Note: This draft is being updated f

## Abstract

Which trading mechanisms are optimal for revenue
bination of the two? Do they resemble institutions
trade be structured if there is no adverse selectio
If there is adverse selection? If traders are hete
sets? If there are dynamics? Which mechanisms offe
are robust to adverse selection? Which information
gaurd against? How well do prices aggregate privat
mechanism is endogenous? I investigate these ques
ting in finance market microstructure: large trad
endowments.

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*I thank Markus Brunnermeier, Briana Chang, Piotr Dworcza
Wilson, Yuliy Sannikov, Wei Xiong, Motohiro Yogo, Anthony Z
UCL Decentralized Financial Markets Conference for insig
Kayla Jiang and Ethan Wang for excellent research assistan
support of the Bradley Graduate Fellowship through a grant

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## 1. Introduction

In the past two decades, trade in financial mark
and fragmented. This has raised concern among re
financial markets may be organized in a way that is
so because most trading institutions are for prof

To help further our understanding of these issu
design analysis of trade for a workhorse model se
In this setting, a finite number of traders are pr
assets in the market and trade to share holding[1] cos
solve for mechanisms that are optimal in that they
and allocative efficiency. A key aspect of my anal
of market conditions including when there is adv
payoffs, multiple assets, and heterogeneity amon
allows me to offer insight into how trade should be
also allows me to relate my findings to several res
fixed trading mechanism (or within a parametric c

I now briefly summarize the main results.

- In general, it is optimal to distort the alloca
  have the highest marginal value.[2] Moreover, un-
  der optimal mechanisms each trader's allocati
  of all traders—it is never optimal to Malamud & them
  Rostek (2017).

- Across each of the environments I study, doubl
  often implement a point on the revenue-efficiency frontie
  tiple assets, this can be done with double auct
  trader's demand schedule can be contingent on
  exchange but not on the prices of securities in

- If an exchange has incentives to maximize reve
  small and has only a small piece of information

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[1] Papers with th Vives (2011), Chen & Duffie (2021), Du & Zhu (2017), Rostek & Weretka (2012), Rostek & Yoon (2021), Sannikov & Skrzypacz (2016), Babus & Parlatore (2022), (2015), al. Glosten (1994), Wittwer (2021), Rostek & Yoon (2023), and many others.

[2] This is in contrast with the typical “no distortion at the

should be designed to target desirable outcomes an
consider putting in place as a society. Of course
some caution given the stylized nature of the mode

Indeed, the analysis in this paper is not withou
anticipate many qualitative insights hold more g
cost model for tractability. Also, my analysis of
or multiple assets relies on restrictive distrib
nical challenges associated with multidimensional[4] In
optimal mechanisms for the model with adverse sel
ture that is "worst case" for allocative efficien
in the literature on robust mechanism design). Th

[3] I demonstrate this for the case of no private information

[4] The analysis for these settings seems close to the limits
multidimensional screening.

## 2. Related Literature

This paper is at the intersection of finance mark
To my knowledge, this paper is among the first to
for objectives other than allocative efficiency
a finite number of traders, and 3) traders who may
endogenously based on the terms of trade.

I characterize mechanisms that maximize linear
efficiency for a model setting where traders have
costs that are quadratic in their asset positions
finance market microstructure in the past fifteen
this environment Vives (2011); Chen & Duffie (2021); Du & Zhu (2017); Rostek & Weretka (2012); Rostek & Yoon (2021); Sannikov & Skrzypacz (2016); Babus & Parlatore (2022); Biais et al. (2005); Glosten (1994); Wittwer (2021); Rostek & Yoon (2023) and many others.[7] Existing work typically fixes a trading m
mechanisms in a narrow parametrized class.[8] The most commonly assumed me
is the (uniform-price) double auction. In contra
ing mechanism as being chosen by a planner who plac
efficiency. As discussed in the Introduction, my
ison for existing work to see how results may chan
(e.g., chosen by a revenue-maximizing exchange)
to see which inefficiencies arise because of the p

This paper is also related to work in finance mar
pact of exchange fees (Malinova & Park, 2015; Foucault et al., 2013; Colliard &
Foucault, 2012; Jantschgi et al., 2022) show that trading fees can i
optimal mechanism design. Jantschgi et al. (2022) provide a theore
analysis of transaction costs fixing a uniform-p
et al. (2022), I do not make parametric assumptions on th

[7] Variants include single and multiperiod models, single a
out private information about asset payoffs, models with s

[8] This is also true of other papers in finance market design, Budish
et al. (2019) and Budish et al. (2023), and many others.

exchange mechanisms when designer may ha

motive. Furthermost of these papers consider env
designated buyers or sellers and have linear uti
extended to settings in finance market microstru
allow for multiple dimensions of private informa

The most closely related paper is Biais et al. (2000) who study a single asset
competing mechanisms in a finance setting with a s
may choose to be a buyer or seller. Biais et al. (2000) study the
case of a single trader they do not consider exchan
not absorb or supply any net quantity of the asset.
both of our papers share many properties such as a
constraints and kinked transfer rules. One impor
be distorted for extreme types in my paper but not i
similarities, the focuses of our papers are quite
are not directly comparable.

Also related is Lu & Robert (2001) which studies optimal exchang
allows for traders to be buyers or sellers but stu
timal mechanisms are shown to depend on delicate
involve randomization. This is in sharp contrast
do not have these properties. Also, in Lu & Robert (2001) there are no disto
extremes whereas I find the opposite. Andreyanov & Sadzik (2017). T
studies optimal exchange mechanisms in a closely
locative efficiency with a robust objective and m
restrict attention to quadratic subsidies in a do

## 3. Basic Model

### 3.1. Environment

In what follows, all random variables are defined on $(\Omega, \ldots, \ldots)$. There is a single asset with a payoff that is a random variable. There are $\geq 2$ traders indexed by $\{1, 2, \ldots, \}$. Traders are endowed with some of the asset prior to trade; they know the endowment. The endowment is a random variable with variance distribution function (CDF) that has a continuous density $> 0$. Traders' endowments are jointly independent.

If trader purchases units of the asset at the total price, then his utility is

$$( + \;, ) = ( + \;) - \frac{1}{2}( + \;)^2 \qquad (1)$$

> 0. The quadratic cost represents the dis
or any other costs associated with holding a net po
as a trader's holding capacity.

As rlvilwld io Slctioo 2, variaots of tmis sltup
marklt microstructurl 1) Pmralvflla rplplkla (1989) Ctmol formi
& Duf (2017) Du & Zmu (2017) Rostlk & Wlolr (2012) Rostlk & Yoo (2021) Sao-
oikov & Sk (2015) Babcuzs & Pa (2022) Wtiortl (2021) aod maoy otmlrs.
mavl adoptld a baslliol modll tmat is tml simpllst
plrs. Subslpulot slctioos will systlmatically i
io tml litlraturl.

## 3.2. Trading Mechanisms

I seek to conduct a mechanism design analysis of
I first define the concept of a trading mechanism.

Definition 3.1. A trading mechanism ({}, {}) consists of

- a message space for trader,
- an allocation rule $\prod_{=1} \mapsto$ mapping profiles of reported
  quantity purchased by trader,
- and a transfer rule $\prod_{=1} \mapsto$ mapping profiles of reported
  price paid to the mechanism by trader,

for each trader.[9]

The most commonly assumed trading mechanism in t
I study in this paper is the (uniform-price) doubl

Example 1. In a double auction, ... message space is the set of measur
functions $\to$ specifying how much to purchase for each realiz
asset's price. Given a vector of reported demand functions $(\,)_{=1}$, ... are such that,

$$(\quad)(=)$$

[9] Formally, the mechanism ... measurable ... that $(\prod_{=1}, \; _{=1}) \to (\,,\,)$ and $(\prod_{=1}, \; _{=1}) \to (\,,\,)$ where is the Lebesgue algebra.

and transfer rules are such that

$$( \quad ) = ( \quad )$$

for each whenever there is a unique demanded by market clearing pr

$$( \quad ) = 0 .$$
$$= 1$$

If no such exists, then trade ( s h) u (= t s ) d, = f 0 o r i e a c h that

The double auction will serve as a useful benchm come. An important property of the double auction or supply any net quantity of the asset. $\sum_{=1} = 0$. I call this a trading mechan property exchange mechanism. of this paper analyzes l that are optimal in a sense that I now specify.[10]

## 3.3. Objective

I seek to derive exchange mechanisms that maxim binations of revenue generated by the mechanism a the negative of the sum of traders' holding costs rium of the mechanism.[11] By the revelation principle, it is wi restrict attention to direct mechanisms in which trader's message spa space and such that 1) it is individually rational fo anism, 2) incentive compatible for each trader to the mechanism is an exchange mechanism.

[10] In some cases, it may be of interest to allow the mechanism this channel down for this paper, though it is easy to extend amount of the asset. For example, if the designer also has a optimal mechanisms is largely the same.

[11] That is, I allow the designer to select the equilibrium (p out to be without loss because the optimal direct mechanism

[0, 1]. The objective is to solve

$$\max_{\{\ \},(\ )} \underbrace{\sum_{=1} (\ + (\ )(1 - \ )}_{\text{Revenue}} \underbrace{- \frac{1}{2} \sum_{=1} + (\ )^2 )}_{\text{Allocative Efficiency}} \qquad (2)$$

such that,

$$\text{(P)} \quad + (\ (\ ),\ ) \geq (\ , 0), \quad ,$$

$$\text{(IC)} \quad \arg\max + (\ ,\ _{-}),(\ ,\ _{-}) \quad , \quad ,$$

$$\text{(E)} \quad \sum_{=1} (\ ) = 0,$$

where $= \{\ \}_{=1}$ is the vector of traders' endowments.

In the constraint (IC), with a trader's report as ... as the first argument and ... the vector of the other tr... I will continue to do so whenever convenient.

By solving (2) for each ... [0, 1] I am able to identify the revenue... The revenue-efficiency frontier ... combioatioos of lxp... plctld allocativl lfficilocy sucm tmat tmlrl dol... acmilvls amigmlr llvll of botm.

I focus on the ... (2) for ... is zero. The objective ... the preferences of a social planner (such as the go... efficiency, incurs no fixed costs of running the e... in illiquid markets. On the other hand, ... > 0 ... planner who has a binding budget constraint. The p... to run by ... level that satisfies ... 0 ... her co... is useful to analyze is because the planner may wa... use it for some other productive purpose ... > 0 ... the marginal social value ... = 1 ... the planner may ... When ... maximizing exchange (most exchanges in practice a... motives). In this case, the analysis sheds light o...

### 3.4. Model Discussion

Despite the prevalence of the model setting in th
little analysis is using mechanism or information[12] d
related literature. Below, I briefly discuss mod

- To ease the exposition, I restrict attention t
  is no residual uncertainty in allocations or tr
  I formally show in Appendix A.2 that this is without loss
  achieve a higher value for the objective using
- The quadratic holding cost (1) and CARA utility are equiva
  monotone transformation) when the allocation
  known to her conditional on her message in a dir
  are generally not equivalent. Typically, in an
  a trader is not known to her given her report bec
  the other traders' endowments. Quadratic util
  in this case and is why I am able to solve the mod
  why I do not need to restrict attention to dete
  turn out that the allocation rules I identify a
  are implementable when traders instead have C
  whether they are necessarily optimal in that ca
- The model assumes that endowments are indepen
  appears often in the literature (Chen & Duffie (2021), for examp
  pendence allows me to avoid the Crémer & McLean mechanism
  (1988). There is broad consensus that such mechan
  their sensitivity to common knowledge assumpt
- Other assumptions will be relaxed in subsequen
  tion about the fundamental in Section 5, multiple assets in Section 6, heterogeneity
  among traders in Section 7, and dynamics in Section 8. However, I obtain the
  characterizations with the least conditions f

[12] Indeed, classical auction theory focuses on revenue maxi
revenue-maximizing designs of exchange mechanisms.

## 4. Optimal Mechanisms

I now give an informal sketch of the derivation of t... nique, I build on methods developed for abstract s... constraints in Jullien (2000) when there is a single agent. The ana... of Biais et al. (2005), though they study the CARA setting.[13] Relative to th... these papers, the setting I consider involves sev... mechanisms are exchange mechanisms, the problem c... not be solved trader-by-trader. Nevertheless, w... tractable.

### 4.1. Sketch of the Derivation

Step 0: Notation. To start, I define some notation to eas... expected utility, expected trade quantity, and e...

$$\hat{\ }(\ ) \equiv \quad + \ ( \quad (), \ ),$$

$$\hat{\ }(\ ) \equiv [\ ( \quad ]), |$$

and

$$\hat{\ }(\ ) \equiv [\ ( \quad ]) |$$

respectively.

Also, whenever $>$, let $\equiv$ .

Step 1: Incentive Compatibility. Traders solve

$$\max \quad + \ ( \ , \_ \ ), ( \ , \_ \ ) | \ .$$

By standard arguments, local incentive compatab...

$$\hat{\ }(\ ) - \quad , 0 = -\frac{1}{\ } \quad \hat{\ }(\ d) + \hat{\ }(\ ) - \quad , 0, \qquad (3)$$

[13] Also, Biais et al. (2005) use a different method where traders' rents a... attention to deterministic mechanisms.

$$\hat{}\,( ) = -\frac{1}{2}\,( \;^{2}\;) - \frac{1}{-} - \;\hat{}\,( ) + \frac{1}{-}\;\hat{}\,( \; d)$$

$$- \hat{}\,( ) - \quad , 0. \quad (4)$$

Moreover, a mechanism is globally incentive compa
compatible if $\hat{}$ is weakly decreasing.

By inspection, (3) implies that for the types for whom (P) binds expect to
$\hat{}\,( ) = 0$ and these types must fall in some closed inte $[\;,\;]$.

Step 2: Lagrangian Relaxation. The next step is to form the Lagran
version of (2) that ignores global incentive constraints.

Let $\Omega$ be the Lagrange multiplier for the participation constraint $\Omega$ ... constra
bounded variation. With an abuse of notation, let $\Omega$ also denote the dis
bution function of this measure.

Using equations (3) and (4), we can form the Lagrangian, integrate
drop some constant terms to arrive at

$$\max_{\{\},\,\hat{}\,( )} \sum_{=1} -\frac{1}{2} \quad - \frac{\Omega( ) - \quad )}{( )} + \left( \quad \right)^{2} + 1 - \lim_{\to \sup} \Omega( \quad )\,\hat{}\,( \quad )\,( \quad )$$

such that $\sum_{=1} ( ) = E0$ :

Step 3: Candidate Optimal Mechanism. ... Jensen's in
solution for the optimal allocation rule, ignori

$$( \quad ) = - \frac{\Omega( ) - \quad )}{( )} + \frac{1}{-} \sum_{=1} - \frac{\Omega( ) - \quad )}{( )}$$

Next, we observe that for the $\hat{}\,( )$ to be ... it must be that the r

$$\lim_{\to \sup} \Omega( \quad ) = 1.$$

Ω is flat [o,u] ts, we have

$$\Omega(\quad) = 0, \quad <$$
$$\Omega(\quad) = 1, \quad . \quad >$$

The $\{\Omega\}_{=1}$ are constructed so that [ , ] for each the Appendix shows that the same for each so continuous.

Step 4: Verification. The last step is to verify that strong ... Appendix) and to establish conditions for when th... decreasing so that global incentive constraints ... clearly sufficient for this and is analogous to th... Myerson (1981).

Condition 1 holds if $\frac{1-(\ )}{(\ )}$ and $+\frac{(\ )}{(\ )}$ are weakly increasing.

Condition 1 holds when is log-concave, a stronger condi... Many common distributions satisfy Condition 1 including ... and Ga... butions.

## 4.2. Characterization

The following Theorem 1 summarizes the results o... characterization of optimal mechanisms.

Theorem 1. Suppose that Condition 1 holds. Then the unique solutio... tion problem (2) for {} sets

$$(\quad) = ) + \frac{1}{\ }(\quad ( ))_{=1}$$

for each where

$$(\quad) = \frac{(\quad)}{(\quad)}, \quad <$$
$$(\quad) = \frac{1 - (\quad)}{(\quad)}, \quad >$$
$$(\quad) = (\,), \quad , [\ ]$$

for and such that is continuous: $\lim_{\to}(\ ) = (\ )$.

( , , [ ()]). There are many examples for which there
lutions. For instance, this is so when is symmetric (as for the example
Gaussian case). This simplicity Liu & Rosenthal (2011) contr
who study trade in settings with linear utility wh
domization with delicately constructed tiebreak
easy to derive comparative statics and to extend t
important features of the environment and thus f

Corollary 1. Suppose that the conditions of Theorem 1 hold and that
symmetric about the mean of endowments. Then

$$)( \quad = ,$$

and

$$= \quad - ( - \quad ) ,$$

where solves

$$- \frac{1 - \quad ()}{()} = \quad .$$

Thus, the optimal mechanism is characterized in c
in addition is uniform, then the optimal mechanism is c
form.

There are several notable implications of the optimal mechanism. One is that
perhaps surprisingly, of traders in the market is very lim

Corollary 2. The range [ ] of endowments for which the par
strictly (P) binds does not depend on the number of traders. Thus, the exp
quantity for any given trader, is invariant to the nu

A second is that optimal mechanisms typically fea
the extremes.

Remark 1. An important property of the optimal mechanism that distin-
guishes it from optimal mechanisms in prior work

interim allocation of a trader are, in general, bo
with extreme types. That is, though there is no di
extreme types in that $\lim_{\to \sup} - (\ ) = 0$ and $\lim_{\to \inf} - (\ ) = 0$, there is distort
allocations at the extremes. This contrasts with Bernas (2015) & Rost (2011)
and to my knowledge, almost all other models of opt
no distortion in interim allocations when is symmetric about its mean, b
Example 3 in Appendix A, this is not generally so when is asymmetric.

A third notable implication is that it is in fact
cation with ex-ante budget balance.

Proposition 1. The efficient allocation is implementab
in that expected revenue is zero.

In other work, McAfee (1991) also derive conditions for when exp
able with ex-ante budget balance in 1 can also be rel
proven by checking those conditions. However, sub
of Proposition 1 under different model conditions (advers
erogeneity, dynamics etc.) which McAfee (1991) fall outsid
Before, turning to an illustrative example, I p
which show that, even accounting for the designer
revenue and efficiency improve when the market is
when the designer places mroe weight on revenue in

Proposition 2. Suppose that Condition 1 holds. Then under the optima

1. As a function of the number of traders, the expected holding cost re $\frac{1}{2}(\ + (\ )^2 -\ )^2$, expected $[\ (\ )]$, and the expected uti $\hat{}(\ ) - (\ , 0)$ are all proportional to $-1$ when $> 0$. Thus expected allocative efficiency, expected revenue, and increases.

2. If the weight on revenue increases, then $\sum_{=1} \hat{x}(\ )$ expected uti decreases. If is symmetric about its mean, then leads the total expected $\sum_{=1} (\ )$ to decrease.

Part 1 of Proposition 2 implies that the revenue-efficiency
ber of traders increases. Intuitively, with more

$> 0$ the designer can also extract some fractio
expected transfer of each trader also rises.

Part 2 of the proposition states that, even when a
incentives of the designer, traders nonetheless

Part 3 implies that as the revenue-maximizing m
trade volume decreases and each trader is worse of

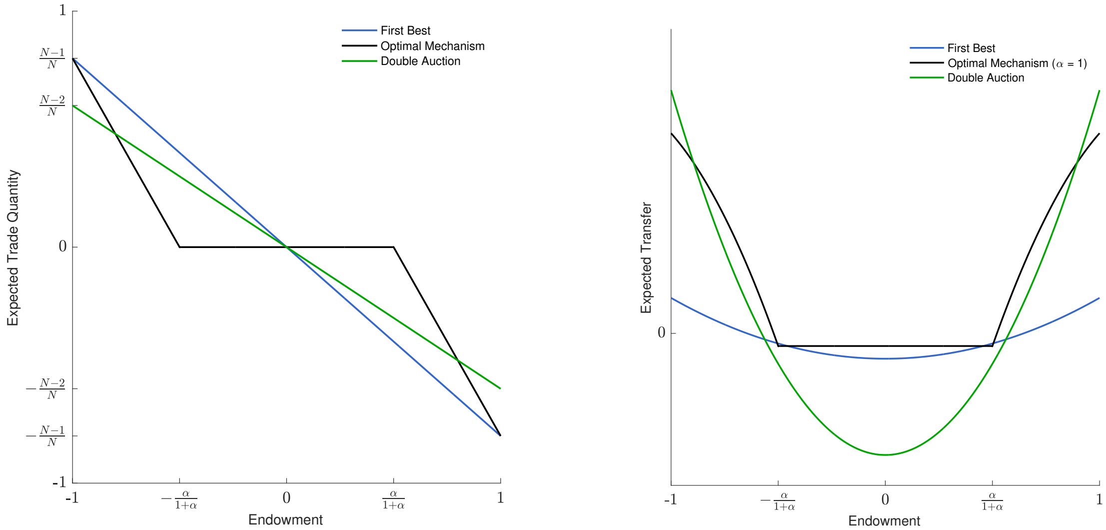
### 4.3. Illustrative Example

To further highlight some properties of the opt
the double auction, I present an illustrative exam $i = U[-1, 1]$, 
as shown in Corollary 1, C.1 is a case when there is a closed-form s
anisms).

Panel (a) in Figure 1 depicts the expected trade quantity f
location (in blue), under the optimal mechanism, and for the m
(symmetric-linear) equilibrium of the double auction, the dou
auction converges to the first best — this is to be e
which is the fundamental source of inefficiency.
bounded away for any given $> 0$. When is higher, the region of bin
tion constraints expands. Compared with the doub
achieved at the extremes. Interestingly, this is
generic property when not symmetric (see the Remark in t
It is, however, a generic property of the virtual e

Panel (b) plots the expected transfer. Because
transformation of the endowments the price sched
expected transfer is more convex under the double
is because of price impact costs (later we will se
adverse selection). Under the optimal mechanism
spreads. A similar feature appears in Biais et al. (2000) in the case of a single
market maker who functions as the designer. The co
discounts and are needed to encourage extreme typ
regions gradually become convex as decreases.

Figure 2 plots the expected revenue per trader in pan
ing. Intuively, there is less randomness in the av



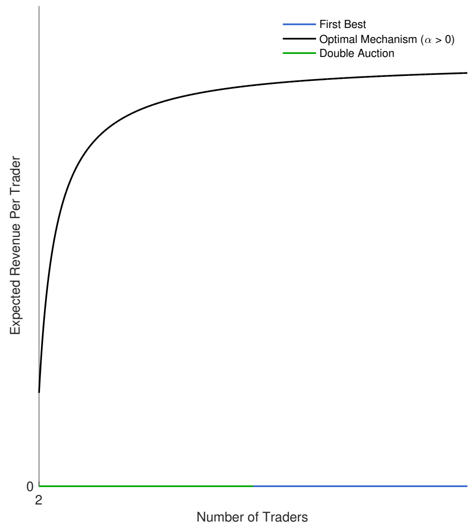

Note: Comparison of interim allocation $e \sim U[-1,1]$, and interim tra

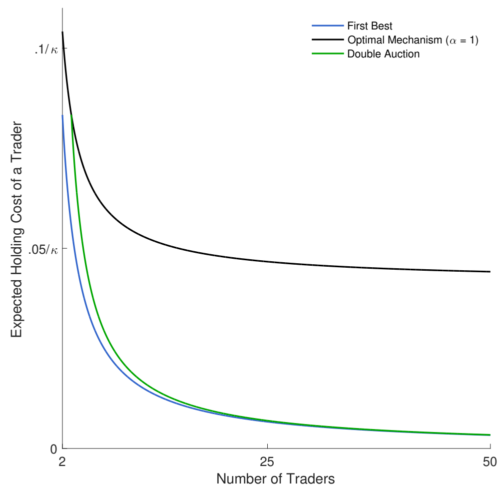
are risk-averse over asset allocations, they lik thicker markets, the designer can extract more fr the expected holding cost of a trader. As seen in t double auction and first best mechanism but it pe mechanism for revenue. In fact $> 0$ Even under revenue max tion, each trader is better off when there are more there are 3 traders is the same for the double auct mechanism.

Thus, the double auction does not lie on the reve in Figure 3. One might wonder whether the mechanisms th indirect implementations that might resemble tr one might be able to alter the double auction to br this next.

## 4.4. Implementation: Double Auction with Transa

It turns out that the double auction can be altere simply by introducing a transaction fee.



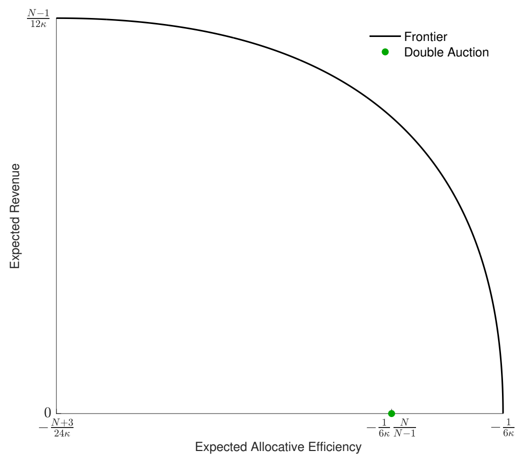

Note: Revenue per trader and expected holding cost per trader when $= U[-1, 1]$

**Definition 4.1.** A double auction with[2] transaction fee is an exchange mechanism that operates in three stages:

1. Each trader submits a demand schedule.
2. The clearing price is computed: $\sum_{=1} (d\ ) = 0$. If there does not exis clearing price, then no trades or transfers are
3. If a unique clearing price exists, trader ( ) pays ( ) + ( ) in return for ( ) units of the asset.

Given the flexibility of the transaction fees or practically anything with them. That is not so. Co consider Malamud & Rostek (2017). Clearly, those allocations a running a single double auction with transaction ples?)

**Proposition 3.** For any given $[0, 1]$, the allocation $\{\ \}_{=1}$ in Theorem 1




Note: The revenue-efficiency frontier when $= U[-1, 1]$

implementable by a double auction with transaction fees defined by $^{2}$

$$( \; , ( \; ) ) = -\frac{}{2} \; ( \; ^{2} ) \; - \; ( \; ) \frac{1}{} - \; -$$

where

$$= \frac{-}{- 1},$$

$\rightarrow$ satisfies

$$( \; ) = \frac{- 1}{} \; \frac{1}{2} \; ^{2} - \; ^{-1} ( \; \mathrm{d} ) , \qquad ,$$

and is a constant set sufficiently high so that par types of all traders.

Before I offer intuition for Proposition 3, I remark on some of implications. Proposition 3 suggests that acheiving desirable require a major overhaul of existing market infra brating transaction fees a planner can target any practice, exchanges often charge transaction fe ject of much policy debateð these fees may not be “j

Proof Sketch. Recall that

$$( \quad ) = - \quad )( + \frac{1}{\quad} \quad ( \quad ) .$$

$= 1$

common to all

In a double auction, each trader submits $= \sum_{= 1}$ ( )

$$( \quad ) = - ) + ( \quad . \qquad (5)$$

Reverse engineer: this is optimal for each trader

$$( \quad , \quad ) = \frac{}{2} {}^{2} + + ( \quad ( \quad ) + )$$

for some , , and ( . )

The function affects incentives for the intercept of efficient affects incentives for the slope of the demand so that each trader submits the demand schedule (5) and individually rational to participate. □

## 5. Adverse Selection

For trade of many securities, adverse selection Kyle 1985; 1989; Du & Zhu, 2017. In this section, I investigate the presence of adverse selection. Loosely speaking a characterization of the worst-case information characterization of optimal mechanisms for these 3) a designer's motives for revenue maximization prices. Exactly what is meant by "worst-case info below.

## 5.1. Environment

I retain the setup of the basic model except now private signals {} of the asset. These signals are random vari correlated with. I will not take a stance, yet, on the dist spaces where they live. I continue to assume that {} are independent both of each other and of the asset's payoff, that they are in signal {}. This assumption simplifies the analysis wh adverse selection. Also, to ease the exposition, = 0, though all results extend to the case of nonzero.

Mechanism design in this environment is difficu each trader has two dimensions of private information. One cannot mean-ingfully simplify the model by removing one dimension {} because movements are the source of gains from trade. Thus, th is a multidimensional screening problem, which is special cases. Second, it is unclear what particu model. Most of the market microstructure literat sional and equal to the fundamental plus additive noise that traders. However, because these signals are corr mation structure were Crémer & McLean (1988) in which are very sens to the fine details of the environment and not robu cumvent these issues, I first take a detour and co structures for the double auction.

## 5.2. Worst-Case Information Structures: Double

The analysis here is in the spirit of the litera "information design" (Bergemann & Morris, 2013; Bergemann et al., 2017; Lambert et al., 2014). I assume that each trader's value is the sum of the trader's signal and her posterior expectation of $= E[\,|\,]$.

I assume that it is known that

1. $( , {}^{2})$
2. $_1, \ldots,$ are jointly Gaussian
3. $\mathrm{cor}( , ) \geq 0$ is the same for

and is the same

In what follows, let $\Delta(\,,\,)$ denote the set of information stru
tions 1-4 above. In what follows, I take the persp
the marginal distributions of traders' signals a
signals is weakly positive (which seems to be an e Vives, 2011). The observer also knows that traders' signa
tions 3. and 4. above (an assumption I make for tra
not know how traders' signals combine. That is, she does
not know the nature of the interdependency in trad
traders may have formed their expectations using

In what follows, I seek to derive predictions for
Bayesian equilibria in symmetric linear demand s
submiting a demand function of the form

$$(\quad) = - + \quad - \qquad (6)$$

for some constants , and , and she anticipates that the other tra
that the restriction to symmetric Gaussian envir
in Bergemann & Morris (2013)

Fixing a double auction, what information struc
I say that an information structure $\Delta(\,,\,)$ is worst-case if it minimizes s
ciency among equilibria in symmetric linear dema
symmetric linear equilibrium. Let SLE( ) denote the set of symmetr
libria for the information structure. Formally, $\Delta(\,,\,)$ is worst-case if SLE( ) is nonempty
or

$$\arg\min_{SLE(\;)} - \sum_{=1} ( + (\;))^2$$

where , defined formally in Example 1, is the market
The subscript on the expectation operator indica

The following Theorem 2 characterizes all equilibria for i
and provides conditions for existence and unique
with respect to the correlation among traders' si
is novel for two reasons. First, almost all paper
with errors that are independent across traders w

sum of the private and common components ... Rostek & Weretka, 2012). Thus, the characterization of symmetric ... novel to my knowledge.

Theorem 2. If a symmetric linear equilibrium exists, ... coefficients ... and characterized by (18), (19), and (20) in Appendix B. Letting $c \equiv \rho(r, r)$, the following statements hold:

1. If $= 0$, a symmetric linear equilibrium exists if

$$\frac{2}{2\,\overline{1}\,{}^2 + 2} < \frac{-2\,1}{-1\,2}.$$

The parameter range of equilibrium existence ... increases ...

2. Conditional on equilibrium existence, alloca...

3. The unique worst-case information ... $_1$, ... is independent ...
That is,

$$| = \qquad . \quad {}_{=1}$$

4. Under the condition for existence in Part 1 of t... efficiency, which occurs when $= 0$, is:

$$- \quad (1 + {}^2 - 2 \quad {}^2) + \quad {}^2\,{}^2\,{}^2$$

where

$$= 1 - \frac{2( \quad - 1)}{- 2} \frac{{}^2}{2\,\overline{1}\,{}^2 + 2}.$$

5. Under the condition for existence in Part 1 of th... which occurs when $= 0$, is

$$\Lambda = \frac{1}{- 2}\,\overline{1} + \frac{{}^2}{2\,{}^2 + 2\,2\,2}$$

where is as in Part 5.

Remark. I have assumed that is known (although its me... inferred from ). If $\geq$ 2, then this is without loss. If $\leq$ 2, the worst-case information structure is known, it is determined by the m... such that $\frac{1}{1+(\ -1)}\sum = 1$. This is defined by

$$\frac{\ ^2}{1+(\ -1)} = \ ^2.$$

When 2 is known and $\leq$ 2, under the worst-case informatio... signals together fully reveal .

If the marginal of is known but the marginal of { } are unknown, then the wor... information structure is also fully revealing an... in numerical examples I have computed, the 2 is corre... increases.

Part 1 of Theorem 2 implies that if an equilibrium exists, it exists... for all > 0. Parts 2 and 3, imply that the worst-case i... when = 0. Parts 4 and 5 characterize allocative eff... worst-case information structure, and in doing so... across all information structures in .

Intuitively, = 0 is worst-case becase it is harder for a... will trade when signals are independent. Though... actually more subtle. Indeed, the proof is somew... characterized in terms of a cubic equation which... correlation increases, traders put more weight on thei... schedules as well as on their endowments. The net... rates of increase. The ratio of the weights, in the... Appendix, must contend with these complications...

In light of Theorem 2, several questions come to mind. Whic... well for these information structures and can bes... a sense in which the information structure is... on the assumed mechanism? Are there mechanisms th... tive efficiency than double auctions? What is th...

## 5.3. Optimal Mechanisms

Throughout, I now assume that the prior is a multivariate ... arbitrary distribution with ... variance. Note that I am n... continue to assume that each trader sees a single- ... . I further assume that ... jointly independent and that

$$| = \qquad . \quad {}_{=1}$$

It turns out that, for this information structur... despite the multiple dimensions of private infor...

Lemma 11. In any mechanism, the optimal message in a Bayes Na... is a function of ... $= 1$ ...

Proof. Let $_$ denote the messages chosen by the other tr... mechanisms. The trader's message maximizes

$$-\frac{1}{2} \quad + \quad ( \quad , \_ )^2 + \sum_{=1} \quad + \quad ( \quad , \_ ) - \quad ( \quad , \_ ) \,|, \quad .$$

Rearranging, the objective is equivalent to

$$-\frac{1}{2} \;{}^2 - \;\frac{1}{-} \;- \quad ( \quad , \_ ) - \frac{1}{2} \quad ( \quad , \_ )^2$$

$$+ \sum_{\neq} \quad ( \quad , \_ ) - ( \quad , \_ ) \frac{1}{|} \; - \quad .$$

Thus, the optimal choice of message depends on onl...

and denote the cdf and pdf respectively for any given trader . Let

$$\hat{\ }\ (\ )\ [\ \ |\ \ =\ \ ].$$

The following Theorem 3 characterizes the optimal mechanism ... selection.

**Condition 2.** It holds that $\frac{(\ )}{(\ )}$ and $\frac{1-(\ )}{(\ )}$ are weakly increasi ...
.

**Theorem 3** Suppose that Condition 2 holds. Then the unique solution to problem (1) sets

$$(\ )\ =\ -\ )\ (+\ \frac{1}{}\ \sum_{=1}\ (\ )$$

for each trader where

$$(\ )\ =\ \hat{\ }\ \frac{(\ )}{(\ )}\ (\ ,\ )\ +\ <$$

$$(\ )\ =\ \hat{\ }\ \frac{1-(\ )}{(\ )}\ (\ ,\ -\ )\ >$$

$$(\ )\ [=\ (\ )]\ ,\qquad ,(\ )$$

for and such that is continuous and $\lim_{\to}\ (\ )\ =\ )\ =\ (\ )$.

As in the basic model, optimal mechanisms are ch... know (n, s, [ ( )]) and there are special cases where the u... acterized almost in closed form.

**Corollary 3** Suppose that Condition 2 holds and is symmetric about it... Then under the optimal mechanism,

$$[\ (\ )]=\ \ ,$$

$$\hat{\ }\ )(+\ \frac{(\ )}{(\ )}\ =\ \ ,$$

and

$$\hat{\ } \; )( - \frac{1 - \quad ()}{(\ )} = \quad .$$

Using Cor. 3.1, I am able to prove the following compa

Proposition 4. Suppose that and are Gaussian. Then the follow statics hold:

1. As $^2$ increases, the region of binding participat– increases without bound for any $> 0$. Thus, as $^2 \to \infty$, the probability trade vanishes in that

$$\sum_{=1} |(\ )| > 0 \to 0 .$$

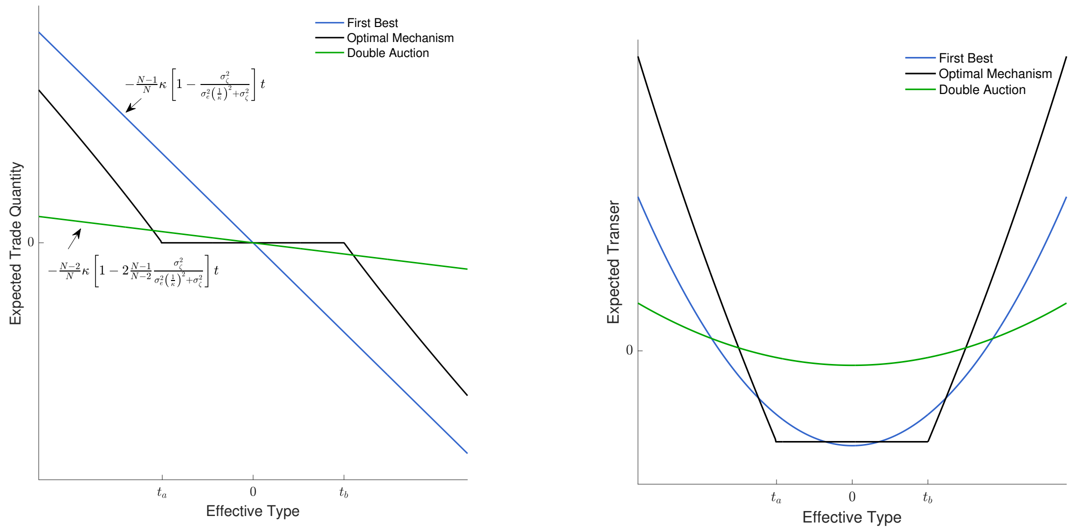
2. As $^2$ increases the revenue-efficiency frontier

3. For any given $> 0$, as $^2$ increases, the expected (Do) utility g any given and the expected $\mathbb{E}[\sum_{=1} |(\ )|]$ trade volume decrease.

Proposition 4 implies that as adverse selection becomes more severe (as $^2$ increases), the region of binding participation constraints incr vanishes, and the revenue-efficiency frontier s finite $^2$ level, there is always some probability of trade the double auction where (symmetric linear) equi $^2$.

## 5.4. Illustrative Example

To further illustrate the properties of optima selection, I now present an example ($^2$ = 10), where Panel (a) of Figure 4 plots the expected trade quantity as a funct see, many qualitative properties are the same as t $^2$ increases, the gap between the double auction and constraints also expands and eventually diverges




Note: Comparison of interim allocation rules (a) $\sigma^2$ and interim tr
( $0^2$ ).

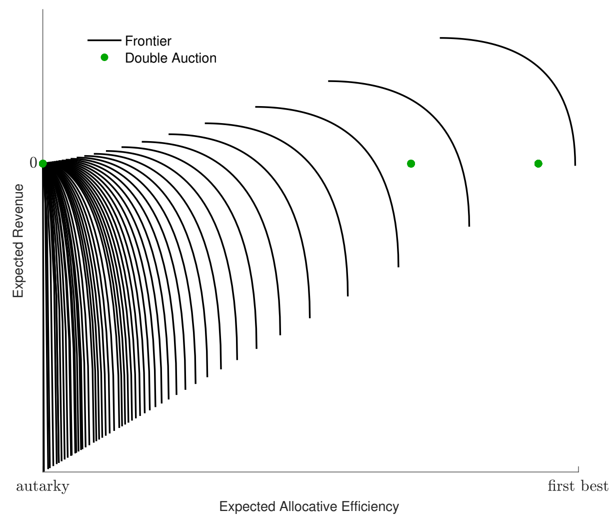
As seen from panel (b), $\sigma^2$ ... shifts up if ... is sufficiently high, with curvature under first best than under the double a
case without adverse selection. This must be so b ... ($\sigma^2$), increases the ratio of the slopes of the blue and g
bounded from above for the case without adverse se

Below in Figure 5, I plot the revenue-efficiency frontier $\sigma^2$ ... of .5. As one can see the fronteir shifts down befo
at autarky. The double auction reaches the autark
mechanisms.

## 5.5. Implementation: Double Auction with Transa

It turns out, that even with adverse selection,
to the revenue-efficiency frontier using transac

Proposition 5. For any given $t \in [0, 1]$, the allocation ... Theorem 1 is implementable by a double auction with ... equations of the Appendix.




Note: The revenue-efficiency frontier for different values of $\sigma^2(0)$ and $(\varepsilon^2)$.

The proof is similar to the case without adverse
other term in the transaction costs. A large litera
ture studies information aggregation in financia
3.2 implies that once one endogenizes the mechanism
of the designer worsen price informativeness.

Corollary 3.2. Suppose that are Gaussian. Whenever a sym
equilibrium of the double auction without transa
librium is more Blackwell informative than in the
transaction fees set optimally for any $\varepsilon > 0$

Corollary 3.2 indicates that conclusions concerning in
erature may depend on the exogeneity of the tradin
prices to aggregate information will not disappe

## 5.6. Robust Mechanisms for Allocative Efficienc

Is there a sense in which the information struct
does not depend on assuming the double auction mec
antee of allocative efficiency that can be achiev

sees a signal of the fundamental where the signal space is an arbitrary measurable set. Let

$$= ( \{ , \; )$$

denote the information structure, where $\Delta( \times \Pi )$ is the joint distribution ... and the fundamental.

I assume that the designer knows some set of ... expectation of the common component of the payoff ... that is known a ... and that is otherwise stochastically mono... realizations of ... an increase in the ... of first-order stochastic dominance ... to assume that it is known that traders' endowmen... $\{ \}$ and .

That is, the designer may have some sense of trad... tal of the asset, but does not know exactly how tho... signal structure) and in particular, how they are ... know how these signals combine to give the best es...

$$( _1 , \ldots ) = [ \; | _1 , \ldots ] ,$$

or even what spaces these signals live in. An alte... formation necessarily, but that values are interd... simply has no idea how values are interdependent.

Here are two examples of information structures ... guity set (though there are of course many other ex...

Example 2. Suppose that signals are just traders' pos... $_1 , \ldots$ and that these are independent random variables (and ... earlier). Then both

$$( _1 , \ldots ) = \sum_{=1}$$

and

$$( _1 , \ldots ) = -1 + \prod_{=1} ( + 1 )$$

can be are consistent with information structure

In the face of this ambiguity, the designer look
sense that it delivers the highest guarantee acro
sistent with the marginals.

To state the objective a bit more formally, I firs
design and information structure, let ... be the set of Bayes Nash Equi
( , . )

Let

$$AE( \ , \ , \ ) = -\frac{1}{2} \sum_{,=1} ( + ( ))^2$$

where $\{r_i\}$ is the allocation rule and where the expectation is ta
according to the probability distribution indu

I adopt the solution concept of Brooks and Du (2021): maxmin so

Definition 5.1 (strong maxmin solution). consists of a mechanism
formation structure and a Bayes Nash equilibrium

1. For any mechanism and any equilibrium ... it holds that AE( ) ≤ AE( ′, ′)

2. For any information structure and equilibrium ... it holds that AE( , ′, ′) ≥ AE( , , )

3. is an equilibrium of

The following theorem assumes compact support b
ample, suppose (...) is Gaussian, then the result app

Theorem 4. Suppose that has compact support and that Condit
2 holds when = 0. Then there is a strong maxmin solution
allocation rule with = 0 and where nature selects the in
assumed in Subsection 5.3.

Proof. Recall that the designer solves

$$\max -\frac{1}{2} \quad + \quad ( \quad , \_ )^2 + \qquad + \qquad , \_ ) \quad - ( \quad , \_ ) \,|, \quad .$$

Now suppose we set the transfer rule

$$( \qquad ) = \hat{-} ( \quad ) + \quad \hat{} ( \; d) - \frac{1}{2} \quad ( \;\; {}^2 + ) \qquad (7)$$

For the transfer rules in (7), all terms involving ... drop out from the obje...

Thus, each trader submits the same message ... under any information struc...

Now, I directly verify that allocative efficien...

worst-case information structure where signals a...

$$\sum_{=1} \quad + \quad ( \quad )^2 = \quad {}^2 - \frac{-1}{} \sum_{=1} \quad () + \sum_{=1} {}^2$$

$$= \quad {}^2 - \frac{-1}{} \sum_{=1} \quad ()$$

$$+ \qquad - \frac{-1}{} \quad () + \frac{1}{} \sum_{\neq} \quad ()^2$$

$$= \quad {}^2 - ( \quad - 1 ) \quad () + \frac{-1}{} + \frac{1}{2} \quad ()^2$$

$$- \frac{1}{} \sum_{=1} \sum_{\neq} \quad \mathrm{cov}\,( \; , \; )( \quad .$$

Because the coefficient on covariance pairs ... is ... set to ... each ...

tically monotone, it follows that independence i...

Theorem 4 illustrates the sense in which the informa...

far are worst-case without fixing, at the outset, ...

weakness of the result is that it allows for a budg...

with the classic Vickrey (1961), Clarke (1971), Groves (1973) ... n efficient

implementation ... Bergemann & Morris (2012) ... In some cases, the gov...

can enforce participation. Following the classi...

Corollary 4.1. The strong maxmin mechanism is robust to traders' higher-
order beliefs about other traders' beliefs about

In general, deriving robust guarantees when th
intractable. As seen from the proof of Theorem 4, the main difficulty is th
not find it individually rational to participate
conditional on full participation, one can always construct
such that a trader always optimally reports of the informatio
same argument as in the proof of Theorem 4. However, that efficie
are typically quite “aligned,” our analysis of optimal
information structures for efficiency is still i
revenue in the presence of severe adverse selecti

## 6. Multiple Assets

Can the efficient allocation be implemented by
when there are multiple assets? Is it more or less c
when there are more assets? Does this depend on th
among asset endowments? What do revenue-maximiz
be implemented by double auctions with transactio
questions for a multi-asset version of the model.

### 6.1. Environment

There are now multiple assets. Trader i's utility is

$$( \; + \; , \; ) = \; - \frac{1}{2} \; + \; \Sigma \; + - \qquad (8)$$

where

- $=\{\}$ is the vector of asset payoffs
- $=\{ \; \}$ is the mean of
- $\Sigma$ is the covariance matrix of
- $=\{ \; \}$ is the vector of trader's endowments
- $=\{ \; \}$ is the vector of trade quantities of the
- is the net transfer
- $> 0$ is a constant.

I assume that endowments $=\{ \; \}$ are independent across traders
pendent of asset payoffs. However, the endowments of the d
assets may be correlated. I retain all 3 other aspec

### 6.2. Optimal Mechanisms

Proposition 6 states that it is possible to implement th
| separate double auctions with transaction fees
ante budget balance, regardless of correlation a
different assets. This result is perhaps surpris
in one auction contingent on the prices in the oth
would make implementation impossible.

Proposition 6. The efficient allocation is implementable by | separate double auctions with transaction fees. It is implementable
ex-post equilibrium.

Condition 3. The following hold:

Å = {} are independent Gaussian random variables $^2$

Å = {} are independent Gaussian random variables $^2$

Note that, besides Gaussianity, either of the two parts of Condition 3 is without loss on its own and is just a normalization, however, because one can always construct securities so that they are independent. On can also redefine securities by scaling so that the variances of all payoffs or all endowments are the same. However, one cannot simultaneously satisfy both properties. Note also that Condition 3 makes no restrictions on the mean {n} of or for the mean {n} of .

**Proposition 7.** Suppose that Condition 3 holds. Let $\sum ( - )^2$ be the length of for each . For each and , let $\tilde{} = ( - )/$ . Then the unique solution to (32) for the optimal allocation rule is

$$( ) = - \tilde{} + \left( \frac{1}{} \sum_{=1} ( ) \tilde{} \right.$$

for each and where

$$( ) = 0, \quad \sqrt{} \; ) \; [0,$$

$$( ) = \frac{2}{} - , \quad \sqrt{} \; [ \; , \infty)$$

and where $[0,1]$ is the weight on revenue in (32). [illegible] the formal [illegible] Appendix C. The optimal allocation rule is implemented [illegible] Appendix equation (34).

Proposition 7 shows that many properties of optimal mech[illegible] are many assets, as well. To prove the Proposition 3, Condition [illegible] "symmetry" of the optimization problem. Armstrong (1996) building [illegible] Wilson (1993) who study a multiproduct monopolist.

Consider Figure 6 which depicts the case of two asse[illegible] anism, the region of binding participation constraint [illegible] $\sqrt{}$ [illegible] thus larger when the objective places more weight [illegible] region expect to trade zero of both assets. Outsi[illegible] constraints lie along radial lines, are local, an[illegible] type is distorted only radially. That is, the opti[illegible] of a trader's. The proof of Proposition 7 is in Appendix [illegible]

Unfortunately, it is not possible to implement t[illegible] submission strategies using separate double auc[illegible] possible to do so with cross-exchange transaction fees [illegible] made contingent on trades and prices on all exchan[illegible]

**Definition 6.1.** A double auction with cross-exchange tr[illegible] is an exchange mechanism that operates in three st[illegible]

1. Each trader submits a demand schedule for each asset .

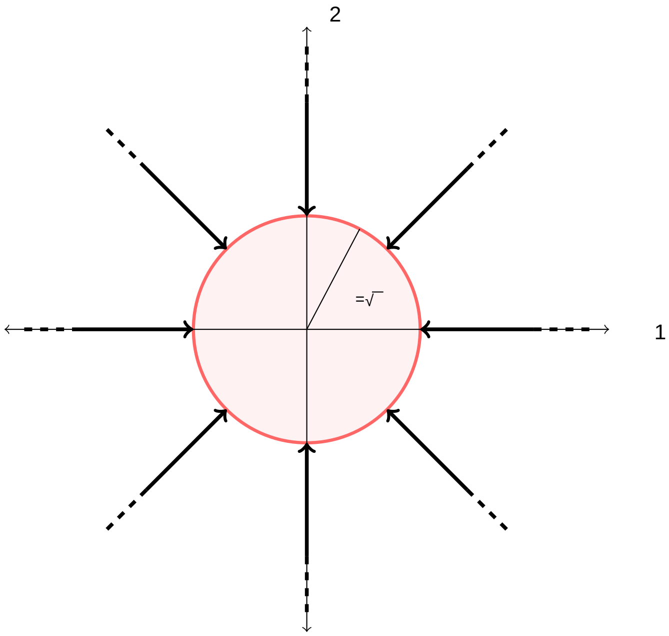



Figure 6: Binding participation and incen

Note: The figure illustrates the binding participa of two assets. In the general case with $|a| \geq 2$ assets, the region of bindi constraints is a ball with radius $\sqrt{\ }$. The binding incentive const point in the direction of the ball and bind only ra

2. The clearing price is computed as $\sum_{=1} (d) = 0$. If there does no exist a unique clearing price for each asset, then no trader transfer

3. If a unique clearing price exists for each asset, then trader $i$ pays $\sum$ tot$(a) + (\{(\ ),\}\ )$ in return for $(n)$ units of each asset.

Note that though cross-exchange transaction fe each exchange nonetheless clears separately in t given trader is contingent on only the price of tha is still relatively simple.[14]

Proposition 8. Suppose that Condition 1 holds. Then the sit 7 for any given $[0, 1]$ can be implemented by a double auc

[14] Most exchanges clear separately, except (Rostek & Yoon, 2021; Chen & Duffie, 2021; Budish et al., 2023).

exchange transaction fee defined by

$$\{(\ ),\ \} = (\ )\frac{1}{}+ {}^{2} + \frac{1}{2}(\ {}^{2}) +$$

where → satisfies

$$(\ ) = -\frac{1^2}{}\int_0^{\sqrt{}} {}^{-1}(\ \mathrm{d}) - \frac{1}{2}, \quad \geq 0,$$

$$= -\frac{{}^2}{(\ -'1)}$$

and is a constant set sufficiently high so that par

types of all traders.

Though Condition 3 is restrictive, Proposition 3 further demonstrates the effectiveness of transaction fees even in a setti the designer may care about revenue. For this set exchange market clearing. That is, there is no ne exchange to be made contingent on the prices in ot mentary perspective to Budish et al. (2023) who argue for a market desig cross-asset market clearing. Though cross-asset Propositions 6 and 8 show it may not be needed in all instance freedom to set transaction fees or to design deriv

## 7. Heterogeneity

Our analysis so far has assumed that traders are to the probability distributions of their endowm ity, these likely differ across traders. Though microstructure literature, some papers allow fo and interesting phenomenona. By allowing for hete questions such as: How should trade be designed in retail traders with low holding capacity receive cated institutional traders who typically have h with transaction fees still optimal?

## 7.1. Environment

I retain all aspects of the baseline model of Section 3 except ... {( )} of endowments and also the fidelity... of traders. The designer's ... objective (2) with these changes is stated formally ... (38) ...

## 7.2. Optimal Mechanisms

The derivation of optimal mechanisms is now ... analogous to Section 3 except ... requires a stronger technical condition.

Condition 4. ... holds that $\frac{1-( )}{( )}$ and $+\frac{( )}{( )}$ are weakly increasing and ... has full support for each trader.

The full support assumption is needed to ensure ... expect to trade.[15] If it is violated, we can simply ... calculate the optimal mechanism, ... taking limits as the tails vanish.

Theorem 5. Suppose that Condition 4 holds. Then the unique solution ... tion problem (38) for ... sets

$$( \quad ) = ( - ) + \frac{}{\sum_{=1}} \quad ( \quad )_{=1}$$

for each trader ... where

$$( \quad ) = \frac{( \quad )}{( \quad )}, \quad <$$

$$( \quad ) = \frac{1-( \quad )}{( \quad )}, \quad >$$

$$( \quad ) = \frac{}{\sum_{\neq}} \quad ( \; ) \quad , \quad , \; []$$

for and such that ... continuous ... $\lim_{\to} ( ) = ( ) - ( )$.

Now, each trader unloads her virtual endowment in ... the aggregate virtual endowment that is proportional ... libria is now significantly more ... because ...

[15] These types need not exist if traders are sufficiently he...

**Corollary 5.** Suppose that Condition 4 holds, $[t_1, \ldots]$ and $= _1[\ldots]$, are colinear and symmetric about $[ ( )]$ its mean, then

$$+ \frac{( )}{( )} = ,$$

and

$$- \frac{1 - ( )}{( )} =$$

for each .

At first thought the colinear requirement seems … but it holds when each has a common mean (but … differ in all higher-order moments) and all trade … for each. The symmetry assumption is … Gaussian, but … a common assumption in the market microstructure …

Let me note that in this case, we can often compu… mechanism in closed form, just as in the basic mode…

**Remark.** If each is uniformly distributed, as long as … trade zero in expectation, then the optimal mechan… a system of linear equations.

## 7.3. Comparative Statics

Using Corollary 5, I am able to derive the following comp… 9 focuses on the special case with Gaussian with zero mean. A… ex-ante would be an ex-ante source of heterogenei… which I seek to shut down.

**Proposition 9.** Suppose that the endowment is Gaussian w… zero and variance $^2$. Then the following comparative statics …

1. The regions of binding $\{p[a, r]\}$ do not depend on …

2. Suppose $^2$ increases. Then for $> 0$, …

(a) The range [ , ] of binding participation constrai

but has no effect on [ , ] for any trader ≠ .

(b) The expected utility gain of trader decreases.

(c) The expected utility gain of trader ≠ is unaffected.

3. If increases for trader :

(a) The expected utility gain decreases for trac

(b) The expected utility gain increases for trader ≠ .

Proposition 9 shows how a trader's utility gains from tr in the market as well as her own characteristics u surprisingly, as seen from Part 2c, for any given other traders' endowments has no effect on her ex proposition shows that a trader benefits when the holding capacities. This is because she will be be

## 7.4. Implementation: Double Auction with Transa

One might wonder whether a double auction with t the revenue-efficiency frontier when there is he as long as the transaction fees can be tailored to

**Proposition 10.** A double auction with trader-specific t any outcome on the revenue-efficiency frontier.

The intuition is the same as for the basic model. ization of the implementing transaction fees whi sition 3 except with coefficients that depend on the i { }. Thus, even in the presence of heterogeneity, t With heterogeneity, exchanges have incentives t whether it be to promote efficiency or revenue. I differ across traders—large instutitutional[16] tra

[16] For example, see https://optiver.com/insights/a-little-under -options-in-the-us/.

## 8. Dynamics

The majority of this paper's analysis is conduct…
a sense in which the results extend to dynamic set…

Suppose that trader $i$'s utility is of the form

$$(\{(\ )(\ ,\ );\ \ \geq 0\}) = \frac{1}{2}(\ +\ (\ )^2) - (\ )d$$

where ( ) denotes the quantity that a trader holds of … index … in h… her initial endowment, $\Delta$( ) and where … is terminal time.

The designer's problem is to set a path of allocations to maximize a convex combination of the net-present … ciency subject to the constraints that 1) it is ind… in the mechanism at each time, 2) it is incentive compatible … port her endowment truthfully, 3) the mechanism d… of the asset at any time. A formal statement of the … E.

Proposition 11 implies that the dynamic problem simplif… the analysis of the one-shot problem considered so far … Section 3 …

Proposition 11. … all trades are executed at … = 0 … (3) …
( ) = 0 and ( ) = 0 for each > 0 for each.

Of course, whenever the weight … is positive, this re… strong commitment power of the designer who must r… in the future and 2) the absence of renewed endowm… that several papers study the setting with a sing… shorter time horizons (Vayanos, 1999; … (2011)).[17] Though the reliance on commitment is an important limita… upper bound on what is achievable with weaker form… a conventional assumption in the l… Bergemann & … dy… Välimäki, 2010; Pavan, … 2014; Athey & Segal, 2013).

When there are renewed endowment shocks over ti… can say. However, I am able to derive conditions w… implementable.

[17] Vayanos (1999) allows for renewed endowments shocks in the mode… focuses attention on the limiting case as the variance of fu…

Proposition 12. Suppose that the cash endowment process of

$$= {}_0 + \quad , \qquad [0, \quad ]$$

where { ; [0, ]} are independent Brownian motions and { $_0$} are independent Gaussian random variables. Then the efficient allocation implemented with an ex-ante budget surplus by a c transaction fees.[18]

Thus, even in a dynamic setting with renewed end with transaction fees can be a powerful policy to achieve efficiency with a budget surplus which wa is because gains from future trades can be used to Indeed, as a trader grows impatient ( $\to 0$), her private information $\{ _0\}$ becomes irrelevent for her gains from participation beca endowment process. Thus, it is as though interim ex-ante participation constraints.

This result is a contrast to Vayanos (1999) who shows that, fixing the tion, as the frequency of trade increases, welfar less aggressively.[19] Proposition 12 implies that opposite can be true troduce transaction fees, which are often employ whether the Vayanos (1999) result holds when the exchange can set an open question and may depend on the strength of maximization.

## 9. Conclusion

The objective of this paper has been to investiga a variety of market conditions for a workhorse mod crostructure. There is only a relatively small li microstructure and mechanism design of trade. Th filling this gap.

---

[18] A version of this result first appeared in Chen & Zhang (2020), which is an earlier version of this work with am a coauthor.

[19] A low frequency of trade approximates a static environmen

exchange mechanisms often leads to tipping onto
oping a cohesive analysis with competing exchang
would have great value. A plausible conjecture is
exchanges may appear similar to outcomes in this p
weight on revenue. Investigating whether this is

# Online Appendix

## A. Omitted Material for Section 4

**Lemma 2.** It is without loss of generality for the des
ministic mechanisms. That is, the designer canno
(2) by selecting a stochastic mechanism.

*Proof.* Because utility is quasilinear in transfers i
transfers to be deterministic. I shall now show it
istic allocation rules.

Let = { } be an implementable allocation rule tha
( , ) = { } denote the random vector of trade quant
reporting and given . We can write

$$( \quad , \quad ) \; (= \quad , + \quad ) \; (| \quad )$$

where ( ) = | 0.

Define the deterministic allocation rule = { } by

$$( \quad ) = ( \quad , \, , \quad ) \; | \quad .$$

Clearly is implementable ... is weakly decreasing ... implemen-
mentable by assumption ... Moreover, transfers and allo
are both higher because they are concave in the allo
also (4) which applies even when mechanism are stocha
individually rational ... the utility gain fr
depends on only the interim ... (3) which applies even w
the mechanism is stochastic).

It thus follows that any stochastic mechanism c
mechanism. □

The rest of this appendix gives formal proofs of ...
prove the following auxiliary Lemma 3.

**Lemma 3.** If > 0, then any solution to (2) is unique up to measure zero
If = 0, then all solutions have the same allocation

Proof. Using equation (4) for the transfer rule and integratin
mization problem (2) as

$$\max_{\{,\hat{}()\}_{=1}} \sum_{=1} -\frac{1}{} - \frac{\mathbb{1}_{\{\geq\}} - ()}{()} \left( \frac{1}{2} \right) - (^2) - \hat{}() \quad (9)$$

such that

$$-\frac{1}{} \int \hat{}(\mathrm{d}) + \hat{}() - \geq 0, \quad 0, \quad ,$$

$$\hat{} \text{ is nondecreasing},$$

$$\sum_{=1} () = 0,$$

where recall the $\equiv - \int$ is concave. It is easy to see th
solution must necessarily be unique because the
objective function is strictly concave in ... and the set of
satisfying the constraints is convex. ... solutions in
solution for the allocation rule is necessarily u
$\hat{}()$ that are optimal. □

Consider the problem

$$\min () \text{ such that } \in \Omega, \quad () \leq \quad (10)$$

where $\Omega$ is a convex subset of a vector space, is a real-valued convex fun
and is a convex mapping into a normed space ... a convex cone in
For, write $\geq$ if $-$ ... . The cone which defines this relat
the positive cone. To complete the proof, we use the result to a
following restatement of Theorem 1 in Luenberger (1969) that appears

Theorem (Luenberger (1969)). Let denote the positive cone of
closed. Suppose there exists $_0$, $\geq _0$, and $_0 \in \Omega$ such that the Lagran

$(\ ,\ ) = (\ ) + $ possesses a saddle point $(\ _0, \ _0)$. That is,

$$(\ _0, \ ) \le \ _0(\ _0) \le (\ _0)\ ,$$

for all $\ \in \Omega$, $\ \ge 0$. Then $\ _0$ solves (10).

Using Lemma 3 and Theorem 6 we can now complete the proof of Theorem 1.

Proof of Theorem 1. The relaxed version of problem (2) without the monotonicity constraint fits into the framework of (10) with the following translation.

1. Let

$$= \frac{1}{\ } - \frac{\mathbb{1}_{\{\ \ge\ \}} - (\ )}{(\ )} \quad (\ ) \frac{1}{2} + (\ ^2 + \ ) \ \hat{}\ (\ )\ .$$

2. Let $\Omega$ be the set of $(\{\ \}_{=1}, \{\ \}_{=1})$ such that $\sum_{=1} \ = 0$ and each is a measurable function from $[\ , \bar{\ }]$ to a compact interval in

3. Let $\Omega \ \ [\ , \bar{\ }] ( )$ be the function that maps

$$(\{\ \}_{=1}, \{\ \}_{=1}) \quad \frac{1}{\ } \quad [\ (\ _{-},\ )]\, d \ - (\ ) \ \frac{1}{2} \ ^2 \quad .$$
$$_{=1}$$

Note that is convex.

4. Let be the set $[\ , \ ]$.

5. Let be the function that is equal to zero

6. The set is the set of bounded variation measures on $[\ , \bar{\ }]$

7. Let be the set of nonnegative continuous functions

It is clear that is nonempty, closed and convex, and contains an interior point. Thus we can apply Theorem 6 to verify the optimality of the relaxed problem which was obtained by solving for a saddle sufficient conditions the monotonicity constraint Theorem 1 must also be a solution of the original problem (2).

Proof of Corollary 2. The proof of is immediate from Theorem 1.

Proof of Corollary 1.1. We verify that the proposed solution in [Coro]llary does indeed satisfy the conditions of Theorem 1.

We have

$$- \quad = \frac{1 - \quad ()}{()},$$

and so, using the above symmetry of , we have

$$- \quad = \frac{( - ( - \quad ))}{( - ( - \quad ))}$$

$$\Leftrightarrow$$

$$- ( - \quad ) + \frac{1 - \quad (- ( - \quad ))}{( - ( - \quad ))} =$$

$$\Leftrightarrow$$

$$- \quad = \frac{()}{()\,)}$$

as desired.

I now verify ( ) = . We have that

$$() - \quad = \quad - - \frac{1 - \quad ( \quad )}{( \quad )}$$

when ≥ and

$$() - \quad = \quad - + \frac{( \quad )}{( \quad )}$$

whenever ≤ . Everywhere else, ( ) − = 0. We have

Because ( − ) = − ( − ), and we indeed have

$$)( \quad =$$

where I have used an symmetry of . □

Proof of Proposition 2. To prove Part 1 recognize that

$$( \; ^2 \; ) = \frac{( \quad - \; ^2 1\,)}{2} + \frac{- 1}{2} \text{ var}[\;()\;] = \frac{- 1}{\quad} \text{ var}[\;()\;] .$$

Moreover,

$$\hat{} ( \quad ) = \frac{- 1}{\quad} () + \frac{- 1}{\quad} [ \quad ]( ]$$

$E[\ ](\ )$ does not depend on . Then by (3) and (4), the expected transfer and expected utility gain are both proportional to $\frac{-1}{\ }$ as claimed. The expected $\hat{\ }(\ ) - (\ )$ ... $(\ ,0) = 0$ under the optimal mechanism.

Similarly the expected holding costs relative t...

$$\frac{1}{2}\ (\ ^2 + )\frac{1}{\ }\ (\ )$$

which also is proportional to $\frac{-1}{\ }$.

I now prove part 2. Clearly, when ... increases, revenue increases a... ciency decreases. Because the expected utility g... change allocative efficiency and expected transf... utility gain must decrease.

To show that the total expected trading volume $(\ )$ d... decreases in the mean-preserving spread ... about its mean. This is ... ( ) ... of $'(\ )$ simply by using the mean-preserving spread d... symmetry of .

□

Proof of Proposition 3. See the proof of Proposition 5 which proves the more gen... where there may be adverse selection.

□

Example 3. Suppose $\sim$ Exp(1) and $= 1$. Then

$$E[\ (\ )] = .86814$$

and so under the optimal mechanism, a trader of typ...

$$-\frac{-1}{\ }\ .86814$$

units in expectation rather than

$$-\frac{-1}{\ }$$

as she would under the efficient mechanism. Under... tation

$$-\frac{-2}{\ }.$$

Thus, the distortion can be higher under the optim
Expected trade volume is trickier.

## B. Omitted Material for Section 5

Formal Statement of the Design Objective. Given [0, 1], the objective is to solve

$$\max_{\{\ \},\ \}\{} \underbrace{\sum_{=1} (\ )}_{\text{Revenue}} + (1 - \ ) \underbrace{\sum_{=1} -\frac{1}{2} \quad + (\ )^2}_{\text{Allocative Efficiency}} \qquad (11)$$

such that,

(P) $\quad + (\ )(\ ) \geq (\ ,\ )\ 0, \quad ,$

(IC) $\quad \arg\max + (\ ,\ _{-}\ ),(\ ,\ _{-}\ ) \quad , \quad ,$

(E) $\quad \sum_{=1} (\ ) = 0,$

where $= \{\ \}_{=1}$ is the vector of traders' effective types.

The rest of this Appendix contains 5.1 followed by an auxiliary Lemma 4, which will be used in the proof of Theorem 2.

Lemma 4. If a symmetric linear equilibrium exists then coefficients, and that are characterized by the following

$$- = \quad 1 - \frac{(\ \ /^2(\ ) - 1)^2}{^2 + (\ \ /^2\ \hat{}^2)} \quad \hat{} \qquad (12)$$

$$= \quad \frac{-2}{-1} - 2\frac{1 + \frac{-2}{2}}{1 + (\ \ - 1)^2} \frac{- \quad ^2\ \hat{}^2}{+ \ - \quad ^2\ \hat{}^2} \qquad (13)$$

$$= \frac{-1\ 1}{-2} - + \quad \frac{2\hat{}}{^2\ ^2 + \ ^2\hat{}^2}^{-1}. \qquad (14)$$

where $\hat{}^2 = {}^2 \quad 1 + (\ \ - 2)^2(\ \ - 1) \quad .$

Proof. I conjecture an equilibrium of the form

$$(\ ) = - + \ - \ .$$

It follows from the rules of conditional Gaussian

$$[\ |\ _1, \ldots] = \ \sum_{=1} \ ,$$

for some which we shall now determine.

Because $= [\ |\ ]$, it must be that

$$+ \ (\ - 1) =$$

which implies that

$$= \frac{1}{1 + (\ - 1)}.$$

By market clearing, if trader submits the demand schedule ( ) while all other trad
follow the conjectured strategy then the price mu

$$= \frac{- \sum_{\neq} + \sum_{\neq} + (\ )}{(\ - 1)}$$

$$\Leftrightarrow$$

$$= \frac{- \sum_{\neq} + \frac{1}{}\hat{} \ \frac{1}{} + (\ )}{(\ - 1)}. \qquad (15)$$

In order ( ) to be optimal, the first-order condition

$$\sup_{\Delta} [\ |\ (,\ )](\ (\ ) + \Delta)(- + (\ ) + \Delta) - \frac{1}{2}(\ + \Lambda)\Delta^2 + \Lambda((\ ) + \Delta) \qquad (16)$$

must be $\Delta = 0$ for any given , where $\Lambda = 1/[(\ - 1)\ ]$ denotes price impa
This can be shown formally by a straightforward ap

The first-order condition is

$$-\frac{1}{}(\ + (\ )) [+ |\ (,\ ),,] = \Lambda(\ ) + \ . \qquad (17)$$

In order to compute the conditional expectatio

$\sum_{\neq}$ condition. We have

$$\text{var}_{\neq} = ( - 1)^2 [1 + ( - 2)]$$

Then, by the rules of conditional Gaussian random

$$\text{var} \;|_{\neq} = ( - 1)^2 [1 + ( - 2)] - \frac{( - 1)^2 \, {}^2}{2}$$

$$= {}^2 ( - 1) \quad 1 + ( - (2) - 1 -) \quad .$$

Therefore,

$$| \quad , \; {}^2 \hat{}^2 ( - 1)$$

where

$$\hat{}^2 \equiv {}^2 \; 1 + ( - 2)^2 ( - - 1.)$$

Again, using the rules for conditional Gaussians

$$|, - \quad + \frac{1}{} = + \frac{{}^2 \hat{}}{{}^2 \; {}^2 + {}^2 \hat{}^2} - \quad + \frac{1}{} - \frac{1}{}$$

$$= + \frac{{}^2 \hat{}}{{}^2 \; {}^2 + {}^2 \hat{}^2} \frac{- \Lambda ( \quad )}{\Lambda} - \quad ( - 1)$$

Substituting into the first-order conditions (17) and matching coefficients yields

$$\Lambda + \frac{1}{} + \frac{{}^2 \hat{}}{{}^2 \; {}^2 + {}^2 \hat{}^2} = \frac{1}{} \tag{18}$$

$$\Lambda + \frac{1}{} + \frac{{}^2 \hat{}}{{}^2 \; {}^2 + {}^2 \hat{}^2} = 1 - \frac{{}^2 ( - 1)^2 \quad \hat{}}{{}^2 \; {}^2 + {}^2 \hat{}^2} \tag{19}$$

$$- \quad \Lambda + \frac{1}{} + \frac{{}^2 \hat{}}{{}^2 \; {}^2 + {}^2 \hat{}^2} = - 1 + \frac{{}^2 \hat{}}{{}^2 \; {}^2 + {}^2 \hat{}^2} ( - 1) \tag{20}$$

where recall that $\Lambda = 1 / [ ( - 1) ]$. Equations (18), (19), and (20) are necessary conditions for a symmetric linear equilibrium.

$$\frac{-2}{-1} = \frac{1}{-} + \frac{2^{\wedge}}{2\ 2 + 2^{\wedge}2}$$

or

$$\Lambda = \frac{1}{-2} \frac{1}{-} + \frac{2^{\wedge}}{2\ 2 + 2^{\wedge}2} . \qquad (21)$$

Substituting (19) and (20) into equations (13) and (14) yields
Dividing (19) by (18) yields (12). By inspection the (12) is a unique solution for (12), for a coefficient character (13) and (14) uniquely.
Thus there can be at most one symmetric linear equ

□

Lemma A5. An increase in the correlation increases in the dem
, , and.

Proof. I first show that increasing . Using (19), we have

$$\frac{-1}{-2}1 + \frac{2(\quad -1)1 + \frac{-2}{2}}{-2\ 1 + (\quad -1^2) +} \frac{2^{\wedge}2}{2^{\wedge}2} = 1 \qquad (22)$$

Consider the second term on the LHS:

$$\frac{2(\quad -1)}{-2}1 + \frac{-2}{2}\ \frac{2(1 - {}^2)}{2 + 2\ 2\,[1 + (\quad -2) \quad - (2]} = 1 - 1) \qquad (23)$$

Suppose for contradiction that . Suppose further the term $2^{\wedge}2$ is increasing. Then (12) implies that is decreasing. But because

$$1 + \frac{-2}{2} (1 - )$$

is decreasing, it follows from (23) that is increasing which is a contradiction that $2^{\wedge}2$ is decreasing. Because

$$\frac{1 + \frac{-2}{2}}{1 + (\quad -1)}$$

is likewise in... For the same reason... Formerase, I... duced equation (18) as below (

$$\Lambda + \frac{1}{} + \frac{2^{\wedge}}{2 \quad 2 + 2^{\wedge}2} = \frac{1}{} .$$

Suppose for contradiction that ... is increasing. Then it must be

$$\frac{2^{\wedge}}{2 + 2^{\wedge}2}$$

is increasing. If it was increasing we would have (12) ... would contradict ... show must be decreasing. But we know ... is increasing ... if ... decreasing ... must be increasing. This contradiction ... that

Finally I now show that ... in ... $\Lambda = 1/[(1 - \ldots)]$ ... decreasing. Recall that

$$\Lambda + \frac{1}{} + \frac{2^{\wedge}}{2 \quad 2 + 2^{\wedge}2} = \frac{1}{} .$$

Suppose for contradiction that $\Lambda$ is increasing. Then by (12) ... it must be that the term in parenthesis ... But ... must be decreasing ... contradiction. □

Proof of Theorem 2. To prove part 1 of the theorem I recogni... when $= 0$ equations (12), (13), (14) can all be solved in closed form, and, in terms of primitives are given by

$$= \frac{1}{} \qquad = \frac{-2}{-1} \quad 1 - \frac{2(\quad -1)^{2}}{-2 \quad 2 \; \frac{1}{}^{2} + 2} .$$

Second-order conditions (16) are satisfied if ...

$$\frac{1}{2} + \Lambda \geq 0 .$$

= 0. If < 0, then in order for (19) to be satisfied, it must be that

$$\Lambda + \frac{1}{\ } \le 0.$$

This implies that

$$\frac{1}{2} + \Lambda < 0$$

which implies second-order conditions are not sat... > 0 then $\Lambda > 0$ and the second-order conditions are always s...

Thus $\ge 0$ is necessary and sufficient for equilibri...

What about > 0? Suppose < 0. Consider (19) again which implies

$$\Lambda + \frac{1}{\ } \quad = 1 - \frac{2\hat{\ }2}{2\ 2 + \ 2\hat{\ }2} > 0.$$

Thus, once again we see that if > 0 is not satisfied then we do no... order conditions is not a concern. Since ... is increasing in ... that the r... equilibrium existence expands.

To prove part 2, I proceed as follows. To start, ... allocative efficiency in terms of only the parame...

By straightforward computations, one can show t... is simply

$$( \quad - 1)(1 + ^2 - 2 \quad ^2) + \ ^2 (1 - \ ^2).$$

I shall now prove that the above is decreasing in

To start, let me rewrite the above as

$$( \quad - 1)(1 - 2 \quad ^2) + ^2 + \ ^2\ ^2 (1 - \ ^2). \qquad (24)$$

I shall minimize the above with respect to ... taking ... is given. The first-order ...

$$(2 \quad - )1^2 + 2\ ^2 (1 - \ ^2) = 0$$

$$= \frac{^2}{^2 (1 - \ ^2) + \ ^2}$$

$$1 + \frac{2\ 2(1 - \quad )}{2} = 1$$

I shall now prove that < in any equilibrium of the bl

In equilibrium we have

$$\frac{-1}{-2} + \frac{2}{2\hat{}2 + 2}\frac{2 \quad -2}{-2} = 1.$$

Thus to prove < it suffices to show

$$\frac{2\ 2(1 - \quad )}{2} \leq \frac{2}{2\hat{}2 + 2}\frac{2 \quad -2}{-2}$$

$$\Leftrightarrow$$

$$\frac{2\ 2(1 - \quad )}{2} \leq \frac{2\ 2(1 - 2) - 2}{2\hat{}2 + 2}\frac{}{-2}$$

$$\Leftrightarrow$$

$$- \leq \frac{2}{2\hat{}2 + 2}\frac{2 \quad -2}{-2}$$

$$\Leftrightarrow$$

$$1 - \frac{2( \quad -1)^2}{2 + 2\hat{}2} \leq \frac{\hat{} \quad 2}{2\hat{}2 + 2}\frac{2 \quad -2}{-2}$$

$$\Leftrightarrow$$

$$2 + 2\hat{}2(1 - ( \quad -1)) \leq 2\frac{2 \quad -2}{-2}$$

$$\Leftrightarrow$$

$$2\hat{}2(1 - ( \quad -1)) \leq 2\frac{}{-2} \tag{25}$$

To do this, observe that

$$2\hat{}2(1 - ( \quad -1)) \leq 2\ 2 \quad 1 + \quad ( \quad -2)(\ ) - 1(1 - ( \quad -1))$$

$$\Leftrightarrow$$

$$2\hat{}2(1 - ( \quad -1)) \leq 2\ 2 \quad 1 + \quad ( \quad -2)(\ ) - 1) - ( \quad -1) \quad (1 - \quad )$$

$$\Leftrightarrow$$

$$^{2\,\hat{}\,2}(1-(\ -1))\leq\ ^{2\ 2}\ 1+\ (\ -2()-1)-(\ -1)\ ^{2}+)(\ -1)$$

$$\Leftrightarrow$$

$$^{2\,\hat{}\,2}(1-(\ -1))\leq\ ^{2\ 2}(1-\ ).$$

To prove (25) it suffices to show

$$^{2\ 2}(1-\ )^{2}\leq\frac{}{-2}.$$

This holds because we know that

$$1-\frac{2(\ -1)}{-2}\frac{^{2}}{^{2}\ \frac{1}{-}\ ^{2}+\ ^{2}}\geq 0$$

$$\Leftrightarrow$$

$$1\geq\frac{2(\ -1)}{-2}\frac{^{2}}{^{2}\ \frac{1}{-}\ ^{2}+\ ^{2}}\geq$$

$$\Leftrightarrow$$

$$\frac{^{2}}{^{2\ 2}}\geq\frac{2(\ -1)}{-2}-1$$

$$\Leftrightarrow$$

$$\frac{^{2}}{^{2\ 2}}\geq\frac{}{-2}.$$

Thus we have shown . Suppose that $^{2}(1-\ )$ is decreasing in . Since is always to the left of the line and is the point that minimizes , and is always lower when is higher, it follows that all locatives e

I now show that $^{2}(1-\ )$ is indeed decreasing in . Consider again the equation :

$$=\ 1-\frac{^{2}(\ -1)^{2}}{^{2}+\ ^{2\,\hat{}\,2}}\ \hat{}$$

$$\Leftrightarrow$$

$$=\ 1-\frac{^{2}(\ -1)\ (1^{2}-\ )}{^{2}+\ ^{2}(1-\ )(1+\ (^{2}\ -1))}$$

$$\Leftrightarrow$$

$$= 1 - \frac{{}^2( - 1)(1^2 - )}{\frac{}{^2} + {}^2(1 - \frac{1}{}) + ( - 1^2)}$$

If $^2(1 - )$ is increasing, it follows from the above equation , a contradiction.

Parts 3, 4, and 5 follow straightforwardly.

Proof: Fixing a direct mechanism $\{ , \}$, the expected utility gain from reporting truthfull is

$$\hat{}() - ( , 0) = -\frac{1}{} () - \frac{1}{2} (^2) - ().$$

if all other traders do so as well.

By the envelope integral formual, it is easy to s

$$\hat{}() - ( , 0) = -[ ( _-, ) d] + \hat{}() - ( , 0)$$

under any incentive compatible mechanism.

This implies that

$$\hat{}( , _-) = - + \sum_{\neq} ( , _-) + ( _-, ) d - \frac{1}{2} ( , _-)^2$$

$$- \hat{}() - ( , 0) .$$

The designerôs objective is the following

$$\sup_{\{ \} \{ \hat{}() \}} \sum_{=1} - + \sum_{\neq} () + ( _-, ) d - \frac{1}{2} (^2)$$

$$- \hat{}() - ( , 0) + (1 - ) \sum_{=1} \frac{1}{2} ( () )^2 . \quad (26)$$

$$( \quad ) = 0$$
$$= 1$$

and for each $= 1, 2$ it holds that

$$\hat{\ } ( \ ) - ( \ , 0 ) = - [ \ ( \ _- , ) d ] + ( \ ) - ( \ , 0 ) \geq 0$$

for all and

$$[ \ ( \ _- , ) \ ]$$

is nondecreasing in .

Simplifying and omitting constant terms, the ob

$$\sup_{\{ \ \} \hat{\{} ( \ ) \}} \ \frac{1}{-} \hat{\ } \sum_{= 1} )( ( \ ) - ( \ _- , ) d - \frac{1}{2} ( \ ^2 ) + ( \ )$$

$$+ ( 1 - \ ) \sum_{= 1} \frac{1}{2} ( \ ^2 ) - \frac{1}{-} \hat{\ } )( ( \ )$$

subject to the same constraints.

Let $\Omega$ be the Lagrange multiplier on the participat

$$\sup_{\{ \ \} \hat{\{} ( \ ) \}} \ \hat{\ } \sum_{= 1} )( ( \ ) - ( \ _- , ) d - \frac{1}{2} ( \ ^2 ) + ( \ )$$

$$+ \sum_{= 1} \int_{-} [ \ ( \ _- , ) ] d - ( \ ) d\Omega ( \ )$$

where I will ignore the monotonicity constraint o

Integrating by parts, we can rewrite the Lagrang

$$\hat{\ } \sum_{= 1} )( ( \ ) + ( \ ) \frac{( ) - \Omega ( \ )}{( )} - \frac{1}{2} ( \ ^2 ) - 1 - \lim_{\to \infty} \Omega ( \ \hat{)} ( \ )$$

$$( \quad ) = 0 .$$
$$=1$$

We define the virtual value as

$$( ) = - \hat{\ } ) + ( \frac{( ) - Q( )}{( )} . \qquad (27)$$

Ignoring the monotonicity constraint, by Jense

$$( \quad ) = ( - ) + \frac{1}{\ } \quad ( ) \qquad (28)$$
$$=1$$

as desired. The last step which is to verify that t
follows the same steps as in the proof of Theorem 1.

Proof. Given $> 0$, let $\hat{\ }(t)$ denote

$$\hat{\ } ( ) = ( + \max\{ \min\{ , \} - , 0 \} .$$

This is the virtual value of the type truncated over [ , ] of the inte
binding participation constraints $\hat{\ }( , \_ )$ at the fu
denote

$$\hat{\ } ( , \_ ) = \hat{\ } ( ) + \frac{1}{\ } \hat{\ } ( ) . \qquad (29)$$
$$=1$$

Then as $\to 0$, $\hat{\ } \to$ pointwise. Also, note that $\sum_{=1} \hat{\ } = 0$ by construc
auctioneer does not retain any net position in the
Suppose that each trader submits the demand schedule given by

$$\hat{\ } ( \quad ) = \hat{\ } ( - ) - \qquad (30)$$

to the double auction. Then the resulting market c

$$= \frac{1}{\ } \hat{\ } ( ) .$$
$$=1$$

can be computed as follo

market clearing,

$$\hat{} + \quad - \hat{} \; ( \; ) \; - ( \quad - 1) \quad = 0 .$$
$$\neq$$

This implies that

$$= \hat{} + \quad - \hat{} \; ( \; ) \; \frac{1}{( \quad - 1)} .$$
$$\neq$$

The price impa $\Lambda$ c= t 1 i / s [ t ( h e − r 1 e ) f o ] r e

Next, I construct the transaction fee such that the demand schedule (3). I consider a side payment rule of the f

$$( \quad ( , \quad ) \; ) \; \hat{} = ( \; {}^2 \; ) \; \frac{}{2} + \hat{} \; ( - \; ( \quad ) \; - ) + \quad | \quad - \Lambda \; ( \quad ) \; \hat{} +$$
$$\neq$$

where the $\hat{}$ constant and function $\hat{}$ are to be derived.

To derive them, we consider the maximization problem. It must be optimal for $\hat{}$ ( ) to pursue $\hat{}$ ( ) to $\Delta$ purchasing be suboptimal $\Delta \neq 0$. Thus $\Delta = 0$ must solve

$$\max_{\Delta} - \frac{1}{2} ( \quad + \hat{} + \Delta^2 ) - \quad + \Delta \frac{1}{( \quad - 1)} \quad ( \hat{} + \Delta )$$
$$+ \hat{} \quad - \hat{} - \Delta - \quad + \Delta \frac{1}{( \quad - 1)} \quad - \frac{\hat{}}{2} ( \hat{} + \Delta^2 )$$

where to ease notation I have. Taking the first order condition,

$$- \frac{1}{} ( \quad + \hat{} + \Delta ) - \hat{} + \Delta ( ) \hat{} - \quad - \Delta \frac{1}{( \quad - 1)} - \frac{}{\quad - 1} ( \hat{} + \Delta )$$
$$+ \hat{} \, ' \quad - \hat{} - \Delta - \quad + \Delta \frac{1}{( \quad - 1)} \quad 1 + \frac{1}{\quad - 1} = 0 .$$

This must hold at $\Delta = 0$ when $\hat{}$ is as in (3). That is,

$$- \frac{1}{} ( \quad - \hat{} \; ( \; ) \; - \quad ) - \hat{} ( - \hat{} \; ( \; ) \; - \quad ) =$$

$$+\frac{}{-1}(-\hat{}\,()-\,)-\hat{}\,'(\hat{}\,())1+\frac{1}{-1}.$$

Gathering the terms involving only

$$\hat{}\qquad\frac{=}{-1}.$$

Thus we may ensure that these terms are consistent

$$\hat{}\qquad=\frac{}{-1}.$$

We next gather only terms involving

$$\hat{}\,()\frac{1}{}-\frac{1}{}=-\hat{}\,'(\hat{}\,())1+\frac{1}{-1}.$$

Writing in place of $\hat{}\,()$ and integrating both sides we have

$$\hat{}\,(\quad)\frac{2}{2}=-\int\hat{}^{-1}(\;\mathrm{d})\frac{-1}{}+$$

for some constant. As shown, $\hat{}^{-1}$ is well-defined as we have perturbed to ensure $\hat{}$ is strictly monotone.

To ensure global optimality of $\hat{}$ it suffices to show that is globally concave. Notice that $\hat{}$ is increasing in . The ignoring this term, by a simple computation taking ignoring this term, we can show that the objective

Thus we have achieved an indirect implementation (29). To derive an exact implementation, we take the limit $\to 0$. One can show that the $\hat{}$ converges pointwise and by obtaining the side payment rule in the statement of the theorem. Incentive constraints were not true, if sufficiently small, one can show that it is violated for the perturbed allocation rule under contradiction. □

Proof of Proposition 4. To prove part 1, using the rules of con

$$\frac{\frac{1}{2}\ {}^2}{\frac{1}{2}\ {}^2 + \ {}^2} + \frac{(\ )}{(\ )} =$$

which is equivalent to

$$\frac{\frac{1}{2}\ {}^2}{\frac{1}{2}\ {}^2 + \ {}^2} + \frac{\Phi\left( ( - ) / \sqrt{\frac{1}{2}\ {}^2 + \ {}^2} \right)}{\left( ( - ) / \sqrt{\frac{1}{2}\ {}^2 + \ {}^2} \right)} =$$

where Φ and are the CDF and PDF of a standard Gaussian ... define $= ( - ) / \sqrt{\frac{1}{2}\ {}^2 + \ {}^2}$. Then we have

$$\frac{\frac{1}{2}\ {}^2}{\frac{1}{2}\ {}^2 + \ {}^2} + \frac{}{\sqrt{\frac{1}{2}\ {}^2 + \ {}^2}} \sqrt{\frac{1}{2}\ {}^2 + \ {}^2} + \frac{\Phi(\ )}{(\ )} =$$

which is equivalent to

$$\frac{\frac{1}{2}\ {}^2}{\sqrt{\frac{1}{2}\ {}^2 + \ {}^2}} + \frac{\Phi(\ )}{(\ )} = 0 .$$

Clearly, because $< 0$, we know $^2$ must decrease without ... Because we know that $< 0$, it follows that must also decrease wi... bound. Because and are symmetric, it follows that $^2$ increases as increases and eventually diverges.

I now prove Part 3. The expected utility gain of a...

$$\hat{}(\ ) - (\ , 0) = -\hat{}(\ \mathrm{d}) + \hat{}(\ ) - (\ , 0). \qquad (31)$$

Note that $\hat{}(\ ) - (\ , 0)$ is always set to $\geq 0$. We have that

$$\frac{\hat{}(\ )}{-1} = -\frac{\frac{1}{2}\ {}^2}{\frac{1}{2}\ {}^2 + \ {}^2} + \frac{\Phi\left( ( - ) / \sqrt{\frac{1}{2}\ {}^2 + \ {}^2} \right)}{\left( ( - ) / \sqrt{\frac{1}{2}\ {}^2 + \ {}^2} \right)} +$$

< . Clearly < when gets less $^2$ increases. When the
inside the hazard rate increases and thus becomes
becomes more negative. On the other hand, when $^2$ increases the
term in brackets decreases and becomes less positive. We can deal >
Therefore the expected utility gain of a trader de

Now I must argue that the expected trade volume
( ) is a mean-preserving contraction of what it onc

Recall that we can have the transformation

$$\frac{\frac{1}{2}\ \ {}^2}{\frac{1}{2}\ \ {}^2 + \ \ {}^2} + \frac{\Phi(\ )}{(\ )} = 0.$$

Now recall the definition of a mean-preserving
see that when $^2$ is higher this is a mean-preserving contr

I now prove Part 2. Consider a mechanism that uses the same alloc
fer rules. Then each trader submits the same mess
based on mean-preserving contraction that alloca
the expected utility gain of all traders summed to
ficiency improves, this can only happen if total r
that (31) is convex. Thus, reducing the variance reduces the expecte
gain.

□

## C. Omitted Material for Section 6

Proof of Proposition 6. Construct derivative securities that
underlying assets such that the payoffs of these d
of generality, suppose that the derivatives are t
traders

$$(\ + \ ,\ ) = \ -\frac{1}{2}\ {}^2(\ + \ )^2 -$$

where $^2$ = var[ r ].

Consider running a separate double auction with th
tion for each asset. Because the utility function is

□

Formal Statement of the Design Objective is to s... Given [0, 1], the objective is to s...

$$\max_{\{\ \},\ \}} \underbrace{\sum_{=1} ( \ + ( )1 - \ )}_{\text{Revenue}} \underbrace{\sum_{=1} -\frac{1}{2}\ ^2 + ( \ ^2)}_{\text{Allocative Efficiency}} \qquad (32)$$

such that,

(P) $\quad + ( \ ( ), \ ) \geq ( \ , \ )0, \quad ,$

(IC) $\quad \arg\max + ( \ , _- ), ( \ , _- ) \quad , \quad ,$

(E) $\quad \sum_{=1} ( \ ) 0, =$

where $=\{\}_{=1}$ is the vector of traders' endowment vecto...

Proof of Proposition 7. I prove the Proposition 7 for the special case $= 0$ to ease the exposition. The general case is analogous.

Trader selects messages $= \{ \}$ to solve

$$\sup -\frac{1}{2} ( \ , _- )^2 - \frac{1}{-} ( \ , _- )_1 + \ _1( \ , _- )$$

$$- ( \ , _- ).$$

I conjecture that only local incentive constra... pose that the designer knows $\{\tilde{e}\}$ and will verify later... reporting is indeed incentive compatible under t...

Let $\tilde{t} = \{ \tilde{} \}$. By the envelope integral formula,

$$\hat{}( , \tilde{}) - ( \tilde{}, 0) = -\int_0^1 \tilde{}\ \hat{}\ ( \tilde{})d + \hat{}(0 \tilde{}) - ( \tilde{}0, 0) \qquad (33)$$

$\hat{\ }(\ ,\tilde{\ })$ denotes the expected utili[ty]

reporting for a $\tilde{\ }$ and where $(\ ,0)$ is the expected utility of a trader of that ty[pe in]

autarky.

This implies the expected transfer is

$$\hat{\ }(\ ,\tilde{\ }) = -\frac{1}{\ }\ \hat{\ }(\ ,\tilde{\ }) + \frac{1}{2}(\ ,\tilde{\ })^2 + \frac{1}{\ }\int_0 \tilde{\ }\ \hat{\ }(\ \tilde{\ })\,d$$

$$-\ \hat{\ }(0,\tilde{\ }) - (\tilde{\ },0,0). \quad (34)$$

Using (33) we can write down the planner's objective

$$-\frac{1}{\ }\ \hat{\ }(\ ,\tilde{\ }) + \frac{1}{2}(\ ,\tilde{\ })^2 + \frac{1}{\ }\int_0 \tilde{\ }\ \hat{\ }(\ \tilde{\ })\,d$$

$$-\ \hat{\ }(0,\tilde{\ }) - (\tilde{\ },0,0)$$

such that (33) is nonnegative and $\sum_{=1} = 0$. such that

Let $\Omega(\tilde{\ })$ denote the Lagrange multiplier on the participation constraint $\hat{\ }(\tilde{\ }),\ -\ (\tilde{\ },0) \geq 0$. Forming the Lagrangian, integrating yields the objective

$$\sup \frac{1}{2}(\ ,\tilde{\ }^2) - \frac{\Omega(|\tilde{\ }) - (\tilde{\ })}{(|\tilde{\ })}\frac{1}{\ }\tilde{\ }(\ ,\tilde{\ })$$

$$-\ -\Omega(\tilde{\ })\ \hat{\ }(0,\tilde{\ }) - (\tilde{\ },0,0) \quad (35)$$

such that $\sum_{=1} = 0$ where $(\tilde{\ })$ denotes the PDF and $(\ )$ denotes the CDF of $\tilde{\ }$, both conditional on $\tilde{\ }$.

The solution to (35) yields

$$(\ ,\tilde{\ }) = -\ -\frac{\Omega(|\tilde{\ }) - (\tilde{\ })}{(\tilde{\ })}\tilde{\ }$$

$$+\frac{1}{\ }\sum_{=1}\ -\frac{\Omega(|\tilde{\ }) - (\tilde{\ })}{(|\tilde{\ })}\tilde{\ }.$$

is the Rayleigh distribution:

$$( \tilde{\ } ) | = 1 - \frac{-\ ^2}{2\ ^2}$$

$$( \tilde{\ } ) | = \frac{}{\ ^2} \frac{-\ ^2}{2\ ^2} .$$

Because and do not depend on the ~, the logical conjecture that the ... on ~ and the region of binding part [0, ] for some constant ... 0. A trader with type in this interval will expect to trade ... observations yield the allocation rule in the sta...

There are two remaining steps. The first step is ... taken is valid. This is straightforward and ana... 1. The last step is to verify that only the incenti... from the proof of Proposition 8, it is shown that a double auction ... transaction fee can indeed implement the allocat... 7.

□

Proof of Proposition 8. Note that, given preferences (8), ... equivalent to a change in notation of ... when $^2 = 1$.

If each trader submits the demand schedule

$$( \ ) = - \ )( \tilde{\ } - \qquad (36)$$

for as ..., then the unique market clearing price is

$$= \frac{1}{\ } \sum_{=1} ( ) \tilde{\ }$$

and so the resulting allocation is of the desired ... 7.

I now engineer the transaction fee (36) is an equilibrium for ... conjecture that the implementing transaction fee ...

$$( \{ ( \ ) , \} \ ) = \sum_{=1} + \frac{1}{\ }\ ^2 + \frac{1}{2}\ ^2 +$$

and constants to be determined.

If it is optimal for trader to sell units when the vector of prices is , i.e. $= \{ \}$ then the solution to

$$\sup_{\{\Delta\}} -\frac{1}{2}(\tilde{\ } + \quad + \Delta)^2 - ( + \Lambda\Delta)( + \Delta) - \frac{1}{2}( + \Delta)^2$$

$$- \sum_{=1} ( + \quad + \Delta(1 + \Lambda))^2 \qquad (37)$$

is to set $\Delta = 0$ for each . Above, $\Lambda = \frac{1}{(\ - 1)}$ is trader's price impact when the traders submit a demand schedule (36). The first-order conditions for optimality are

$$-\frac{1}{\ }(\tilde{\ } + \ ) = \ + \Lambda \ + $$

$$+ 2' \sum_{=1} ( + )^2 ( + )(1 + \Lambda)$$

for each . Using that $\Lambda = \frac{1}{(\ - 1)}$, we equivalently have

$$-\frac{1}{\ }(\tilde{\ } + \ ) = \ + \frac{1}{(\ - 1)} + $$

$$+ 2' \sum_{=1} ( + )^2 ( + ) \left(1 + \frac{1}{\ - 1}\right)$$

for each .

Substituting (36) yields

$$-\frac{1}{\ } \tilde{\ } = - \ + \frac{1}{(\ - 1)} + \frac{1}{\ } (\ () \tilde{\ } + \ )$$

$$- 2' ()^2 () \tilde{\ } \left(1 + \frac{1}{\ - 1}\right).$$

, ,{ $\tilde{}$ } we require that

$$2\,' \quad ()^2 \quad () \; 1 + \frac{1}{-1} = \frac{1}{} - \quad + \frac{1}{(\quad -1)} + \frac{1}{} \qquad ()$$

and

$$+ \frac{1}{(\quad -1)} + \frac{1}{} = 1 .$$

Thus

$$= -\frac{1}{(\quad -1)}$$

and we have a differential equation for

$$2\,' \quad ^2 \quad 1 + \frac{1}{-1} = \frac{1}{} \;{}^{-1}(\quad)\frac{1}{} \,, \qquad > 0 .$$

A technical issue is that ... over a range of values. ... easily dealt with as in ... by perturbation ... strictly monotonic ... thus invertible). Following the arguments above ... perturbed ... then take limits ... implementable ... steps.

Let me define $\tilde{}$ such that $\tilde{}$( ) = $^2$ ( . Then we have

$$1 + \frac{1}{-1} \;\tilde{}\,'(\quad)\frac{1}{} = {}^{-1}(\quad)\frac{1}{} \,, \qquad .$$

Integrating both sides yields

$$\tilde{}\,(\quad) = \frac{1}{1 + \frac{1}{-1}} \frac{1}{} \int_0 {}^{-1}(\quad d) - \frac{1}{2} \;{}^2 + \qquad .$$

Note that $^{-1}$(0) is not well-defined, the value of th... on $^{-1}$(0.)

It is easy to verify that ... is concave and thus the o... (37) is also convex. Therefore, the first-order ne... sufficient ... $\Delta = 0$ ... (37) ... follows ... strategy ( ... show rigorously that this ... to an alternative set of demand schedules is a stra...

$\square$

## D. Omitted Material for Section 7

**Formal Statement of Designer's Objective.** Given [0,1], the objective is to

$$\max_{\{\ \},\{\ \}} \underbrace{\sum_{=1} ( + ( )(1 - ) )}_{\text{Revenue}} \underbrace{- \sum_{=1} \frac{1}{2} \ + ( ^{2} )}_{\text{Allocative Efficiency}} \qquad (38)$$

such that,

$$\text{(P)} \qquad + ( ( ), ) \geq ( , ) 0, \qquad ,$$

$$\text{(IC)} \qquad \operatorname{argmax} + ( , \_ ), ( , \_ ) \qquad , \qquad ,$$

$$\text{(E)} \qquad \sum_{=1} ( ) = 0,$$

where $= \{\ \}_{=1}$ is the vector of traders' endowments.

**Proof of Proposition 9.** Part 1 follows immediately from Corollary 5.

Part 2a follows from Corollary 5 and uses the same argument as in Part 1 of Proposition 4.

To prove Part 2b and 2c, observe that increases when is negative and decreases when it is positive. Thus from the formula for

$$\hat{\ }( ) - \ , 0 = -\frac{1}{\ } \ \hat{\ }( d) + \hat{\ }( ) - \ , 0,$$

trader's expected utility gain must decrease in $^{2}$. On the other hand, the effect on $\hat{\ }$ for any $\neq$ and thus the expected utility gain of $\neq$ is unaffected.

To prove part 3a, I recognize that, as expected, the effect on of is proportional to

$$\frac{1}{\ + \sum_{\neq}} .$$

. Because $\hat{a}$ does not depend on ..., there is no effect on ... $\neq$ ..., which ... any trade ... proves Part 3b.

□

**Proof of Proposition 10.** The proof is analogous to that of Proposition 5.

□

## E. Omitted Material for Section 8

**Formal Statement of the Designer's Problem.** The designer's objective is to ... allocations $\{(\ ,\ )\}$ and transfers $\{\ \}\geq 0$ ... $_{=1}$, to maximize

$$\sum_{=1}^{\infty} {}^{-} \int_{0} -\frac{1}{2}\left(\ +\ (\ )^{2}_{,}\ \mathrm{d}\ \right) + (1 - \ ) \sum_{=1}^{\infty} {}^{-} \int_{0} (\ \mathrm{d}_{,}\ ) \qquad (39)$$

subject to

(P) $\{(\ ,(\ )_{,}\ );\ \geq\ \geq (0,0)\},\ ,$

(IC) $\arg\max \{(\ ,\ _{-}),(\ ,\ _{-});\ \geq 0\}_{,}\ ,$

(E) $\sum_{=1}(\ ,\ )_{,} = \sum_{=1}\ ,$

where, with abuse of notation, $(\ ,0)$ ... indicates the ... (P) ... not participating in trade in which case she retai... makes no transfers.

**Proof of Proposition 11.** Let $(\ ,\ )$ be an implementable allocation ... compatibility of truth telling implies that the t...

$$\hat{}(\ ,\ ) = \sum_{=1}^{\infty} \sum_{=0} - {}^{-} \frac{1}{2}\ +\ (\ {}^{2}_{,}\ )$$

$$- \frac{1}{}\ \hat{}(\ ) + \frac{1}{}\ \int \ (\ _{-},)\ \mathrm{d}\ - \hat{}(\ ) - \ _{-},\ 0$$

$$\bar{\ }(\ ,\ )^{-} \equiv \sum_{=0}^{\infty} (\ ,\ ).$$

In order for global incentive compatibility $\bar{v}(\ ,\ )$ must be non-in- creasing. Consider with respect to its average value. Construct a

$$(\ ,\ \frac{1}{-}) = \sum_{=0}^{\infty} (\ ,\ ).$$

Since $(\ ,\ )$ is implementable $(\ ,\ )$ so since its interim expec be decreasing. By Jensen's inequality this can or Jensen's inequality both allocative efficiency a

□

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