Only one initial value is given

4 Trajectory: $y = -\frac{g}{2V^2}(x\sec\theta)^2 + x\tan\theta$;
Height $= \frac{(V\sin\theta)^2}{2g}$

5. (i) 19.71° (ii) 13.91° (iii) 43.37°

7. $z(t) = \beta\cosh(\sqrt{ab}t) + \frac{b\alpha}{\sqrt{ab}}\sinh(\sqrt{ab}t)$;
$w(t) = \alpha\cosh(\sqrt{ab}t) + \frac{a\beta}{\sqrt{ab}}\sinh(\sqrt{ab}t)$; **No.**

8. $y(x) = \frac{\alpha \sin p(a-x)}{\cos pa}$; $z(x) = \frac{\alpha \cos p(a-x)}{\cos pa}$
Critical length $= \frac{\pi}{2p}$.

9.(a) Let $r = Q^{-1} \quad q^{-1}$

$$\alpha\frac{Qe^{-kra} - qe^{-krx}}{Qe^{-kra} - q}$$
$$Q\alpha\frac{e^{-kra} - e^{-krx}}{Qe^{-kra} - q}$$

(b) a as long as possible; q as high as possible.

10 $i(t) \quad 10(1 - e^{-5t})$

11 $q(t) = 0.25(1 \quad e^{-8t})$ coulomb; $i(t) \quad 2e^{-8t}$ ampère

12. $q(t) = 10 - 5(1 + 0.02t)^{-500}$ coulomb;
$i(t) = 50(1 + 0.02t)^{-501}$ ampère for $0 \leq t \leq 1000$.

13. Current through switch $= 25 - 25e^{-4t} + \frac{3}{2}e^{-t}$
current through inductor $= 25 - 25e^{-4t}$;
charge on capacitor $= \frac{1}{2}(5 - 3e^{-t})$.

14 $i_1(t) = \frac{1}{15}(-11e^{-5t} - 22e^{-20t} + 33\cos 10t + 33\sin 10t)$;
$i_2(t) = \frac{1}{3}(22e^{-5t} - 22e^{-20t} - 33\sin 10t)$.

15.(a) $q_3(t) = 0.581\sin 50t - 0.105\cos 50t - 0.345e^{-20t} + 0.449e^{-80t}$;
$i_2(t) = -0.558\sin 50t - 3.100\cos 50t + 4.598e^{-20t} - 1.498e^{-80t}$

(b) $q_3(t) = 0.04(2 - 3e^{-50t})\sin 50t + 0.04(1 - e^{-50t})\cos 50t$;
$i_2(t) = -8(1 - e^{-50t})\cos 50t + 4(1 + e^{-50t})\sin 50t$.

(c) $q_3(t) = 0. \quad \sin 50t \quad 5te^{-50t}$; $i_2(t) = 5(1 + 50t)e^{-50t} \quad 5\cos 50t$

16.(i)

$$m(t) \qquad \frac{\alpha}{3}(1.103e^{-0.001t} - 0.436e^{-0.036t} + 0.333e^{-0.009t});$$
$$f(t) \qquad \frac{\alpha}{3}(1.36e^{-0.001t} - 0.36e^{-0.036t})$$
$$w(t) \qquad \frac{\alpha}{3}(0.8e^{-0.001t} + 0.2e^{-0.036t})$$

(a) $N(10) \quad 1.040\alpha$;
(b) $(m; f; w) \quad (35.8\%; 35.1\%; 29.9\%)$

16.(ii)

$$\frac{\alpha}{3}(1.123e^{0.018t} - 0.123e^{-0.075t});$$
$$\frac{\alpha}{3}(1.123e^{0.018t} - 0.123e^{-0.075t});$$
$$\frac{\alpha}{3}(0.935e^{0.018t} + 0.065e^{-0.075t})$$

(a) $N(10) = 1.241\alpha;$
(b) $(m; f; w) = (34.6\%; 34.6\%; 30.9\%)$

17 Let $i(t), r(t)$ and $h(t)$ denote the infectious, the recuperating, and the healthy people at time t, respectively. Assume that the duration of the epidemic is short enough so that the natural increase of the population does not play a role, with the result that the total number of people N remains constant. If we also assume that the recuperating people are immune during the duration of the epidemic, then

$$h'(t) = -ah(t)i(t), \quad h(0) = N \quad i(0);$$
$$i'(t) = ah(t)i(t) - bi(t), \quad i(0) = c;$$
$$r'(t) = bi(t), \quad r(0) = 0;$$

where the constants a, b, and c are positive real numbers. If the second assumption is not valid, then an extra term $r(t-d)$ must be added to the first and subtracted from the third equation, where d denotes the fixed period of immunity. Laplace transforms cannot be used because of the product hi in the first and second equations.

19 At time $= \frac{1}{2\delta} \ln(1 - \frac{2\beta\delta}{c\alpha}$ the population of Nerds will be zero

Section 6.5

2. For $0 \leq t \leq b$: $x(t) = \frac{a}{k}[1 - e^{\frac{-ct}{2m}}(\cos\frac{t}{2} + \frac{c}{m}\sin\frac{t}{2})]$
For $t > b$: $x(t) = e^{\frac{-ct}{2m}}(A\cos\frac{t}{2} + B\sin\frac{t}{2})$
where $A = \frac{2a}{k}[e^{bc}2m - (\frac{1}{2}\cos\frac{b}{2} - \frac{c}{2m}\sin\frac{b}{2}) - \frac{1}{2}(\cos\frac{b}{2})^2 + \frac{c^2}{2m^2}(\sin\frac{b}{2})^2] - \frac{4a}{m}(\sin\frac{b}{2})^2$
and $B = \frac{4a}{m}\sin\frac{b}{2}\cos\frac{b}{2} + \frac{2a}{k}[e^{\frac{bc}{2m}} - \cos\frac{b}{2} - \frac{c}{m}\sin\frac{b}{2}][\frac{c}{2m}\cos\frac{b}{2} + \frac{1}{2}\sin\frac{b}{2}]$

4.(a) $10^{-5}(2e^{-500t} - e^{-1000t});$

(b) $10^{-5}(1 \quad 500t)e^{-500t}$

(c) $10^{-5}(\cos 200t + 0.5\sin 200t)e^{-100t}$

5. (i) $q(0.001)$ 1.5468×10^{-6} farad;
(ii) $q(0.01)$ 2.9998×10^{-6} farad; $q \to 3 \times 10^{-6}$ farad

6. $0.11\cos(120\pi t + \delta)$ with $\tan\delta$ $\frac{386}{360\pi}$

9. A solution must satisfy $y = 4x$ for $t \geq 0$. If $y(0)$ 4, then there is the solution $x(t) = e^{\frac{2}{3}t}, y(t) = 4e^{\frac{2}{3}t}$

10. For the cubic $a_3s^3 + a_2s^2 + a_1s + a_0$ with each coefficient positive: $a_1a_2 > a_0a_3$. (This is a special case of a more general result, called the Routh-Hurwitz criterion - see, for example, [5] page 214.)

11. $\frac{80}{69}\cos 5t$

Section 7.5

1.(a) $x(t)$ $1 - t\arcsin t$ $\sqrt{1-t^2}$;

(b) x'(and hence also x) is unique on [0,0.86] by Theorem 3.2.1

(c) No; x'' does not exist at $t =$

3.(i) 5.89 m/s

(ii) 7.93 m/s

5.(a) a_7 $\dfrac{w^2 + 40,320a_3a_5}{5,040(c + 8a_1)}$,

$$a_9 = \frac{w^2 - 3,628,800a_3a_7 - 1,814,400a_5^2}{362,880(c + 10a_1)}$$

(c) No.