Preparing for the Physics GRE: Day 5
Making Difficult Problems Easier

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Dimensional Analysis

- Dimension analysis can be used to gain physical intuition about problems we haven't seen before

95. A beam of 10^{12} protons per second is incident on a target containing 10^{20} nuclei per square centimeter. At an angle of 10 degrees, there are 10^{2} protons per second elastically scattered into a detector that subtends a solid angle of 10^{-4} steradians. What is the differential elastic scattering cross section, in units of square centimeters per steradian?

(A) 10^{-24}
(B) 10^{-25}
(C) 10^{-26}
(D) 10^{-27}
(E) 10^{-28}

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(D) 10^{-27}
(E) 10^{-28}

- Just focus on information given in the problem
- Need units of answer to be cm^2/steradian
- $(10^{20}\ cm^2)^{-1}$ / 10^{-4} steradian = $10^{-16}\ cm^2$/steradian
- Other two numbers given need to combine to be unitless
- => (10^2 proton/sec)/(10^{12} proton/sec) = 10^{-10}
- Answer must be:

  $10^{-16}\ cm^2$/steradian *10^{-10} = $10^{-26} cm^2$/steradian

Particle Physics

96. Which of the following reasons explains why a photon cannot decay to an electron and a positron ($\gamma \rightarrow e^+ + e^-$) in free space?

(A) Linear momentum and energy are not both conserved.
(B) Linear momentum and angular momentum are not both conserved.
(C) Angular momentum and parity are not both conserved.
(D) Parity and strangeness are not both conserved.
(E) Charge and lepton number are not both conserved.

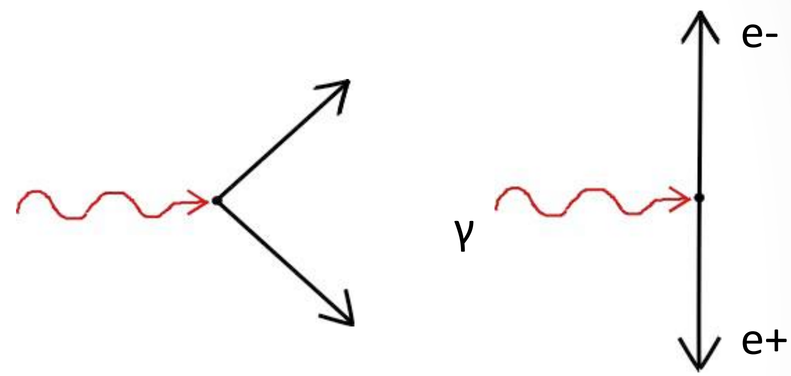
Particle Physics

- Visualize this decay in the center of mass frame of the two electrons
- Since the incident photon carries momentum, clearly momentum cannot be conserved
- Can also work out the mathematics to show that linear momentum and energy cannot be simultaneously conserved here

- (B) & (C): What angular momentum?
- (D): Strangeness? There are no quarks here! This is not a weak interaction.
- (E) Both charge and lepton number *are* conserved

- Similarly, can the photoelectric effect occurring for a free electron?

Boost from lab frame (left) to center of mass frame (right)



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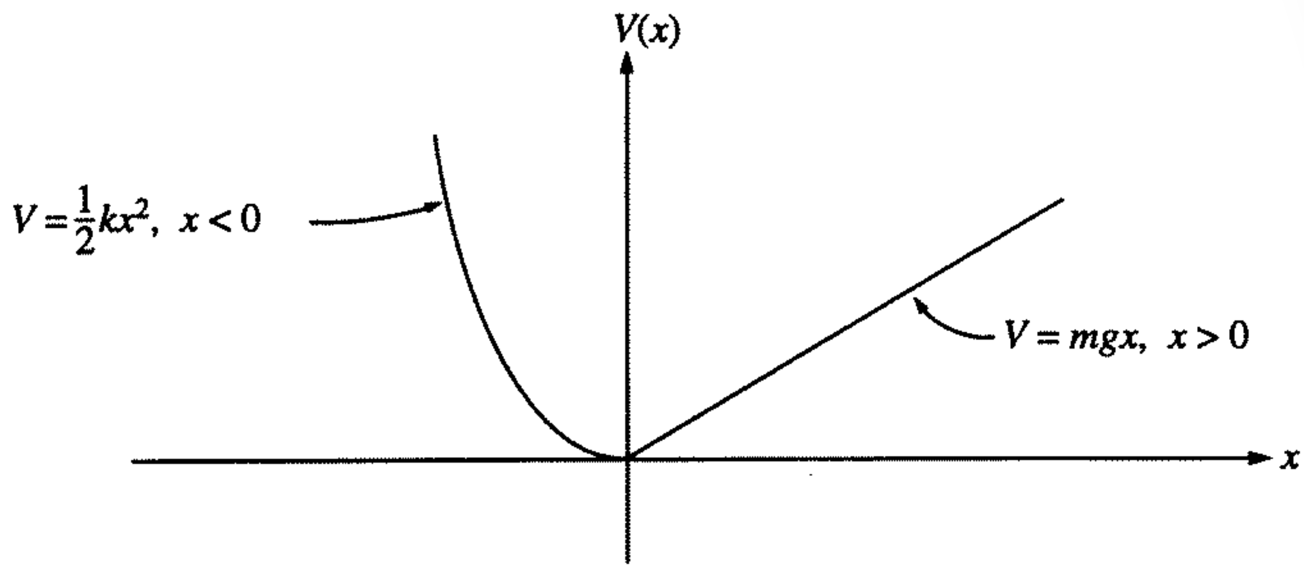
Classical Mechanics - 1



93. A particle of mass m moves in the potential shown above. The period of the motion when the particle has energy E is

(A) $\sqrt{k/m}$

(B) $2\pi\sqrt{m/k}$

(C) $2\sqrt{2E/mg^2}$

(D) $\pi\sqrt{m/k} + 2\sqrt{2E/mg^2}$

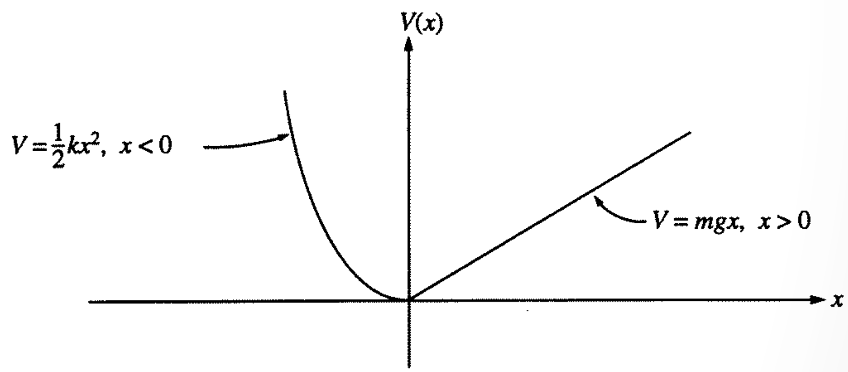
(E) $2\pi\sqrt{m/k} + 4\sqrt{2E/mg^2}$

Classical Mechanics - 1

- Strangely-shaped well is half harmonic, half linear
- Find the period of a harmonic well $\omega = \sqrt{k/m}, \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{m/k}$
- Find the period of a linear well $x_{max} = E/mg$

$$-x_{max} = -\frac{1}{2}g\left(\frac{T}{4}\right)^2 \Rightarrow T/4 = \sqrt{2E/mg^2}$$

- Each half of the well contributes T/2 to the total period
  - $T_{total} = T_{linear}/2 + T_{harmonic}/2$



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Classical Mechanics - 2

92. A particle of mass m moves in a one-dimensional potential $V(x) = -ax^2 + bx^4$, where a and b are positive constants. The angular frequency of small oscillations about the minima of the potential is equal to

(A) $\pi(a/2b)^{1/2}$
(B) $\pi(a/m)^{1/2}$
(C) $(a/mb)^{1/2}$
(D) $2(a/m)^{1/2}$
(E) $(a/2m)^{1/2}$

Classical Mechanics - 2

- Sketch a picture of the potential:
- Imagine a small amplitude oscillation about one of the two minima:
  - Find the minima

$$\frac{\partial V}{\partial x} = -2ax + 4bx^3 = 0 \;\Rightarrow\; x^* = \pm\sqrt{a/2b}$$

  - Add a small perturbation

$$x^* \longrightarrow x^* + \epsilon(t)$$

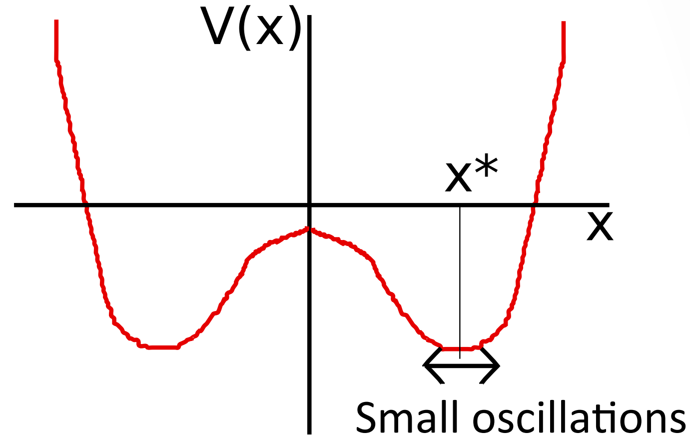
  - Obtain equation of motion

$$m\,(\ddot{x^*+\epsilon}) = 2a(x^*+\epsilon) - 4b(x^*+\epsilon)^3$$

$$m\,\ddot{\epsilon} = 2ax^* - 4b(x^*)^3 + 2a\epsilon - 4b\cdot 3(x^*)^2\epsilon$$

  - Extract frequency

$$m\,\ddot{\epsilon} = -4a\epsilon \;\Rightarrow\; \omega = 2\sqrt{a/m}$$



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Classical Mechanics - 3

22. The curvature of Mars is such that its surface drops a vertical distance of 2.0 meters for every 3600 meters tangent to the surface. In addition, the gravitational acceleration near its surface is 0.4 times that near the surface of Earth. What is the speed a golf ball would need to orbit Mars near the surface, ignoring the effects of air resistance?

(A) 0.9 km/s
(B) 1.8 km/s
(C) 3.6 km/s
(D) 4.5 km/s
(E) 5.4 km/s

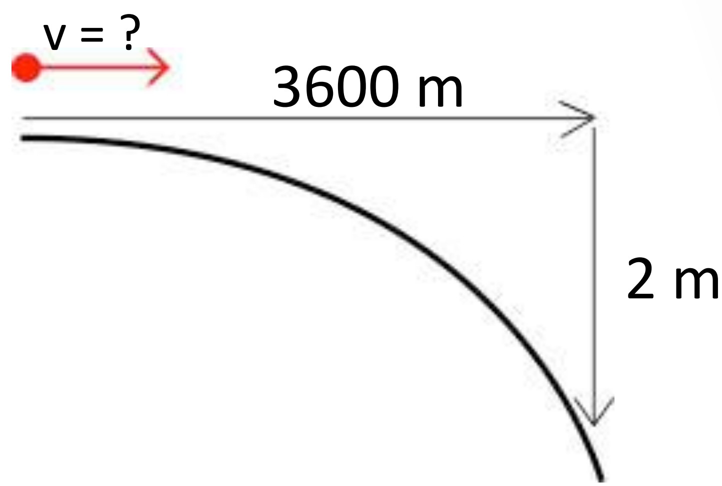
Classical Mechanics - 3

- Imagine a circular orbit using this (very not to scale) image:
- It takes a certain amount of time to fall 2 m under gravity=.4g
- In that same amount of time, the golf ball goes3600 m
- The rest is just kinematics:

$$\Delta y = \frac{1}{2}.4g\left(\frac{\Delta x}{v}\right)^2, \quad \Delta x = v\Delta t$$

$$\Delta y = \frac{1}{2}.4g\left(\frac{\Delta x}{v}\right)^2, \quad v^2 = \frac{1}{2}.4g\frac{\Delta x^2}{\Delta y}$$

$$v^2 = \frac{1}{2}4\frac{(3600)^2}{2}, \quad v = 3600 \;\; m/s$$



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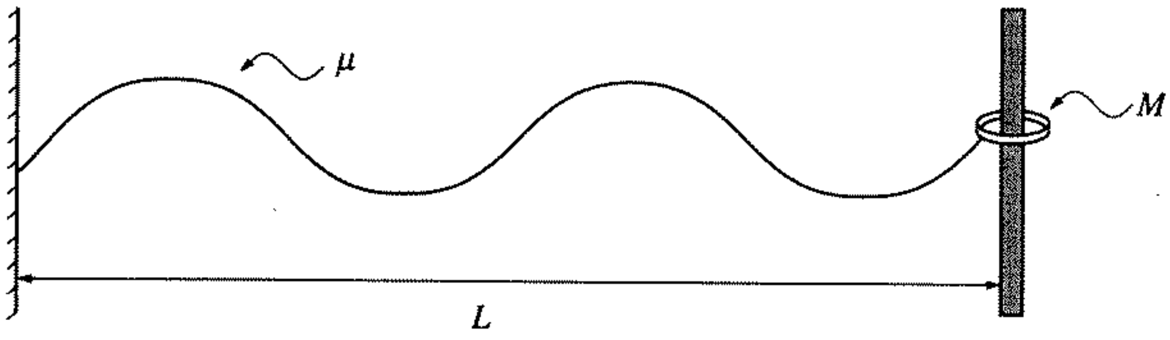
Waves and Optics - 1



85. Small-amplitude standing waves of wavelength λ occur on a string with tension T, mass per unit length μ, and length L. One end of the string is fixed and the other end is attached to a ring of mass M that slides on a frictionless rod, as shown in the figure above. When gravity is neglected, which of the following conditions correctly determines the wavelength? (You might want to consider the limiting cases $M \to 0$ and $M \to \infty$.)

(A) $\mu/M = \frac{2\pi}{\lambda} \cot \frac{2\pi L}{\lambda}$

(B) $\mu/M = \frac{2\pi}{\lambda} \tan \frac{2\pi L}{\lambda}$

(C) $\mu/M = \frac{2\pi}{\lambda} \sin \frac{2\pi L}{\lambda}$

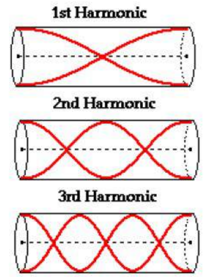
(D) $\lambda = 2L/n, \quad n = 1, 2, 3, \ldots$

(E) $\lambda = 2L/(n + \frac{1}{2}), \quad n = 1, 2, 3, \ldots$

Waves and Optics - 1

- Treat as standing waves
- Take limits! How does the condition at the boundary change when…
  - M -> ∞: closed boundary
  - M-> 0: open boundary
- So, what wavelengths are and are not allowed in each case?
- M -> ∞
  - Right hand side must go to zero at allowed frequencies
- M -> 0
  - Right hand side must go to ∞ at allowed frequencies

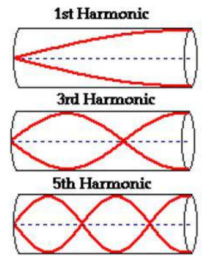
Open at Both Ends



Harmonic	Wavelength λ	Frequency f
1st	$2L$	f_1
2nd	L	$2f_1$
3rd	$2L/3$	$3f_1$

Odd and Even Harmonics

Closed at One End



1st	$4L$	f_1
3rd	$4L/3$	$3f_1$
5th	$4L/5$	$5f_1$

Odd Harmonics

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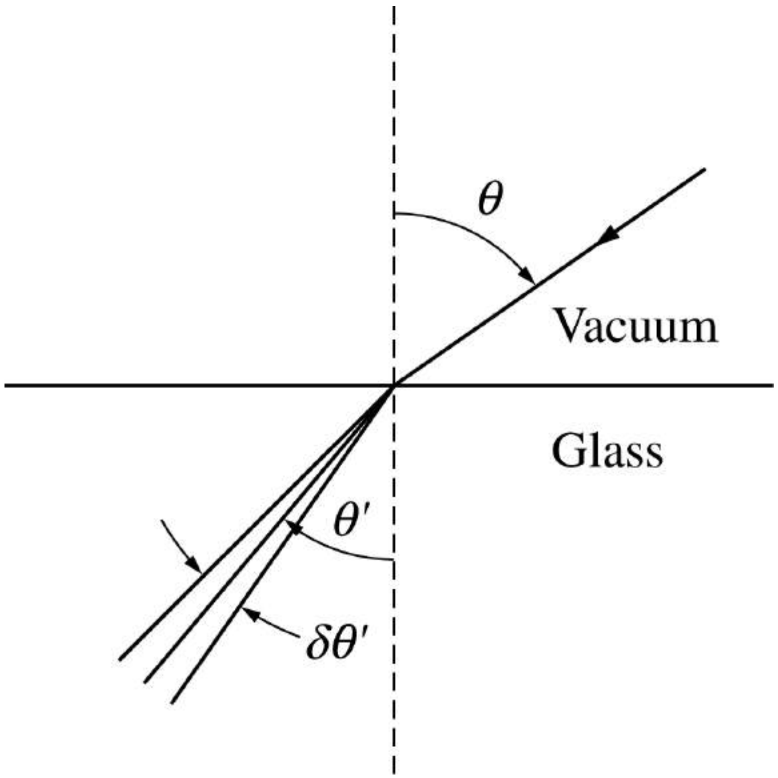
Waves and Optics - 2



97. A beam of light has a small wavelength spread $\delta\lambda$ about a central wavelength λ. The beam travels in vacuum until it enters a glass plate at an angle θ relative to the normal to the plate, as shown in the figure above. The index of refraction of the glass is given by $n(\lambda)$. The angular spread $\delta\theta'$ of the refracted beam is given by

(A) $\delta\theta' = \left| \frac{1}{n} \delta\lambda \right|$

(B) $\delta\theta' = \left| \frac{dn(\lambda)}{d\lambda} \delta\lambda \right|$

(C) $\delta\theta' = \left| \frac{1}{\lambda} \frac{d\lambda}{dn} \delta\lambda \right|$

(D) $\delta\theta' = \left| \frac{\sin\theta}{\sin\theta'} \frac{\delta\lambda}{\lambda} \right|$

(E) $\delta\theta' = \left| \frac{\tan\theta'}{n} \frac{dn(\lambda)}{d\lambda} \delta\lambda \right|$

Waves and Optics - 2

- Looking at all the answers, we can think of angular dispersion as being a partial differential
- Start with Snell's law

$$sin(\theta_i)=n\cdot sin(\theta_o)$$

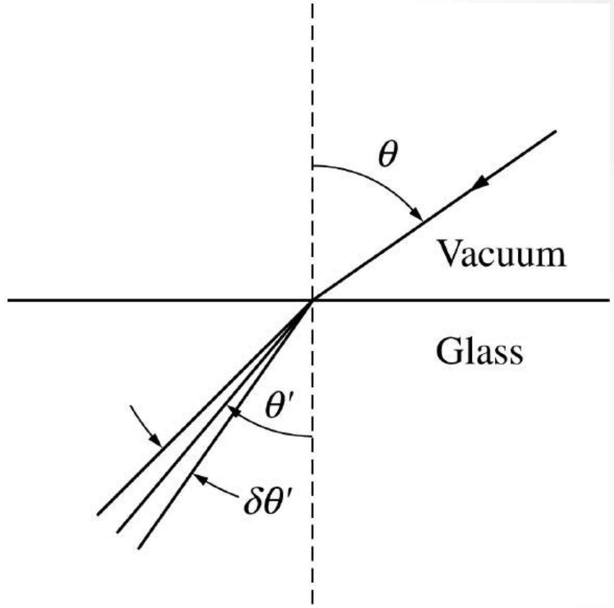
- Think: what variables will depend on the wavelength of light?

$$sin(\theta_i)=n(\lambda)\cdot sin(\theta_o(\lambda))$$

$$\frac{\partial}{\partial\lambda}sin(\theta_i)=\frac{\partial}{\partial\lambda}n(\lambda)\cdot sin(\theta_o(\lambda))$$

$$0=\frac{\partial n}{\partial\lambda}sin(\theta_o(\lambda))+n\cdot cos(\theta_o)\frac{\partial\theta_o}{\partial\lambda}$$

- Rearrange to find the answer



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Electromagnetism - 1

In case you were worried about waveguides:

34. A conducting cavity is driven as an electromagnetic resonator. If perfect conductivity is assumed, the transverse and normal field components must obey which of the following conditions at the inner cavity walls?

(A) $E_n = 0, B_n = 0$

(B) $E_n = 0, B_t = 0$

(C) $E_t = 0, B_t = 0$

(D) $E_t = 0, B_n = 0$

(E) None of the above

Electromagnetism - 1

- Recall Maxwell's equations :

$$1.\ \oiint \vec{E}\cdot\hat{n}\,dS = Q_f \qquad 2.\ \oiint \vec{B}\cdot\hat{n}\,dS = 0$$

$$3.\ \oint \vec{E}\cdot d\ell = -\frac{d}{dt}\iint \vec{B}\cdot\hat{n}\,dA \qquad 4.\ \oint \vec{B}\cdot d\ell = I_f + \frac{1}{c^2}\iint \frac{\partial}{\partial t}\vec{E}\cdot\hat{n}\,dA$$

- Draw a small Amperian loop containing the interface
  - Using Faraday's law (#3), as the loop becomes very small the flux vanishes, we find that the tangential electric field must be continuous at the boundary

$$\vec{E}_{out}\cdot\vec{l} - \vec{E}_{in}\cdot\vec{l} = 0 \quad\Rightarrow\quad E^{||}_{out} - E^{||}_{in} = 0$$

- Draw a small Gaussian pillbox containing the interface
  - Applying (#2), we find that the perpendicular magnetic field must be continuous across the boundary

$$\vec{B}_{out}\cdot\hat{n} - \vec{B}_{in}\cdot\hat{n} = 0 \quad\Rightarrow\quad B^{\perp}_{out} - B^{\perp}_{in} = 0$$

Electromagnetism - 1

- Returning to the problem, we can remember that electromagnetic waves are shielded by conductors, so we set E and B to 0 on the inside.
- We can now obtain a rule for the boundary condition at the cavity wall

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(D) $E_t = 0, B_n = 0$

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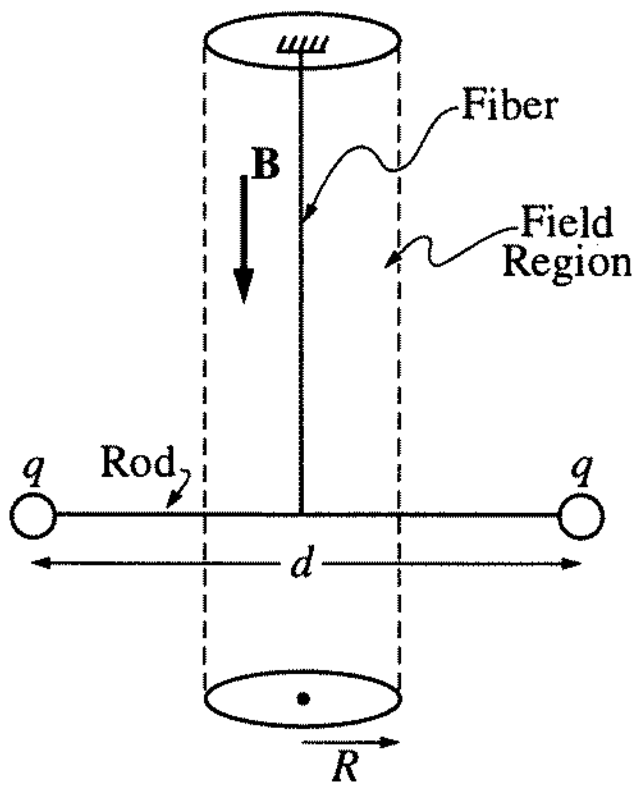
- In general, even in the presence of sources, we can use all four of Maxwell's equations to obtain four possible boundary conditions:

$$1.\quad \epsilon_{out} E^{\perp}_{out} - \epsilon_{in} E^{\perp}_{in} = \sigma_f \qquad 2.\quad B^{\perp}_{out} - B^{\perp}_{in} = 0$$

$$3.\quad E^{\parallel}_{out} - E^{\parallel}_{in} = 0 \qquad 4.\quad \frac{1}{\mu_{out}} B^{\parallel}_{out} - \frac{1}{\mu_{in}} B^{\parallel}_{in} = \vec{K}_f \cdot \hat{n}$$

Griffiths, *Introduction to Electrodynamics*, 7.3.6: "Boundary Conditions"

Electromagnetism - 2



87. Two small pith balls, each carrying a charge q, are attached to the ends of a light rod of length d, which is suspended from the ceiling by a thin torsion-free fiber, as shown in the figure above. There is a uniform magnetic field $\mathbf{B}$, pointing straight down, in the cylindrical region of radius R around the fiber. The system is initially at rest. If the magnetic field is turned off, which of the following describes what happens to the system?

(A) It rotates with angular momentum qBR^2.

(B) It rotates with angular momentum $\frac{1}{4}qBd^2$.

(C) It rotates with angular momentum $\frac{1}{2}qBRd$.

(D) It does not rotate because to do so would violate conservation of angular momentum.

(E) It does not move because magnetic forces do no work.

Electromagnetism - 2

- Electromagnetic fields do carry angular momentum: turning off the magnetic field will cause the apparatus to rotate
- Which Maxwell equations help us here?

$$1.\ \oiint \vec{E} \cdot \hat{n}\, dS = Q_f \qquad 2.\ \oiint \vec{B} \cdot \hat{n}\, dS = 0$$

$$3.\ \oint \vec{E} \cdot d\ell = -\frac{d}{dt} \iint \vec{B} \cdot \hat{n}\, dA \qquad 4.\ \oint \vec{B} \cdot d\ell = I_f + \frac{1}{c^2} \iint \frac{\partial}{\partial t} \vec{E} \cdot \hat{n}\, dA$$

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- Faraday's law: a changing magnetic flux through an area enclosed by a loop induces an electric field around that loop
- The magnetic fields are not doing work: it is the field induced by the changing magnetic field that does work on the charges

Electromagnetism - 2

- Loop is a circle with radius d/2, total flux is $B\pi R^2$
- Evaluate integrals on both sides: $E\pi d = -\frac{d}{dt}(B\pi R^2)$
- A change in angular momentum implies an applied torque, which comes from the induced electric field

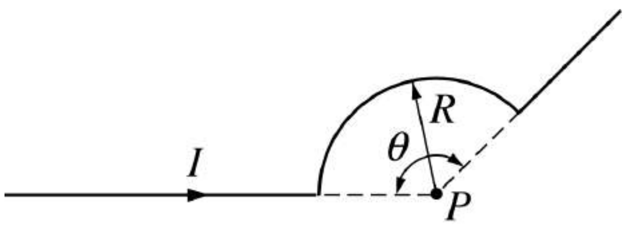
$$\Delta L = \int \tau dt = \int 2qE(d/2)dt = \int qEddt$$

- Using the expression for the electric field in terms of change of flux:

$$\Delta L = \int -\frac{d}{dt}(qBR^2)dt = qBR^2$$

- We can also solve by taking limits, without any integrals:
  - How should ΔL change as R->0?
  - How should ΔL change as d-> ∞?

Electromagnetism - 3



88. A segment of wire is bent into an arc of radius R and subtended angle θ, as shown in the figure above. Point P is at the center of the circular segment. The wire carries current I. What is the magnitude of the magnetic field at P?

(A) 0

(B) $\dfrac{\mu_0 I\theta}{(2\pi)^2 R}$

(C) $\dfrac{\mu_0 I\theta}{4\pi R}$

(D) $\dfrac{\mu_0 I\theta}{4\pi R^2}$

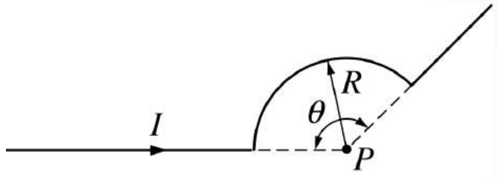
(E) $\dfrac{\mu_0 I}{2\theta R^2}$

Electromagnetism - 3

- The net magnetic field should be some fraction of the magnetic field at the center of a loop of current with radius R
- Take limits: the answer should scale with θ
- Dimensional analysis:

  $[\mu I/R] = Tm^2/(Am)A/m = T = [B]$
- Leaves a choice between (B) and (C)

- To solve exactly:
  - Recall expression for magnetic field at center of a loop $B = \dfrac{\mu_0 I}{2R}$
  - Use Biot-Savart Law (last resort!) $\vec{B} = \dfrac{\mu_0}{4\pi}\int \dfrac{I d\ell \times \hat{r}}{r^2} = \dfrac{\mu_0 I}{4\pi R}\int_0^\theta \hat{z}\, d\theta'$



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Electromagnetism - 4

52. A cube has a constant electric potential V on its surface. If there are no charges inside the cube, the potential at the center of the cube is

(A) zero (B) $V/8$ (C) $V/6$
(D) $V/2$ (E) V

Electromagnetism - 4

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- Which electrostatics equation applies here?

- What would it mean if the potential were *not* V?

Electromagnetism - 4

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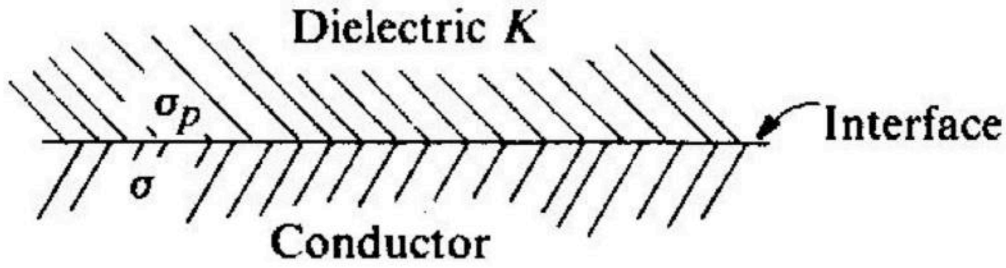
(A) zero (B) $V/8$ (C) $V/6$
(D) $V/2$ (E) V

- Which electrostatics equation applies here?
  - Laplace's equation
  - Averages everything according to boundary conditions

$$\nabla^2 \phi = 0$$

- What would it mean if the potential were *not* V?
  - Nonzero gradient in potential inside the cube
  - Electric field inside the cube
  - But there are no charges

Electromagnetism - 5



54. A dielectric of dielectric constant K is placed in contact with a conductor having surface charge density σ, as shown above. What is the polarization (bound) charge density σ_p on the surface of the dielectric at the interface between the two materials?

(A) $\sigma \frac{K}{1-K}$ (B) $\sigma \frac{K}{1+K}$ (C) σK

(D) $\sigma \frac{1+K}{K}$ (E) $\sigma \frac{1-K}{K}$

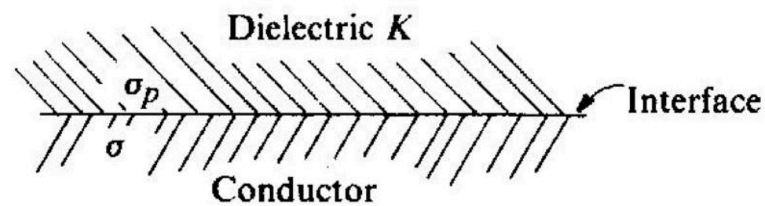
Electromagnetism - 5

- Definition of dielectric constant K
  - Permittivity increases in matter

$$\varepsilon = K\varepsilon_0 > \varepsilon_0$$

- Bound vs. Free Charge
  - Bound charges induced in matter
  - Free charges added from outside

$$\rho_{total} = \rho_{bound} + \rho_{free}$$



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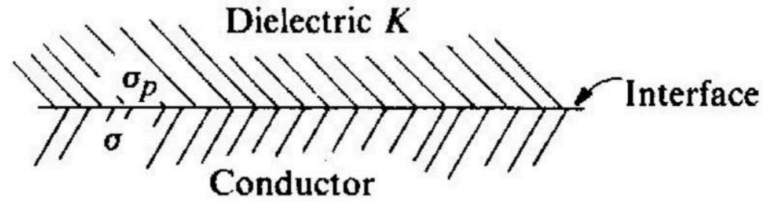
Electromagnetism - 5

- Definition of dielectric constant K
  - Permittivity increases in matter

$$\varepsilon = K\varepsilon_0 > \varepsilon_0$$

- What is the *sign* of the bound charge induced in the dielectric?
- Try taking limits:
  - What happens if K=1?
  - No dielectric, just vacuum
  - No polarization, no bound charge!



54. A dielectric of dielectric constant K is placed in contact with a conductor having surface charge density σ, as shown above. What is the polarization (bound) charge density σ_p on the surface of the dielectric at the interface between the two materials?

(A) $\sigma \dfrac{K}{1-K}$ (B) $\sigma \dfrac{K}{1+K}$ (C) σK

(D) $\sigma \dfrac{1+K}{K}$ (E) $\sigma \dfrac{1-K}{K}$

Electromagnetism - 5

$$\rho_{total} = \rho_{bound} + \rho_{free}$$

- Using Gauss's law on the boundary

$$\nabla \cdot \vec{E} = \rho_{total}/\varepsilon_0$$

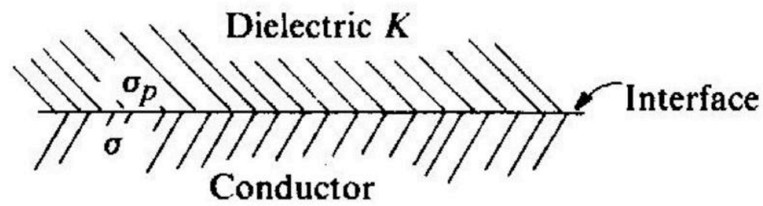
$$\nabla \cdot \vec{D} = \rho_{free}$$

$$\nabla \cdot \vec{P} = -\rho_{bound}$$

- Use definition of K in matter:

$$\vec{D} \equiv \varepsilon_0 \vec{E} + \vec{P} = K\varepsilon_0 \vec{E}$$

- Combine to get relationship between total and free charge, solve for bound charge



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(A) $\sigma \frac{K}{1-K}$ (B) $\sigma \frac{K}{1+K}$ (C) σK

(D) $\sigma \frac{1+K}{K}$ (E) $\sigma \frac{1-K}{K}$

Electromagnetism - 6

- More dielectrics

93. A parallel-plate capacitor has plate separation d. The space between the plates is empty. A battery supplying voltage V_0 is connected across the capacitor, resulting in electromagnetic energy U_0 stored in the capacitor. A dielectric, of dielectric constant κ, is inserted so that it just fills the space between the plates. If the battery is still connected, what are the electric field E and the energy U stored in the dielectric, in terms of V_0 and U_0?

	E	U
(A)	$\frac{V_0}{d}$	U_0
(B)	$\frac{V_0}{d}$	κU_0
(C)	$\frac{V_0}{d}$	$\kappa^2 U_0$
(D)	$\frac{V_0}{\kappa d}$	U_0
(E)	$\frac{V_0}{\kappa d}$	κU_0

- Why do we put dielectrics in capacitors?

$$C = \varepsilon_0 \frac{A}{d}$$

Electromagnetism - 6

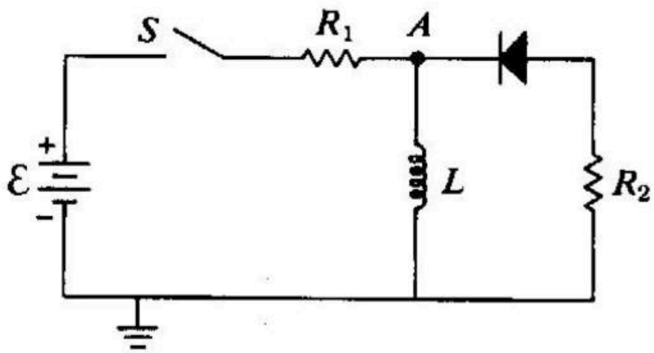
- Electric field change: the battery is constant so the voltage drop is constant
  - What should happen to the electric field if the voltage drop is the same?
- Energy change: it takes work to add a dielectric to a capacitor – need to move extra charges around

$$U_{field} = \varepsilon_0 \int E^2 dV$$

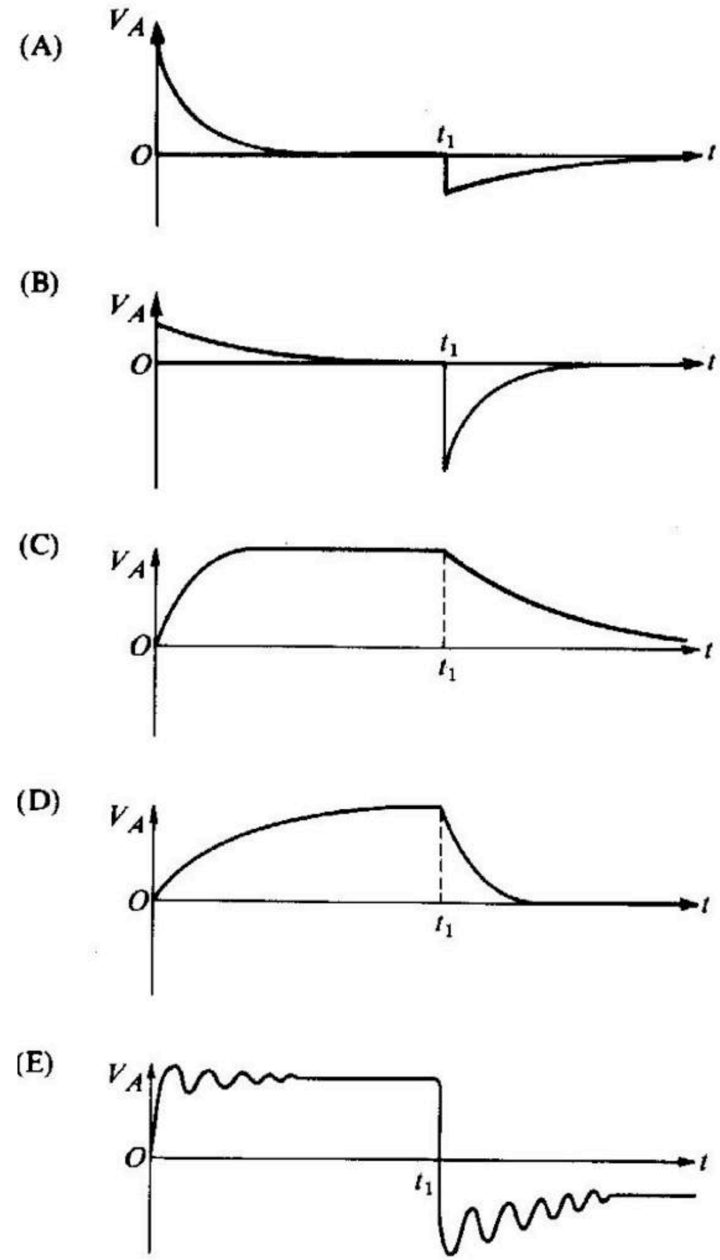
93. A parallel-plate capacitor has plate separation d. The space between the plates is empty. A battery supplying voltage V_0 is connected across the capacitor, resulting in electromagnetic energy U_0 stored in the capacitor. A dielectric, of dielectric constant κ, is inserted so that it just fills the space between the plates. If the battery is still connected, what are the electric field E and the energy U stored in the dielectric, in terms of V_0 and U_0?

	E	U
(A)	$\frac{V_0}{d}$	U_0
(B)	$\frac{V_0}{d}$	κU_0
(C)	$\frac{V_0}{d}$	$\kappa^2 U_0$
(D)	$\frac{V_0}{\kappa d}$	U_0
(E)	$\frac{V_0}{\kappa d}$	κU_0

Circuits - 1

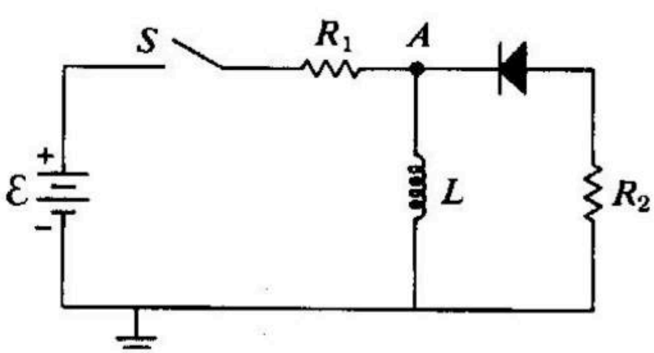


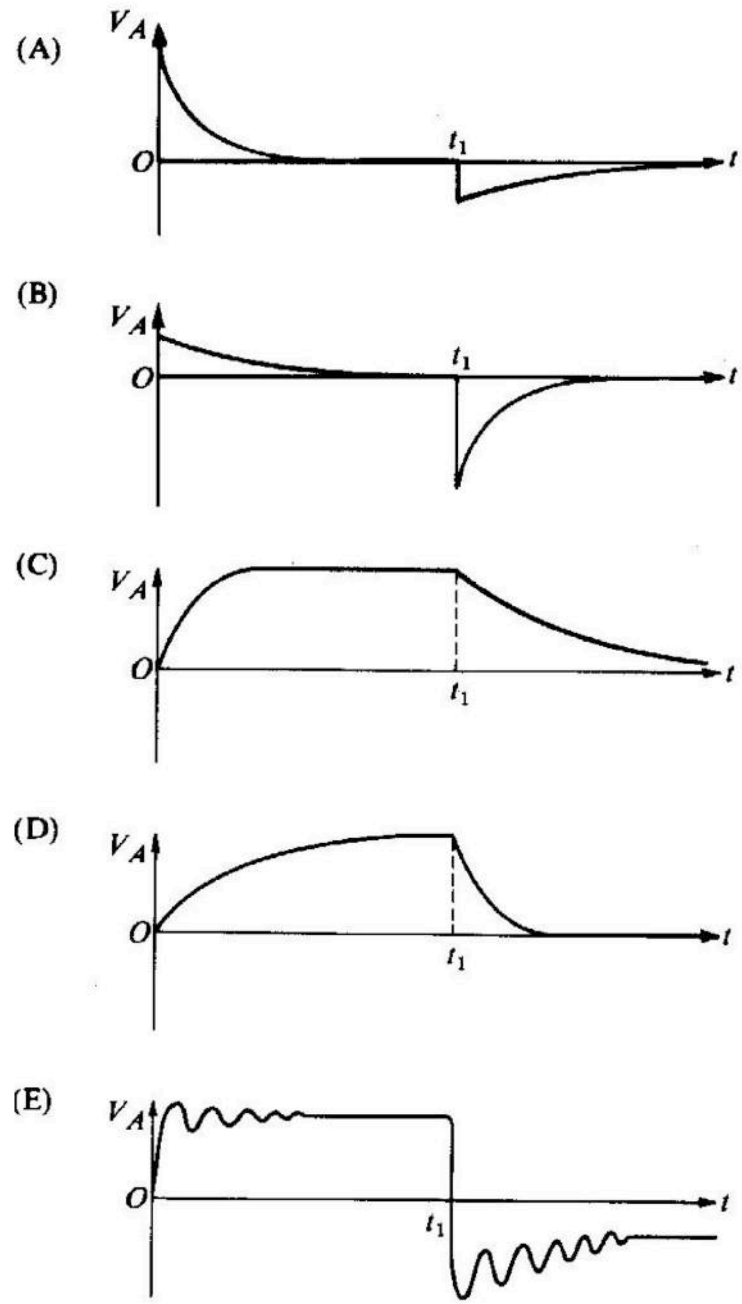
94. In the circuit shown above, $R_2 = 3R_1$ and the battery of emf $\mathcal{E}$ has negligible internal resistance. The resistance of the diode when it allows current to pass through it is also negligible. At time $t = 0$, the switch S is closed and the currents and voltages are allowed to reach their asymptotic values. Then at time t_1, the switch is opened. Which of the following curves most nearly represents the potential at point A as a function of time t?



Circuits - 1

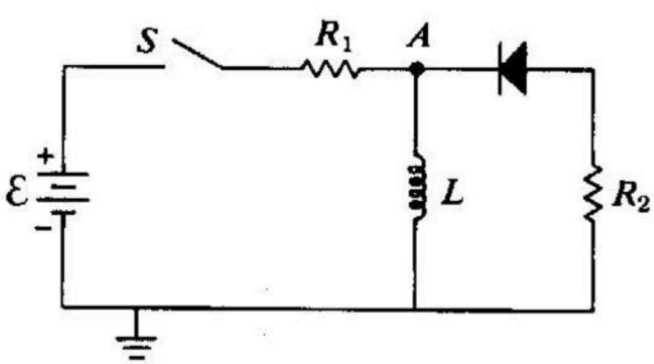
- How does voltage change with time in an LR circuit?
- What does the inductor do at t = 0?
- What does the inductor do at t = t1?

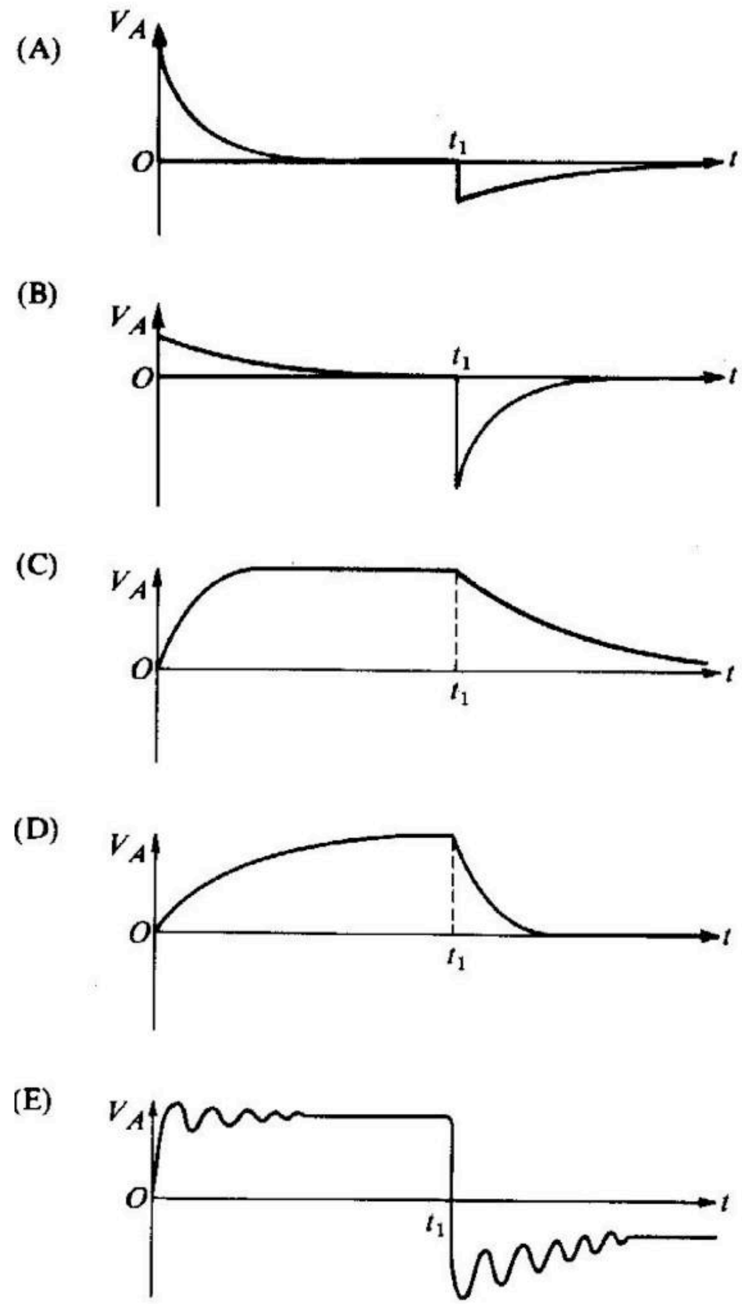




Circuits - 1

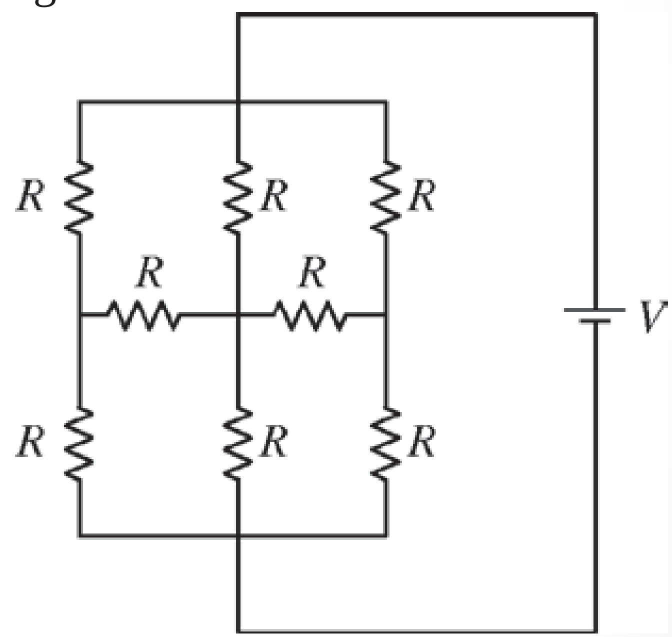
- How does voltage change with time in an LR circuit?
  - Exponential decay, time constant L/R
- What does the inductor do at t = 0?
  - Sudden change in current, large resistance
  - Voltage at A is nonzero
- What does the inductor do at t = t1?
  - Powers circuit, but LESS resistance now





Circuits - 2

68. The circuit shown in the figure above consists of eight resistors, each with resistance R, and a battery with terminal voltage V and negligible internal resistance. What is the current flowing through the battery?



(A) $\frac{1}{3}\frac{V}{R}$

(B) $\frac{1}{2}\frac{V}{R}$

(C) $\frac{V}{R}$

(D) $\frac{3}{2}\frac{V}{R}$

(E) $3\frac{V}{R}$

Circuits - 2

- All resistors are equal – use symmetry
- All currents are equal in each set of three parallel legs
- Voltage is constant all along the middle
- Can ignore the two resistors in the middle

68. The circuit shown in the figure above consists of eight resistors, each with resistance R, and a battery with terminal voltage V and negligible internal resistance. What is the current flowing through the battery?

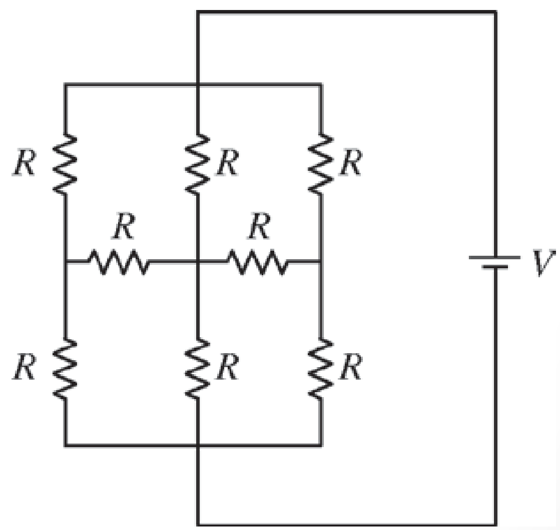
(A) $\frac{1}{3}\frac{V}{R}$

(B) $\frac{1}{2}\frac{V}{R}$

(C) $\frac{V}{R}$

(D) $\frac{3}{2}\frac{V}{R}$

(E) $3\frac{V}{R}$



Quantum Mechanics - 1

98. A particle of mass m is acted on by a harmonic force with potential energy function $V(x) = m\omega^2 x^2/2$ (a one-dimensional simple harmonic oscillator). If there is a wall at $x = 0$ so that $V = \infty$ for $x < 0$, then the energy levels are equal to

(A) $0, \ \hbar\omega, \ 2\hbar\omega, \ \ldots$

(B) $0, \ \frac{\hbar\omega}{2}, \ \hbar\omega, \ \ldots$

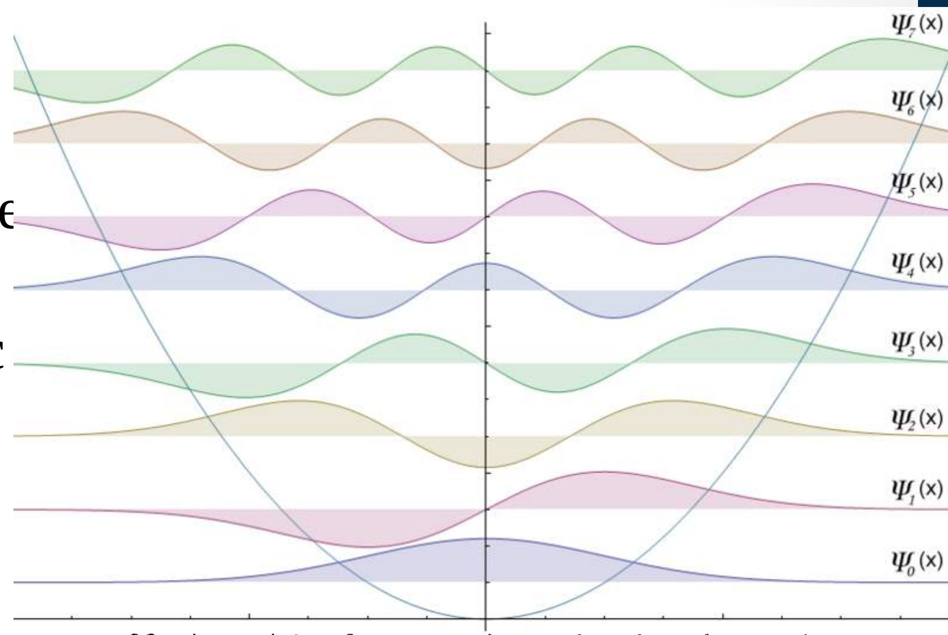
(C) $\frac{\hbar\omega}{2}, \ \frac{3\hbar\omega}{2}, \ \frac{5\hbar\omega}{2}, \ \ldots$

(D) $\frac{3\hbar\omega}{2}, \ \frac{7\hbar\omega}{2}, \ \frac{11\hbar\omega}{2}, \ \ldots$

(E) $0, \ \frac{3\hbar\omega}{2}, \ \frac{5\hbar\omega}{2}, \ \ldots$

Quantum Mechanics - 1

- (This is a common question)
- Know what the solutions to the Schroedinger equation look like for the quantum harmonic oscillator



- How does placing an infinite potential boundary at x=0 change what solutions are allowed?
  - What is the probability of a particle being found in a region with infinite potential?

98. A particle of mass m is acted on by a harmonic force with potential energy function $V(x) = m\omega^2 x^2/2$ (a one-dimensional simple harmonic oscillator). If there is a wall at $x = 0$ so that $V = \infty$ for $x < 0$, then the energy levels are equal to

(A) $0, \hbar\omega, 2\hbar\omega, \ldots$

(B) $0, \frac{\hbar\omega}{2}, \hbar\omega, \ldots$

(C) $\frac{\hbar\omega}{2}, \frac{3\hbar\omega}{2}, \frac{5\hbar\omega}{2}, \ldots$

(D) $\frac{3\hbar\omega}{2}, \frac{7\hbar\omega}{2}, \frac{11\hbar\omega}{2}, \ldots$

(E) $0, \frac{3\hbar\omega}{2}, \frac{5\hbar\omega}{2}, \ldots$

http://en.wikipedia.org/wiki/Quantum_harmonic_oscillator

Quantum Mechanics - 2

93. The solution to the Schrödinger equation for the ground state of hydrogen is

$$\psi_0 = \frac{1}{\sqrt{\pi a_0{}^3}} e^{-r/a_0},$$

where a_0 is the Bohr radius and r is the distance from the origin. Which of the following is the most probable value for r?

(A) 0
(B) $a_0/2$
(C) a_0
(D) $2a_0$
(E) ∞

Quantum Mechanics - 2

- Recall how spherical wave functions are normalized:

$$1 = \iint_0^{\infty} |\psi_0|^2 r^2 dr d\Omega$$

- So we can write the marginal probability of finding the particle at radius r (inside a tiny window of dr):

$$\frac{dP}{dr} = 4\pi |\psi_0|^2 r^2 = \frac{4r^2}{a_0^3} e^{-2r/a_0}$$

- So all we need to do is maximize this expression!

$$0 = \frac{4}{a_0^3}\left(2re^{-2r/a_0} - 2r^2/a_0 \, e^{-2r/a_0}\right), \quad r_{max} = a_0$$

- (Can also recall historical definition of Bohr radius)

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where a_0 is the Bohr radius and r is the distance from the origin. Which of the following is the most probable value for r?

(A) 0
(B) $a_0/2$
(C) a_0
(D) $2a_0$
(E) ∞

Quantum Mechanics - 3

90. The spacing of the rotational energy levels for the hydrogen molecule H_2 is most nearly

(A) 10^{-9} eV
(B) 10^{-3} eV
(C) 10 eV
(D) 10 MeV
(E) 100 MeV

Quantum Mechanics - 3

- The rotational kinetic energy operator: $L^2/2I$
- Moment of inertia: $I = 2mr^2$
  - m is atomic weight of $H_2 \approx 10^{-27}$kg
  - r is Bohr radius: $.5\ 10^{-10}$ m
- L^2 operator eigenvalues: $\hbar^2 l(l+1)$
  - So the difference between the l=0 and l=1 states is about $\hbar^2$
- Altogether, the energy levels are around:
- $10^{-68}/10^{-27}/10^{-19}=10^{-22}$ J = 10^{-3} eV
- (Important to know the conversion between eV and J very quickly: 1eV = 10^{-19} J)

90. The spacing of the rotational energy levels for the hydrogen molecule H_2 is most nearly

(A) 10^{-9} eV
(B) 10^{-3} eV
(C) 10 eV
(D) 10 MeV
(E) 100 MeV

Special Relativity - 1

80. A tube of water is traveling at 1/2 *c* relative to the lab frame when a beam of light traveling in the same direction as the tube enters it. What is the speed of light in the water relative to the lab frame? (The index of refraction of water is 4/3.)

    (A) 1/2 *c*
    (B) 2/3 *c*
    (C) 5/6 *c*
    (D) 10/11 *c*
    (E) *c*

Special Relativity - 1

- In the moving frame of the tube, the light is only moving c/n = 3/4c<c, so let's just treat this like an ordinary velocity addition problem (c = 1):

$$v' = \frac{u+v}{1+uv}$$

- u is the velocity of the tube with respect to the lab frame
- v is the velocity of light within the tube

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(A) 1/2 *c*
(B) 2/3 *c*
(C) 5/6 *c*
(D) 10/11 *c*
(E) *c*

- Since u and v have the same sign in the lab frame, they have the same sign when using this formula (they would have opposite sign if the tube were moving in opposite direction of light

Special Relativity - 2

49. The infinite xy-plane is a nonconducting surface, with surface charge density σ, as measured by an observer at rest on the surface. A second observer moves with velocity $v\hat{\mathbf{x}}$ relative to the surface, at height h above it. Which of the following expressions gives the electric field measured by this second observer?

(A) $\dfrac{\sigma}{2\epsilon_0}\ \hat{\mathbf{z}}$

(B) $\dfrac{\sigma}{2\epsilon_0}\sqrt{1 - v^2/c^2}\ \hat{\mathbf{z}}$

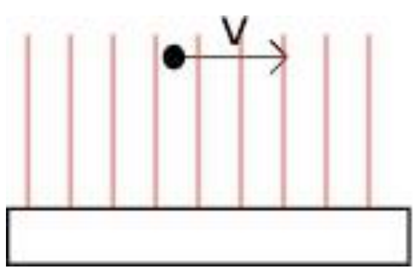
(C) $\dfrac{\sigma}{2\epsilon_0\sqrt{1 - v^2/c^2}}\ \hat{\mathbf{z}}$

(D) $\dfrac{\sigma}{2\epsilon_0}\left(\sqrt{1 - v^2/c^2}\ \hat{\mathbf{z}} + v/c\ \hat{\mathbf{x}}\right)$

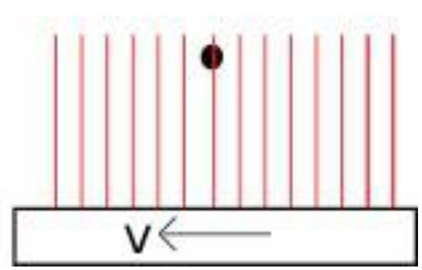
(E) $\dfrac{\sigma}{2\epsilon_0}\left(\sqrt{1 - v^2/c^2}\ \hat{\mathbf{z}} - v/c\ \hat{\mathbf{y}}\right)$

Special Relativity - 2

- Imagine drawing the electric field lines in the lab frame as being spaced evenly apart
- Now imagine boosting to the moving observer's frame
- Now the charged plane appears to be moving at -v**x**
- How do the distances between the evenly spaced electric field lines change in the moving frame?
  - Lengths contract, so field lines are *denser* and the electric field must appear *stronger* to the moving observer
  - Effectively, the observer observes the presence of more charge in a given amount of time if the plane is moving than if the plane is stationary



Lab frame



Observer's Frame

Special Relativity - 2

- The answer must reflect a stronger field
- Also, no symmetry breaking: the field is not bent in the y or x directions

- Can also remember how electromagnetic fields Lorentz transform under a boost in the x direction:

$$E'_x = E_x \qquad B'_x = B_x$$

$$E'_y = \gamma(E_y - \beta B_z) \qquad B'_y = \gamma(B_y + \beta E_z)$$

$$E'_z = \gamma(E_z + \beta B_y) \qquad B'_z = \gamma(B_z - \beta E_y)$$

49. The infinite xy-plane is a nonconducting surface, with surface charge density σ, as measured by an observer at rest on the surface. A second observer moves with velocity $v\hat{\mathbf{x}}$ relative to the surface, at height h above it. Which of the following expressions gives the electric field measured by this second observer?

(A) $\dfrac{\sigma}{2\epsilon_0}\,\hat{\mathbf{z}}$

(B) $\dfrac{\sigma}{2\epsilon_0}\sqrt{1 - v^2/c^2}\;\hat{\mathbf{z}}$

(C) $\dfrac{\sigma}{2\epsilon_0\sqrt{1 - v^2/c^2}}\;\hat{\mathbf{z}}$

(D) $\dfrac{\sigma}{2\epsilon_0}\left(\sqrt{1 - v^2/c^2}\;\hat{\mathbf{z}} + v/c\;\hat{\mathbf{x}}\right)$

(E) $\dfrac{\sigma}{2\epsilon_0}\left(\sqrt{1 - v^2/c^2}\;\hat{\mathbf{z}} - v/c\;\hat{\mathbf{y}}\right)$

Special Relativity - 3

- Velocity Addition

23. Two spaceships approach Earth with equal speeds, as measured by an observer on Earth, but from opposite directions. A meterstick on one spaceship is measured to be 60 cm long by an occupant of the other spaceship. What is the speed of each spaceship, as measured by the observer on Earth?

    (A) $0.4c$
    (B) $0.5c$
    (C) $0.6c$
    (D) $0.7c$
    (E) $0.8c$

Special Relativity - 3

- Step 1: How fast is ship 2 moving with respect to ship 1 in ship 1's frame of reference?
  - Use length contraction
- Step 2: boost ship 2 from earth's frame of reference into ship 1's frame of reference
  - Use velocity addition
  - Same as adding ship 2's speed to itself (u=v in equation)

$$v' = \frac{u+v}{1+uv}$$

23. Two spaceships approach Earth with equal speeds, as measured by an observer on Earth, but from opposite directions. A meterstick on one spaceship is measured to be 60 cm long by an occupant of the other spaceship. What is the speed of each spaceship, as measured by the observer on Earth?

(A) 0.4c
(B) 0.5c
(C) 0.6c
(D) 0.7c
(E) 0.8c

Special Relativity - 4

94. An observer O at rest midway between two sources of light at $x = 0$ and $x = 10$ m observes the two sources to flash simultaneously. According to a second observer O', moving at a constant speed parallel to the x-axis, one source of light flashes 13 ns before the other. Which of the following gives the speed of O' relative to O?

(A) $0.13c$
(B) $0.15c$
(C) $0.36c$
(D) $0.53c$
(E) $0.62c$

Special Relativity - 4

- (This problem is hard)
- Draw picture of geometry
- Derive expression for time delay in moving frame
- Solving for velocity
- Use Taylor Expansions to approximate

94. An observer O at rest midway between two sources of light at $x = 0$ and $x = 10$ m observes the two sources to flash simultaneously. According to a second observer O', moving at a constant speed parallel to the x-axis, one source of light flashes 13 ns before the other. Which of the following gives the speed of O' relative to O?

(A) $0.13c$
(B) $0.15c$
(C) $0.36c$
(D) $0.53c$
(E) $0.62c$

$$v^2 = \frac{(39/100)^2}{1+(39/100)^2}$$

$$v^2 \approx (39/100)^2(1-(39/100)^2) \approx (39/100)^2$$

Statistical Mechanics

16. The mean free path for the molecules of a gas is approximately given by $\frac{1}{\eta\sigma}$, where η is the number density and σ is the collision cross section. The mean free path for air molecules at room conditions is approximately

(A) 10^{-4} m
(B) 10^{-7} m
(C) 10^{-10} m
(D) 10^{-13} m
(E) 10^{-16} m

Statistical Mechanics

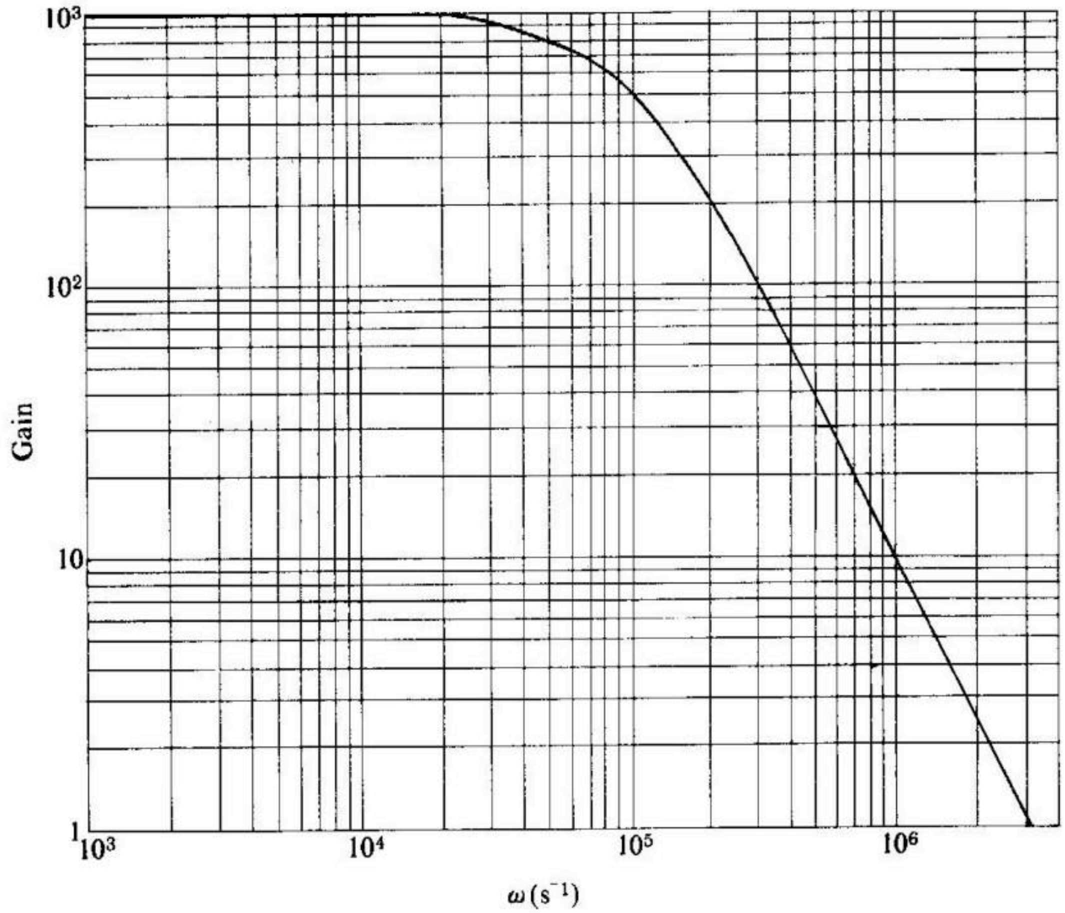
- Really this is a problem of knowing your length scales:
  - 10^{-9} - 10^{-10} m is the size of an atom
  - 10^{-3} m is a millimeter, 10^{-5} m is the width of a human hair
  - So plausibly, what is the likeliest given answer?
- Can also solve this knowing the number density of air, and estimating the collision cross section as atomic cross sectional area:

$$[\eta]=atoms/volume$$

$$\eta=\frac{n}{V}=\frac{P}{RT}=\frac{1atm}{\left(8\cdot 10^{-5}\frac{m^3\cdot atm}{Mol\cdot K}\right)300K}\frac{6\cdot 10^{23}atom}{1Mol}\approx 10^{25}\frac{atom}{m^3}$$

$$\sigma\approx(10^{-9})^2m^2 \qquad \frac{1}{\eta\sigma}\approx 10^{-25}\cdot 10^{18}m=10^{-7}m$$

Laboratory Methods - 1



39. The gain of an amplifier is plotted *versus* angular frequency ω in the diagram above. If K and a are positive constants, the frequency dependence of the gain near $\omega = 3 \times 10^5$ second^{-1} is most accurately expressed by

(A) $Ke^{-a\omega}$ (B) $K\omega^2$ (C) $K\omega$ (D) $K\omega^{-1}$ (E) $K\omega^{-2}$