Memo: Tutorial 4

March 12, 2004

Question 1: Exercise 1.17*b*:
Assume that $L = \{www|w \in \{a,b\}^*\}$ is regular and let p be the pumping length.

Let $s = 0^p10^p10^p1$. Then $s \in L$ and $|s| = 3p \geq p$.

From the pumping lemma we can write s as xyz such that:

1. for each $i \geq 0$, $xy^iz \in L$,
2. $|y| > 0$,
3. $|xy| \leq p$.

As $|xy| \leq p$ we know that xy must form part of the first p symbols of s. Thus xy consists of 0's. It now follows from 2) that y is a string of one or more 0's. From 1) $xz = xy^0z \in L$, but $xz = 0^j10^p10^p1$, with $j < p$. Thus xz is not of the form www and is therefore not in L. It now follows by contradiction that A is not regular.

Question 2: Exercise 1.17*c*:
Assume that $L = \{a^{2^n}|n \geq 0\}$ is regular and let p be the pumping length.

Let $s = a^{2^p}$, then $s \in L$ and $|s| = s^p \geq p$.

From the pumping lemma we can write s as xyz such that:

1. for each $i \geq 0$, $xy^iz \in L$,
2. $|y| > 0$,
3. $|xy| \leq p$.

From 2) it follows that $y = 0^k$ with $k > 0$. From 1) the strings $xyz = 0^{2^p}, xy^2z = 0^{2^p+k}, xy^3z = 0^{2^p+2k}, xy^4z = 0^{2^p+3k}, ...$ are all in L. For this to happen, $2^p, 2^p+k, 2^p+2k, 2^p+3k, ...$ must all be powers of 2. Notice that the difference between two consecutive numbers in this sequence is k, but $2^{n+1} - 2^n = 2^n$ which tends to infinity as n tends to infinity. We conclude that $2^p, 2^p+k, 2^p+2k, 2^p+3k, ...$ can not all be powers of 2 and therefore not all of

$xyz, xy^2z, xy^3z, ..$ are in L. It now follows by contradiction that L is not regular.

Question 3:
Assume $L = \{w \in \{a,b,c\}^*|w$ is a palindrome$\}$ is regular and let p be the pumping length.

Let $s = a^pba^p$. Thus $s \in L$ and $|s| = 2p+1 \geq p$.

From the pumping lemma we can write s as xyz such that:

1. for each $i \geq 0$, $xy^iz \in L$,
2. $|y| > 0$,
3. $|xy| \leq p$.

From 3) $|xy| \leq p$, thus xy forms part of the first p symbols of $s = a^pba^p$. From 2) $|y| > 0$, thus $y = a^k$ with $k > 0$. We conclude that $xz = xy^0z = a^jba^p$ with $j < p$, which is not a palindrome. But according to 1) xz should be a palindrome. It now follows by contradiction that L is not regular.

Question 4:
Assume $PAREN$ is regular and let p be the pumping length.

Let $s = (^p)^p$. Thus $s \in L$ and $|s| = 2p \geq p$.

From the pumping lemma we can write s as xyz such that:

1. for each $i \geq 0$, $xy^iz \in PAREN$,
2. $|y| > 0$,
3. $|xy| \leq p$.

From 3) $|xy| \leq p$, thus xy forms part of the first p symbols of $s = (^p)^p$. From 2) $|y| > 0$, thus $y = (^k$ with $k > 0$. We conclude that $xz = xy^0z = (^j)^p$ with $j < p$, which is not in $PAREN$. But according to 1) xz should be in $PAREN$. We conclude by contradiction that $PAREN$ is not a regular language.

Question 5:
Minimize:

	a	b
→1	1	4
2	3	1
3F	4	2
4F	3	5
5	4	6
6	6	3
7	2	4
8	3	1

States 7 and 8 are not reachable and so can be removed.
Following Method From Class:

1. Construct Table of all pairs unmarked:

1					
-	2				
-	-	3			
-	-	-	4		
-	-	-	-	5	
-	-	-	-	-	6

2. Mark $\{p,q\}$ if one is final and the other not:

1					
-	2				
X	X	3			
X	X	-	4		
-	-	X	X	5	
-	-	X	X	-	6

3. Repeat until no more changes:

   (a) Checking unmarked pair $\{1,2\}$, $\{\delta(1,a), \delta(2,a)\} = \{1,3\}$ which is marked so mark $\{1,2\}$

1					
X	2				
X	X	3			
X	X	-	4		
-	-	X	X	5	
-	-	X	X	-	6

   (b) Checking unmarked pair $\{1,5\}$, $\{\delta(1,a), \delta(5,a)\} = \{1,4\}$ which is marked so mark $\{1,5\}$

1					
X	2				
X	X	3			
X	X	-	4		
X	-	X	X	5	
-	-	X	X	-	6

   (c) Checking unmarked pair $\{2,6\}$, $\{\delta(2,a), \delta(6,a)\} = \{3,6\}$ which is marked so mark $\{2,6\}$

1					
X	2				
X	X	3			
X	X	-	4		
X	-	X	X	5	
-	X	X	X	-	6

   (d) Checking unmarked pair $\{5,6\}$, $\{\delta(5,a), \delta(6,a)\} = \{4,6\}$ which is marked so mark $\{5,6\}$

1					
X	2				
X	X	3			
X	X	-	4		
X	-	X	X	5	
-	X	X	X	X	6

   (e) No more changes occur

4. The states that are equivalent are:

$$1 \equiv 6$$
$$2 \equiv 5$$
$$3 \equiv 4$$

5. This creates the automata in figure 1.

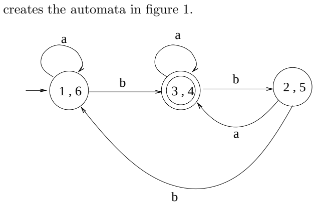

Figure 1: Minimal DFA from Question 4

Question 6:
Form the string-matching automaton for $P = aabab$.

The automaton has 6 states, with state 0 being the start state and state 5 the final state.

Using P_q as the string consisting of the first q letters of P_q, the transition function is calculated as $\delta(q,a) = \max\{k|P_k$ is a suffix of $P_qa\}$ and similar for $\delta(q,b)$. This results in:

δ	a	b
0	1	0
1	2	0
2	2	3
3	4	0
4	2	5
5	1	0

The automaton is shown in figure 2.

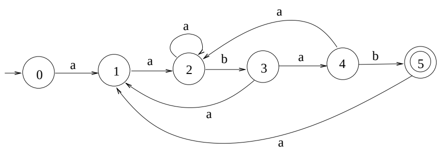

Figure 2: Automaton for Question 6

Using the generated automaton on test string $T = aaababaabaababaab$:

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
T[i]		a	a	a	b	a	b	a	a	b	a	a	b	a	b	a	a	b
state	0	1	2	2	3	4	5	1	2	3	4	2	3	4	5	1	2	3

Matches at shift $= 1, 9$

Question 7: Exercise 2.4:

1. contains at least three 1's

$$S \rightarrow A1A1A1A$$
$$A \rightarrow 0A|1A|\varepsilon$$

2. starts and ends with the same symbol

$$S \rightarrow 0A0|1A1$$
$$A \rightarrow 0A|1A|\varepsilon$$

3. odd length

$$S \rightarrow SSS|0|1$$

4. odd length and middle symbol is a 0

$$S \rightarrow ASA|0$$
$$A \rightarrow 0|1$$

5. contains more 1's then 0's

$$A \rightarrow AAB|ABA|BAA|1$$
$$B \rightarrow 0|\varepsilon$$

6. palindrome

$$S \rightarrow 0S0|1S1|1|0|\varepsilon$$

7. emptyset

$$S \rightarrow S$$