Universiteit van Stellenbosch
Departement Rekenaarwetenskap

Rekenaarwetenskap 324: Semestertoets

19 April 2004

Tyd: 2 uur
VolPunte: 50 (55 beskikbaar)
Beantwoord al die vrae.
Answer all the questions.

Dosent:Brink vd Merwe
Naam:.....
Studentenr:.....

Question 1 [31 marks]

(a) [3] True or False: (Points will be deducted for an incorrect answer.)
(i) If $L_1 \subseteq L_2$ and L_1 is not regular, then L_2 is not regular.
(Indien $L_1 \subseteq L_2$ en L_1 is nie regulêr, dan is L_2 ook nie regulêr.)

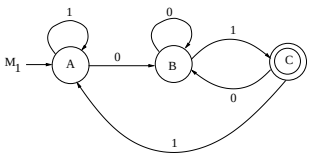
(ii) If L_1 and L_2 is nonregular, then $L_1 \cup L_2$ is nonreguler.
(Indien L_1 en L_2 nie regulêr is nie, dan is $L_1 \cup L_2$ ook nie regulêr nie.)

(iii) If L is nonregular, then the complement of L is nonregular.
(Indien L nie regulêr is, dan is die komplement van L ook nie regulêr nie.)

(b) [1] Fill in the blank. Suppose $L \subseteq \Sigma^*$ is a regular language. If every DFA accepting L has at least n states, then every NFA accepting L has at least ..... states.
(Voltooi die oop spasie. Gestel $L \subseteq \Sigma^*$ is 'n regulêre taal. Indien elke DFA wat L aanvaar ten minste n states het, dan sal elke NFA wat L aanvaar ten minste ..... state hê.)

(c) [3] Draw a DFA recognizing the language of all strings in $\{0,1\}^*$ that begin or end with 00 or 11.
(Teken 'n DFA wat die taal bestaande uit alle stringe in $\{0,1\}^*$ wat met 00 of 11 begin of eindig, herken.)

(d) [6] The DFA M_1 below recognizes the language L_1 and M_2 recognizes the language L_2.
(Die DFA M_1 herken L_1 en M_2 herken L_2.)

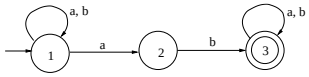


(i) Draw a DFA that will recognize $L_1 \cup L_2$.
(Teken 'n DFA wat $L_1 \cup L_2$ sal herken.)

(ii) Draw a DFA that will recognize $L_1 \cap L_2$.
(Teken 'n DFA that wat $L_1 \cap L_2$ sal herken.)

(iii)Draw a DFA that will recognize $L_1 - L_2$. (In other words, this DFA should recognize the strings that are L_1 but not in L_2.)
(Teken 'n DFA wat $L_1 - L_2$ sal herken. (Dus, hierdie DFA moet stringe wat in L_1 maar nie in L_2 is nie, herken.))

(e) [3] Use the subset construction to draw a DFA that is equivalent to the following NFA.
(Gebruik die subversameling konstruksie om 'n DFA wat ekwivalent is aan die volgende NFA te teken.)

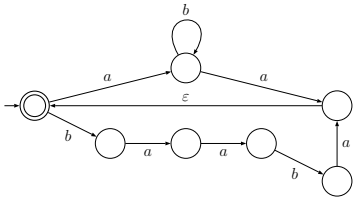


(f) [3] Draw a DFA equivalent to the regular expression $0 + 10^* + 01^*0$
(Teken 'n DFA wat ekwivalent is aan die regulêre uitdrukking $0+10^*+01^*0$)

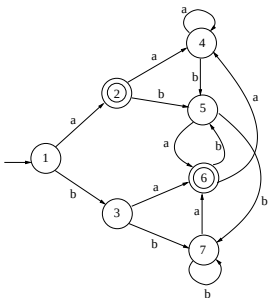
(g) [4] Find regular expressions corresponding to each of the following subsets of $\{0,1\}^*$.
(Gee regulêre uitdrukkings vir elk van die volgende subversamelings van $\{0,1\}^*$.)
(i) The language of all strings that do not end with 01.
(Die taal bestaande uit strings wat nie met 01 eindig nie.)

(ii) The language of all strings in which the number of 0's is even.
(Die taal bestaande uit alle stringe met 'n ewe aantal 0'le.)

(h) [4] Find a regular expression corresponding to the following NFA:
(Gee 'n regulêre uitdrukking wat ekwivalent is aan die volgende NFA:)



(i) [4] Find the minimal DFA equivalent to the following DFA:
(Gee die minimale DFA ekwivalent aan die volgende DFA:)



1
... 2
... ... 3
... ... 4
... ... 5
... ... 6
... ... 7

Mark in here all pairs of states that are not equivalent.
(Dui alle pare van nie−ekivalente state in hierdie diagram aan.)

Draw now the minimal DFA (Teken nou die minimale DFA)

Question 2 [11 marks]

(a) [4] Find CFG's for the following languages:
(Gee CFG's vir die volgende tale:)
(i) The language of odd-length strings in $\{a, b\}^*$ with middle symbol a.
(Die taal van stringe in $\{a, b\}^*$ met onewe lengte en middel simbool 'n a.)

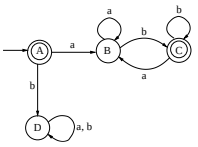
(ii) $\{a^i b^j c^k | i = j + k; \quad j, k \geq 0\}$

(b) [2] Show that the CFG with productions
$S \rightarrow a \mid Sa \mid bSS \mid SSb \mid SbS$
is ambiguous by considering the string $abaa$.
(Wys dat die CFG
$S \rightarrow a \mid Sa \mid bSS \mid SSb \mid SbS$
dubbelsinnig is deur van die string $abaa$ gebruik te maak.)

(c) [3] Draw a state diagram diagram for a pushdown automaton that recognizes the language generated by the following CFG. You may use transitions where you push more than one symbol on the stack at a time.
(Teken 'n PDA wat die taal herken wat die volgende CFG genereer. Jy mag van transisies gebruik maak wat meer as een simbool op die stapel plaas.)

$$S \rightarrow zMNz$$
$$M \rightarrow aMa|z$$
$$N \rightarrow bNb|z$$

(d) [2] Let L be the language recognized by the DFA below. Give a CFG without ε-productions that generates the language $L - \{\varepsilon\}$.
(Gegee die volgende DFA wat die taal L herken. Gee 'n CFG sonder ε-produksies wat die taal $L - \{\varepsilon\}$ genereer.)



Question 3 [13 marks]

(a) [4] State the pumping lemma for regular languages.
(Gee sonder bewys die pomp lemma vir regulêre tale.)

(b) [4] State the pumping lemma for context-free languages.
(Gee sonder bewys die pomp lemma vir konteksvrye tale.)

(c) [5] Decide whether the language $L = \{xcx | x \in \{a, b\}^*\}$ is context-free, and prove your answer.
(Besluit of die taal $L = \{xcx | x \in \{a, b\}^*\}$ konteksvry is en bewys jou antwoord.)