TW324 **Tutoriaaltoets #2** **2004**

NAAM: ____ US NR: _______________

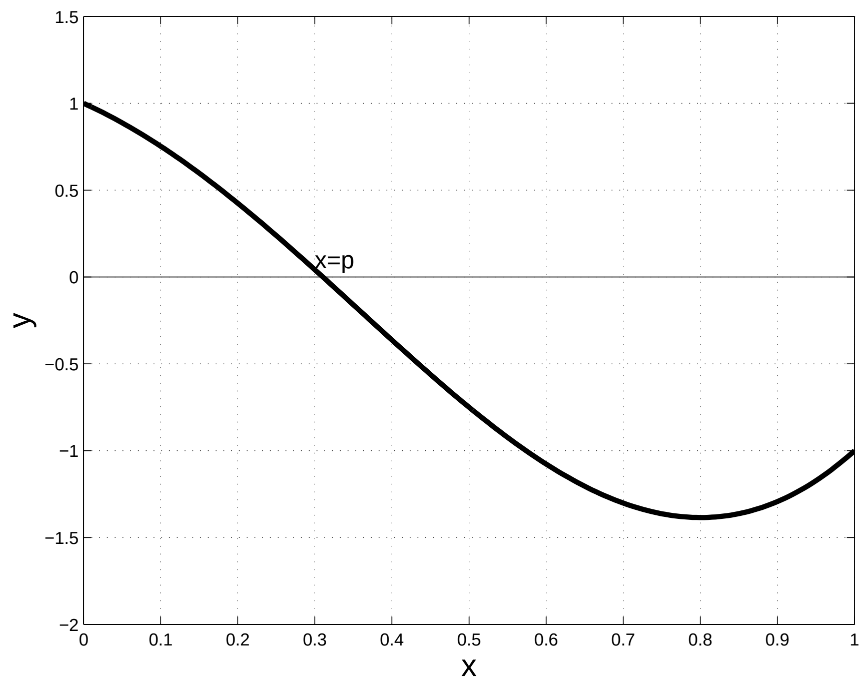
INSTRUKSIES: Drie probleme, 20 punte, 45 minute. Toon alle berekenings by Probleem 3.

INSTRUCTIONS: *Three problems, 20 marks, 45 minutes. Show all work on Problem 3.*

Probleem 1 ($3 + 2 = 5$ punte)

Die figuur toon die grafiek van 'n funksie $y = f(x)$, met wortel $x = p$ soos aangetoon.

The figure shows the graph of a function $y = f(x)$, with root $x = p$ as shown.



(a) Gestel die halveringsmetode word gebruik om die wortel p te benader, met aanvanklike interval $[a_0, b_0] = [0, 1]$. Toon aan hoe die metode sal vorder, deur die tabel hier onder te voltooi.

Suppose the bisection method is used to approximate the root p, with initial interval $[a_0, b_0] = [0, 1]$. Indicate how the method will proceed, by completing the table below.

(b) Herhaal deel (a) met die Regula-Falsi metode (slegs benaderde waardes verlang).

Repeat part (a) with the Regula-Falsi method (only approximate values required).

Halvering/*Bisection*

n	a_n	b_n
0	0	1
1		
2		
3		

Regula-Falsi

n	a_n	b_n
0	0	1
1		
2		

Probleem 2 (*6 punte*)

'n Sekere vergelyking $f(x) = 0$ het eksakte wortel $p = \pi/3$. Wanneer 'n spesifieke numeriese metode op hierdie vergelyking toegepas word, word die volgende drie benaderings, x_n, verkry. (Die absolute foute word ook vir jou gerief getoon.)

A certain equation $f(x) = 0$ has the exact root $p = \pi/3$. When a particular numerical method is applied to this equation, the following three approximations, x_n, are obtained. (The absolute errors are also shown for your convenience.)

x_n	$\lvert x_n - p \rvert$
1.04789506304527	6.9751e-04
1.04719769147403	1.4028e-07
1.04719755119660	5.7732e-15

(a) Skat die orde van konvergensie, α, deur gebruik te maak van die definisie

Estimate the order of convergence, α, by using the definition

$$\lim_{n\to\infty} \frac{|e_{n+1}|}{|e_n|^\alpha} = C.$$

(Omsirkel die beste skatting.)

(Circle the best estimate.)

(A) $\alpha = 1$ (B) $\alpha = 1.5$ (C) $\alpha = 2$ (D) $\alpha = 2.5$ (E) $\alpha = 3$

(b) Skat ook die foutkonstante, C. (Omsirkel die beste skatting.)

Also estimate the error constant, C. (Circle the best estimate.)

(A) $C = 0.03$ (B) $C = 0.06$ (C) $C = 0.1$ (D) $C = 0.3$ (E) $C = 1.1$

(c) Wanneer 'n ander numeriese metode op hierdie vergelyking toegepas word, word 'n waarde $\alpha = 1.6$ verkry. Watter tipe konvergensie is hierdie?

When a different numerical method is applied to this equation a value of $\alpha = 1.6$ is obtained. What type of convergence is this?

(A) lineêr (*linear*) (B) super-lineêr (*super-linear*) (C) kwadraties (*quadratic*)

(D) super-kwadraties (*super-quadratic*) (E) kubies (*cubic*)

(d) Die metode in deel **(c)** is heel waarskynlik

The method in part **(c)** *is most likely*

(A) Secant (B) Newton (C) Halley (D) Heron (E) Horner

(e) Die metode in deel **(a)** is heel waarskynlik

The method in part **(a)** *is most likely*

(A) Secant (B) Newton (C) Halley (D) Brent (E) Horner

Probleem 3 $(3+3+3=9$ *punte*)

Beskou die vergelyking (x in radiale)

Consider the equation (x in radians)

$$x^2 = 1 - \sin x.$$

(a) Gebruik 'n grafiese metode om vas te stel hoeveel reële wortels daar is.

Use a graphical method to determine how many real roots there are.

(b) Gebruik Newton se metode met aanvanklike skatting $x_0 = 0$ om 'n wortel te benader. Voer twee stappe van die metode uit, d.w.s., bereken x_1 en x_2. *Wenk:*

Use Newton's method with an initial guess $x_0 = 0$ to approximate a root. Execute two steps of the method, i.e., compute x_1 and x_2. Hint:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

(c) Indien mens die iterasie van deel (b) verder sou voer, watter orde van konvergensie verwag jy om waar te neem? Lineêr, kwadraties, kubies? Dalk iets anders? *Wenk:*

Suppose one continues the iteration of part (b). What order of convergence do you expect to observe? Linear, quadratic, cubic? Perhaps something else? Hint:

$$x_{n+1} - p = \frac{f''(\xi_n)}{2f'(x_n)}(x_n - p)^2$$