Note: This draft is being updated f

Abstract

Which trading mechanisms are optimal for revenue
bination of the two? Do they resemble institutions
trade be structured if there is no adverse selectio
If there is adverse selection? If traders are hete
sets? If there are dynamics? Which mechanisms of fe
are robust to adverse selection? Which informatio
gaurd against? How well do prices aggregate privat
mechanism is endogenous? I investigate these ques
ting in finance market microstructure: large trad
endowments.

*I thank Markus Brunnermeier, Briana Chang, Piotr Dworcza
Wilson, Yuliy Sannikov, Wei Xiong, Motohiro Yogo, Anthony Z
UCL Decentralized Financial Markets Conference for insig
Kayla Jiang and Ethan Wang for excellent research assistan
support of the Bradley Graduate Fellowship through a grant

À dtchen@princeton.edu; Department of Economics, Princet

1. Introduction

In the past two decades, trade in financial mark
and fragmented. This has raised concern among re
financial markets may be organized in a way that is
so because most trading institutions are for prof

To help further our understanding of these issu
design analysis of trade for a workhorse model se
In this setting, a finite number of traders are pr
assets in the market and trade to share[1] holding cos
solve for mechanisms that are optimal in that they
and allocative efficiency. A key aspect of my anal
of market conditions including when there is adv
payoffs, multiple assets, and heterogeneity amon
allows me to offer insight into how trade should be
also allows me to relate my findings to several res
fixed trading mechanism (or within a parametric c

I now briefly summarize the main results.

- In general, it is optimal to distort the alloca
  have the highest marginal value.[2] Moreover, un-
  der optimal mechanisms each trader's allocati
  of all traders—it is never optimal to Malamud & Rostek (2017).

- Across each of the environments I study, doubl
  often implement a point on the revenue-efficiency frontie
  tiple assets, this can be done with double auct
  trader's demand schedule can be contingent on
  exchange but not on the prices of securities in

- If an exchange has incentives to maximize reve
  small and has only a small piece of information

[1] Papers with th Vives (2011), Chen & Duffie (2021), Du & Zhu (2017), Rostek & Weretka (2012), Rostek & Yoon (2021), Sannikov & Skrzypacz (2016), Babus & Parlatore (2022), et al. Glosten (1994), Wittwer (2021), Rostek & Yoon (2023) and many others.

[2] This is in contrast with the typical "no distortion at the

should be designed to target desirable outcomes an
consider putting in place as a society. Of course
some caution given the stylized nature of the mode

Indeed, the analysis in this paper is not withou
anticipate many qualitative insights hold more g
cost model for tractability. Also, my analysis of
or multiple assets relies on restrictive distrib
nical challenges associated with multidimensional[4] In
optimal mechanisms for the model with adverse sel
ture that is "worst case" for allocative efficien
in the literature on robust mechanism design). Th

[3] I demonstrate this for the case of no private information

[4] The analysis for these settings seems close to the limits
multidimensional screening.

2. Related Literature

This paper is at the intersection of finance mark
To my knowledge, this paper is among the first to
for objectives other than allocative efficiency
a finite number of traders, and 3) traders who may
endogenously based on the terms of trade.

I characterize mechanisms that maximize linear
efficiency for a model setting where traders have
costs that are quadratic in their asset positions
finance market microstructure in the past fifteen
this environment include Vives (2011), Chen & Duffie (2021), Du & Zhu (2017), Rostek & Weretka (2012), Rostek & Yoon (2021), Sannikov & Skrzypacz (2016), Babus & Parlatore (2022), Eriais et al. (2018), Glosten (1994), Wittwer (2021), Rostek & Yoon (2023) and many others.[7] Existing work typically fixes a trading m
mechanisms in a narrow parameterized class.[8] The most commonly assumed me
is the (uniform-price) double auction. In contra
ing mechanism as being chosen by a planner who plac
efficiency. As discussed in the Introduction, my
ison for existing work to see how results may chan
(e.g., chosen by a revenue-maximizing exchange)
to see which inefficiencies arise because of the p

This paper is also related to work in finance mar
pact of exchange Malinova & Park (2015), Foucault et al. (2013), Colliard & Foucault (2012), Jantschgi et al. (2022) show that trading fees can i
optimal mechanism design. Jantschgi et al. (2022) develop a theore
analysis of transaction costs fixing a uniform-p Jantschgi
et al. (2022). I do not make parametric assumptions on th

---

[7] Variants include single and multiperiod models, single a
out private information about asset payoffs, models with sy

[8] This is also true of other papers in finance market design Budish et al. (2019) and Budish et al. (2023) and many others.

exchange mechanisms when designer may ha

motive. Furthermost of these papers consider env designated buyers or sellers and have linear uti extended to settings in finance market microstru allow for multiple dimensions of private informa

The most closely-related paper is Biais (2000) who study a single asset t competing mechanisms in a finance setting with a s may choose to be a buyer or seller. Biais et al. (2000) study only the case of a single trader they do not consider exchar not absorb or supply any net quantity of the asset. both of our papers share many properties such as a constraints and kinked transfer rules. One impor be distorted for extreme types in my paper but not i similarities, the focuses of our papers are quite are not directly comparable.

Also related is Lu & Robert (2001) which studies optimal exchang allows for traders to be buyers or sellers but stu timal mechanisms are shown to depend on delicate involve randomization. This is in sharp contrast do not have these properties. Also, Lu & Robert (2001) find there are no disto extremes whereas I find the opposite. Andreyanov & Sadzik (2017). T studies optimal exchange mechanisms in a closely locative efficiency with a robust objective and m restrict attention to quadratic subsidies in a do

3. Basic Model

3.1. Environment

In what follows, all random variables are defined on (Ω, ,). There is a single fina[illegible] with a payoff that is a random variable . There are ≥ 2 traders indexed by $\{1, 2, \ldots\}$. Traders are endowed with some of the asset prior to trade. They know [illegible] asset prior to trade. The endowment is a random variable [illegible] distribution function (CDF) that has a continuous density [illegible] > 0. Traders' endowments are jointly independent [illegible] If trader purchases units of the asset at a total price , their utility is

$$(+ ,) = (+) - - \frac{1}{2}(+)^2 \qquad (1)$$

> 0. The quadratic cost represents the dis
or any other costs associated with holding a net po
as a trader's holding capacity.

Asrlvilwld io Slctioo 2, variaots of tmis sltup
marklt microstructurl litlraturl, iocludiog Kylr (1989); Vayaoos (1999); Malamud & Rostlk (2017); Rostlk & Wlrltka (2012); Du & Zmu (2017); Rostlk & Wlrltka (2015); Rostlk & Yooo (2021); Sao-
oikov & Skrzypacz (2016); Babus & Parlatorl (2022); Wittwlr (2021) aod maoy otmlrs.
mavl adoptld a baslliol modll tmat is tml simpllst
plrs. Subslpulot slctioos will systlmatically io
io tml litlraturl.

3.2. Trading Mechanisms

I seek to conduct a mechanism design analysis of
I first define the concept of a trading mechanism.

Definition 3.1. A trading mechanism $(\{c\}, \{a\}, \{p\})$ consists of

Å a message space for trader

Å an allocation rule $\prod_{=1} \mapsto$ mapping profiles of reported
quantity purchased by trader

Å and a transfer rule $\prod_{=1} \mapsto$ mapping profiles of reported
price paid to the mechanism by trader

for each trader.[9]

The most commonly assumed trading mechanism in t
I study in this paper is the (uniform-price) doubl

Example 1. In a double auction, the message space is the set of measur
functions $\to$ specifying how much trader i wants for each realiz
asset's price. Given a vector of reported demand functions
are such that,

$$(\quad) (=)$$

[9] Formally, the mechanism is a triple consisting of measurable spaces such that $(\prod_{=1}, \quad _{=1}) \to (\, ,)$ and $(\prod_{=1}, \quad _{=1}) \to (\, ,)$ where is the Lebesgue algebra.

and transfer rules are such that

$$(\quad) = (\quad)$$

for each whenever there is a unique market-clearing pr defined by

$$\sum_{=1} (\quad) = 0.$$

If no such exists, then trades ( ) = 0 for each such that

The double auction will serve as a useful benchm

come. An important property of the double auction

or supply any net quantity of the asset: $\sum_{=1} = 0$. I call a trading mechan

property an exchange mechanism. The focus of this paper is to analyze

that are optimal in a sense that I now specify.[10]

3.3. Objective

I seek to derive exchange mechanisms that maxim

binations of revenue generated by the mechanism a

the negative of the sum of traders' holding costs

rium of the mechanism.[11] By the revelation principle, it is wi

restrict attention to direct mechanisms in which each trader's report is his type,

space and such that 1) it is individually rational fo

anism, 2) incentive compatible for each trader to

the mechanism is an exchange mechanism.

[10] In some cases, it may be of interest to allow the mechanism

this channel down for this paper, though it is easy to extend

amount of the asset. For example, if the designer also has a

optimal mechanisms is largely the same.

[11] That is, I allow the designer to select the equilibrium (p

out to be without loss because the optimal direct mechanism

[0, 1], the objective is to solve

$$\max_{\{\},\{\}} \underbrace{\sum_{=1} (+ () 1 -)}_{\text{Revenue}} - \underbrace{\frac{1}{2} \sum_{=1} + (^{2})}_{\text{Allocative Efficiency}} \qquad (2)$$

such that,

$$\text{(P)} \quad + (() ,) \geq (,) 0, \quad ,$$

$$\text{(IC)} \quad \arg\max + (, _{-}) , (, _{-}) \quad , \quad ,$$

$$\text{(E)} \quad \sum_{=1} ( ) = 0 ,$$

where $= \{\}_{=1}$ is the vector of tradersô endowments.

In the constraint (IC), with a tranôs preport abus
as the first argument in the vector of the other tr
I will continue to do so whenever convenient.

By solving (2) for [0, 1] able to identify the revenue
The revenue-efficiency frontier combioatioos of lxp
plctldallocativl lfficilocy sucmtmat tmlrldol
acmilvlsamigmlr llvll of botm.

I focus on two special cases. The objective
the preferences of a social planner (such as the go
efficiency, incurs no fixed costs of running the e
in illiquid markets. On the other hand, the case wh
planner who has a binding budget constraint. The p
to run by collecting the lowest level that satisfies her c
is useful to analyze is because the planner may wa
use it for some other productive purposes for
the marginal social value of the planner may When
maximizing exchange (most exchanges in practice a
motives). In this case, the analysis sheds light o

3.4. Model Discussion

Despite the prevalence of the model setting in th little analysis is using mechanism or information[12] d related literature. Below, I briefly discuss mod

- To ease the exposition, I restrict attention t is no residual uncertainty in allocations or tr I formally show in Appendix A.2 that this is without loss achieve a higher value for the objective using

- The quadratic holding cost (1) and CARA utility are equiva monotone transformation) when the allocation known to her conditional on her message in a dir are generally not equivalent. Typically, in an a trader is not known to her given her report bec the other traders' endowments. Quadratic util in this case and is why I am able to solve the mod why I do not need to restrict attention to dete turn out that the allocation rules I identify a are implementable when traders instead have C whether they are necessarily optimal in that ca

- The model assumes that endowments are indepen appears often in the lit Chen & Duffie (2021), for examp pendence allows me to avoid de Crémer & McLean mechanism (1988). There is broad consensus that such mechan their sensitivity to common knowledge assumpt

- Other assumptions will be relaxed in subsequen tion about the fundamental in Section 5, multiple assets in Section 6, het among traders in Section 7, and Section 8. However, I obtain the characterizations with the least conditions f

[12] Indeed, classical auction theory focuses on revenue maxi revenue-maximizing designs of exchange mechanisms.

4. Optimal Mechanisms

I now give an informal sketch of the derivation of
nique, I build on methods developed for abstract s
constr Jullien (2000) when there is a single agent. The ana
of Biais (2005) although they study the CARA setting.[13] Relative to
these papers, the setting I consider involves sev
mechanisms are exchange mechanisms, the problem c
not be solved trader-by-trader. Nevertheless, w
tractable.

4.1. Sketch of the Derivation

Step 0: Notation. To start, I define some notation to eas
expected utility, expected trade quantity, and e

$$\hat{\ }(\ ) \equiv \quad + \ (\quad (), \ ),$$

$$\hat{\ }(\ ) \equiv [\ (\quad]), |$$

and

$$\hat{\ }(\ ) \equiv [\ (\quad]) \ |$$

respectively.

Also, whenever $>$, I define $= \int$.

Step 1: Incentive Compatibility. Trader i solves

$$\max \quad + \ (\ , _) , (\ , _) \ | \quad .$$

By standard arguments, local incentive compatab

$$\hat{\ }(\ ) - \quad , 0 = -\frac{1}{\ } \quad \hat{\ }(\ \mathrm{d}) + \hat{\ }(\ ) - \quad , 0, \qquad (3)$$

[13] Also Biais (2005) uses a different method where traders' rents a
attention to deterministic mechanisms.

$$\hat{}(\ ) = -\frac{1}{2}\ (\ ^{2}\ ) - \frac{1}{}\ -\ \hat{}(\ ) + \frac{1}{}\ \hat{}(\ d)$$

$$-\ \hat{}(\ ) -\ ,\ 0. \quad (4)$$

Moreover, a mechanism is globally incentive compa
compatible if $\hat{}$ is weakly decreasing.

By inspection, (3) implies that for the types where (P) binds expect to
$\hat{}(\ ) = 0$ and these types must fall in some closed inte $[\ ,\ ]$.

Step 2: Lagrangian Relaxation. The next step is to form the Lagran
version of (2) that ignores global incentive constraints

Let Ω be the Lagrange multiplier on the participation constra
bounded variation. With abuse of notation, let Ω also denote the dis
bution function of this measure.

Using eq (3) and (4), we form the Lagrangian, integrate
drop some constant terms to arrive at

$$\max_{\{\ \},\hat{}(\ )} \sum_{=1} -\frac{1}{2}\ -\ \frac{\Omega(\ ) -\ }{(\ )} + (\ )^{2} + 1 - \lim_{\to \sup} \Omega(\ )\ \hat{}(\ )(\ )$$

such that $\sum_{=1} (=E0)$:

Step 3: Candidate Optimal Mechanism. Following ensen's in
solution for the optimal allocation rule, ignori

$$(\ ) = -\ \frac{\Omega(\ ) -\ }{(\ )} + \frac{1}{}\sum_{=1} -\ \frac{\Omega(\ ) -\ }{(\ )}$$

Next, we observe that for the $\hat{}(\ )$ to be interior, the multiplier must satisfy

$$\lim_{\to \sup} \Omega(\ ) = 1.$$

Ω is flat[o,u]t) s, iwdeehave

$$\Omega(\quad) = 0, \quad <$$
$$\Omega(\quad) = 1, \quad . \quad >$$

The{eΩ} $_{=1}$ are const[r,u]cttoednisnuˆr=e 0tohm[a t,] for e.ach the AppenAd,iIxshow Ωthaaret tlhee same fdo areeaachlso continuous.

Step 4: VerTihfeicLastiosntep is to verify that strong c Appendix) and to establish conditions for when th decreasing so that global incentive constraints clearly sufficient for this and iMsyeamr(al1o9g8o1u)s to th

Conditiltohno1l.ds $-\frac{1-(\ )}{(\ )}$ aand $+\frac{(\ )}{(\ )}$ are weakly in.creasing

Condi1thiod ns wheinselvoegr-concave, a stronger condi Many common distribu1tiiocnl suadtinsgfy Cthoenudniitfoiromn and G bu tions.

4.2. Characterization

The following Theorem 1 summarizes the results o characterization of optimal mechanisms.

TheoreSmu1ppose tha1h tC obnsd.i Thieonn the unique solutio tion pr(2o)bfo{rlem}se ts

$$(\quad) =) + \frac{1}{}(\quad (\,)) \quad {}_{=1}$$

for eachwtrearder

$$(\quad) = \frac{(\quad)}{(\quad)}, \quad <$$
$$(\quad) = \frac{1 - (\quad)}{(\quad)}, \quad >$$
$$(\quad) = (\,), \quad ,[\ ]$$

for and such thsactontliinmuo(s:) =) = ( ).

(, , [( )]). There are many examples for which there

lutions. For instance, this is so when is symmetric (as for the exp

Gaussian case). This simplicity Lu & Robert (2001) k contr

who study trade in settings with linear utility wh

domization with delicately constructed tiebreak

easy to derive comparative statics and to extend t

important features of the environment and thus fl

Corollary 1. Suppose that the conditions of Theorem 1 is satisfied and that the

symmetric about the mean of the endowments. Then

$$)(\quad = ,$$

and

$$= \quad - (- \quad) ,$$

where solves

$$- \frac{1 - \quad ()}{( )} = \quad .$$

Thus, the optimal mechanism is characterized in c

in addition is uniform, then the optimal mechanism is in c

form.

There are several notable implications of Theorem 1. One is that the optimal mechanism

perhaps surprisingly, depends on the number of traders only very lim

Corollary 2. The range [,] of endowments for which the par

strictly (P) binds does not depend on the number . Thus, the exp

quantity for any given trader, is invariant to the nu

A second is that optimal mechanisms typically fea

the extremes.

Remark 1. An important property of the optimal mechanism that distin-

guishes it from optimal mechanisms in prior work

interim allocation of a trader are, in general, bo
with extreme types. That is, though there is no di
extremes in that $\lim_{\to \sup} ( ) - ( ) = 0$ and $\lim_{\to \inf} ( ) - ( ) = 0$, there is distort
allocations at the extremes. This is in contrast w Bernas (2015) & Rio (2001)
and to my knowledge, almost all other models of opt
no distortion in interim allocations when is symmetric about its mean, b
Example 3 in Appendix A, this is not generally true when is asymmetric.

A third notable implication is that it is in fact
cation with ex-ante budget balance.

Proposition 1. The efficient allocation is implementab
in that expected revenue is zero.

In other McAfee (1991) also derive conditions for when exp
able with ex-ante budget balance. Proposition 1 can also be rel
proven by checking those conditions. However, sub
of Proposition 1 under different model conditions (advers
erogeneity, dynamics etc.) which fall outside McAfee (1991).

Before, turning to an illustrative example, I p
which show that, even accounting for the designer
revenue and efficiency improve when the market is
when the designer places mroe weight on revenue in

Proposition 2. Suppose that Condition 1 holds. Then under the optima

1. As a function of the number of traders, the expected trading cost re $\frac{1}{2}(+ ( )^2 -)^2$, expected trade $[( )]$, and the expected uti $\hat{ }( ) - (, 0)$ are all proportional to $1/$ when > 0. Thus expected allocative efficiency, expected revenue, and increases.

2. If the weight on revenue increases, then $\sum_{=1} \hat{x}( )$ expected uti decreases. If is symmetric about its mean also, then leads the total expected traded volume $\sum_{=1} | ( ) |$ to decrease.

Part 1 of Proposition 2 implies that the revenue-efficiency
ber of traders increases. Intuitively, with more

> 0 the designer can also extract some fractio

expected transfer of each trader also rises.

Part 2 of the proposition states that, even when a

incentives of the designer, traders nonetheless

Part 3 implies that as the revenue-maximizing m

trade volume decreases and each trader is worse of

4.3. Illustrative Example

To further highlight some properties of the opt

the double auction, I present an $i = U[-1, 1]$ (which, the exam

as shown in Corollary 1, is a case when there is a closed-form s

anisms).

Panel (a) in Figure 1 depicts the expected trade quantity f

location (in blue), under the optimal mechanism, and for the m

(symmetric-linear) equilibrium of the double auction; n increases, the d

auction converges to the first best ð this is to be

which is the fundamental source of inefficiency.

bounded away for any given > 0. When is higher, the region of bin

tion constraints expands. Compared with the doub

achieved at the extremes. Interestingly, this is

generic property when is not symmetric (see the Remark in t

It is, however, a generic property of the virtual e (p.e)

Panel (b) plots the expected transfer. Because

transformation of the endowments the price sched

expected transfer is more convex under the double

is because of price impact costs (later we will se

adverse selection). Under the optimal mechanism

spreads. A similar feature appears in Biais et al. (2000) for the case of a single

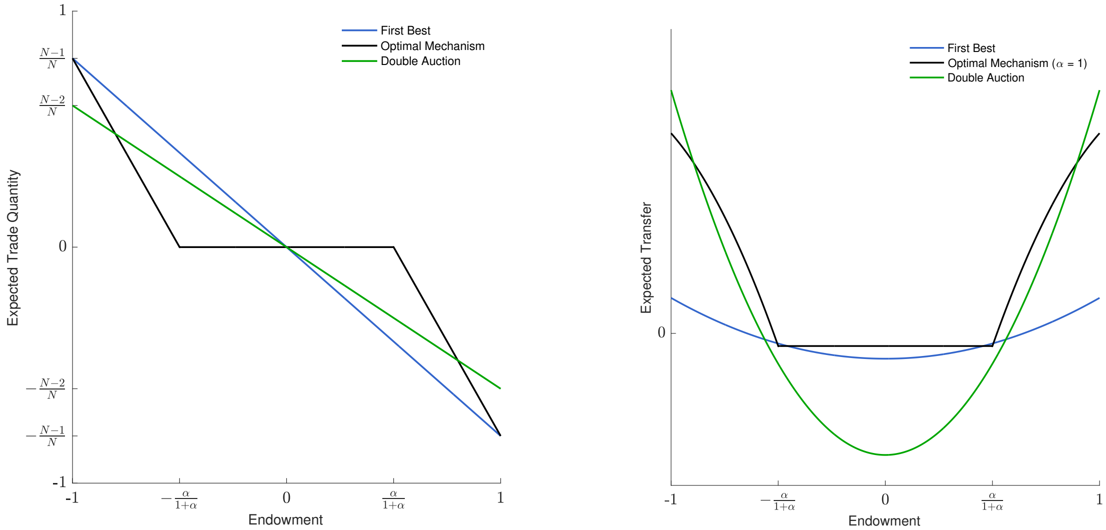
market maker who functions as the designer. The co

discounts and are needed to encourage extreme typ

regions gradually become convex as decreases.

Figure 2 plots the expected revenue per trader in pan

ing. Intuively, there is less randomness in the av



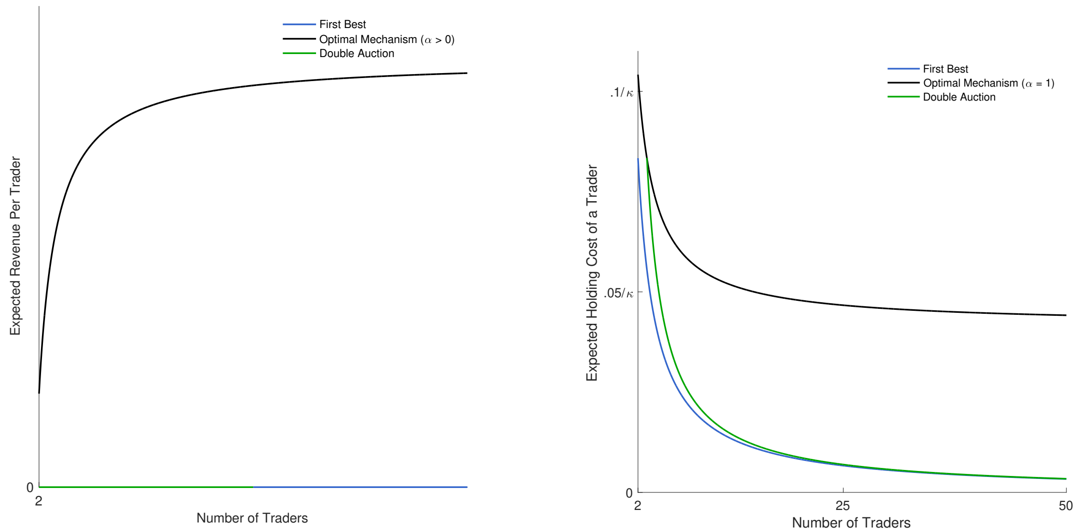
Note: Comparison of interim allocation $= U[-1, 1]$ and interim tra

are risk-averse over asset allocations, they lik
thicker markets, the designer can extract more fr
the expected holding cost of a trader. As seen in t
double auction and first best mechanism but it pe
mechanism for revenue. In > 0 Even under revenue max
tion, each trader is better off when there are more
there are 3 traders is the same for the double auct
mechanism.

Thus, the double auction does not lie on the reve
in Figure 3. One might wonder whether the mechanisms th
indirect implementations that might resemble tr
one might be able to alter the double auction to br
this next.

4.4. Implementation: Double Auction with Transa

It turns out that the double auction can be altere
simply by introducing a transaction fee.



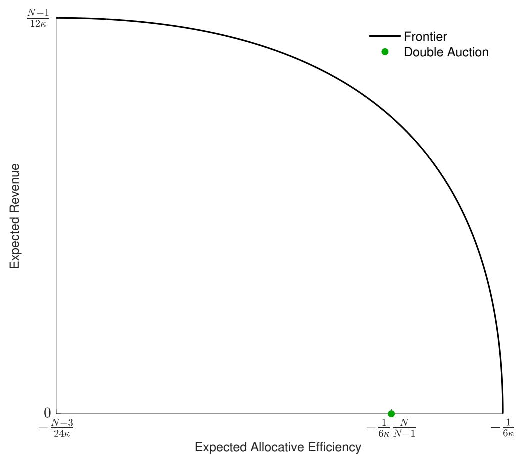
Note: Revenue per trader and expected holding cost per trader where $= U[-1, 1]$

Definition 4.1. A double auction with[2] transaction fees is an exchange mechanism that operates in three stages:

1. Each trader submits a demand schedule.
2. The clearing price is computed: $\sum_{=1} (\;) = 0$. If there does not exist a clearing price, then no trades or transfers are
3. If a unique clearing price exists, trader pays $(\;) + (\;)$ in return for $(\;)$ units of the asset.

Given the flexibility of the transaction fees one practically anything with them. That is not so. Co considered in Malamud & Rostek (2017). Clearly, those allocations a running a single double auction with transaction ples?)

Proposition 4.1. For any given $\in [0, 1]$, the allocation $\{\}_{=1}$ in Theorem



Note: The revenue-efficiency frontier when $= U[-1, 1]$

implementable by a double auction with transacti[2] defined by

$$(\ ,(\ )) = -\frac{\ }{2}(\ ^2) - \ - (\ )\frac{1}{\ } - \ -$$

where

$$= \frac{\ }{-1},$$

$\rightarrow$ satisfies

$$(\ ) = \frac{-1}{\ }\frac{1}{2}\ ^2 - \ ^{-1}(\ d), \qquad ,$$

and is a constant set sufficiently high so that par

types of all traders.

Before I offer intuition for Proposition 3, let me remark on some of

implications. Proposition 3 suggests that acheiving desirable m

require a major overhaul of existing market infra

brating transaction fees a planner can target any

practice, exchanges often charge transaction fe

ject of much policy debate—these fees may not be “j

Proof Sketch. Recall that

$$(\quad) = - \quad)(+ \frac{1}{\quad} \quad \underbrace{\sum_{=1} \quad (\quad)}_{\text{common to all}} .$$

In a double auction each trader submits

$$(\quad) = -) + (\quad . \qquad (5)$$

Reverse engineering that this is optimal for each trader:

$$(\quad , \quad) = -\frac{}{2} \,^{2} + + (\; (\quad) +)$$

for some , and (.)

The function affects incentives for the intercept of … efficiency affects incentives for the slope of the demand … so that each trader submits the demand schedule (5) … individually rational to participate. □

5. Adverse Selection

For trade of many securities, adverse selection … Kyle 1985; … 1989; Du & Zhu 2017). In this section, I investigate the … presence of adverse selection. Loosely speaking … a characterization of the worst-case information … characterization of optimal mechanisms for these … 3) a designer's motives for revenue maximization … prices. Exactly what is meant by "worst-case info… below.

5.1. Environment

I retain the setup of the basic model except now
private signals $\{s_i\}$ of the asset. The signals are random vari
correlated. I will not take a stance, yet, on the dist
spaces where they live. I continue to assume that $\{e_i\}$ are independent
both of each other and the asset's payoff, and they are in
signals $\{s_i\}$. This assumption simplifies the analysis wh
adverse selection. Also, to ease the exposition, $=0$
though all results extend to the case of nonzero .

Mechanism design in this environment is difficu
each trader has two dimensions of private information. One can not mean-
ingfully simplify the model by removing one of the two, because $\{e_i\}$ endowments are the source of gains from trade. Thus, th
is a multidimensional screening problem, which is
special cases. Second, it is unclear what particu
model. Most of the market microstructure literat
sional and equal to the fundamental plus additive noise that
traders. However, because these signals are corr
mation structure where Crémer & McLean (1988) hold, are very sens
to the fine details of the environment and not robu
cumvent these issues, I first take a detour and co
structures for the double auction.

5.2. Worst-Case Information Structures: Double

The analysis here is in the spirit of the litera
"information design" (Bergemann & Morris, 2013; Bergemann et al., 2017; Lambert et al.,
2014). I assume that each trader's signal is her posterior expectation of θ: $s_i = E[\theta \mid s_i]$.

I assume that it is known that

1. $\theta \sim (\, , \,^2)$
2. $\theta, s_1, \ldots, s_n$ are jointly Gaussian
3. $\mathrm{cor}(s_i, s_j) \geq 0$ is the same for (i, j)

and is the same

In what follows, let $\Delta(\ ,\ )$ denote the set of information stru tions 1-4 above. In what follows, I take the persp the marginal distributions of traders' signals a signals is weakly positive (which seems to be an e Vives, 2011). The observer also knows that traders' signa tions 3. and 4. above (an assumption I make for tra not know how traders' signals com . That is, the obse not know the nature of the interdependency in trad traders may have formed their expectations using

In what follows, I seek to derive predictions for Bayesian equilibria in symmetric linear demand s submiting a demand function of the form

$$(\quad) = - + \quad - \qquad (6)$$

for some , , and if she anticipates that the other tra that the restriction to symmetric Gaussian envir in Bergemann & Morris (2013)

Fixing a double auction, what information struc I say that an information structure $\Delta(\ ,\ )$ is worst case if it minimizes s ciency among equilibria in symmetric linear dema symmetric linear equilibria. Let SLE( ) denote the set of symmetr libria for the information structure . Formally, SLE( ) is nonempty or

$$\underset{SLE(\ )}{\arg\min} - \quad (+ ( ))^2 \quad {}_{=1}$$

where , defined formally in Example 1, is the market . The subscript on the expectation operator indica

The following Theorem 2 characterizes all equilibria for i and provides conditions for existence and unique with respect to the correlation among traders' si is novel for two reasons. First, almost all paper with errors that are independent across traders w sum of the private and common components (Rostek & ...

Weretka, 2012). Thus, the characterization of symmetric ...

novel to my knowledge.

Theorem 2. If a symmetric linear equilibrium exists, ...

coefficients, and characterized by (18), (19), and (20) in Appendix B.

Letting $\rho = \rho(r, r)$, the following statements hold:

1. If $= 0$, a symmetric linear equilibrium exists if ...

$$\frac{\quad^{2}}{\quad^{2}\,\frac{1}{-}\,\quad^{2}+\quad^{2}} < \frac{-2}{-1}\,\frac{1}{2}.$$

   The parameter range of equilibrium existence ... increases ...

2. Conditional on equilibrium existence, alloca...

3. The unique worst-case information ... $r_1, \ldots, r$ ... is independent ...

   That is,

$$| = \quad_{=1} .$$

4. Under the condition for existence in Part 1 of t...

   efficiency, which occurs when $= 0$, is:

$$- \quad (1 + \quad^{2} - 2 \quad^{2}) + \quad^{2}\quad^{2}\quad^{2}$$

   where

$$= 1 - \frac{2(\quad - 1)}{-2}\,\frac{\quad^{2}}{\quad^{2}\,\frac{1}{-}\,\quad^{2}+\quad^{2}} .$$

5. Under the condition for existence in Part 1 of th...

   which occurs when $= 0$, is:

$$\Lambda = \frac{1}{-2}\,\frac{1}{-} + \frac{\quad^{2}}{\quad^{2}\quad^{2}+\quad^{2}\quad^{2}\quad^{2}}$$

wheries as in Part 5.

Remark. I have assumed that the ... is known (although its me... inferred from ...). If $\,^2 \geq \,^2$, then this is without loss. If ... worst-case information structure is known; it is determined by the m... such that $\frac{1}{1+(\;-1)}\sum = 1$. This is defined by

$$\frac{\;^{2}}{1+(\;-1)} = \;^{2}.$$

When ^{2} is known and $\,^2 \leq \,^2$, under the worst-case informatio... signals together fully reveal .

If the marginal of ... is known but the marginal of $\{\}$ is not, then the wor... information structure is also fully revealing an... in numerical examples I have computed, th... ^{2} is corre... increases.

Part 1 of Theorem 2 implies that if an equilibrium exists ... $= 0$ ... for all > 0. Parts 2 and 3, imply that the worst-case i... when $= 0$. Parts 4 and 5 characterize allocative eff... worst-case information structure, and in doing so... across all information structures in .

Intuitively, $= 0$ is worst-case becase it is harder for a... will trade when signals are independent. Though... actually more subtle. Indeed, the proof is somew... characterized in terms of a cubic equation which... correlation increases, traders put more weight on thei... schedules as well as on their endowments. The net... rates of increase. The ratio of the weights ... in th... Appendix, must contend with these complications.

In light of Theorem 2, several questions come to mind. Whic... well for these information structures and can bes... a sense in which the information structure ... on the assumed mechanism? Are there mechanisms th... tive efficiency than double auctions? What is th...

5.3. Optimal Mechanisms

Throughout, I now assume that the prior is a multivariate normal with arbitrary distribution ... variance. Note that I am n

continue to assume that each trader sees a single-

. I further assume $=$ § jointly independent and that

$$| = \qquad .$$
$$=1$$

It turns out that, for this information structur

despite the multiple dimensions of private infor

Lemma 1. In any mechanism, optimal message in a Bayes Na

is a function of $= 1 -$ of

Proof. Let $\_$ denote the messages chosen by the other tr mechanisms. Then trader's message maximize

$$-\frac{1}{2} \quad + \quad (\quad , \_)^2 + \qquad\qquad + \quad (\quad , \_) - \quad (\quad , \_) \,|, \quad .$$
$$=1$$

Rearranging, the objective is equivalent to

$$-\frac{1}{2} \;^2 - \;\frac{1}{-} \; - \qquad (\quad , \_) - \frac{1}{2} \; (\quad , \_)^2$$

$$+ \qquad (\quad , \_) - (\quad , \_) \frac{1}{|} \; - \quad .$$
$$\neq$$

Thus, the optimal choice of message depends on onl

and denote
the cdf and pdf respectively for any given trader.

$$\hat{} \quad (\quad) [\quad | \quad = \quad].$$

The following Theorem 3 characterizes the optimal mechanism ... selection.

Condition 2 holds that $\frac{(\;)}{(\;)}$ + ... and $\hat{}$ ... $\frac{1 - (\;)}{(\;)}$ are weakly increasing ...
.

Theorem 3 Suppose that Condition 2 holds. Then the unique solution ... problem (1) sets

$$(\quad) = -) (+ \frac{1}{} \sum_{=1} ( )$$

for each trader where

$$(\quad) = \hat{} \; \frac{(\quad)}{(\quad)} , \;) + \quad <$$

$$(\quad) = \hat{} \; \frac{1 - (\quad)}{(\quad)} , -) \quad >$$

$$(\quad) [= ( )] , \qquad , ( )$$

for and such that is continuous: $\lim_{\to} ( ) =) = (\;)$.

As in the basic model, optimal mechanisms are ch... know (n , s , [( )]) and there are special cases where the u... acterized almost in closed form.

Corollary 3 Suppose that Condition 2 holds and is symmetric about it... Then under the optimal mechanism,

$$[\; ( )] = \quad ,$$

$$\hat{} \quad) (+ \frac{( )}{( )} = \quad ,$$

and

$$\hat{\ }\ )(- \frac{1 - \ ()}{(\ )} = \ .$$

Using Cor. 3.1, I am able to prove the following compa

Proposition 4. Suppose and are Gaussian. Then the follow statics hold:

1. As ^{2} increases, the region of binding participat increases without bound for $^2 \to \infty$. Thus, as $\to 0$, the probability trade vanishes in that

$$\sum_{=1} |\ (\ )\ | > 0 \to 0 .$$

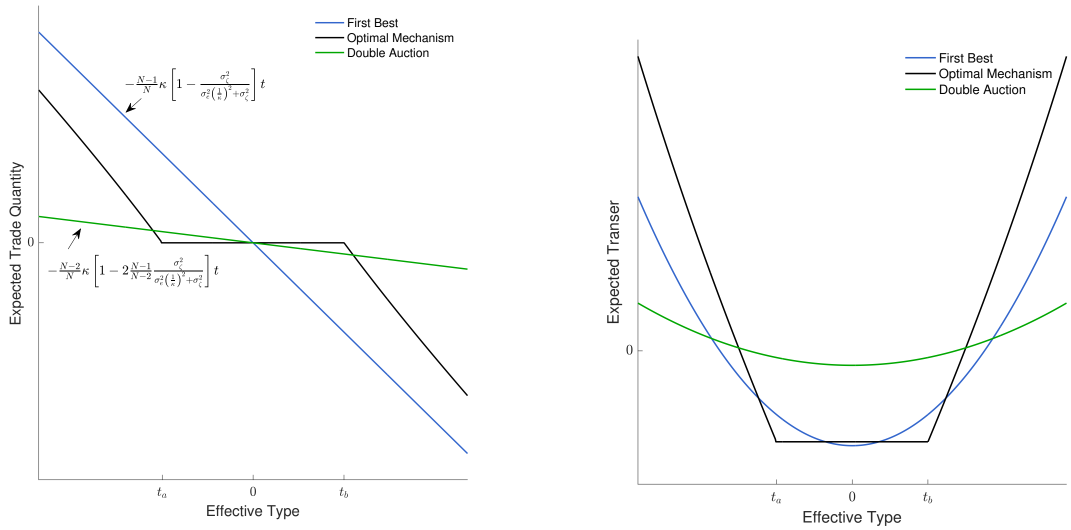
2. As ^{2} increases the revenue-efficiency frontier

3. For any given > 0, as ^{2} increases, the expected utility g any given and the expected $\sum_{=1} (\ )$ decrease.

Proposition 4 implies that as adverse selection becomes more, the region of binding participation constraints incr vanishes, and the revenue-efficiency frontier s finite ^{2} there is always some probability of trade the double auction where (symmetric linear) equi ^{2}.

5.4. Illustrative Example

To further illustrate the properties of optima selection, I now present an example (Panel (a) of Figure 4 plots the expected trade quantity as a funct see, many qualitative properties are the same as t increases, the gap between the double auction and constraints also expands and eventually diverges



Note: Comparison of interim allocation rules (a) and interim tr ... (σ^2 ... 0).

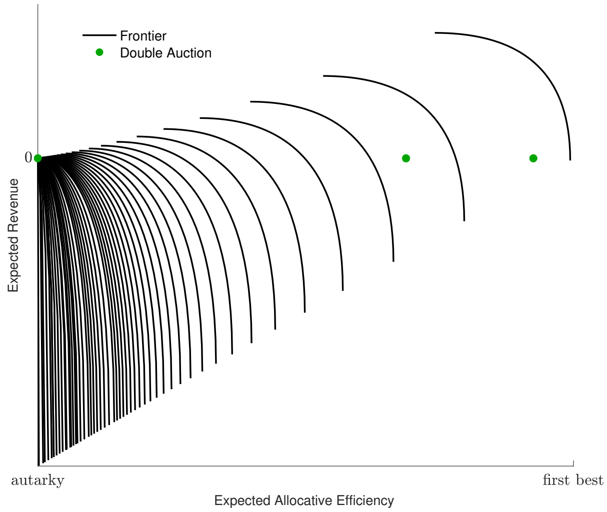
As seen from panel (b), ... σ^2 ... curvature under first best than under the double a... case without adverse selection. This must be so b... increases the ratio of the slopes of the blue and g... bounded from above for the case without adverse se...

Below in Figure 5, I plot the revenue-efficiency frontier ... σ^2 ... of .5. As one can see the fronteir shifts down befo... at autarky. The double auction reaches the autark... mechanisms.

5.5. Implementation: Double Auction with Transa...

It turns out, that even with adverse selection, ... to the revenue-efficiency frontier using transac...

Proposition 5. For any given $t \in [0,1]$ the allocation ... is implementable by a double auction with ... equations of the Appendix.



Note: The revenue-efficiency frontier for different values of when $r(0)$ and (θ^2).

The proof is similar to the case without adverse ... other term in the transaction costs. A large litera-ture studies information aggregation in financia... 3.2 implies that once one endogenizes the mechanism ... of the designer worsen price informativeness.

Corollary 3.2. Suppose that ... are Gaussian. Whenever a sym... equilibrium of the double auction without transa... librium is more Blackwell informative than in the ... transaction fees set optimally for any > 0

Corollary 3.2 indicates that conclusions concerning in... erature may depend on the exogeneity of the tradin... prices to aggregate information will not disappe...

5.6. Robust Mechanisms for Allocative Efficienc...

Is there a sense in which the information struct... does not depend on assuming the double auction mec... antee of allocative efficiency that can be achiev...

sees a signal of the fundamental where the signal space is an arbitrary measurable set. Let

$$= ( ) \{ , \}$$

denote the information structure, where $\Delta(\times \Pi_{=1})$ is the joint distribution of signals and the fundamental.

I assume the designer knows some aspects of the expectation of the common component of the payoff that is known and that it is otherwise stochastically monotone: realizations of an increase in the given signal. It is a form of first-order stochastic dominance, for example, it is to assume that it is known that traders' endowments $\{ \}$ and.

That is, the designer may have some sense of traders' capital of the asset, but does not know exactly how they are (the signal structure) and in particular, how they are know how these signals combine to give the best est

$$(_1 , . .) = [|_1 , . .] . ,$$

or even what spaces these signals live in. An alternative: formation necessarily, but that values are interdependent; simply has no idea how values are interdependent.

Here are two examples of information structures in the ambiguity set (though there are of course many other ex

Example 2. Suppose that signals are just $_1$, . . . and traders' pos that these are independent random variables (and earlier). Then both

$$(_1 , . .) = \sum_{=1}$$

and

$$(_1 , . .) = -1 + \Pi_{=1} (+ 1)$$

can be are consistent with information structure

In the face of this ambiguity, the designer look
sense that it delivers the highest guarantee acro
sistent with the marginals.

To state the objective a bit more formally, I firs
design and information, let the structure of Bayes Nash Equi
(,.)

Let

$$AE(\quad , \quad , \quad) = -\frac{1}{2} \sum_{, =1} (+ ( ))^2$$

where {r} is the allocation rule where the expectation is ta
according to the probability distribution indu

I adopt the solution concept of Brooks and Du (2021): maxmin so

Definition 5.1. A strong maxmin solution consists of a mechanism, in-
formation structure and Bayes-Nash equilibrium

1. For any mechanism and any equilibrium (,) it holds that $AE \leq AE(', ')$

2. For any information structure and equilibrium (', ') it holds that
$AE(', ') \geq AE(, ,)$

3. is an equilibrium of ,

The following theorem assumes compact support b
ample, suppose ( ) is Gaussian, then the result app

Theorem 4. Suppose that has compact support and that Condit
2 holds when = 0. Then there is a strong maxmin solution
allocation rule with = 0 in which nature selects the in
assumed in Subsection 5.3.

Proof. Recall that the aggregate error solves

$$\max \; -\frac{1}{2} \quad + \quad (\quad , \_)^2 + \quad + \quad , \_) \quad -(\quad , \_) \,|, \quad .$$

Now suppose we set the transfer rule

$$(\quad) = \hat{-}(\quad) + \quad \hat{} (\quad d) - \frac{1}{2} \quad (\quad ^2 +) \qquad (7)$$

For the transfer rule in (7), all terms ... drop out from the obje
Thus, each trader submits the same message under any information struc
Now, I directly verify that allocative efficien
worst-case information structure where signals a

$$\sum_{=1} \quad + \quad (\quad)^2 = \quad ^2 - \frac{-1}{} \sum_{=1} \quad () + \sum_{=1} \quad ^2$$

$$= \quad ^2 - \frac{-1}{} \sum_{=1} \quad ()$$

$$+ \quad - \frac{-1}{} \quad () + \frac{1}{} \sum_{\neq} \quad () \quad ^2$$

$$= \quad ^2 - (\quad - 1) \quad () + \frac{-1}{} + \frac{1}{2} \quad ()^2$$

$$- \frac{1}{} \sum_{=1 \neq} \quad \mathrm{cov}(\quad , \quad)(\quad .$$

Because the coefficient on covariance ... pairwise ... each
tically monotone, it follows that independence i

Theorem 4 illustrates the sense in which the informa
far are worst-case without fixing, at the outset,
weakness of the result is that it allows for a budg
with the classical Vickrey (1961), Clarke (1971), Groves (1973) ... n efficient
implementation ... Bergemann and Välimäki (2002) ... In some cases, the gov
can enforce participation. Following the classi

Corollary 4.1. The strong maxmin mechanism is robust to traders' higher-order beliefs about other traders' beliefs about

In general, deriving robust guarantees when th intractable. As seen from the main proof of Theorem 4, th not find it individually rational to participate conditional on full participation, one can always construct such that traders optimally report of the information same argument as in the proof of Theorem 4. However, that efficie are typically quite “aligned,” our analysis of information structures for efficiency is still i revenue in the presence of severe adverse selecti

6. Multiple Assets

Can the efficient allocation be implemented by when there are multiple assets? Is it more or less when there are more assets? Does this depend on th among asset endowments? What do revenue-maximiz be implemented by double auctions with transactio questions for a multi-asset version of the model.

6.1. Environment

There are now multiple assets. Trader i's utility is

$$(\ +\ ,\ ) = \quad -\frac{1}{2} \quad + \quad \Sigma \quad + \ - \qquad (8)$$

where

- $= \{\}$ is the vector of asset payoffs
- $= \{\ \}$ is the mean of
- Σ is the covariance matrix of
- $= \{\ \}$ is the vector of trader's endowments
- $= \{\ \}$ is the vector of trade quantities of the
- is the net transfer
- > 0 is a constant.

I assume that endowments $= \{\}$ are independent across traders
pendent of asset payoffs. However, the endowments of the d
assets may be correlated. I retain all other aspec

6.2. Optimal Mechanisms

Proposition 6 states that it is possible to implement th
| separate double auctions with transaction fees
ante budget balance, regardless of correlation a
different assets. This result is perhaps surpris
in one auction contingent on the prices in the oth
would make implementation impossible.

Proposition 7. The efficient allocation is implementable
tions with transaction fees. It is implementable
ex-post equilibrium.

Condition 3. The following hold:

Å = {} are independent Gaussian random variable

Å = {} are independent Gaussian random variable

Note that, besides Gaussianity, each of the 3 in Condition 3 is without loss on its own and is just a normalization, howev because one can always construct securities so th pendent. On can also redefine securities by scali of all payoffs or all endowments are the same. How simultaneously satisfy both properties. Notice also that Condition 3 makes no r on the mean = {n} or for the mean = {n} of.

Proposition 7. Suppose that Condition 3 holds. Let $\|(-)\|^2$ be the length of for each. For each and, let $\tilde{} = (-)/$. Then the unique solution to the optimal allocation rule (32) is

$$(\quad) = (\quad) \tilde{} + \frac{1}{} \sum_{=1} ( ) \tilde{}$$

for each and where

$$(\quad) = 0, \quad \sqrt{\ } \in [0, \ )$$

$$(\quad) = \frac{2}{\ } - , \quad \sqrt{\ } \in [\ , \infty)$$

and where $\in [0, 1]$ is the weight on revenue in the objective (32), and the formula for is in Appendix C. The optimal allocation rule is implemented ... Appendix C, equation (34).

Proposition 7 shows that many properties of optimal mech... are many assets, at least under Condition 3. To prove the proposition, le... symmetry" of the optimization problem, as in Armstrong (1996), building ... Wilson (1993) who study a multiproduct monopolist.

Consider Figure 6, which depicts the case of two asse... anism, the region of binding participation constraints is $\sqrt{\ }$ ... thus larger when the objective places more weight ... region expect to trade zero of both assets. Outsi... constraints lie along radial lines, are local, an... type is distorted only radially. That is, the opti... of a trader's. The proof of Proposition 7 is in Appendix C.

Unfortunately, it is not possible to implement t... submission strategies using separate double auc... possible to do so with cross-exchange transaction fees ... made contingent on trades and prices on all exchan...

Definition 6. A double auction with cross-exchange tr... is an exchange mechanism that operates in three st...

1. Each trader submits a demand schedule for each asset.

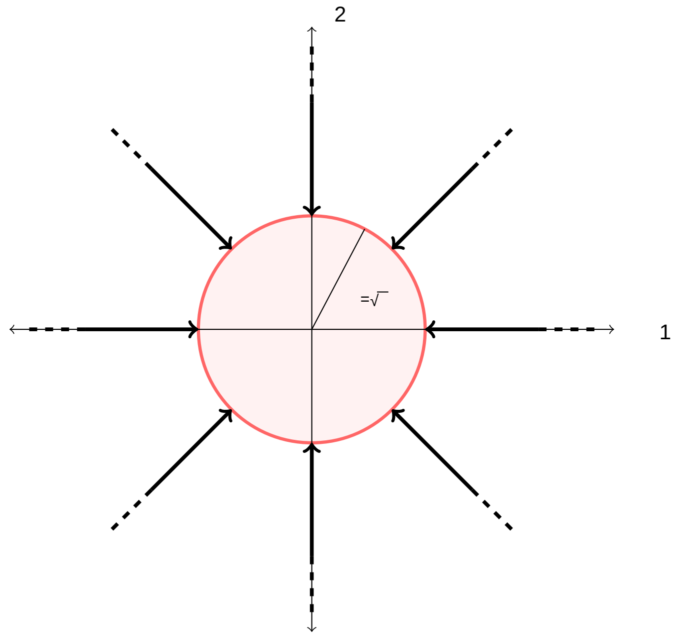


Figure 6: Binding participation and incen

Note: The figure illustrates the binding participa
of two assets. In the general case, with $|\ | \geq 2$ assets, the weight on of bindi
constraints is a ball with radius $\sqrt{\ }$. The binding incentive const
point in the direction of the ball and bind only ra

2. The clearing price is computed as $\sum_{=1}(\ ) = 0$ if there does no
exist a unique clearing price, no trade occurs for each asset for

3. If a unique clearing price exists for each asset, trader pays in total $(\ ) +$
$(\{(\ ), \}\ )$ in return for $(\ )$ units of each asset.

Note that though cross-exchange transaction f
each exchange nonetheless clears separately in t
given trader is contingent on only the price of tha
is still relatively simple.[14]

Proposition 8. Suppose that Condition 1 holds. Then the
sit 7 for any given $\in [0, 1]$ can be implemented by a double auc

[14] Most exchanges clear separately, though Chen & Duffie excep
2021; Budish et al., 2023.

exchange transaction fee is defined by

$$\{(\ ),\} = (\ ) \frac{1}{} + {}^{2} + \frac{1}{2} (\ {}^{2}) +$$

where $\rightarrow$ satisfies

$$(\ ) = \frac{-}{}\frac{1^{2}}{} \int_{0}^{\sqrt{}} {}^{-1}(\ \mathrm{d}) - \frac{1}{2}, \qquad \geq 0,$$

$$= -\frac{{}^{2}}{(\ -{}'1)}$$

and is a constant set sufficiently high so that par
types of all traders.

Though Condition 3 is strict, Proposition 3 further demonstrates the
effectiveness of transaction fees even in a setti
the designer may care about revenue. For this set
exchange market clearing. That is, there is no ne
exchange to be made contingent on the prices in ot
mentary per Budish et al. (2023) who argue for a market desig
cross-asset market clearing. Though cross-asset
Propositions 6 and 8 show it may not be needed in all instance
freedom to set transaction fees or to design deriv

7. Heterogeneity

Our analysis so far has assumed that traders are
to the probability distributions of their endowm
ity, these likely differ across traders. Though
microstructure literature, some papers allow fo
and interesting phenomenona. By allowing for het
questions such as: How should trade be designed in
retail traders with low holding capacity receive
cated institutional traders who typically have h
with transaction fees still optimal?

7.1. Environment

I retain all aspects of the baseline model of Section 3 except ... $\{l\}$ ... of endowments and also $\{s\}$ ... traders. The designer's objective (2) ... changes is stated formally (38) ...

7.2. Optimal Mechanisms

The derivation of optimal mechanisms is now analogous to Section 3 except ... require a stronger technical condition.

Condition 4 holds that $-\frac{1-(\ )}{(\ )}$ and $+\frac{(\ )}{(\ )}$ are weakly increasing ... has full support for ... each trader.

The full support assumption is needed to ensure ... expect to trade is ...[15] If ... is violated, we can ... calculate the optimal mechanism, ... Theorem 5 ... tails vanish.

Theorem 5. Suppose that Condition 4 holds. Then the unique solution to the ... problem (38) for $\{r\}$ sets

$$(\quad) = (\ -) + \frac{\quad}{\sum_{=1}} \ (\ )_{=1}$$

for each trader where

$$(\quad) = \frac{(\quad)}{(\quad)} + , \quad <$$

$$(\quad) = \frac{1 - (\quad)}{(\quad)} -, \quad >$$

$$(\quad) = \frac{\quad}{\sum_{\neq}} \ (\ )_{\neq}, \quad , \quad , []$$

for and such that ... continuous: $(\ ) = (\ ) - (\ )$.

Now, each trader unloads her virtual endowment ... the aggregate virtual endowment that is proportional ... libria is now significantly more ... $\{\ ,\ [\ (\ )]\}_{=1}$ because ...

[15] These types need not exist if traders are sufficiently he...

Corollary 5. Suppose that Condition 4 holds, $[t_1, \ldots]$, and $= {}_1[\ldots]$, are colinear and symmetric about $[u(\ )] =$ its mean, then

$$+ \frac{(\ )}{(\ )} = \quad ,$$

and

$$- \frac{1 - (\ )}{(\ )} =$$

for each .

At first thought the colinear requirement seems ... but it holds when ... have zero mean and each has a common mean (but ... differ in all higher-order moments) and all trade ... for each. The symmetry assumption is $\{r\}$ are Gaussian, but ... a common assumption in the market microstructure ...

Let me note that in this uniform case, we can often compute ... mechanism in closed form, just as in the basic mode...

Remark. If each is uniformly distributed, as long as t... trade zero in expectation, then the optimal mechan... a system of linear equations.

7.3. Comparative Statics

Using Corollary 5, I am able to derive the following comp... 9 focuses on the special case where each is Gaussian with zero mean. A... ex-ante would be an ex-ante source of heterogenei... which I seek to shut down.

Proposition 9. Suppose that each trader's endowment is Gaussian w... zero and variance ^{2}. Then the following comparative static...

1. The regions of binding $\{p[a, r]\}$ ... constraints ...

2. Suppose ^{2} increases. Then for ... > 0 ...

(a) The range [,] of binding participation constrai
but has no effect on trader $\neq$.

(b) The expected utility gain of trader decreases.

(c) The expected utility gain of trader $\neq$ is unaffected.

3. If increases for trader :

(a) The expected utility gain decreases for trad

(b) The expected utility gain increases for trad $\neq$.

Proposition 9 shows how a trader's utility gains from tr
in the market as well as her own characteristics u
surprisingly, as seen from Part 2c, for any given
other traders' endowments has no effect on her ex
proposition shows that a trader benefits when the
holding capacities. This is because she will be be

7.4. Implementation: Double Auction with Transa

One might wonder whether a double auction with t
the revenue-efficiency frontier when there is he
as long as the transaction fees can be tailored to

Proposition 10. A double auction with trader-specific t
any outcome on the revenue-efficiency frontier.

The intuition is the same as for the basic model.
ization of the implementing transaction fees whi
sition 3 except with coefficients that depend on the i
{ }. Thus, even in the presence of heterogeneity, t
With heterogeneity, exchanges have incentives t
whether it be to promote efficiency or revenue. I
differ across traders—large institutional tra[16]

[16] For example, see https://optiver.com/insights/a-little-under
-options-in-the-us/.

8. Dynamics

The majority of this paper's analysis is conduct... a sense in which the results extend to dynamic set...

Suppose that a trader's utility is of the form

$$(\{(\ )(\ ,\ );\ \geq 0\})=\int_0 \frac{1}{2}(\ +(\ )^2)-(\ )\,d$$

where ( ) denotes the quantity that a trader holds in ... her initial endowment, and where ( ) is the terminal time.

The designer's problem is to set a path of allocations ... maximize a convex combination of the net-present ... ciency subject to the constraints that 1) it is ind... in the mechanism at each time, 2) it is incentive compatible ... port her endowment truthfully, 3) the mechanism d... of the asset at any time. A formal statement of the ...

Proposition 1 implies that the dynamic problem simplif... the analysis of the one-shot problem considered in Section ...

Proposition 11. (3) solution to trades are executed at ... $(\ )=0$ and $(\ )\neq 0$ for each >0 for each.

Of course, whenever the weight is positive, this re... strong commitment power of the designer who must r... in the future and 2) the absence of renewed endowm... that several papers study the setting with a sing... shorter time horizons (Vayanos, 1999; Amin & Chen, 2011).[17] Though the reliance on commitment is an important limita... upper bound on what is achievable with weaker form... a conventional assumption in the literature on dy... (Bergemann & Välimäki, 2010; Pavan, 2014; Athey & Segal, 2013).

When there are renewed endowment shocks over ti... can say. However, I am able to derive conditions w... implementable.

---

[17] Vayanos (1999) allows for renewed endowments shocks in the mode... focuses attention on the limiting case as the variance of fu...

Proposition 12. Suppose that each trader's endowment process of

$$= {}_0 + \quad , \qquad [0, \quad]$$

where { ; [0,]} are independent Brownian motions and { } are independent Gaussian random variables. Then the efficient allocation implemented with an ex-ante budget surplus by a c transaction fees.[18]

Thus, even in a dynamic setting with renewed en with transaction fees can be a powerful policy to achieve efficiency with a budget surplus which wa is because gains from future trades can be used to Indeed, as a trader's $\to 0$, her private information becomes irrelevent for her gains from participation beca endowment process. Thus, it is as though interim ex-ante participation constraints.

This result is in contrast to Vayanos (1999) who shows that, fixing the tion, as the frequency of trade increases, welfar less aggressively.[19] Proposition 12 implies that opposite can be true troduce transaction fees, which are often employ whether the Vayanos (1999) intuition holds when the exchange can set an open question and may depend on the strength of maximization.

9. Conclusion

The objective of this paper has been to investiga a variety of market conditions for a workhorse mod crostructure. There is only a relatively small li microstructure and mechanism design. This paper this gap.

[18] A version of this result first appeared in Chen & Zhang (2020), preliminary work with am a coauthor.

[19] A low frequency of trade approximates a static environmen

exchange mechanisms often leads to tipping onto
oping a cohesive analysis with competing exchang
would have great value. A plausible conjecture is
exchanges may appear similar to outcomes in this p
weight on revenue. Investigating whether this is

Online Appendix

A. Omitted Material for Section 4

Lemma 2. It is without loss of generality for the des
ministic mechanisms. That is, the designer canno
(2) by selecting a stochastic mechanism.

Proof. Because utility is quasilinear in transfers i
transfers to be deterministic. I shall now show it
istic allocation rules.

Let = { } be an implementable allocation rule tha
(,) = { } denote the random vector of trade quant
reporting and giv ... We can write

$$(\quad , \quad) (= \quad , + \quad) (| \quad)$$

where ( ) = | 0.

Define the deterministic allocation rule = { } by

$$(\quad) = (\quad , , \quad) \quad | \quad .$$

Clearly, is implementable ... is weakly decreasing ... imple-
mentable by assumption ... expected transfers and allo
are both higher ... because they are concave in the allo
also (4) which applies even when mechanism are stocha
individually rational ... the utility gain fro
depends on only the interim ... (3) ... which applies even w
the mechanism is stochastic).

It thus follows that any stochastic mechanism c
mechanism. □

The rest of this appendix gives ... proof of
prove the following auxilary Lemma 3.

Lemma 3. If > 0, then any solution (2) is unique up to measure zero
If = 0, then all solutions have the same allocation

Proof. Using equation (4) for the transfer rule and integratin
mization problem (2) as

$$\max_{\{\,,\hat{}(\,)\}_{=1}} \sum_{=1} -\frac{1}{} - \frac{\mathbb{1}_{\{\geq\}} - (\,)}{(\,)} \quad (\quad) - \frac{1}{2} - (\,^2\,)$$

$$- \hat{}(\,) \qquad (9)$$

such that

$$-\frac{1}{} \int \hat{}(\,\mathrm{d}) + \hat{}(\,) - \quad, \; 0 \geq 0, \quad ,$$

$$\hat{} \text{ is nondecreasing},$$

$$\sum_{=1} (\quad) = 0,$$

where recall the $\equiv$ ... It is easy to see th
solution must necessarily be unique because the
objective function is strictly concave in the ...
satisfying the constraint $= 0$ is a convex set ...
solution for the allocation rule is necessarily u
$\hat{}(\,)$ that are optimal. □

Consider the problem

$$\min \quad \text{such that} \quad \in \Omega, \quad (\quad) \leq \qquad (10)$$

where Ω is a convex subset of a vector space, is a real-valued convex fur
and is a convex mapping from Ω to a normed space. ... convex cone in
For, write $\geq$ if $-$. The cone which defines this relat
the positive cone. To complete the proof we will use the following
following restatement of Theorem 1 of Luenberger (1969) that appears

Theorem 6 (Luenberger (1969)). Let denote the positive cone of
closed. Suppose there exists an $_0 \in \Omega$ such that $(_0) <$, and that the Lagran

$(\ ), = (\ ) + $ possesses a saddle point at $_0$, $_0$. That is,

$$(_0, \ ) \le {}_0(_0) \le (_0) ,$$

for all Ω, $_0 \ge$. Thus $_0$ solves (10).

Using Lemma 3 and Theorem 6 we can now complete the proof of Theorem 1.

Proof of Theorem 1. The relaxed version of problem (2) with the monotonicity constraint fits into the framework of (10) with the following translation.

1. Let

$$= \frac{1}{} - \frac{\mathbb{1}_{\{ \ge \}} - ()}{()} \quad (\quad) \frac{1}{2} + (^2 +) \ \hat{} (\ ) .$$

2. Let Ω be the set of $(\{\}_{=1}, \{\}_{=1})$ such that $\sum_{=1} = 0$ and each is a measurable function from $[\ , \bar{}\ ]$ to a compact interval in .

3. Let $\Omega \to$ $\bar{}$, $(\ ]$ be the function that maps

$$(\{\}_{=1}, \{\}_{=1}) \quad \frac{1}{} \quad [\ (\ ,)] \mathrm{d} \ - (\ ) \frac{1}{2} \ ^2 \quad .$$
$$_{=1}$$

Note that is convex.

4. Let be the set $[\ , \ ]$.

5. Let be the be the function that is equal to zero

6. The set is the set of bounded variation measures on $[\ , \bar{}\ ]$

7. Let be the set of nonnegative continuous functi

It is clear that is nonempty, convex, closed and contains an interior point. Thus we can apply Theorem 6 to verify the optimality of the relaxed problem which was obtained by solving for a saddle point; the sufficient conditions the monotonicity constraint in Theorem 1 must also be a solution of the original problem (2).

Proof of Corollary 2. The proof is immediate from Theorem 1.

Proof of Corollary 1.1. We verify that the proposed solution in Corol- lary does indeed satisfy the conditions of Theorem 1.

We have

$$- \; = \frac{1 - \quad ()}{( )},$$

and so, using the symmetry of , we have

$$- \; = \frac{(- (- \;))}{(- (- \;))}$$

$$\Leftrightarrow$$

$$- (- \;) + \frac{1 - \quad (- (- \;))}{(- (- \;))} =$$

$$\Leftrightarrow$$

$$- \; = \frac{( )}{( ))}$$

as desired.

I now verify that ( ) = . We have that

$$( ) - \; = \; - - \frac{1 - \quad (\quad)}{(\quad)}$$

when $\geq$ and

$$( ) - \; = \; - + \frac{(\quad)}{(\quad)}$$

whenever $\leq$. Everywhere else ( ) − = 0. We have

Because (−) = − (−) and we indeed have

$$)(\; =$$

where I have used the symmetry of and an argument of . $\square$

Proof of Proposition 2. To prove Part 1 recognize that

$$(\; ^2 \;) = \frac{(\quad - 1)^2}{2} + \frac{- 1}{2} \text{var}[( )] = \frac{- 1}{} \text{var}[( )] .$$

Moreover,

$$\hat{} (\;) = \frac{- 1}{} ( ) + \frac{- 1}{} [\quad]( )$$

[ )( ] does not depend on ( ). Thus, by (4) and (30), the expected transfer and expected utility gain are both proportional to $\frac{1}{-1}$ as claimed ... (, 0) = 0 under the optimal mechanism.

Similarly the expected holding costs relative to

$$\frac{1}{2} \quad (\; ^2 +) \frac{1}{-} \quad (\;)$$

which also is proportional to $\frac{-1}{-}$.

I now prove part 2. Clearly, when revenue increases allocative efficiency decreases. Because the expected utility gain ... change allocative efficiency and expected transfer ... utility gain must decrease.

To show that the total expected trading volume ( ) decreases in the mean-preserving spread ... about its mean. This is ... of ′( ) simply by using the mean-preserving spread ... symmetry of

□

Proof of Proposition 5: The proof of 3 ... the more general case where there may be adverse selection.

□

Example 3: Suppose Exp(1) and = 1. Then

$$[\;(\;)] = .86814$$

and so under the optimal mechanism, a trader of type

$$-\frac{-1}{\quad} .86814$$

units in expectation rather than

$$-\frac{-1}{\quad}$$

as she would under the efficient mechanism. Under ... tation

$$-\frac{-2}{\quad}.$$

Thus, the distortion can be higher under the optim

Expected trade volume is trickier.

B. Omitted Material for Section 5

Formal Statement of the Design Problem. Given [0, 1], the Objective is to solve

$$\max_{\{\ \},\ \}\{} \underbrace{\sum_{=1} (\ )}_{\text{Revenue}} + (1 - \ ) \underbrace{\sum_{=1} -\frac{1}{2} \quad + \quad (\ )^2}_{\text{Allocative Efficiency}} \tag{11}$$

such that,

$$\text{(P)} \qquad + \quad (\ )(\ ) \geq (\ ,\ )0, \qquad ,$$

$$\text{(IC)} \qquad \arg\max + \quad (\ ,\_\ ),(\ ,\_\ ) \qquad , \qquad ,$$

$$\text{(E)} \qquad \sum_{=1} (\ ) = 0,$$

where $= \{\ \}_{=1}$ is the vector of traders' effective types.

The rest of this Appendix contains 5. In all proofs, I first provide 4 auxiliary Lemma which will be used in the proof of Theorem 2.

Lemma 4. If a symmetric linear equilibrium exists, the coefficients that are characterized by the following

$$- = \quad 1 - \frac{(\quad /^2(\ ) - 1)^2}{{}^2 + (\quad /^2\hat{}^2)} \quad \hat{} \tag{12}$$

$$= \quad \frac{-2}{-1} - 2\frac{1 + \frac{-2}{2}}{1 + (\quad - 1)^2} \frac{- \quad {}^2\hat{}^2}{+ \ - \ {}^2\hat{}^2} \tag{13}$$

$$= \frac{-1\ 1}{-2} - + \quad \frac{2\hat{}}{{}^2\ {}^2 + {}^2\hat{}^2}^{-1}. \tag{14}$$

where $\hat{r}^2 = {}^2 \quad 1 + (\quad - 2)^2(\quad - 1) \quad .$

Proof. I conjecture an equilibrium of the form

$$(\quad) = - + \quad - \quad .$$

It follows from the rules of conditional Gaussian

$$[\quad | _1 , \ldots] = \quad , \quad _{=1}$$

for some which we shall now determine.

Because $= [\quad |]$, it must be that

$$+ \quad (\quad - 1) =$$

which implies that

$$= \frac{1}{1 + (\quad - 1)} .$$

By market clearing, if trader submits demand schedule ( ) and all other trad follow the conjectured strategy then the price mu

$$= \frac{- \sum_{\neq} \quad + \sum_{\neq} \quad + (\quad)}{(\quad - 1)}$$

$$\Leftrightarrow$$

$$= \frac{- \sum_{\neq} \quad + \frac{1}{} \hat{} \quad \frac{1}{} \quad + (\quad)}{(\quad - 1)} . \qquad (15)$$

In order for ( ) to be optimal, the first-order condition

$$\sup_{\Delta} [\quad | \quad (, \quad)] (\; (\quad) + \Delta) (- + \; (\quad) + \Delta) - \frac{1}{2} (\quad + \Lambda) \Delta^2 ((\quad) + \Delta) \qquad (16)$$

must be $\Delta = 0$, for any given , where $\Lambda = 1 / [(\quad - 1)]$ denotes price impa This can be shown formally by a straightforward ap

The first-order condition is

$$- \frac{1}{} (\quad + \; (\quad)) \; [+ | \quad (, \quad) , ,] = \Lambda \; (\quad) + \quad . \qquad (17)$$

In order to compute the conditional expectatio

$\sum_{\neq}$ conditio.We have

$$\mathrm{var}_{\neq} = (- 1)^2 [1 + (-]2)$$

Then, by the rules of conditional Gaussian random

$$\mathrm{var}_{\neq} | = (- 1)^2 [1 + (-]2) - \frac{(- 1)^2}{2}$$

$$= {}^2 (- 1) \quad 1 + (\; - (2) - 1 -)^2 \quad .$$

Therefore,

$$| \quad , {}^2 \hat{}^2 (- 1)$$

where

$$\hat{}^2 \equiv {}^2 \; 1 + (- 2)^2 (- - 1 .)$$

Again, using the rules for conditional Gaussians

$$| , - _{\neq} + \frac{1}{} = + \frac{{}^2 \hat{}}{{}^2 \; {}^2 + {}^2 \hat{}^2} - _{\neq} + \frac{1}{} - \frac{1}{}$$

$$= + \frac{{}^2 \hat{}}{{}^2 \; {}^2 + {}^2 \hat{}^2} \frac{- \Lambda (\;)}{\Lambda} - \quad (- 1)$$

Substituting into the first-order conditions (1)–(7) and matching coefficients yields

$$\Lambda + \frac{1}{} + \frac{{}^2 \hat{}}{{}^2 \; {}^2 + {}^2 \hat{}^2} = \frac{1}{} \tag{18}$$

$$\Lambda + \frac{1}{} + \frac{{}^2 \hat{}}{{}^2 \; {}^2 + {}^2 \hat{}^2} = 1 - \frac{{}^2 (- 1)^2 \; \hat{}}{{}^2 \; {}^2 + {}^2 \hat{}^2} \tag{19}$$

$$- \quad \Lambda + \frac{1}{} + \frac{{}^2 \hat{}}{{}^2 \; {}^2 + {}^2 \hat{}^2} = - 1 + \frac{}{{}^2 \; {}^2 + {}^2 \hat{}^2} (- 1) \tag{20}$$

where recall that Equations (18), (19), and (20) are necessary conditions; $\Lambda = 1 / [(- 1)] ; \, ( ) \, ( )$
for a symmetric linear equilibrium.

$$\frac{-2}{-1} = \frac{1}{} - + \frac{2\hat{}}{2\ 2 + 2\hat{}2}$$

or

$$\Lambda = \frac{1}{-2} \frac{1}{} - + \frac{2\hat{}}{2\ 2 + 2\hat{}2} . \qquad (21)$$

Substituting (19) and (20) into equations (13) and (14) yields (1 ...). Dividing (19) by (18) yields (12). By inspection the (12) is a unique ... Given the solution to (12) for , coefficients are characterized uniquely by (13) and (14). Thus there can be at most one symmetric linear equ

□

Lemma 5. An increase in ... corresponds to an increase in the dem... , , and .

Proof. I first show that increasing ... Using (19) we have

$$\frac{-1}{-2} \frac{1}{} + \frac{2(-1)}{-2} \frac{1) + \frac{-2}{2}}{1 + (-1)^2) +} \frac{2\hat{}2}{2\hat{}2} = 1 \qquad (22)$$

Consider the second term on the LHS:

$$\frac{2(-1)}{-2} \frac{1}{} + \frac{-2}{2} \frac{2(1 - {}^2)}{2 + {}^2\ {}^2 [1 + (-2) - (^2]} = \frac{1}{} - 1) \qquad (23)$$

Suppose for contradiction that ... is ... Suppose further the term ... $2\hat{}2$ is increasing. Then (12) implies that ... is decreasing. But because

$$1 + \frac{-2}{2} (1 -)$$

is decreasing, it follows from (23) that ... is decreasing, which is a contradiction ... that $2\hat{}2$ is decreasing. Because

$$\frac{1 + \frac{-2}{2}}{1 + (-1)}$$

is likewise in. For the same reason increase, I
duced equation (18) below (

$$\Lambda + \frac{1}{} + \frac{2^{\wedge}}{2\;2 + 2^{\wedge}2} = \frac{1}{}.$$

Suppose for contradiction that it is not. Then the term must be

$$\frac{2^{\wedge}}{2 + 2^{\wedge}2}$$

is increasing. If it was increasing we would have a contradiction (12), so it must be decreasing. But then since ... was increasing, ... if decreasing, must then be increasing. This contradiction must be that
Finally I now show that the equation Λ = 1/[(1 − ... 1)] is decreasing. Recall that

$$\Lambda + \frac{1}{} + \frac{2^{\wedge}}{2\;2 + 2^{\wedge}2} = \frac{1}{}.$$

Suppose for contradiction Λ is increasing. Then by (12) it must be that that the term in parenthesis must be decreasing, a contradiction. □

Proof of Theorem 2. To prove part 1 of the theorem I recognize when $n = 0$ equations (12), (13), and (14) can all be solved in closed form and, in terms of primitives are given by

$$= \frac{1}{} = \frac{-2}{-1}\,1 - \frac{2(\;\;-1)}{-2}\;\frac{2}{2\;\frac{1}{}\;^{2} + \;2}.$$

Second-order conditions (16) are satisfied if

$$\frac{1}{2} + \Lambda \geq 0.$$

must be that $= 0$. If < 0, then in order for (19) to be satisfied

$$\Lambda + \frac{1}{\ } \leq 0.$$

This implies that

$$\frac{1}{2} + \Lambda < 0$$

which implies second-order conditions are not sat > 0
that $\Lambda > 0$ and the second-order conditions are always s

Thus ≥ 0 is necessary and sufficient for equilibri

What about > 0? Suppose < 0. Consider (19) again which implies

$$\Lambda + \frac{1}{\ } = 1 - \frac{2\hat{}2}{2\ 2 + 2\hat{}2} > 0.$$

Thus, once again we see that if > 0 is not satisfied then we do no
order conditions is not satisfied. Since ... increasing in ..., it shows that the r
equilibrium existence expands.

To prove part 2, I proceed as follows. To start,
allocative efficiency in terms of only the parame

By straightforward computations, one can show t
is simply

$$(\ - 1)(1 + {}^2 - 2 \ {}^2) + {}^2 (1 - {}^2).$$

I shall now prove that the above is decreasing in

To start, let me rewrite the above as

$$(\ - 1)(1 - 2 \ {}^2) + {}^2 + {}^2 {}^2 (1 - {}^2). \qquad (24)$$

I shall minimize the above with respect to . The first-order

$$(2 \ -)1^2 + 2 {}^2 (1 - {}^2) = 0$$

$$= \frac{{}^2}{{}^2 (1 - {}^2) + {}^2}$$

$$1 + \frac{2\ 2(1 - \ )}{2} = 1$$

I shall now prove that < in any equilibrium of the bl... meaning ... tuo ... the ... e ... rfm

In equilibrium we have

$$\frac{-1}{-2} + \frac{2}{2\,\hat{}\,2 + 2}\frac{2 \quad -2}{-2} = 1 .$$

Thus to prove < it suffices to show

$$\frac{2\ 2(1 - \ )}{2} \leq \frac{2}{2\,\hat{}\,2 + 2}\frac{2 \quad -2}{-2}$$

$$\Leftrightarrow$$

$$\frac{2\ 2(1 - \ )}{2} \leq \frac{2\ 2(1 - 2) - 2}{2\,\hat{}\,2 + 2}\frac{}{-2}$$

$$\Leftrightarrow$$

$$- \leq \frac{2}{2\,\hat{}\,2 + 2}\frac{2 \quad -2}{-2}$$

$$\Leftrightarrow$$

$$1 - \frac{2(\ -1)^2}{2 + 2\,\hat{}\,2} \leq \frac{\hat{}\ 2}{2\,\hat{}\,2 + 2}\frac{2 \quad -2}{-2}$$

$$\Leftrightarrow$$

$$2 + 2\,\hat{}\,2(1 - (\ -1)) \leq 2\frac{2 \quad -2}{-2}$$

$$\Leftrightarrow$$

$$2\,\hat{}\,2(1 - (\ -1)) \leq 2\frac{}{-2} \qquad (25)$$

To do this, observe that

$$2\,\hat{}\,2(1 - (\ -1)) \leq 2\ 2\ 1 + (\ -2)(\ -1)(1 - (\ -1))$$

$$\Leftrightarrow$$

$$2\,\hat{}\,2(1 - (\ -1)) \leq 2\ 2\ 1 + (\ -2)(\ -1) - (\ -1)\ (1 - \ )$$

$$\Leftrightarrow$$

$$2^{\wedge 2}(1-(\quad-1)) \le 2\ 2\ 1+(\quad-2()\ -1)-(\quad-1)\ 2+)(\quad-1)$$

$$\Leftrightarrow$$

$$2^{\wedge 2}(1-(\quad-1)) \le 2\ 2\,(1-\quad).$$

To prove (25) it suffices to show

$$2\ 2\,(1-\quad)^2 \le \frac{}{-2}.$$

This holds because we know that

$$1-\frac{2(\quad-1)}{-2}\frac{2}{2\ \frac{1}{-}\ 2+\ 2} \ge 0$$

$$\Leftrightarrow$$

$$1 \ge \frac{2(\quad-1)}{-2}\frac{2}{2\ \frac{1}{-}\ 2+\ 2} \ge$$

$$\Leftrightarrow$$

$$\frac{2}{2\ 2} \ge \frac{2(\quad-1)}{-2}-1$$

$$\Leftrightarrow$$

$$\frac{2}{2\ 2} \ge \frac{}{-2}.$$

Thus we have shown that 2(1 −) is decreasing in . Suppose this . Since is always to the left of the and is point the minimum and is always lower . Since it follows that all locative e

I now show that 2(1 −) is indeed decreasing in . Consider again the equ
:

$$= \quad 1-\frac{2(\quad-1)^2}{2+\ 2^{\wedge 2}} \quad \wedge$$

$$\Leftrightarrow$$

$$= \quad 1-\frac{2(\quad-1)\quad(1\ 2-\quad)}{2+\ 2(1-\quad)(1+(2\quad-1))}$$

$$\Leftrightarrow$$

$$= 1 - \frac{^2(- 1)(1^2 -)}{\overline{}^2 + {}^2(1 - \frac{1}{-}) + (- 1^2)}$$

If $^2(1 -)$ is increasing, it follows from the above equation is above increasing in, a contradiction.

Parts 3, 4, and 5 follow straightforwardly.

Proof 5: Fixing a direct mechanism $\{, \}$, the expected utility gain from reporting truthfull is

$$\hat{}() - (, 0) = -\frac{1}{} \quad () - \frac{1}{2} \quad (^2) \quad () .$$

if all other traders do so as well.

By the envelope integral formual, it is easy to s

$$\hat{}() - (, 0) = - [(_{-},) d] + \hat{}() - (, 0)$$

under any incentive compatible mechanism.

This implies that

$$\hat{}(, _{-}) = - + \sum_{\neq} (, _{-}) + (_{-},) d - \frac{1}{2} (, _{-})^2$$

$$- \hat{}() - (, 0) .$$

The designer's objective is the following

$$\sup_{\{\}\{\hat{}()\}} \sum_{=1} - + \sum_{\neq} () + (_{-},) d - \frac{1}{2} (^2)$$

$$- \hat{}() - (, 0) + (1 - \sum_{=1}) \frac{1}{2} (())^2 . \quad (26)$$

$$(\quad) = 0$$
$$= 1$$

and for each $= 1, 2$, it holds that

$$\hat{\ } (\ ) - (\ , 0) = - [\ (\ _{-},) d] + (\ ) - (\ , 0) \geq 0$$

for all and

$$[\ (\ _{-},)]$$

is nondecreasing in .

Simplifying and omitting constant terms, the ob

$$\sup_{\{ \ \} \hat{\{} (\ ) \}} \ - \frac{1}{\ } \hat{\ } \sum_{= 1})((\ ) - (\ _{-},) d - \frac{1}{2} (\ ^2) + (\ )$$

$$+ (1 - \ ) \sum_{= 1} - \frac{1}{2} (\ ^2) - \frac{1}{\ } \hat{\ })((\ )$$

subject to the same constraints.

Let Ω be the Lagrange multiplier on the participat

$$\sup_{\{ \ \} \hat{\{} (\ ) \}} \hat{\ } \sum_{= 1})((\ ) - (\ _{-},) d - \frac{1}{2} (\ ^2) + (\ )$$

$$+ \sum_{= 1} \int_{-} [\ (\ _{-},)] d - (\ ) d\Omega (\ )$$

where I will ignore the monotonicity constraint o

Integrating by parts, we can rewrite the Lagrang

$$\hat{\ } \sum_{= 1})((\ ) + (\ ) \frac{( ) - \Omega(\ )}{( )} - \frac{1}{2} (\ ^2) - 1 - \lim_{\to \infty} \Omega(\ \hat{)} (\ )$$

$$(\quad) = 0 .$$
$$= 1$$

We define the virtual value as

$$( ) = - \hat{}) + (\frac{( ) - \Omega( )}{( )} . \qquad (27)$$

Ignoring the monotonicity constraint, by Jense

$$(\quad) = _{(} -) + \frac{1}{} \quad ( ) \qquad (28)$$
$$= 1$$

as desired. The last step which is to verify that t
follows the same steps as in the proof of Theorem 1.

Proof. Given > 0, let $\hat{}$ ( ) denote

$$\hat{} ( ) = \quad (+ \max\{ \min\{ , \} - \quad , 0 \} .$$

This is the virtual value of a trader with type truncated to [,] of the inte
binding participation constraints so that the fu
denote

$$\hat{} (, \_) = \hat{-} ( ) + \frac{1}{} \quad \hat{} ( ) . \qquad (29)$$
$$= 1$$

Then as $\rightarrow 0$, $\hat{} \rightarrow$ pointwise. Also, note that $\sum_{=1} \hat{} = 0$ by construction so that the auc
auctioneer does not retain any net position in the
Suppose that each trader submits the demand schedule given by

$$\hat{} (\quad) = \hat{} (-) - \qquad (30)$$

to the double auction. Then the resulting market c

$$= \frac{1}{} \quad \hat{} ( ) .$$
$$= 1$$

can be computed as follo
market clearing,

$$\hat{} + \quad - \hat{} \; (\;) \; - (\quad - 1) \quad = 0 .$$
$$\neq$$

This implies that

$$= \hat{} + \quad - \hat{} \; (\;) \; \frac{1}{(\quad - 1)} .$$
$$\neq$$

The price impact is therefore $\Lambda = 1/[t(\quad - 1)]$.

Next, I construct the transaction fee such that the demand schedule (30). I consider a side payment rule of the f

$$(\quad (, \quad)) = \frac{\hat{}}{2} (\quad ^2) + \hat{} (- \quad (\quad) -) + \quad | \quad - \Lambda \; (\quad) \hat{} +$$
$$\neq$$

where the constants and functions are to be derived.

To derive them, we consider the maximization problem. It must be optimal to bid $\hat{}$ (or its analog), thus any deviation from it must be suboptimal $\Delta \neq 0$. Thus it must solve

$$\max_{\Delta} - \frac{1}{2} (\quad + \hat{} + \Delta^2) - \quad + \frac{\Delta}{(\quad - 1)} \quad (\hat{} + \Delta)$$

$$+ \hat{} \quad - \hat{} - \Delta - \quad + \frac{\Delta}{(\quad - 1)} \quad - \frac{\hat{}}{2} (\hat{} + \Delta^2)$$

where to ease notation I have taken the first order condition

$$- \frac{1}{} (\quad + \hat{} + \Delta) - \hat{} + \Delta ( ) \hat{} - \quad \frac{- \Delta}{(\quad - 1)} - \frac{}{\quad - 1} (\hat{} + \Delta)$$

$$+ \hat{} \, ' \quad - \hat{} - \Delta - \quad + \frac{\Delta}{(\quad - 1)} \quad 1 + \frac{1}{\quad - 1} = 0 .$$

This must hold at $\Delta = 0$ when $\hat{}$ is as in (30). That is,

$$- \frac{1}{} (\quad - \hat{} \; (\;) \; - \quad) - \hat{} (- \hat{} \; (\;) \; - \quad) =$$

$$+ \frac{}{- 1}(-\hat{}\ () -) - \hat{}'(\hat{}\ ())1 + \frac{1}{- 1} .$$

Gathering the terms involving only

$$\hat{} \quad \frac{= -}{- 1} .$$

Thus we may ensure that these terms are consistent

$$\hat{} \quad = \frac{-}{- 1} .$$

We next gather only terms involving

$$\hat{}\ ()\frac{1}{-} - \frac{1}{-} = -\hat{}'(\hat{}\ ())1 + \frac{1}{- 1} .$$

Writing in place of $\hat{}\ ()$ and integrating both sides we have

$$\hat{}\ (\quad)\frac{^2}{2} = - \quad \hat{}^{-1}(\ \mathrm{d}) \frac{- 1}{} +$$

for some constant. Also $\hat{}^{-1}$ is well-defined as we have perturb

to ensure $\hat{}$ is strictly monotone.

To ensure global optimality of $= 0$ and $\hat{}$, it suffices to show that t

is globally concave. Notice that the $\hat{}$ is increasing. Th

ignoring this term, by a simple computation taki

ignoring this term, we can show that the objective

Thus we have achieved an indirect implementatio

(29). To derive an exact implementation, we take the li

the $\hat{}$ converges pointwise and by obtain the side pay

rule in the statement of the theorem. Incentive co

were not true for sufficiently small, one can show that i

violated for the perturbed allocation rule under

contradiction. □

Proof of Proposition 4. To prove part 1, using the rules of con

$$\frac{\frac{1}{2}\ \ ^2}{\frac{1}{2}\ \ ^2 + \ \ ^2} + \frac{(\ )}{(\ )} =$$

which is equivalent to

$$\frac{\frac{1}{2}\ \ ^2}{\frac{1}{2}\ \ ^2 + \ \ ^2} + \frac{\Phi\ (\ \ -\ \ )\ /\sqrt{\frac{1}{2}\ \ ^2 + \ \ ^2}}{(\ \ -\ \ )\ /\sqrt{\frac{1}{2}\ \ ^2 + \ \ ^2}} =$$

where Φ and are the CDF and PDF of a standard Gaussian. define $= (\ \ -\ \ )\ /\sqrt{\frac{1}{2}\ \ ^2 + \ \ ^2}$. Then we have

$$\frac{\frac{1}{2}\ \ ^2}{\frac{1}{2}\ \ ^2 + \ \ ^2} + \frac{}{\sqrt{\frac{1}{2}\ \ ^2 + \ \ ^2}}\sqrt{\frac{1}{2}\ \ ^2 + \ \ ^2} + \frac{\Phi(\ \ )}{(\ \ )} =$$

which is equivalent to

$$\frac{\frac{1}{2}\ \ ^2}{\sqrt{\frac{1}{2}\ \ ^2 + \ \ ^2}} + \frac{\Phi(\ \ )}{(\ \ )} = 0\ .$$

Clearly, because < 0, we know ^{2} increases must decrease without bound. Because we know < 0, it follows that must also decrease without bound. Because and are symmetric, it follows that increases as increases and eventually diverges.

I now prove Part 3. The expected utility gain of a

$$\hat{\ }(\ \ ) - (\ \ ,0) = -\hat{\ }(\ \ d) + \hat{\ }(\ \ ) - (\ \ ,0)\ . \qquad (31)$$

Note that $\hat{\ }(\ \ ) - (\ \ ,0)$ is always set to ≥ 0. We have that

$$\frac{\hat{\ }(\ \ )}{\ \ -1} = -\frac{\frac{1}{2}\ \ ^2}{\frac{1}{2}\ \ ^2 + \ \ ^2} + \frac{\Phi\ (\ \ -\ \ )\ /\sqrt{\frac{1}{2}\ \ ^2 + \ \ ^2}}{(\ \ -\ \ )\ /\sqrt{\frac{1}{2}\ \ ^2 + \ \ ^2}} +$$

< . Clearly wthhiens gets less ^{2} pionscirteiavse.s When te
inside the hazard rate increases and thus becomes
becomes more negative. On the other hand, ^{2} in the term ... negative
term in brackets becomes less positive. We can deal a
Therefore the expected utility gain of a trader de
Now I must argue that the expected trade volume
( ) is a mean-preserving contraction of what it onc
Recall that we can have the transformation

$$\frac{\frac{1}{2}\ {}^2}{\frac{1}{2}\ {}^2 + \ {}^2} + \frac{\Phi(\ )}{(\ )} = 0.$$

Now recall the definition of a mean-preserving
see that when ^{2} is higher this is a mean-preserving contr
I now prove Part 2. Consider ^{2} but maintaining the same alloc
fer rules. Then each trader submits the same mess
based on mean-preserving contraction that alloca
the expected utility gain of all traders summed to
ficiency improves, this can only happen if total r
that (31) is convex. Thus, reducing must reduce the expecte
gain.

□

C. Omitted Material for Section 6

Proof of P 6. Construct derivative securities that
underlying assets such that the payoffs of these c
of generality, suppose that the derivatives are t
traders

$$(\ +\ ,\ ) = -\frac{1}{2}\ {}^2(\ +\ )^2 -$$

where ^{2} = var[r].

Consider running a separate double auction with
tion 3 for each asset. Because the utility function is

□

Formal Statement of Design Given [0, 1], the Objective is to s

$$\max_{\{\ \},\ \{\ \}} \underbrace{\sum_{=1} (+ ()(1 -)}_{\text{Revenue}} \underbrace{\sum_{=1}) - \frac{1}{2}\ ^2 + (\ ^2)}_{\text{Allocative Efficiency}} \quad (32)$$

such that,

$$\text{(P)} \qquad + \ (\ () \ , \ ) \geq (\ , \ )0, \qquad ,$$

$$\text{(IC)} \qquad \arg\max + (\ , \_ \ ), (\ , \_ \ ) \qquad , \qquad ,$$

$$\text{(E)} \qquad \sum_{=1} (\ ()) = \ ,$$

where = $\{\ \}_{=1}$ is the vector of traders' endowment vecto

Proof of Proposition 7. I prove the Proposition 7 for the special case = 0 to ease the exposition. The general case is analogous.

Trader selects messages = $\{\ \}$ to solve

$$\sup -\frac{1}{2} (\ , \_ \ )^2 - \frac{1}{} (\ , \_ \ )_1 + \ (_1 \ , \_ \ )$$

$$- (\ , \_ \ ).$$

I conjecture that only local incentive constra pose that the designer knows $\{\tilde{e}\}$ and I will verify later reporting is indeed incentive compatible under t

Let $\tilde{t} = \{\ \tilde{}\ \}$. By the envelope integral formula,

$$\hat{}(\ ,\tilde{}\ ) - (\tilde{}\ , 0) = -\frac{1}{}\int_0 \tilde{}\ \hat{}\ (\ \tilde{}\ )d + \hat{}\ (0\tilde{}\ ) - (\tilde{}\ 0, 0) \quad (33)$$

$\hat{}(,\tilde{})$ denotes the expected utili[ty]

reporting for a ,$\tilde{}$ and where $(\tilde{}, 0)$ ... trader of that ty[pe]

autarky.

This implies the expected transfer is

$$\hat{}(,\tilde{}) = -\frac{1}{}\ \hat{}(,\tilde{}) + \frac{1}{2}(,\tilde{})^2 + \int_0 \frac{1}{}\ \tilde{}\ \hat{}(\tilde{})\, d$$

$$- \hat{}(0,\tilde{}) - (\tilde{}, 0, 0). \quad (34)$$

Using (33) we can write down the planner's objective

$$-\frac{1}{}\ \hat{}(,\tilde{}) + \frac{1}{2}(,\tilde{})^2 + \int_0 \frac{1}{}\ \tilde{}\ \hat{}(\tilde{})\, d$$

$$- \hat{}(0,\tilde{}) - (\tilde{}, 0, 0)$$

such that (33) is nonnegative and $\sum_{=1} = 0$. such that

Let $\Omega(\tilde{})$ denote the Lagrange multiplier on ...

$\hat{}(\tilde{}) - (\tilde{}, 0) \geq 0$. Forming the Lagrangian, integratin[g] ...

yields the objective

$$\sup \frac{1}{2}(,\tilde{}^2) - \frac{\Omega(|\tilde{}) - (\tilde{})}{(|\tilde{})}\frac{1}{}\tilde{}\ (,\tilde{})$$

$$- \quad - \Omega(\tilde{})\ \hat{}(0,\tilde{}) - (\tilde{}, 0, 0) \quad (35)$$

such that $\sum_{=1} = 0$ where $(\tilde{})$ denotes the PDF and $(\tilde{})$ denotes the CDF of ..., both conditional on $\tilde{}$.

The solution to (35) yields

$$(,\tilde{}) = - - \frac{\Omega(|\tilde{}) - (\tilde{})}{(\tilde{})}\ \tilde{}$$

$$+\frac{1}{}\sum_{=1} - \frac{\Omega(|\tilde{}) - (\tilde{})}{(|\tilde{})}\ \tilde{}.$$

is the Rayleigh distribution:

$$(\tilde{\ })| = 1 - {}^{-\frac{2}{2\ 2}}$$

$$(\tilde{\ })| = \frac{}{2}\ {}^{-\frac{2}{2\ 2}}.$$

Because and do not depend on ˜ this is a logical 0 conjecture that the ... on ˜ and the region of binding part [0,] at some construc... 0. A trader with this type interval will expect to trade ... observations yield the allocation rule in the sta...

There are two remaining steps. The first step is ... taken is valid. This is straightforward and ana... 1. The last step is to verify that only the incenti... from the proof of Proposition 8, it is shown that a double auction ... transaction fee can indeed implement the allocation 7.

□

Proof of Proposition 8. Note that, given preferences 8), for each ... equivalent to a change in notation of Proposition 8, or the special ca... when $^2 = 1$.

If each trader submits the demand schedule

$$(\ \ ) = -\ )(\tilde{\ } - \qquad (36)$$

for all , then the unique market clearing price is

$$= \frac{1}{\ }\sum_{=1} (\ )\ \tilde{\ }$$

and so the resulting allocation is of the desired 7. ...

I now engineer the transaction fee (36) is an equilibrium. For each ... conjecture that the implementing transaction fee ...

$$(\{(\ \ ),\}\ \ ) = \sum_{=1} + \frac{1}{\ }\ {}^{2} + \frac{1}{2}\ {}^{2} +$$

and constants to be determined.

If it is optimal for trader to hold units when the vector of prices is , i.e. $= \{ \}$ then the solution to

$$\sup_{\{\Delta\}} -\frac{1}{2}(\tilde{} + + \Delta)^2 - (+ \Lambda\Delta)(+ \Delta) - \frac{1}{2}(+ \Delta)^2$$

$$- \sum_{=1} (+ + \Delta(1 + \Lambda))^2 \quad (37)$$

is that $\Delta = 0$ for each. Above $\Lambda = \frac{1}{(-1)}$ is trader's price impact when the other traders submit a demand schedule (36). The first-order conditions for optimality are

$$-\frac{1}{}(\tilde{} +) = + \Lambda +$$

$$+ 2' \sum_{=1} (+)^2 (+)(1 + \Lambda)$$

for each. Using that $\Lambda = \frac{1}{(-1)}$, we equivalently have

$$-\frac{1}{}(\tilde{} +) = + \frac{1}{(-1)} +$$

$$+ 2' \sum_{=1} (+)^2 (+)\left(1 + \frac{1}{-1}\right)$$

for each.

Substituting (36) yields

$$-\frac{1}{}\tilde{} = - + \frac{1}{(-1)} + \frac{1}{}(()\tilde{} +)$$

$$- 2' ()^2 ()\tilde{}\left(1 + \frac{1}{-1}\right).$$

, ,{ ~} we require that

$$2 ' ( )^2 ( ) 1 + \frac{1}{-1} = \frac{1}{} - + \frac{1}{(-1)} + \frac{1}{} \quad ()$$

and

$$+ \frac{1}{(-1)} + \frac{1}{} = 1 .$$

Thus

$$= - \frac{1}{(-1)}$$

and we have a differential equation for

$$2 ' \, ^2 \; 1 + \frac{1}{-1} = \frac{1}{} \; ^{-1}( ) \frac{1}{} , \quad > 0 .$$

A technical issue ... over a range of values. ... easily dealt with as in the proof of Proposition 3 ... thus invertible). Following the arguments above ... perturbed ... then take limits ... implementable ... steps.

Let me define ... such that ... ^{2}), (. Then we have

$$1 + \frac{1}{-1} \; \tilde{}' ( ) \frac{1}{} = \; ^{-1}( ) \frac{1}{} , \quad .$$

Integrating both sides yields

$$\tilde{} ( ) = \frac{1}{1 + \frac{1}{-1}} \; \frac{1}{} \int_0 \; ^{-1}(\; d) \; \frac{1}{2} \; ^2 + \quad .$$

Note that ... $^{-1}(0)$ is not well-defined, the value of th... on $^{-1}(0.)$

It is easy to verify that ... is concave and thus the o... (37) is also convex. Therefore, the first-order ne... sufficient ... $= 0$ ... (37) ... follows ... (30) ... strategy (... show rigorously that this ... to an alternative set of demand schedules is a stra...

□

D. Omitted Material for Section 7

Formal Statement of Design Objective. Given $\in [0, 1]$, the objective is to

$$\max_{\{\},\{\}} \underbrace{\sum_{=1} (\;+\;(\;)(1-\;)}_{\text{Revenue}} \underbrace{\sum_{=1} -\frac{1}{2}\;+\;(\;^2)}_{\text{Allocative Efficiency}} \tag{38}$$

such that,

(P) $+ (\;(\;),\;) \geq (\;, 0),\;,$

(IC) $\in \arg\max\; + (\;, \_\;), (\;, \_\;)\;,\;,$

(E) $\sum_{=1} (\;) = 0,$

where $= \{\}_{=1}$ is the vector of traders' endowments.

Proof of Proposition 9. Part 1 follows immediately from Corollary 5.1.

Part 2a follows from Corollary 5.1 and uses the same argument as in Part 1 of Proposition 4.

To prove Part 2b and 2c, observe that $\hat{}$ increases when it is negative and decreases when it is positive when ^{2} increases. Thus from the formula for

$$\hat{}(\;) - \;, 0 = -\frac{1}{}\;\hat{}(\;d) + \hat{}(\;) - \;, 0,$$

trader's expected utility gain must decrease as ^{2} increases. On the other hand, $\hat{}$ has no effect on $\;$ for any $\neq$ and thus the expected utility gain of $\neq$ is unaffected.

To prove part 3a, I recognize that, as a function of $\;$, the expected utility gain of $\;$ is proportional to

$$\frac{1}{+\sum_{\neq}}.$$

. Because $\hat{a}$ does not depend on , there is no effect on $\neq$ which any trad
proves Part 3b.

□

Proof of Proposition 10. The proof is analogous to that of Proposition 5 to that of Propo

□

E. Omitted Material for Section 8

Formal Statement of the Designer's Problem. The designer's objective is to
allocations and transfers $\{(\ ,(\ )\ ,\ )\}_{=1}$, ≥ 0, to maximize

$$\sum_{=1}^{\infty}\int_0^{\ } - \left(-\frac{1}{2}(\ +\ (\ )^2_{,}\ \mathrm{d}\ ) + (1-\ )\right)\int_0^{\ } - \ (\ \mathrm{d},\ ) \quad (39)$$

subject to

(P) $\{(\ ,(\ )\ ,\ )\ ;\ \geq\ \geq (0,\ ) 0,\ ,$

(IC) $\arg\max\{(\ ,\ )\ ,(\ ,\ )\ ;\ \geq 0\}\ ,$

(E) $\sum_{=1}(\ ,\ ) = \sum_{=1}\ ,$

where, with abuse of notation, $(\ ,0)$ indicates the agent's utility from
not participating in trade in which case she retai
makes no transfers.

Proof of Proposition 11. Let $(\ ,\ )$ be an implementable allocation r
compatibility of truthtelling implies that the t

$$\hat{\ }(\ ,\ ) = \sum_{=1}^{\infty}\sum_{=0} - \ \left(-\frac{1}{2}\ +\ (\ )^2_{,}\ \right)$$

$$-\frac{1}{\ }\hat{\ }(\ ) + \frac{1}{\ }\int (\ ,\ )\ \mathrm{d}\ - \hat{\ }(\ ) - \ ,\ 0$$

$$\bar{\ } (\quad , \quad)^{-} \equiv \sum_{=0}^{\infty} (\quad , \quad) .$$

In order for global incentive compatibility to hold, $\bar{v}(c_{-})$ must be non-in-
creasing. Consider its average value with respect to . Construct a

$$(\quad , \frac{1}{\ }) = \sum_{=0}^{\infty} (\quad , \quad) .$$

Since (,) is implementable, since (,) is most interim expec
be decreasing. By Jensenôs inequality this can on
Jensenôs inequality both allocative efficiency a

□

References

Almgren, R. & Chriss, N. (2001). Optimal executio… Journal of Risk, 3, 5–40.

Andreyanov, P., Park, J., & Sadzik, T. (2023). Optimal auctio… Technical report, Working paper.

Andreyanov, P. & Sadzik, T. (2017). Robust mechanism design of exch… Working Paper.

Armstrong, M. (1996). Multiproduct nonlinear pricing. Econometrica: Journal of the Econometric Society, pp. 51–75.

Athey, S. & Segal, I. (2013). An efficient dynam… Econometrica, 81(6), 2463–2485.

Babus, A. & Parlatore, C. (2022). Strategic fragmented… Journal of Financial Economics, 145(3), 876–908.

Bergemann, D., Brooks, B., & Morris, S. (2017). F… mation structures: Implic… Econometrica, 85(1), 107–143.

Bergemann, D., Heumann, T., & Morris, S. (2015). Information and market pow… Working Paper.

Bergemann, D. & Morris, S. (2013). Robust predict… mati… Econometrica, 81(4), 1251–1308.

Bergemann, D. & Välimäki, J. (2002). Information… design. Econometrica, 70(3), 1007–1033.

Bergemann, D. & Välimäki, J. (2010). The dynamic… Econometrica, 78(2), 771–789.

Biais, B., Glosten, L., & Spatt, C. (2005). Market… dations, empirical results, … Journal of Financial Markets, 8(2), 217–264.

Econometrica, 68(4), 799–837.

Brooks, B. & Du, S. (2021). Optimal auction desig
tionally robust approach. Econometrica, 89(3), 1313–1360.

Budish, E., Cramton, P., Kyle, A. S., Lee, J., & Ma Flow Trading. Technical
report, National Bureau of Economic Research.

Budish, E., Lee, R. S., & Shim, J. J. (2019). Will
stock exchange competition and innovation.

Chen, D. & Duffie, D. (2021). Market fragmentation. American Economic Review,
2247–74.

Chen, D. & Zhang, A. L. (2020). Subsidy schemes. Available at SSRN
3668394.

Clarke, E. H. (1971). Multipart pricing of public goods. Public choice, (pp. 17–33).

Colliard, J.-E. & Foucault, T. (2012). Trading f
The Review of Financial Studies, 25(11), 3389–3421.

Crémer, J. & McLean, R. P. (1988). Full extraction
strategy auctions. Econometrica: Journal of the Econometric Society, (pp. 1247–1257).

Dasgupta, P. & Maskin, E. (2000). Efficient auctions. The Quarterly Journal of E
115(2), 341–388.

Du, S. & Zhu, H. (2017). What is the Optimal Tradin
The Review of Economic Studies, 84(4), 1606–1651.

Foucault, T., Kadan, O., & Kandel, E. (2013). Li
electronic markets. The Journal of Finance, 68(1), 299–341.

Glosten, L. R. (1994). Is the electronic open limit The Journal of
Finance, 49(4), 1127–1161.

Groves, T. (1973). Incentives in teams. Econometrica: Journal of the Ec
(pp. 617–631).

Econometrica, 69(5), 1237–1259.

Jullien, B. (2000). Participatio... Journal of Economic Theory, 93(1), 1–47.

Kyle, A. S. (1985). Continuous auctions and ins... Econometrica, 53(6), 1315–1335.

Kyle, A. S. (1989). Informed Specu... The Review of Economic Studies, 56(3), 317–355.

Lambert, N. S., Ostrovsky, M., & Panov, M. (2014) ... complex envi... Proceedings of the fifteenth ACM co... and computation (pp. 3–4).

Lu, H. & Robert, J. (2001). Optimal trading mechan... Journal of Economic Theory, 97(1), 50–80.

Luenberger, D. G. (1997). Optimization by vec... John Wiley & Sons.

Malamud, S. & Rostek, M. (2017). Decentralized Ex... American Economic Review, 107(11), 3320–3362.

Malinova, K. & Park, A. (2015). Subsidizing liqu... market qu... The Journal of Finance, 70(2), 509–536.

McAfee, R. P. (1991). Efficient al... Journal of Economic Theory, 53(1), 51–74.

Myerson, R. B. (1981). Optimal auc... Mathematics of operations r... 6(1), 58–73.

Myerson, R. B. & Satterthwaite, M. A. (1983). Effi... Journal of economic theory, 29(2), 265–281.

Pavan, A., Segal, I., & Toikka, J. (2014). Dynam... approach. Econometrica, 82(2), 601–653.

Econometrica, 80(2), 687–711.

Rostek, M. & Yoon, J. H. (2021). Exchange design … Econometrica, 89(6), 2887–2928.

Rostek, M. J. & Yoon, J. H. (2023). Financial proc… Available at SSRN 3631479.

Sannikov, Y. & Skrzypacz, A. (2016). Dynamic trad… Preprint.

Vayanos, D. (1999). Strategic Trading and Welf… The Review of Economic Studies, 66(2), 219–254.

Vickrey, W. (1961). Counterspeculation, auctio… The Journal of Finance, 16(1), 8–37.

Vives, X. (2011). Strategic Supply Function Compe… Econometrica, 79(6), 1919–1966.

Wilson, R. B. (1993). Nonlinear pricing. Oxford University Press, USA.

Wittwer, M. (2021). Connecting … American Economic Journal: Microeconomics, 13(1), 252–282.