NAAM: **Oplossings** US NR. ____________

INSTRUKSIES:

(a) 45 minute, 5 probleme, 26 punte, toeboek.

(b) Probleem 1,2 en 3: Omsirkel die korrekte antwoord. Geen motivering verlang nie.

(c) Probleme 4 en 5: Toon al jou bewerkings en motiveer alle stappe. Korrekte antwoorde verdien nie volpunte sonder die nodige verduideliking nie.

(d) **Let Wel:** Die wenke hieronder enige plek in die vraestel **sonder bewys** gebruik word.

(e) Moenie omblaai voordat u aangesê word om dit te doen nie.

INSTRUCTIONS:

(a) *45 minutes, 5 problems, 26 marks, closed book.*

(b) *Problem 1,2 and 3: Circle the correct answer. No justification required.*

(c) *Problems 4 and 5: Calculations are to be shown and all steps must be justified. Correct answers do not earn full marks without the necessary explanation.*

(d) **Note:** *The hints below may be used* **without proof** *anywhere in the paper.*

(e) *Do not turn the page until you are told to do so.*

Wenke/*Hints*:

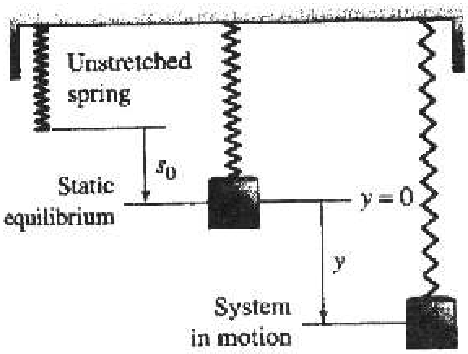
$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B), \quad \sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$

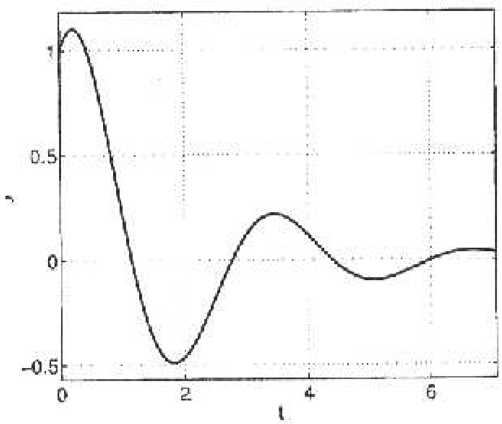
$$L[f'] = sL[f] - f(0), \qquad L[f''] = s^2L[f] - sf(0) - f'(0)$$

Vraag 1 (4 punte) **Question 1** *(4 marks)*

Beskou die model van die massa wat aan 'n veer ossileer (figuur links), asook een tipiese grafiek van die oplossing $y = y(t)$ (figuur regs).

Consider the model of a mass oscillating on a spring (left figure), as well as the graph of one typical solution $y = y(t)$ (right figure).





(a) Watter beskrywing pas die grafiek regs die beste?

(i) Geen demping.
(ii) Ligte demping.
(iii) Kritiese demping.
(iv) Swaar demping.
(v) Resonansie.

(a) *Which description fits the graph on the right best?*

(i) No damping.
(ii) Light damping.
(iii) Critical damping.
(iv) Heavy damping.
(v) Resonance.

(b) Wat was die aanvangsvoorwaardes? Omsirkel die mees waarskynlike keuse.

(b) *What were the initial values? Circle the most likely choice.*

(i) $y(0) = 0,\ y'(0) = 1$ (ii) $y(0) = 1,\ y'(0) = -1$ (iii) $y(0) = 0,\ y'(0) = 0$

(iv) $y(0) = -1,\ y'(0) = 1$ (v) $y(0) = 1,\ y'(0) = 1$

(c) Gegee dat $m = 2$ en $k = 8$, wat is die beste skatting vir c?

(c) *Given that $m = 2$ and $k = 8$, what is the best estimate of c?* $\boxed{4mk = 4 \times 2 \times 8 = 64}$

(i) $c = 0$ (ii) $c = 4$ $\boxed{4^2 = 16 < 64}$ (iii) $c = 6$ $\boxed{6^2 = 36 < 64}$ (iv) $c = 8$ $\boxed{8^2 = 64 = 4mk}$ (v) $c = 10$

(d) Gestel die oplossing in die figuur regs word in amplitude-fase vorm geskryf as $y = Ae^{-\lambda t}\cos(\omega t - \theta)$. Wat is die beste skatting vir ω in die volgende lys?

(d) *Suppose the solution in the figure on the right is written in amplitude-phase form as $y = Ae^{-\lambda t}\cos(\omega t - \theta)$. What is the best estimate for ω in the following list?*

(i) $\omega = 0.1$ (ii) $\omega = 1$ (iii) $\omega = 2$ (iv) $\omega = 3$ (v) $\omega = 4$

Vraag 2 (3 punte)

Vir elk van die volgende drie DVs, kies uit die Figure (A)–(D) die beste oplossingskurwe wat daarmee ooreenstem.

Question 2 *(3 marks)*

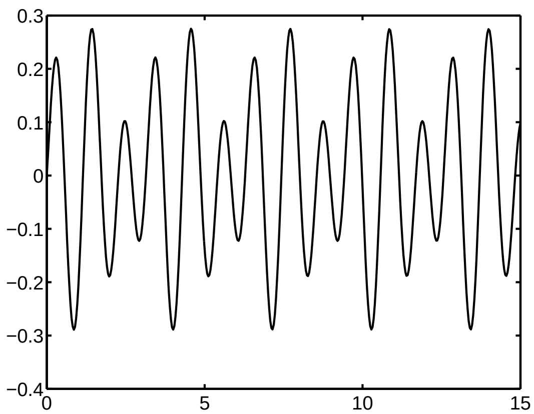
For each of the following three DEs, choose from the Figures (A)–(D) the best corresponding solution curve.

(i) $x'' + 36x = 0$ Fig. **D**

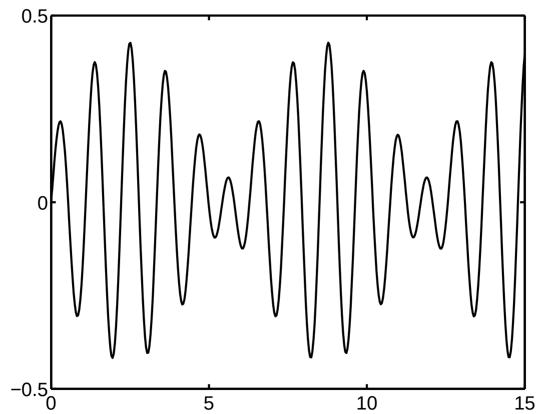
(ii) $x'' + 36x = 2\cos 5t$ Fig. **B**

(iii) $x'' + 36x = 2\cos 6t$ Fig. **C**

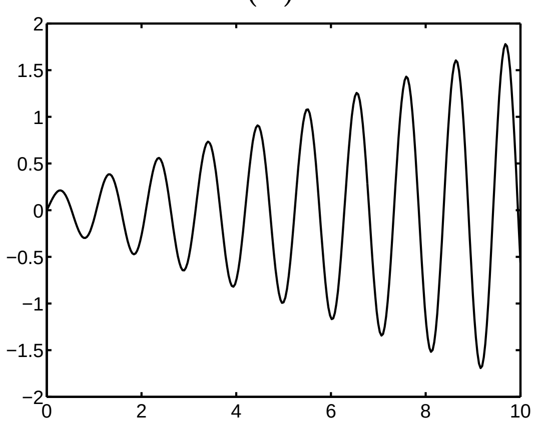
(A)



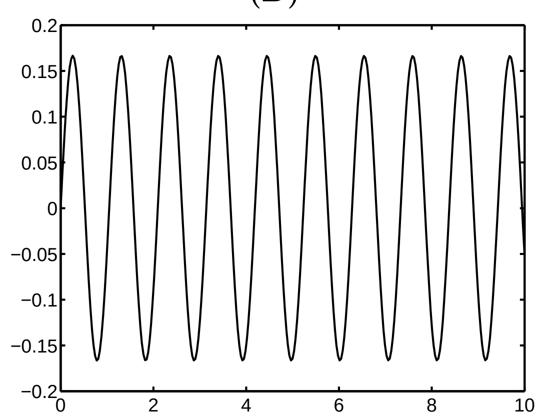
(B)



(C)



(D)



Vraag 3 (3 punte) **Question 3** *(3 marks)*

Beskou die aanvangswaardeprobleem *Consider the initial-value problem*

$$x'' + cx' + \tfrac{9}{4}x = 0, \quad x(0) = \alpha, \quad x'(0) = 0.$$

Vir elk van die volgende drie waardes van c, kies uit die oplossings (A)–(D) die een wat daarmee ooreenstem. *For each of the following values of c, choose from (A)–(D) the corresponding solution.*

(i) $c = 1$ Oplossing/*Solution* C $c^2 = 1 < 4(\tfrac{9}{4}) = 9 = 4mk$ ligte demping

(ii) $c = 3$ Oplossing/*Solution* A

(iii) $c = 5$ Oplossing/*Solution* B

(A) $x(t) = \alpha e^{-\frac{3}{2}t}\left(1 + \frac{3}{2}t\right)$

(B) $x(t) = \alpha\left(\frac{9}{8}e^{-\frac{1}{2}t} - \frac{1}{8}e^{-\frac{9}{2}t}\right)$

(C) $x(t) = \alpha e^{-\frac{1}{2}t}\left(\cos\sqrt{2}t + \frac{1}{2\sqrt{2}}\sin\sqrt{2}t\right)$

(D) $x(t) = \alpha\cos\frac{3}{2}t$

$c=3 \Rightarrow c^2 = 9 = 9 = 4mk$

kritiese demping $\Rightarrow$ A

$c=5 \Rightarrow c^2 = 25 > 9 = 4mk$

swaar demping $\Rightarrow$ B

D het geen deming dus $c=0$.

Vraag 4 (6 punte) — **Question 4** *(6 marks)*

Gegee die differensiaalvergelyking — *For the differential equation*

$$x'' + 6x' + 13x = 0,$$

onderhewig aan die aanvangsvoorwaardes — *with initial conditions*

$$x(0) = 1, \quad x'(0) = 3.$$

Aanvaar, sonder bewys, dat die oplossing gegee word deur — *Assume, without proof, that the solution is given by*

$$x(t) = e^{-3t}\Big(\cos 2t + 3\sin 2t\Big)$$

en skryf hierdie oplossing in die vorm — *and write this solution in the form*

$$x(t) = A\cos(\omega t - \phi).$$

Gee eksplisiete uitdrukkings vir A, ω en ϕ. — *Give explicit expressions for A, ω and ϕ.*

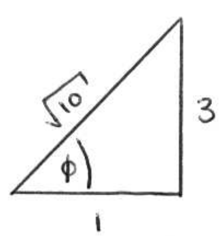
$$\Rightarrow \quad x(t) = \sqrt{10}\, e^{-3t}\left(\frac{1}{\sqrt{10}}\cos 2t + \frac{3}{\sqrt{10}}\sin 2t\right)$$

$$= \sqrt{10}\, e^{-3t}\left(\cos\phi \cos 2t + \sin\phi \sin 2t\right)$$

$$= \sqrt{10}\, e^{-3t} \cos(2t - \phi)$$

met $\cos\phi = \frac{1}{\sqrt{10}} > 0$ en $\sin\phi = \frac{3}{\sqrt{10}} > 0$

$\Rightarrow$



$$\tan\phi = 3$$
$$\phi = \arctan 3$$
$$\approx 1.249 \text{ rad.}$$

$$\Rightarrow \quad A = \sqrt{10}\, e^{-3t} \qquad \omega = 2 \qquad \phi \approx 1.249 \text{ rad}$$

Let op dat ϕ in die eerste kwadrant is omdat $\cos\phi > 0$ en $\sin\phi > 0$.

Vraag 5 $(2+7+1=10$ punte)

Question 5 *$(2+7+1=10$ marks)*

Beskou weer die probleem van die massa wat aan die veer ossileer, soos in die figuur links in Probleem 1. Gestel die konstantes is sodanig dat die aanvangswaardeprobleem reduseer na

Consider again the problem of the mass oscillating at the end of a spring as in the figure on the left in Problem 1. Suppose the constants are such that the initial value problem reduces to

$$y'' + \left(\tfrac{11}{4}\right)^2 y = \tfrac{5}{4}\cos\left(\tfrac{9}{4}t\right), \qquad y(0) = 0 = y'(0).$$

Aanvaar sonder bewys dat die oplossing van hierdie aanvangswaardeprobleem gegee word deur

Assume without proof that the solution to this initial value problem is given by

$$y = \tfrac{1}{2}\Big[\cos\left(\tfrac{9}{4}t\right) - \cos\left(\tfrac{11}{4}t\right)\Big].$$

(a) Beskryf, in woorde, die fisiese betekenis van die gegewe aanvangsvoorwaardes.

(b) Herskryf die gegewe oplossing in 'n vorm wat meer geskik is vir grafiese voorstelling, en maak dan 'n goeie vryhandskets van die grafiek van die oplossing. Hoe meer relevante besonderhede (soos periode, amplitude, ens.) op die skets aangedui word hoe meer punte word verdien.

(c) Noem die fisiese verskynsel wat deur hierdie oplossing verteenwoordig word.

(a) *Describe, in words, the physical meaning of the given initial conditions.*

(b) *Rewrite the given solution in a form that is more suitable for graphical representation, and then draw a good freehand sketch of the graph of the solution. The more relevant details (like period, amplitude, etc.) you indicate on your graph, the more marks you will earn.*

(c) *Name the physical phenomenon represented by this solution.*

(a) Massa begin beweeg met 'n verplasing van 1 meter en 'n snelheid van 1 meter per sekonde.

(a) $y = \frac{1}{2}\left[\cos\frac{9}{4}t - \cos\frac{11}{4}t\right]$

$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$
$$\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$

Trek af

$$\cos(\theta-\phi) - \cos(\theta+\phi) = 2\sin\theta\sin\phi$$

Laat $\theta - \phi = \frac{9}{4}t$

$\theta + \phi = \frac{11}{4}t$

$$\Rightarrow \quad \theta = \frac{5}{2}t \,, \quad \phi = \frac{1}{4}t$$

$$y = \frac{1}{\cancel{2}} \cdot \cancel{2} \sin\frac{5}{2}t \sin\frac{1}{4}t$$

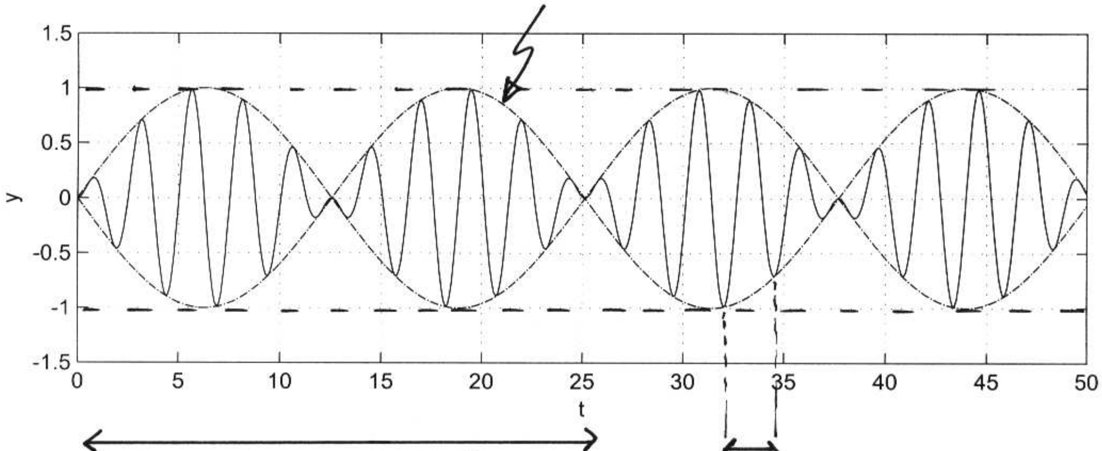
Grafiek hieronder

(b) Swewinge "beats"

Oplossing bestaan uit

* vinnig ossillerende golf (draer golf) met (pseudo-) periode $2\pi / \frac{5}{2} = \frac{4\pi}{5}$, soos getoon deur soliede lyn.

* hierdie draer golf het 'n stadig ossillerende omhulsel $\sin \frac{1}{4} t$, met periode $2\pi / \frac{1}{4} = 8\pi$. Hierdie omhulsel word met stippellyn aangedui.



periode v. omhulsel $= 8\pi \approx 25.$

pseudo-periode v. draer golf $= \frac{4\pi}{5} \approx 2.5.$