NAAM/*Name*: ______________________________ US NR. __________

INSTRUKSIES:

(a) 45 minute, toeboek, 4 probleme, 20 punte.

(b) Probleem 1 is van die kortvraag/veelvuldige keuse tipe. Hier tel slegs die antwoord punte so dit is nie nodig om berekenings te toon of stappe te motiveer nie.

(c) Probleme 2, 3 en 4 is van die uitskryf tipe. Hier moet alle berekenings getoon word en alle stappe gemotiveer word. 'n Korrekte antwoord verdien nie volpunte sonder die nodige verduideliking nie.

(d) U mag die rugkant van die bladsye vir rofwerk gebruik. Maak egter seker dat u alles deurhaal wat u nie gemerk wil hê nie.

(e) Wanneer u klaar is moet al hierdie blaaie ingehandig word. U moet daarna voortgaan met Huiswerk #3.

(f) Moenie omblaai voordat u aangesê word om dit te doen nie.

INSTRUCTIONS:

(a) 45 minutes, closed book, 4 problems, 20 marks.

(b) Problem 1 is of the short question/multiple choice type. Here marks are given for the answer only, so it is not necessary to show calculations or motivate the steps.

(c) Problems 2, 3 and 4 are of the written type. Here all calculations must be shown and all steps must be motivated. A correct answer without the necessary explanation will not earn full marks.

(d) You may use the back of the pages for rough work, though it is important that you indicate which is rough work and not intended for marking.

(e) When you have completed this test, all these pages must be handed in and you should continue with Homework #3.

(f) Do not turn this page before you are told to do so.

Probleem 1 (3 punte)

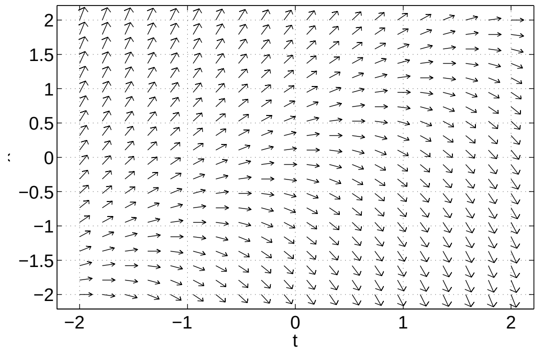
Vir elk van die volgende drie DVs, kies uit die Figure (A)–(D) die rigtingsveld wat daarmee ooreenstem.

Problem 1 *(3 marks)*

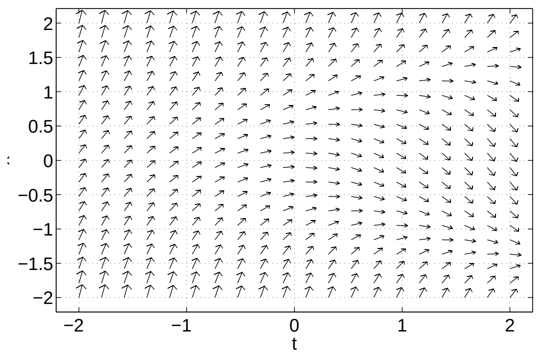
For each of the following three DEs, choose the corresponding direction field from the Figures (A)–(D).

(a) $\dfrac{dx}{dt} = x - t$ Fig.(**A**) (b) $\dfrac{dx}{dt} = 1 - xt$ Fig.(**D**) (c) $\dfrac{dx}{dt} = x^2 - t$ Fig.(**B**)

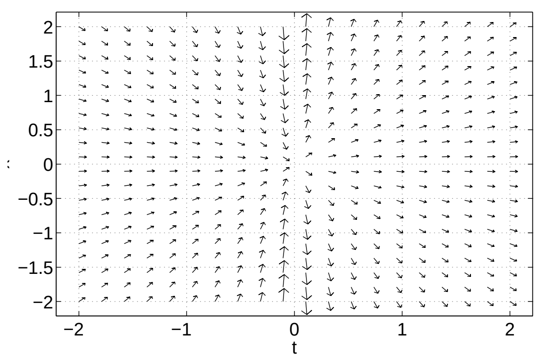
(A)



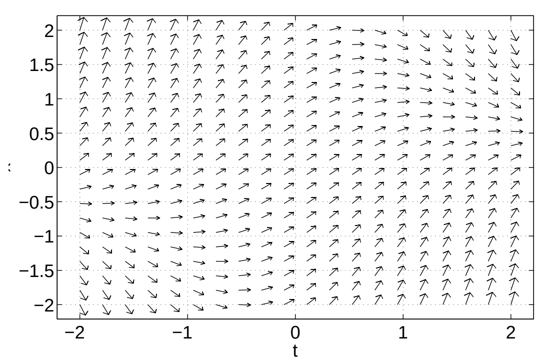
(B)



(C)



(D)



Probleem 2 (7 punte) / **Problem 2** *(7 marks)*

Die rigtingsveld van die DV / *The direction field of the DE*

$$\frac{dx}{dt} = \frac{x}{t} + 1$$

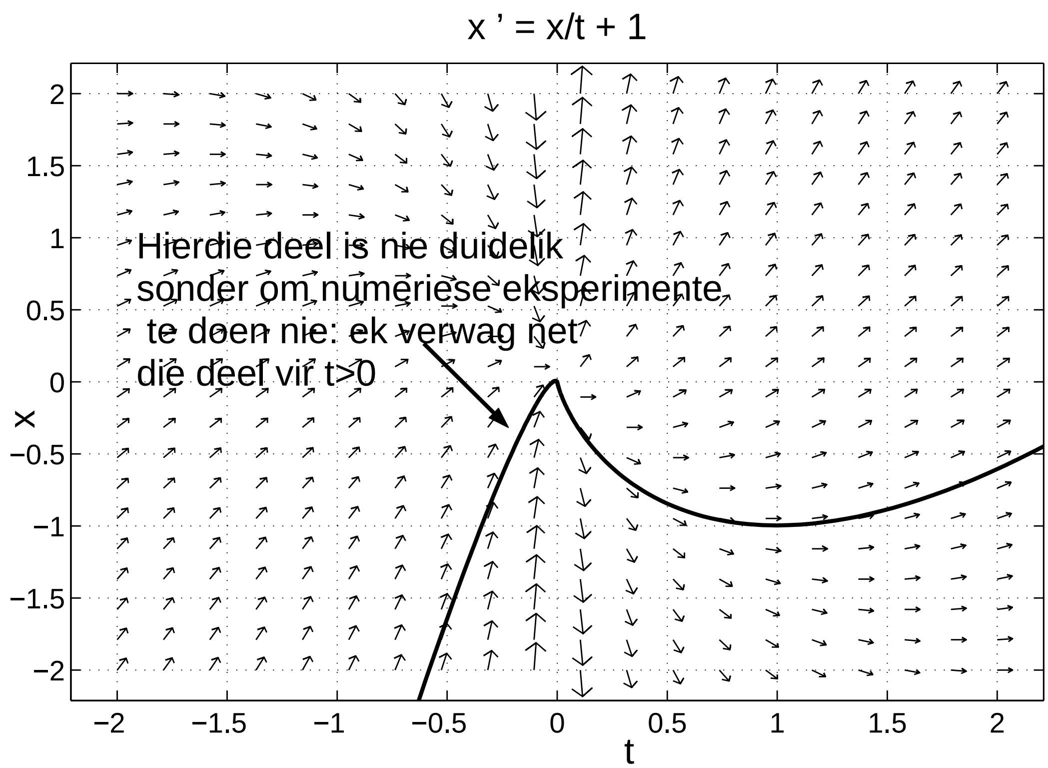
word gegee in die figuur hieronder. / *is given in the figure below.*

(a) Op die figuur, maak 'n akkurate skets van die oplossingskromme wat ooreenstem met die randwaarde $x(1) = -1$.

(a) Make an accurate sketch, on the figure, of the solution curve that corresponds to the boundary value $x(1) = -1$.

(b) Gebruik skeiding van veranderlikes of 'n integrasie faktor om die DV met beginwaarde soos in (a) gegee, analities op te los.

(b) Use separation of variables or an integration factor to solve the DE, with initial values as given in (a), analytically.



$$\frac{dx}{dt} - \frac{1}{t}x = 1$$

Int. faktor $\quad I(t) = e^{-\int 1/t \, dt} = e^{-\ln t} = \frac{1}{t}$

Maal deur : $\quad \frac{1}{t}\frac{dx}{dt} - \frac{1}{t^2}x = \frac{1}{t}$

$$\frac{d}{dt}\left(\frac{1}{t}x\right) = \frac{1}{t}$$

$$\frac{1}{t}x = \int \frac{1}{t}\, dt$$

$$= \ln t + k$$

$$x(t) = t(\ln t + k)$$

$x(1) = -1$

$\Rightarrow \quad -1 = \ln(1) + k \qquad k = -1$

$$\boxed{x(t) = t(\ln t - k)}$$

Probleem 3 (3 punte) *Problem 3 (3 marks)*

Toets deur **differensiasie** dat *Test by means of* **differentiation** *that*

$$y(x) = \tan\left(\frac{x^3}{3} + \frac{\pi}{4}\right)$$

die volgende randwaardeprobleem bevredig *satisfies the boundary value problem*

$$y' = x^2(1 + y^2), \qquad y(0) = 1.$$

Oplossing:
Afgeleide van y is

$$\begin{aligned} y'(x) &= \sec^2\left(\frac{x^3}{3} + \frac{\pi}{4}\right) \cdot x^2 \\ &= \left[1 + \tan^2\left(\frac{x^3}{3} + \frac{\pi}{4}\right)\right] x^2 \\ &= x^2(1 + y^2). \end{aligned}$$

Toets nou die randwaarde

$$y(0) = \tan\left(0 + \frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1.$$

Dus y bevredig die gegewe randwaardeprobleem.

Probleem 4 (1 + 2 + 4 = 7 punte)

'n Sekere modelleringsprobleem benodig die oplossing van die volgende vergelyking (x in radiale!)

Problem 4 *(1 + 2 + 4 = 7 marks)*

A certain modelling problem requires the solution of the following equation (x in radian!)

$$e^{-x} - \cos x = 0. \qquad (1)$$

(a) Skryf die Newton-Raphson rekursieformule neer vir die oplossing van vergelyking (1).
Wenk: $x_{n+1} = x_n - f(x_n)/f'(x_n)$.

(b) Deur geskikte grafieke te skets, stel vas hoeveel reële nulpunte vergelyking (1) het.

(c) Gebruik die skets van deel (b) om aan te toon dat $x_0 = \frac{\pi}{2}$ 'n goeie skatting is vir 'n nulpunt. Gebruik die Newton-Raphson formule van deel (a) om 'n beter skatting te bereken. Staak die iterasie sodra twee opeenvolgende benaderings binne 10^{-5} vanmekaar is. (Doen alle bewerkings tot tenminste sewe beduidende syfers.)

(a) Write the Newton-Raphson recursion formula for the solution of equation (1).
Hint: $x_{n+1} = x_n - f(x_n)/f'(x_n)$.

(b) By sketching appropriate graphs, determine how many real roots equation (1) has.

(c) Use the sketch from (b) to show that $x_0 = \frac{\pi}{2}$ is a good estimate of the root. Use the Newton-Raphson formula from (a) to calculate a better estimate. Terminate the iteration process as soon as two consecutive approximations are within 10^{-5} from each other. (Do all calculations to at least 7 significant digits.)

Oplossing:

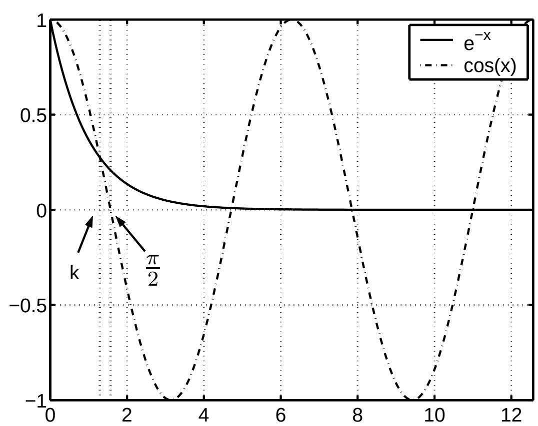
(a) Vir hierdie probleem is

$$f(x) = e^{-x} - \cos x \quad \Rightarrow \quad f'(x) = -e^{-x} + \sin x.$$

Dan is die Newton-Raphson rekursieformule

$$x_{n+1} = x_n - \frac{e^{-x_n} - \cos x_n}{-e^{-x_n} + \sin x_n}, \quad n = 0, 1, \ldots.$$

(b)



Oneindig aantal nulpunte.

(c)

n	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}
0	$\frac{\pi}{2}$	0.20787958	0.7921204	1.308362
1	1.308362	0.01083000	0.6954989	1.292790
2	1.292790	6.499744e-05	0.6871008	1.292696
3	1.292696	2.456288e-09	0.6870489	1.292696

$\Rightarrow$ skatting 1.292696.