Probleem 1:

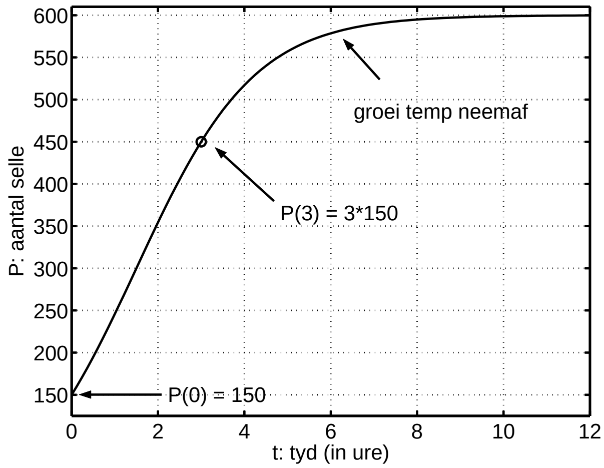
$$\begin{aligned}
\frac{dP}{dt} &= k(600-P)P \\
\int \frac{1}{(600-P)P}\,dP &= \int k\,dt \\
\int \Big(\frac{\frac{1}{600}}{600-P} + \frac{\frac{1}{600}}{P}\Big)dP &= kt + C \\
-\frac{1}{600}\ln(600-P) + \frac{1}{600}\ln(P) &= kt + C \\
\frac{1}{600}\ln\Big(\frac{P}{600-P}\Big) &= kt + C \\
\ln\Big(\frac{600-P}{P}\Big) &= -600(kt+C) \\
\frac{600-P}{P} &= e^{-600(kt+C)} = e^{-600kt}c \\
\frac{600}{P} &= e^{-600kt}c + 1 \\
P(t) &= \frac{600}{e^{-600kt}c+1}
\end{aligned}$$

$$P(0) = 150 \quad \Rightarrow \quad c \ = \ 3.$$

$$P(3) = 450 \quad \Rightarrow \quad k \ = \ \frac{\ln 9}{600(3)}.$$

$$\Rightarrow \quad P(t) = \frac{600}{3e^{-600\frac{\ln 9}{600(3)}t}+1} = \frac{600}{3e^{-\frac{1}{3}(\ln 9)t}+1}$$

$$P(6) = \frac{600}{3e^{-\frac{1}{3}(\ln 9)6}+1} = 578 \text{ selle.}$$



Probleem 2

(a) Laat $m(t)$ die massa sout in tenk op tyd t wees. Dan is die tempo van toename in m

$$\frac{dm}{dt} = \text{tempo van sout in} - \text{tempo van sout uit},$$

waar

$$\text{tempo van sout in} = (0.2)(30) = 6$$

en

$$\text{tempo van sout uit} = \left(\frac{m}{3000 - 10t}\right)(40).$$

Die aanvangswaardeprobleem is dus

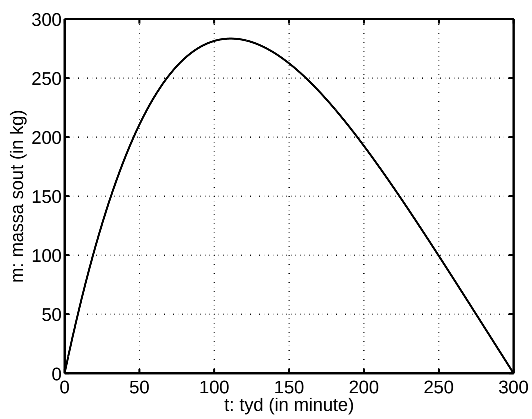
$$\Rightarrow \quad \frac{dm}{dt} - \frac{4m}{t - 300} = 6, \quad m(0) = 0.$$

Hierdie is 'n lineêre DV, met

$$\text{integrasie faktor} = (t - 300)^{-4},$$

dus is die oplossing

$$m(t) = 2(300 - t) - \frac{2}{(300)^3}(300 - t)^4.$$



(b) Leeg na 300 minute, $m(300) = 0$.

(c) Maksimum hoeveelheid sout waar

$$\begin{aligned} \frac{dm}{dt} = 0 \quad \Rightarrow \quad (300-t)^3 &= \frac{300^3}{4} \\ \Rightarrow \quad t &= 300\left(1 - \frac{1}{4^{\frac{1}{3}}}\right) \approx 111 \text{ minute} \\ \Rightarrow \quad m(111) &= 2(300-111) - \frac{2}{(300)^3}(300-111)^4 \approx 283\,kg. \end{aligned}$$

Probleem 3

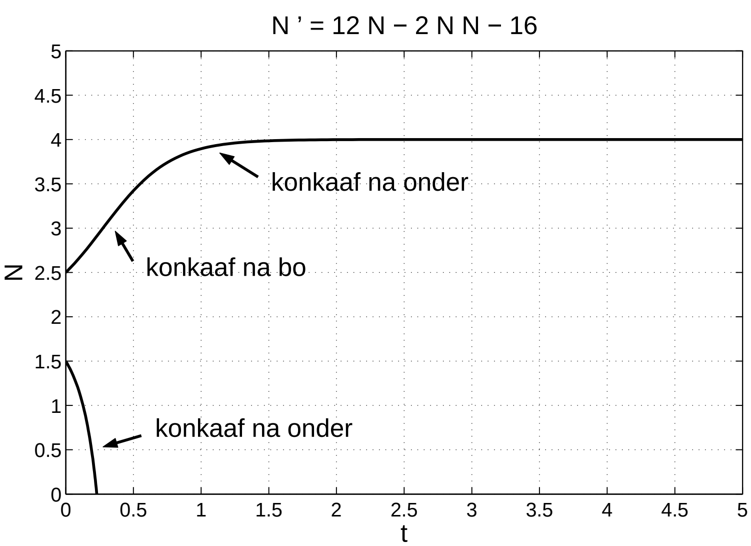
(a) $\dfrac{dN}{dt} = 12N - 2N^2 - 16 = -2(N-2)(N-4)$

$$\begin{array}{lll} \dfrac{dN}{dt} > 0 & \text{as} & 2 < N < 4 \\ \dfrac{dN}{dt} < 0 & \text{as} & N < 2 \text{ of } N > 4. \end{array}$$

(b) $\dfrac{dN^2}{dt^2} = 12\dfrac{dN}{dt} - 4N\dfrac{dN}{dt} = -2(12-4N)(N-2)(N-4) = 6(N-3)(N-2)(N-4)$

$$\begin{array}{lll} \dfrac{dN^2}{dt^2} > 0 & \text{as} & 2 < N < 3 \quad \text{of} \quad N > 4 \\ \dfrac{dN^2}{dt^2} < 0 & \text{as} & N < 2 \quad \text{of} \quad 3 < N < 4. \end{array}$$

(c) Kwalitatiewe skets vir $\alpha = \frac{5}{2}$ en $\alpha = \frac{3}{2}$



(d) Die kolonie oesters sal 'n limietgrootte bereik van 40 000.

(e) Die kolonie oesters sal uitsterf in 'n eindige tyd.

Probleem 4

(a) Die gegewe DV is skeibaar

$$\begin{aligned} \frac{dN}{dt} &= 2N - N^2 - 1 = -(N-1)^2 \\ \int \frac{1}{(N-1)^2}\, dN &= -\int dt + K \\ \Rightarrow \quad N(t) &= \frac{1}{t+K} + 1. \end{aligned}$$

Stel $N(0) = \alpha$ in om te kry

$$N(t) = \frac{\alpha - 1}{(\alpha - 1)t + 1} + 1.$$

(b) en (c) Vir $\alpha = 1$ is die DV triviaal en die limiet populasie is $N = 1$ (oftewel 1000 visse), dus kan ons verder vir $\alpha \neq 1$ neem.

$$N(t) = \frac{\alpha - 1}{(\alpha - 1)t + 1} + 1 = \frac{1}{t + \frac{1}{\alpha - 1}} + 1, \qquad (\star)$$

As $\alpha > 1$ is die nommer in $(\star)$ positief, so $N \neq 0$ vir alle t, wat beteken dat die skool nie kan uitsterf in 'n eindige tyd nie. In so geval geld dat $N \to 1$ and $t \to \infty$, sodat 'n limiet populasie van 1000 visse bereik word.

As $0 < \alpha < 1$ is $N = 0$ moontlik vir 'n eindige tyd t, sê $t = T$. Stel $N = 0$, $t = T$ in $(\star)$, dan

$$T = \frac{\alpha}{1 - \alpha}$$

wat die uitsterf tyd is.