Preparing for the Physics GRE: Day 4
Atomic and Particle Physics

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Outline

- Atomic Physics
  - The Periodic Table
  - The Hydrogen atom – size and energy eigenstates
  - Atomic Spectra and Ionization
  - Atomic Orbitals
  - Spectroscopic Notation
  - Selection Rules
- Particle Physics
  - The Standard Model
  - Conserved quantities
    - Leptons
    - Baryons (and Mesons)
  - Weak interactions

Atomic Physics

The Periodic Table

- A few things worth memorizing:
  - Fact: Z ranges from 1 to (about) 100
  - Noble gases, in order by weight
  - Alkali metals
  - Z for Carbon, Iron (as well as H, He)

Group → / ↓ Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba		72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra		104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo

Lanthanides	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
Actinides	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

The Hydrogen Atom

- Most complicated system completely solved by quantum: one electron and one proton
- Bohr radius: most probable distance between p and e in hydrogen ground state

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} \cong .5\cdot 10^{-10}m$$

- Allowed energies, where n is the quantum number of the radial state of the atom:

$$E_n = -\left[\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\right]\frac{1}{n^2} = \frac{-13.6eV}{n^2}$$

- It is worth knowing these formulae so you know how they scale as mass or charge is changed (or at least be able to reason them out using dimensional analysis)

Positronium

- Electron-positron bound state
- Mass that appears in H spectrum is *reduced* e-p mass:

$$m_{reduced} = \left(\frac{1}{m_p} + \frac{1}{m_e}\right)^{-1}$$

- Since $m_p >> m_e$, the reduced mass is set to m_e
- But for positronium, what is the new reduced mass?
- How does that change the "Bohr" radius and energy spectrum?

99. The positronium "atom" consists of an electron and a positron bound together by their mutual Coulomb attraction and moving about their center of mass, which is located halfway between them. Thus the positronium "atom" is somewhat analogous to a hydrogen atom. The ground-state binding energy of hydrogen is 13.6 electron volts. What is the ground-state binding energy of positronium?

(A) $\left(\frac{1}{2}\right)^2 \times 13.6$ eV

(B) $\frac{1}{2} \times 13.6$ eV

(C) 13.6 eV

(D) 2×13.6 eV

(E) $(2)^2 \times 13.6$ eV

Spectrum for Hydrogen (and for Higher-Z atoms)

- Photons are emitted when atomic electrons fall from high n to low n states
- Photon energy is equal to the energy difference between the two energy states
- As a first approximation, can treat high-Z atoms as having effective charge Ze

$$\hbar\omega = -13.6eV \cdot Z^2 \left(\frac{1}{n_f^{\,2}} - \frac{1}{n_o^{\,2}} \right)$$

9. In the spectrum of hydrogen, what is the ratio of the longest wavelength in the Lyman series ($n_f = 1$) to the longest wavelength in the Balmer series ($n_f = 2$) ?

(A) 5/27
(B) 1/3
(C) 4/9
(D) 3/2
(E) 3

Spectrum for Hydrogen (and for Higher-Z atoms)

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(A) 5/27
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(D) 3/2
(E) 3

$$\hbar\omega = -13.6eV \cdot Z^2\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_o^{\,2}}\right)$$

- Long wavelength => small frequency
- n_f and n_0 must be close to one another
- Balmer $n_0 = 3$, $n_f = 2$: -5/36
- Lyman $n_0 = 2$, $n_f = 1$: -3/4
- Take the ratio…

Ionization energy

- How much energy does it take to ionize an atom, so that one (or more) electrons are removed from their orbits completely?
- Think of the energy required as an atomic transition from n_0 to n_f -> ∞:

$$E_{ionization} = \frac{13.6eV \cdot Z^2}{n_o^{\ 2}}$$

18. The energy required to remove both electrons from the helium atom in its ground state is 79.0 eV. How much energy is required to ionize helium (i.e., to remove one electron) ?

(A) 24.6 eV
(B) 39.5 eV
(C) 51.8 eV
(D) 54.4 eV
(E) 65.4 eV

Helium Ionization Energy

18. The energy required to remove both electrons from the helium atom in its ground state is 79.0 eV. How much energy is required to ionize helium (i.e., to remove one electron) ?

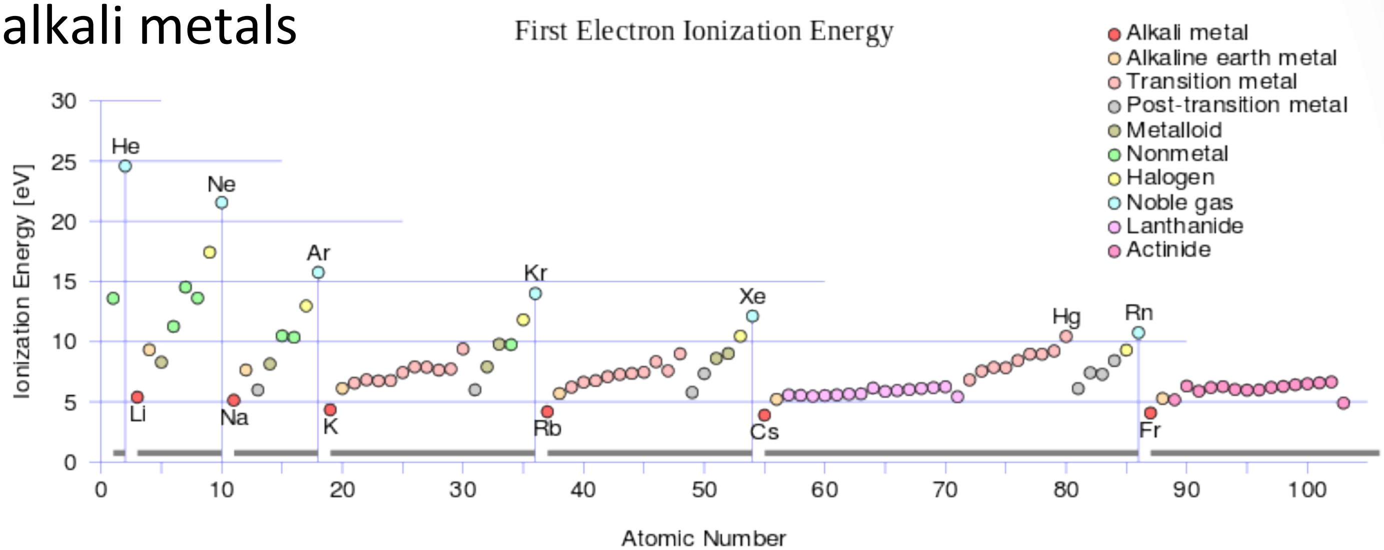
(A) 24.6 eV
(B) 39.5 eV
(C) 51.8 eV
(D) 54.4 eV
(E) 65.4 eV

$$E_{ionization} = \frac{13.6eV \cdot Z^2}{n_o^{\;2}}$$

- The energy to remove two He electrons equals the energy required to ionize He (remove the first) plus the energy required to remove the second
- Treat ionized Helium as a hydrogen-like atom with Z = 2
- What is the ionization energy of ionized Helium?
  - 4 * 13.6 eV = 54.4 eV
  - => 79 eV-54.4 eV = 24.6 eV

Ionization energy

- Generally, ionization energy is highest for noble gases, lowest for alkali metals



39. Which of the following atoms has the lowest ionization potential?

(A) $^{2}_{4}$He

(B) $^{7}_{14}$N

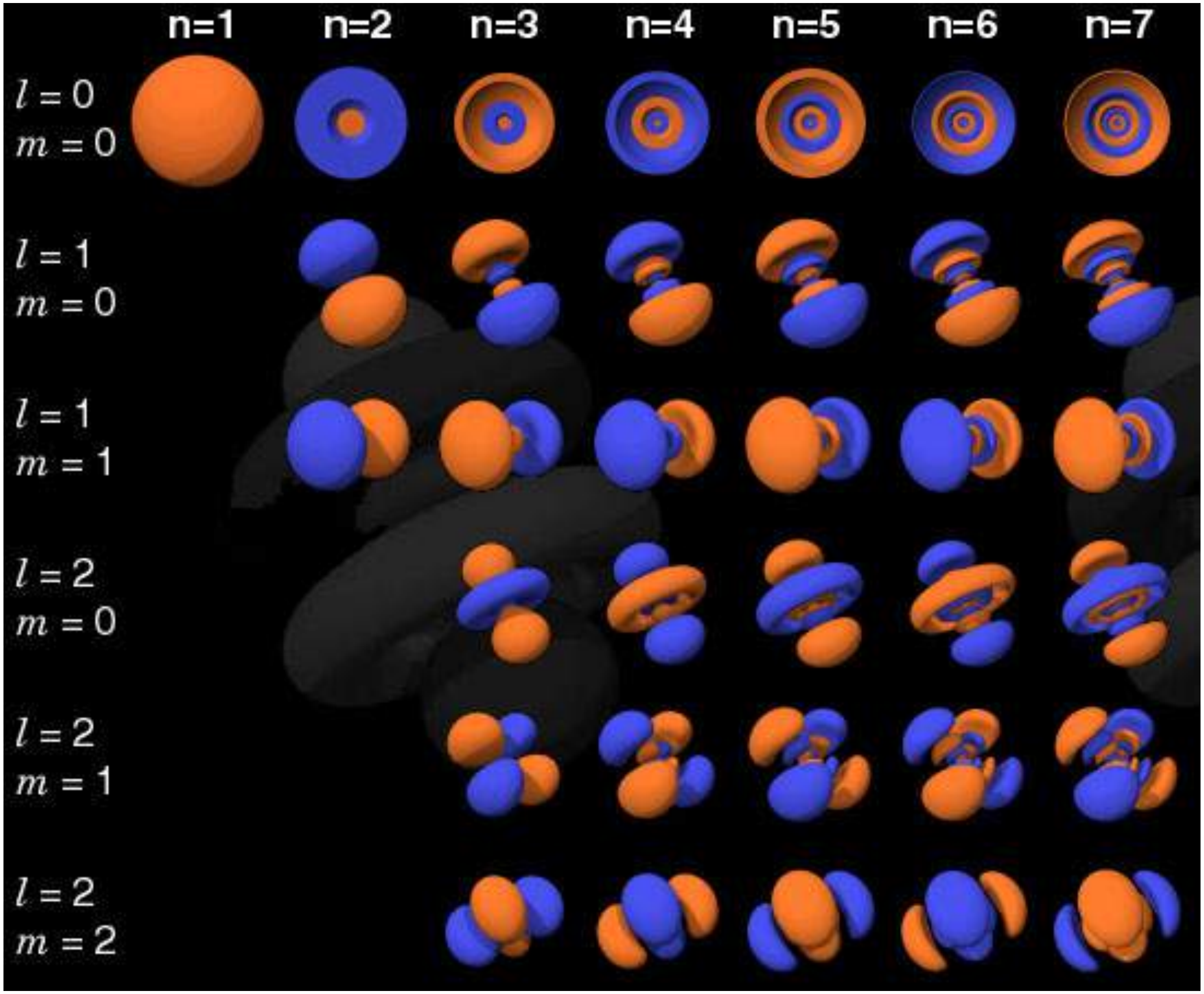
(C) $^{8}_{16}$O

(D) $^{18}_{40}$Ar

(E) $^{55}_{133}$Cs

http://en.wikipedia.org/wiki/Ionization_energy

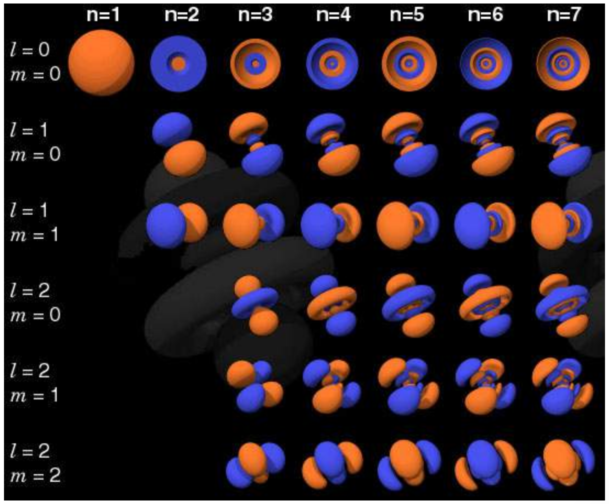
Orbitals



http://chemistry.beloit.edu/Stars/pages/orbitals.html

Orbitals

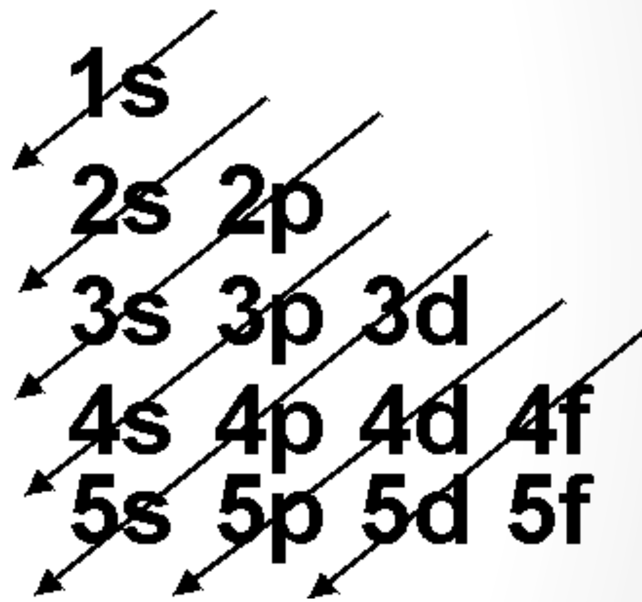
- Orbitals are denoted by their quantum numbers
- n is the radial quantum number, determines energy eigenstate
- l is the azimuthal quantum number: 0, 1, 2, … n-1
- m is the magnetic quantum number: -l, -l+1, … l-1, l
  - l > 0 states are 2l+1-fold degenerate
- l, m = 0 states are spherically symmetric: nonzero l, m states have nonzero angular momentum and break angular symmetry
- For each n, there are n^2 - n possible states to fill
- We distinguish these angular momentum states as follows:
  - s: l = 0 (**s**harp)
  - p: l = 1 (**p**rincipal)
  - d: l = 2 (**d**iffuse)
  - f: l = 3 (**f**undamental)



How ground state orbitals are filled

- I have Z electrons in an atomic ground state, which orbitals do they fill?
- Notation: (n, l-state label, number of electrons in state, up to 2)
- Filling is as follows, from left to right:

  1s2 2s2 2p6 3s2 3p6 4s2 3d10 4p6
- (Note, for example, that 4s are filled before 3d)
- After Helium, noble gases occur when p-orbitals are filled:
  - He: 1s2, Ne: 1s2 2s2 2p6,
  - Ar: 1s2 2s2 2p6 3s2 3p6, etc.
- Sometimes this notation is abbreviated:
  - Ca: 1s2 2s2 2p6 3s2 3p6 = Ar 4s2



How ground state orbitals are filled

- Notation: (n, l-state label, number of electrons in state, up to 2)

  1s2 2s2 2p6 3s2 3p6 4s2 3d10 4p6

- (A) What about 3d?
- (B) Only 1 electron in 4s
- (C) No l = 4 electrons here
- (D) I count 19 orbitals filled
- (E) s-orbitals have 0 net angular momentum and are spherically symmetric

30. The configuration of the potassium atom in its ground state is $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^1$. Which of the following statements about potassium is true?

    (A) Its $n = 3$ shell is completely filled.
    (B) Its $4s$ subshell is completely filled.
    (C) Its least tightly bound electron has $\ell = 4$.
    (D) Its atomic number is 17.
    (E) Its electron charge distribution is spherically symmetrical.

6. An atom has filled $n = 1$ and $n = 2$ levels. How many electrons does the atom have?

   (A) 2
   (B) 4
   (C) 6
   (D) 8
   (E) 10

Spectroscopic notation $^{2S+1}L_J$

- There's yet another notation for the state of atoms, that can also account for states above the ground state
- S is the total spin momentum of the electrons
  - Electrons paired together in the same state have 0 net spin
  - Spin-up and spin-down half-shells are filled separately
  - That is to say, three p states (px, py, pz) are filled as follows: (1,0,0), (1,1,0), (1,1,1), (1,1,2), (1,2,2), (2,2,2)
  - Only unpaired electrons contribute to S
- L is the l-state label of the atom (S, P, D, F, etc)
  - Each half-shell is filled in order: l, l-1, l-2, … 1-l, -l
- J is the total angular momentum, ranging from L+S to |L-S|
  - |L-S| if outermost subshell is only half filled
  - L+S if outermost subshell is completely filled

Spectroscopic notation $^{2S+1}L_J$

- S is the total spin momentum of the electrons
  - Electrons paired together in the same state have 0 net spin
- L is the l-state label of the atom (S, P, D, F, etc)
  - Each half-shell is filled in order: l, l-1, l-2, … 1-l, -l
- J is the total angular momentum, ranging from L+S to |L-S|
  - |L-S| if outermost subshell is only half filled
  - L+S if outermost subshell is completely filled
- Examples:
- Mg: 1s2 2s2 2p6 3s2 -> s = 0, l = 0, j = 0 => $^{1}S_{0}$
  - Example of an atom with all shells filled
- C: He 2p2 -> s = 1, l = 1 + 0, j = l-s = 0 => $^{1}P_{0}$
- F: He 2p5 -> s = ½, l = 1+0-1+1+0 = 1, j = l+s = 3/2 => $^{2}P_{3/2}$
- P: Ne 3s2 3p3 -> s = 3/2, l = 1+0-1= 0, j = l-s = 3/2 => $^{4}S_{3/2}$

Spectroscopic notation $^{2S+1}L_J$

- Two examples:
- Note the atom is out of its ground state
  - S = 3/2
  - l = 0+1+1 = 2

76. The configuration of three electrons ls2p3p has which of the following as the value of its maximum possible total angular momentum quantum number?

(A) $\frac{7}{2}$

(B) 3

(C) $\frac{5}{2}$

(D) 2

(E) $\frac{3}{2}$

- Na: Fill up to 11 electrons:
  - 1s2 2s2 2p6 3s1
  - l = 0
  - S = ½
  - J = |l-s|

84. Sodium has eleven electrons and the sequence in which energy levels fill in atoms is $1s$, $2s$, $2p$, $3s$, $3p$, $4s$, $3d$, etc. What is the ground state of sodium in the usual notation $^{2S+1}L_J$?

(A) $^1S_0$ (B) $^2S_{\frac{1}{2}}$ (C) $^1P_0$

(D) $^2P_{\frac{1}{2}}$ (E) $^3P_{\frac{1}{2}}$

Selection Rules

- For a hydrogen-like atom to undergo a transition from a higher energy state to a lower energy state (thus emitting a photon), there are restrictions on the transitions that are allowed
- Specifically, the angular momentum quantum numbers must change:
  - $\Delta l = +1$ or -1
  - $\Delta m = +1, -1,$ or 0
- When working in spectroscopic notation:
  - $\Delta J = +1, -1,$ or 0
  - $\Delta s = 0$ (no restrictions due to spin, outside of Pauli)
- "Electric dipole transition"

Selection Rules - Examples

48. A transition in which one photon is radiated by the electron in a hydrogen atom when the electron's wave function changes from ψ_1 to ψ_2 is forbidden if ψ_1 and ψ_2

(A) have opposite parity
(B) are orthogonal to each other
(C) are zero at the center of the atomic nucleus
(D) are both spherically symmetrical
(E) are associated with different angular momenta

- Two examples:
- Spherically symmetric wave functions are both S-type orbitals
- There must be some change in angular momentum for radiation to occur

41. A $3p$ electron is found in the $^3P_{3/2}$ energy level of a hydrogen atom. Which of the following is true about the electron in this state?

(A) It is allowed to make an electric dipole transition to the $^2S_{1/2}$ level.
(B) It is allowed to make an electric dipole transition to the $^2P_{1/2}$ level.
(C) It has quantum numbers $\ell = 3$, $j = 3/2$, $s = 1/2$.
(D) It has quantum numbers $n = 3$, $j = \ell$, $s = 3/2$.
(E) It has exactly the same energy as it would in the $^3D_{3/2}$ level.

- Need to use spectroscopic notation here as well!
  - S = 1/2, L = 1, J = L+S = 3/2
- (A) Seems possible
- (B) Angular momentum hasn't changed
- (C) Not true
- (D) An electron with spin 3/2?
- (E) Need an n=4 state to have a d-orbital

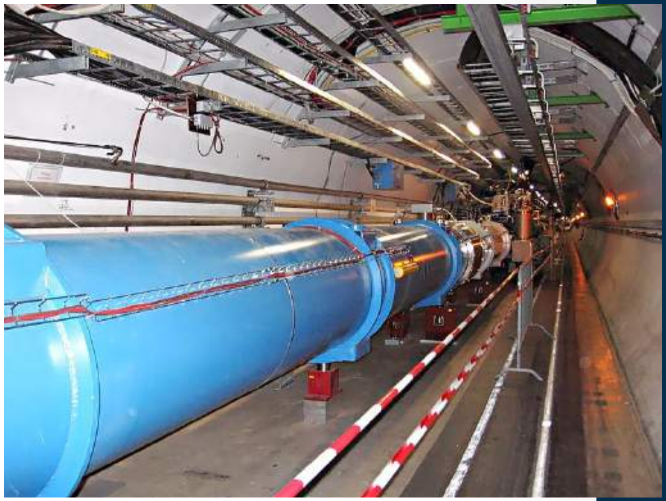
Atomic Physics Summary

- Know parts of the periodic table
- Understand and be able to recognize the spectrum and orbitals of hydrogen-like atoms
- Be able to calculate spectrum and ionization energies
- Know the order in which orbitals are filled
- Recognize spectroscopic notation
- Remember selection rules for dipole transitions

Particle Physics

Some Experimental Context

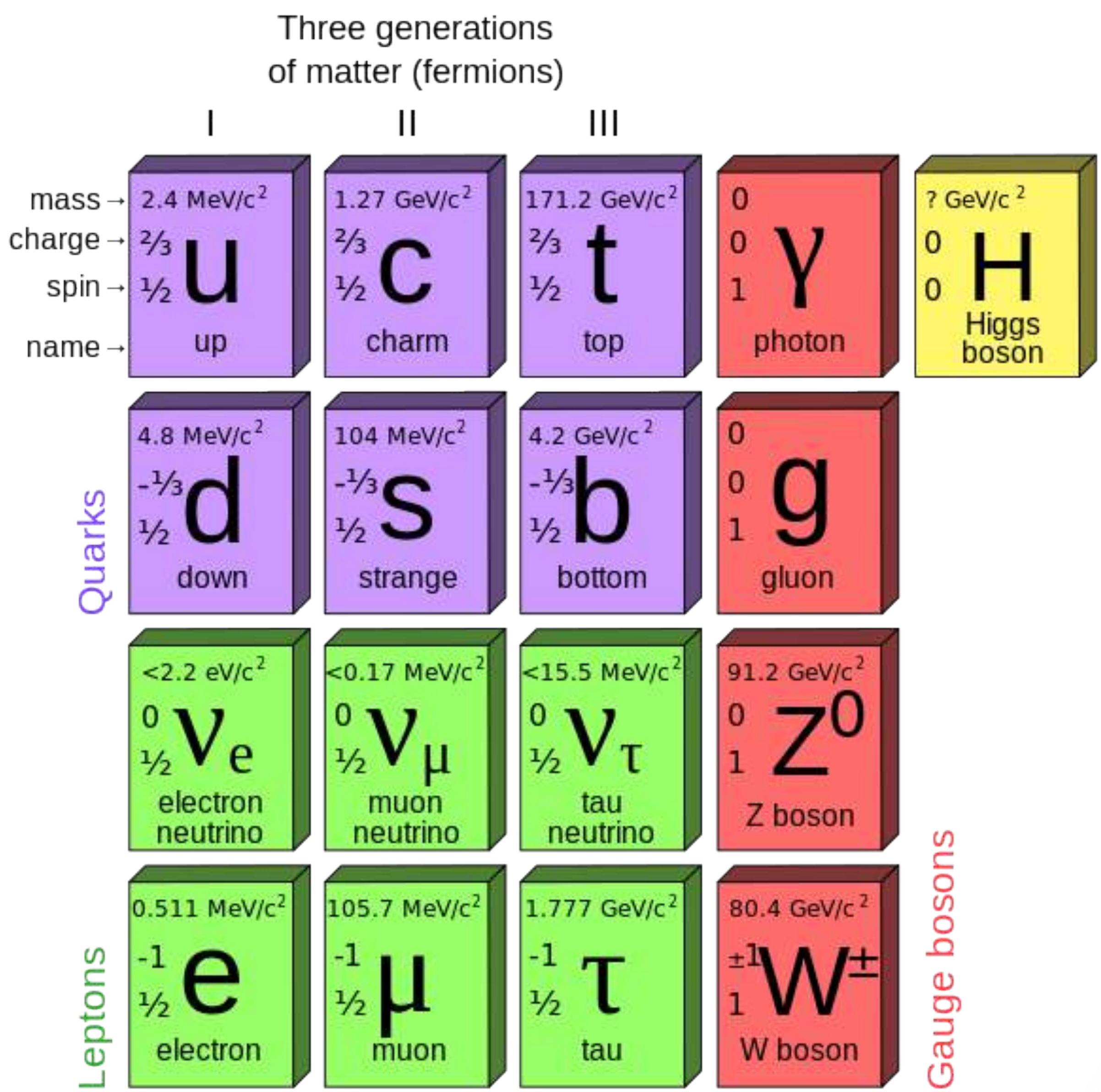
- Where do "particles" come from?
  - Radioactive decay (n, e^+, etc)
  - The environment, such as products from Cosmic Rays (muons, etc)
  - Accelerators, such as the LHC (quarks, etc)
- How do we "see" them?
  - Cloud chambers
  - Bubble chambers
  - Scintillators
- How do we measure their properties?
  - Interactions with matter
  - How do they decay?



http://en.wikipedia.org/wiki/File:CERN_LHC_Tunnel1.jpg

http://en.wikipedia.org/wiki/Cloud_chamber

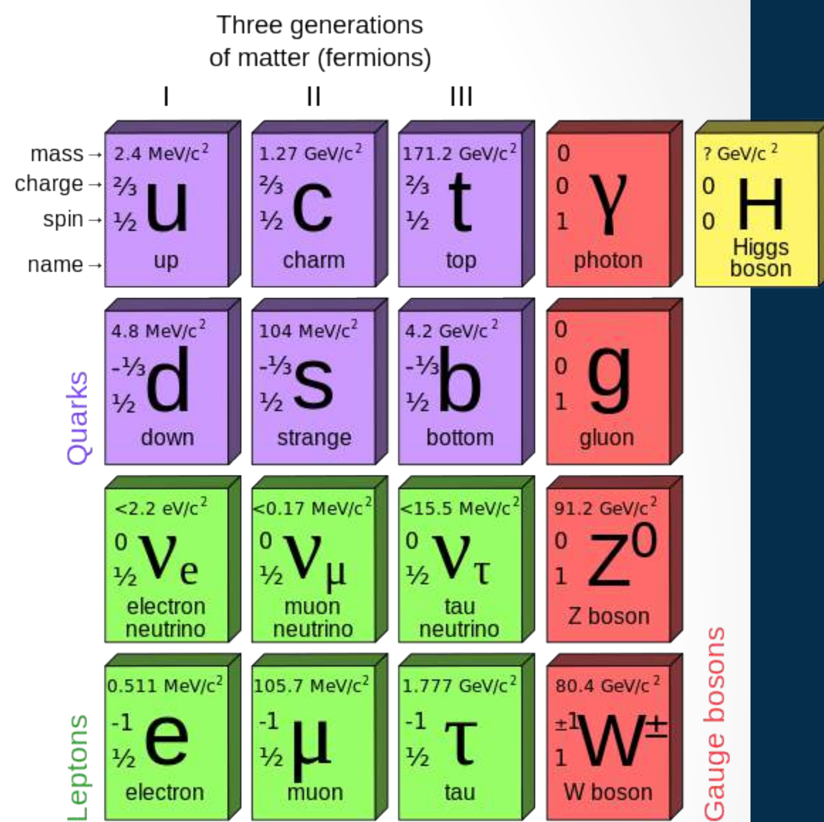
The Standard Model



http://en.wikipedia.org/wiki/File:Standard_Model_of_Elementary_Particles.svg

The Standard Model

- All elementary, non-composite particles that we know of (so far)
- Quarks
  - 6 flavors
  - Fractional charge
  - Fermions
  - Up/Down make up protons, neutrons
- Leptons
  - 3 flavors
  - Fermions
  - Electrons and heavier electrons
  - Neutrinos, chargeless and (practically) massless
- Force Carriers
  - Bosons
  - Photons – carry Electromagnetic Force
  - Gluons – carry Strong Nuclear Force
  - W/Z – carry Weak Nuclear Force
- (Probably don't need Higgs physics for test)



63. According to the Standard Model of elementary particles, which of the following is NOT a composite object?

(A) Muon
(B) Pi-meson
(C) Neutron
(D) Deuteron
(E) Alpha particle

Conserved Quantities

- Most particle physics GRE questions deal with nuclear reactions and other types of decay processes
- Typically, reactions obey **conservation laws**
- What you probably already know:
  - Momentum and Energy
  - Charge
  - Angular momentum (spin)
- New concepts from particle physics:
  - Lepton Number
  - Baryon Number
  - Other quantities violated in Weak interactions only:
    - Strangeness
    - Parity
    - Charge-Parity

Lepton Number

- 3 flavors of lepton (electron, muon, tauon)
  - Note: each electron-like particle has a corresponding neutrino with the same flavor, eg muon and muon neutrino
- The number of particles belonging to each flavor of lepton is conserved
- **Note**: anti-particles contribute *negative* lepton number
  - Example: anti-electrons (e^+) have electron number -1

98. Which of the following is the principal decay mode of the positive muon μ^+ ?

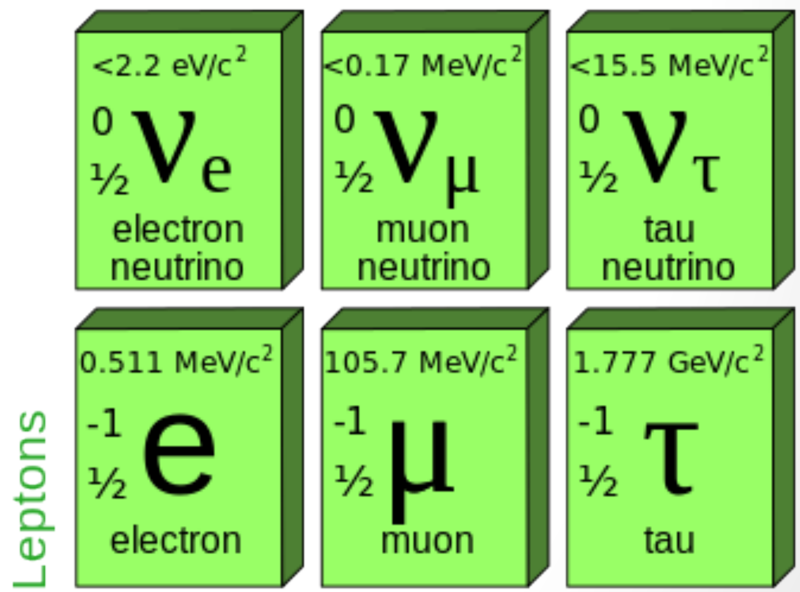
(A) $\mu^+ \to e^+ + \nu_e$

(B) $\mu^+ \to p + \nu_\mu$

(C) $\mu^+ \to n + e^+ + \nu_e$

(D) $\mu^+ \to e^+ + \nu_e + \overline{\nu}_\mu$

(E) $\mu^+ \to \pi^+ + \overline{\nu}_e + \nu_\mu$



Lepton Number Example

- Need to conserve charge (doesn't eliminate any results)
- Need to conserve muon number and electron number:
  - (A) -1 mu -> -1 e + 1 e
  - (B) -1 mu -> proton? + 1 mu
  - (C) -1 mu -> neutron? -1 e + 1 e
  - (D) -1 mu -> -1 e + 1 e -1 mu
  - (E) -1 mu -> pion? - 1 e + 1 mu

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(D) $\mu^+ \to e^+ + \nu_e + \overline{\nu}_\mu$

(E) $\mu^+ \to \pi^+ + \overline{\nu}_e + \nu_\mu$

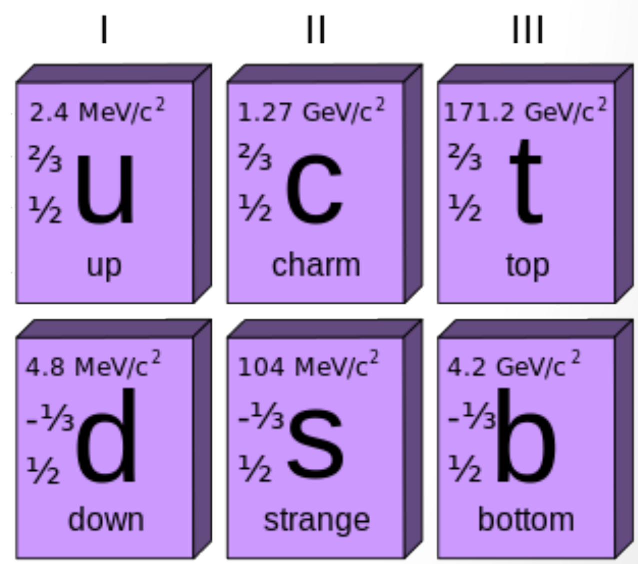
- Additional example:

78. The muon decays with a characteristic lifetime of about 10^{-6} second into an electron, a muon neutrino, and an electron antineutrino. The muon is forbidden from decaying into an electron and just a single neutrino by the law of conservation of

(A) charge
(B) mass
(C) energy and momentum
(D) baryon number
(E) lepton number

Baryons

- Composite particles made up of **3 quarks**
- Examples:
  - Proton = 2 up + 1 down
  - Neutron = 2 down + 1 up
  - Most matter consists of baryons
- All baryons are **fermions**
- Baryon number = (number of quarks – number of antiquarks)/3
  - So protons and neutrons have B = +1
  - Anti-protons have B = -1
- Other (more exotic examples)
  - Δ (3 up/down quarks)
  - Λ, Σ (2 up/down quarks)
  - Ξ (1 up/down quark)
  - Ω (0 up/down quarks)



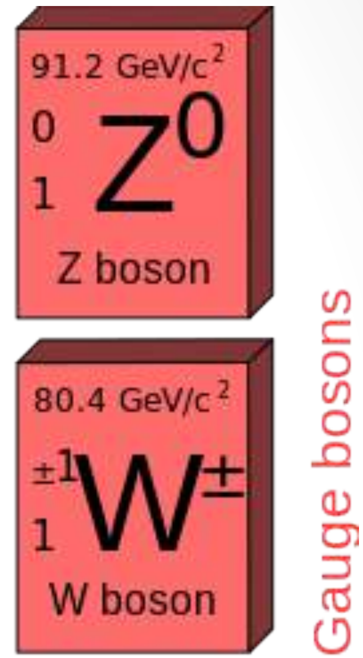
… as opposed to Mesons

- Composite particles made up of one quark and one antiquark
  - These particles appear as decay products (for example from cosmic rays)
- Examples:
  - Pions: π^+ π^- π^0, consist of up/down quarks
  - Kaons: K^+ K^- K^0 , consist of one up/down quark, one strange quark
- Baryon number is 0
- (Side note: Older literature may refer to muons as mesons, though now we know they are leptons.)

The Weak Interaction

- Interacts with all fermions
- Mediated by Z, W^+, W^- bosons
- Responsible for all decay of subatomic particles
  - Produces a whole zoo of possible interactions
- Certain symmetries are violated by Weak Interactions:
  - **Quarks change flavor**
  - Parity (also charge-parity)
- Example: Beta decay
  - Nuclear scale: $n \to p^+ + e^- - \nu_e$
  - Sub-nuclear (quark) scale:

    $d \to u + e^- - \nu_e$



$$^{13}\text{N} \rightarrow {}^{13}\text{C} + e^+ + \nu_e$$

63. The nuclear decay above is an example of a process induced by the

(A) Mössbauer effect
(B) Casimir effect
(C) photoelectric effect
(D) weak interaction
(E) strong interaction

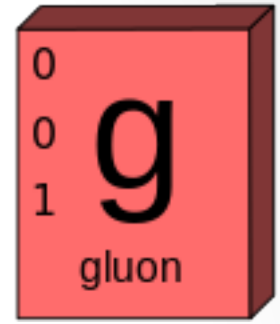
Weak Interaction Example

91. The particle decay $\Lambda \rightarrow p + \pi^-$ must be a weak interaction because

(A) the π^- is a lepton
(B) the Λ has spin zero
(C) no neutrino is produced in the decay
(D) it does not conserve angular momentum
(E) it does not conserve strangeness

- (A) Pions are mesons, not leptons
- (B) Weak interactions only affect fermions (Λ has spin ½)
- (C) Only need neutrinos to conserve lepton number, not necessary for every weak interaction
- (D) Looks like angular momentum *is* conserved
- (E) Strangeness counts number of strange quarks. Since quarks can change flavor under weak interactions, this could be right

Strong Interaction



- Responsible for holding quarks together
  - Hadrons: includes mesons and baryons
- Mediated by gluons
- “Massive photons” follow the Yukawa potential

$$V(r) \propto \frac{e^{-kr}}{r}$$

- Same as charged screened potential for electrons in matter

How do particles interact with matter?

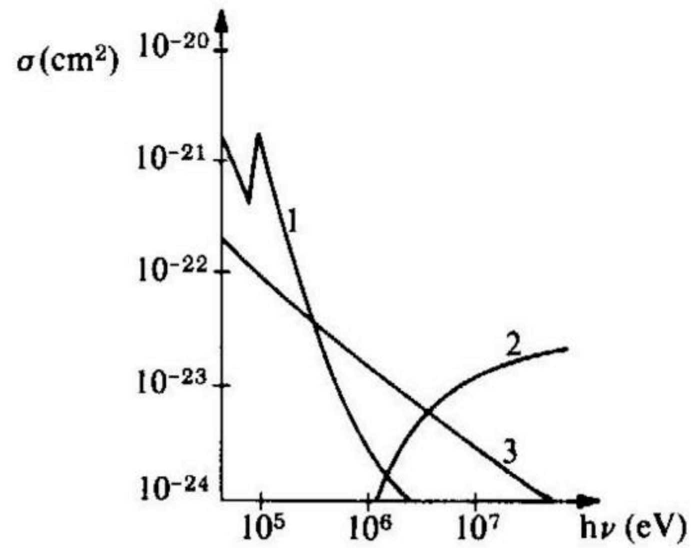
- Treat the interaction between incident particles and matter probabilistically, with some probability of scattering occurring
  - Probability: Cross sections are measured in units of **area**
- Not totally necessary to memorize rules for cross sections, but can list some rules of thumb that build on physical intuition
  - *Charged* particles interact with electrons in matter, so the higher Z of the matter, the more likely they are to interact
  - *Lighter* particle mass scatter more easily (less inertia => easier to change momentum)

25. In experiments located deep underground, the two types of cosmic rays that most commonly reach the experimental apparatus are

(A) alpha particles and neutrons
(B) protons and electrons
(C) iron nuclei and carbon nuclei
(D) muons and neutrinos
(E) positrons and electrons

How do photons interact with matter?

- Photons primarily interact with atomic electrons
- Three primary processes (which you **need** to know for the test)
  - Compton Scattering
  - Photoelectric effect
  - Pair production
- Important to know:
  - Why does the photoelectric effect only occur with atomic electrons (as opposed to free)?
  - Why can't pair production occur in vacuum?



85. The figure above shows the photon interaction cross sections for lead in the energy range where the Compton, photoelectric, and pair production processes all play a role. What is the correct identification of these cross sections?

(A) 1 = photoelectric, 2 = Compton, 3 = pair production

(B) 1 = photoelectric, 2 = pair production, 3 = Compton

(C) 1 = Compton, 2 = pair production, 3 = photoelectric

(D) 1 = Compton, 2 = photoelectric, 3 = pair production

(E) 1 = pair production, 2 = photoelectric, 3 = Compton

Particle Physics Summary

- Begin with the Standard Model
  - Know the different properties of fundamental particles
- What quantities are conserved?
  - Charge, spin angular momentum
  - Lepton number
  - Baryon number
- Weak interactions
  - Interacts with fermions
  - Mediates particle decay
  - Quarks can change flavor
- Interactions between particles and matter
- Other topics:
  - Additional symmetries: Parity, Charge-parity
  - Zoo of subatomic mesons and baryons
  - Interaction cross sections