

# Preparing for the Physics GRE: Day 3 Optics and Waves

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2/20/14

<http://pages.physics.cornell.edu/~dcitron/GREPrep2014.html>

# Outline

- Geometric optics
- Wave model of light: diffraction and interference
- Properties of solutions to the wave equation

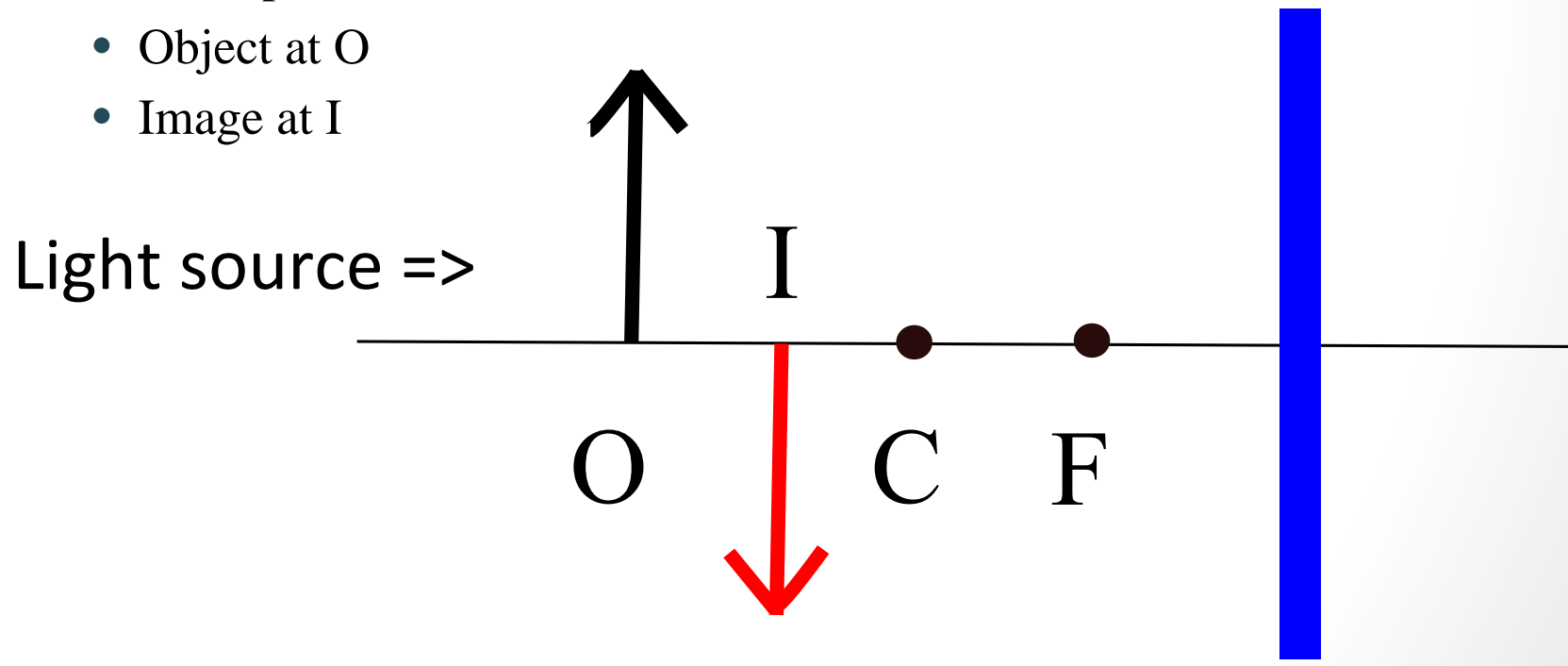
# Geometric Optics

# Geometric Optics

- Treat light classically as a ray
- Materials cause light rays to bend or reflect at interfaces
  - Snell's law:  $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$
- Three cases:
  - Reflection by mirrors
  - Refraction by curved surfaces
  - Refraction by lenses
- Two ways to understand:
  - 1. Draw a ray diagram
  - 2. Thin lens formula
    - This formula requires memorization of a few rules of thumb
- Other concepts and formulae:
  - Magnification
  - Virtual images/Real images

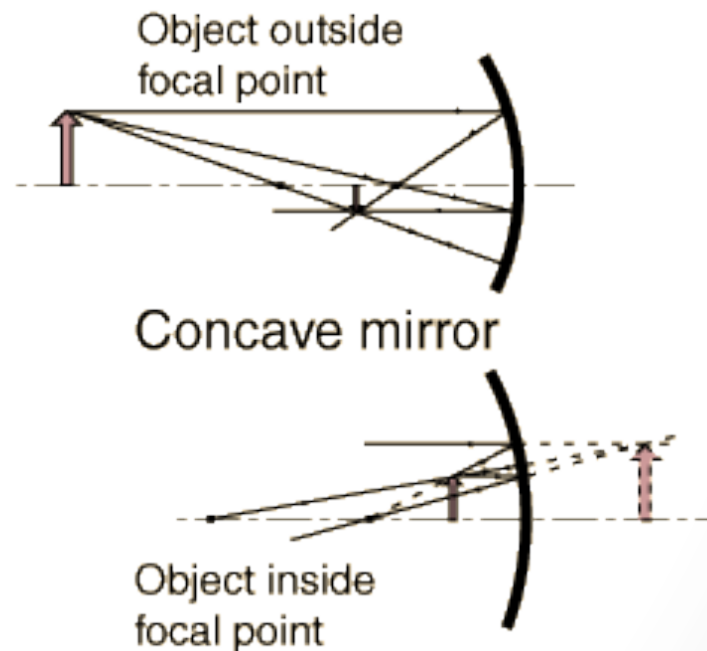
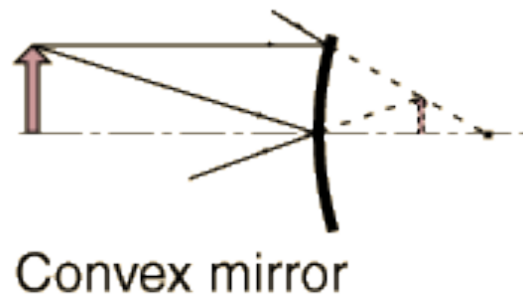
# Geometric Optics: Ray Diagrams

- Work with mirrors first, can generalize to lenses
- Given: light source, object, mirror, focal point
- Want to find where the image will be:
- Parameters to know:
  - Center at C
  - Focal point at  $f = C/2$
  - Object at O
  - Image at I



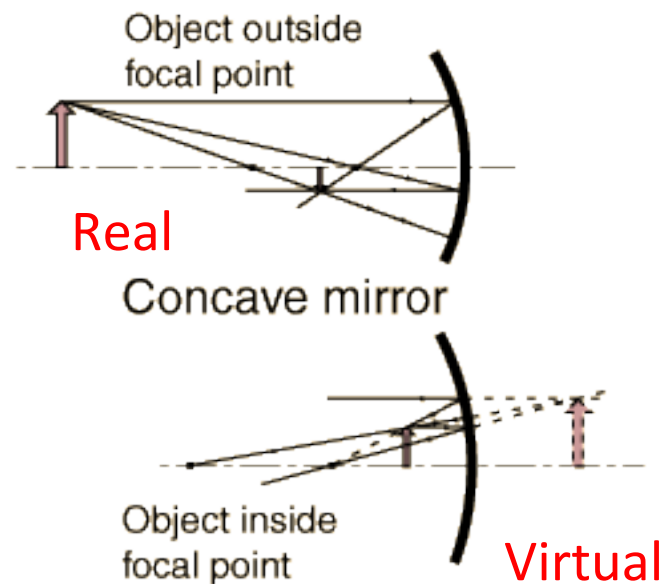
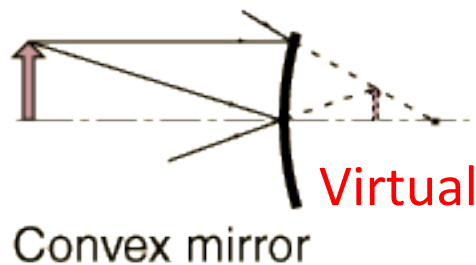
# Geometric Optics: Ray Diagrams

- Light rays scatter off the object at all angles: the image forms because the lens/mirror focuses them the right way
- Draw 3 rays (only need 2 to see where image appears)
  - (Incident light) => (Emerging light)
  - 1. Parallel to axis => Passes through focus
  - 2. Passes through focus => Parallel to axis
  - 3. Points to middle of lens => Does not bend



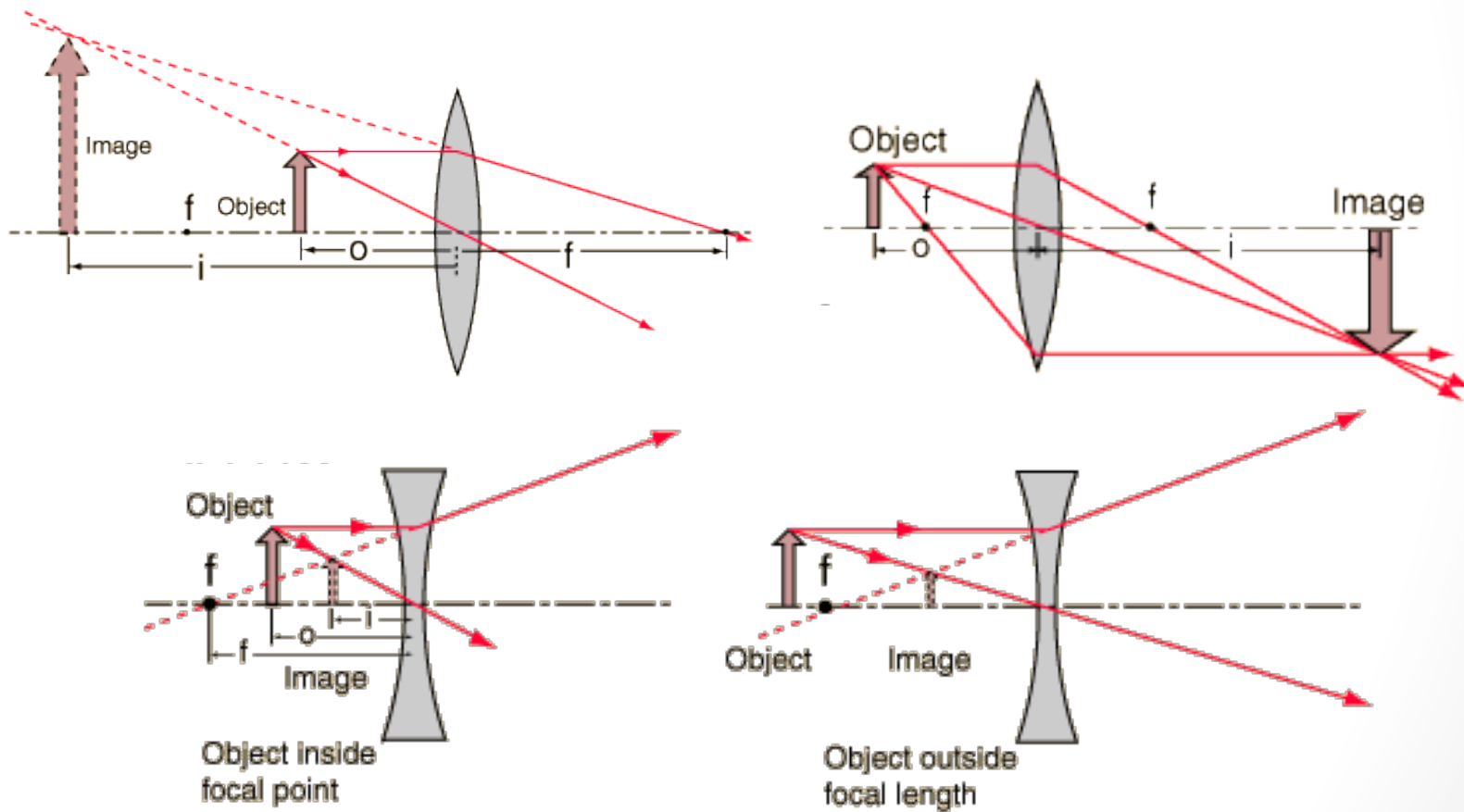
# Geometric Optics: Ray Diagrams

- Many possible cases, but not necessary to memorize each
- Practice by drawing a few
- Image can appear on either side of surface:
  - Real image – light passes through the image
  - Virtual image – light does not pass through the image



# Geometric Optics: Lenses

- Same as mirrors, except light passes through them
- Can draw the same rays to find the image:





# Geometric Optics: Thin Lens Equation

- Ray diagrams yield intuition, but rarely appear on the test
- To actually make calculations:

$$\frac{1}{O} + \frac{1}{I} = \frac{1}{F}$$

- Sign conventions (why this is nontrivial)
  - A is where light **comes from**, B is where light **passes to**
  - Note side B is different for mirrors and lenses

$$O \begin{cases} >0 & \text{if on side A} \\ <0 & \text{if on side B} \end{cases} \quad I, F, C \begin{cases} <0 & \text{if on side A} \\ >0 & \text{if on side B} \end{cases}$$

- Suggestion: memorize the signs of O, I, F for a single case
  - Lens – where are A and B?
  - Mirror – where are A and B?

# Geometric Optics: Thin Lens Equation

- Ray diagrams yield intuition, but rarely appear on the test
- To actually make calculations:

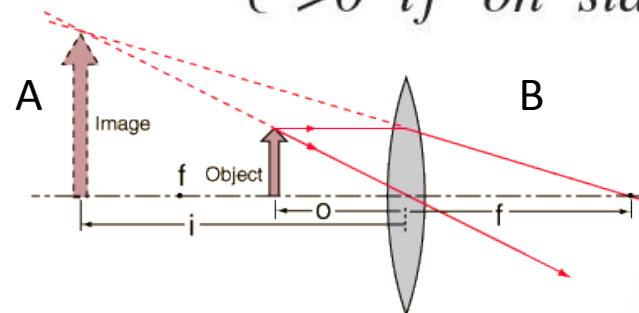
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$$I, F, C \begin{cases} <0 & \text{if on side A} \\ >0 & \text{if on side B} \end{cases}$$

- Try example: what are the signs of O, I, F, C?



- Convex lenses converge light: F is on side B and positive
- Concave lenses diverge light: F is on side A and negative

# Geometric optics: Magnification

- Compare size of image to size of object:

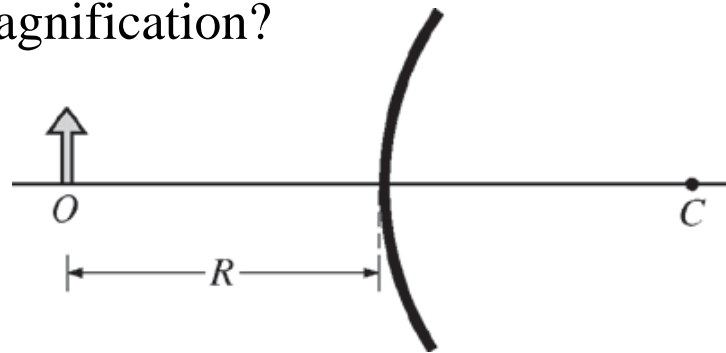
$$M = -I/O = F/(F-O)$$

- Again, need to be careful about signs of I and O
  - Negative magnification means the image is inverted

# Geometric Optics: Example problem

- Try drawing ray diagram
- Check your answer with thin lens equation
  - What are the signs of  $O$ ,  $I$ ,  $F$ ,  $C$ ?
  - What is the magnification?

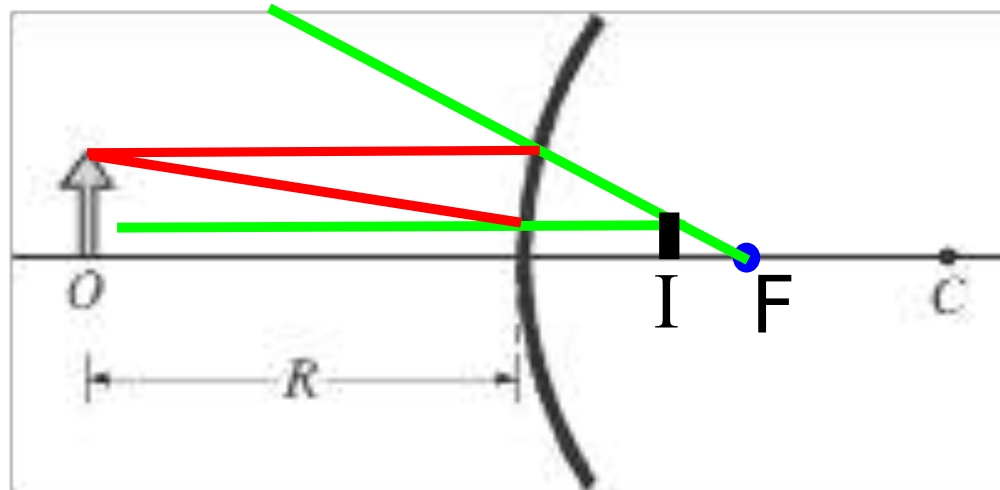
$$\frac{1}{O} + \frac{1}{I} = \frac{1}{F}$$



74. The figure above shows an object  $O$  placed at a distance  $R$  to the left of a convex spherical mirror that has a radius of curvature  $R$ . Point  $C$  is the center of curvature of the mirror. The image formed by the mirror is at
- (A) infinity
  - (B) a distance  $R$  to the left of the mirror and inverted
  - (C) a distance  $R$  to the right of the mirror and upright
  - (D) a distance  $\frac{R}{3}$  to the left of the mirror and inverted
  - (E) a distance  $\frac{R}{3}$  to the right of the mirror and upright

# Geometric Optics: Example problem

- Try drawing ray diagram:



- Using lens equation:

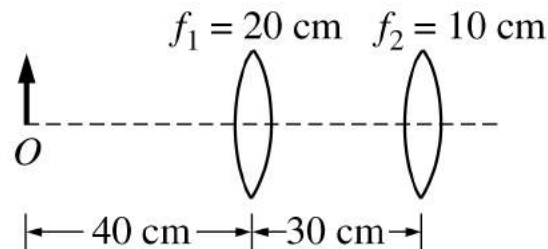
$F = -O/2$  (On opposite side from where light points to)

$$1/I = 1/F - 1/O \Rightarrow I = -2/3 F = -1/3 R$$

- Check: is I inverted or not?
- Magnification:  $-I/O = 1/3 R / R = 1/3$

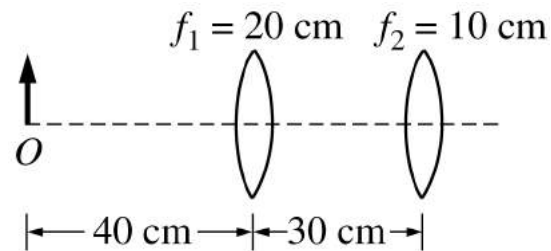
# Geometric Optics: Multiple Lenses

- Treat the image from the first lens as a virtual object
  - 1. Find the image from the first lens
  - 2. Use geometry to find O for the second lens
  - 3. Apply lens equation a second time
- Example:



11. An object is located 40 centimeters from the first of two thin converging lenses of focal lengths 20 centimeters and 10 centimeters, respectively, as shown in the figure above. The lenses are separated by 30 centimeters. The final image formed by the two-lens system is located
- (A) 5.0 cm to the right of the second lens
  - (B) 13.3 cm to the right of the second lens
  - (C) infinitely far to the right of the second lens
  - (D) 13.3 cm to the left of the second lens
  - (E) 100 cm to the left of the second lens

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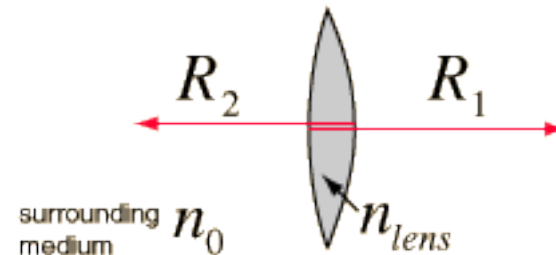
- First lens:  $O = 40 \text{ cm}$ ,  $F = 20 \text{ cm}$ ,  $I = ?$
- Second lens:  $F = 10 \text{ cm}$ ,  $O = ?$ ,  $I = ?$

$$\frac{1}{O} + \frac{1}{I} = \frac{1}{F}$$

# Geometric Optics: Other equations

- Lensmaker's formula
  - Find the focal point, given
    - The radii of the left and right surfaces  $R_1$ ,  $R_2$
    - The index of refraction of the lens:

$$\frac{1}{f} = \left( \frac{n}{n_{out}} \right) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$



- Telescope angular magnification
  - $f_E$ : Focal length of eyepiece
  - $f_O$ : Focal length of objective
  - Note: The two lenses share a focal point

$$M = f_O / f_E$$



- 22 A simple telescope consists of two convex lenses, the objective and the eyepiece, which have a common focal point  $P$ , as shown in the figure above. If the focal length of the objective is 1.0 meter and the angular magnification of the telescope is 10, what is the optical path length between objective and eyepiece?

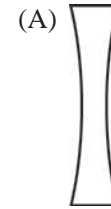
- (A) 0.1 m
- (B) 0.9 m
- (C) 1.0 m
- (D) 1.1 m
- (E) 10 m



# Geometric Optics: Summary

- Ray diagrams give intuition
- Thin lens equation:
  - Sign conventions for all variables
- Magnification definition
- Other equations:
  - Lensmaker's formula
  - Telescopic magnification

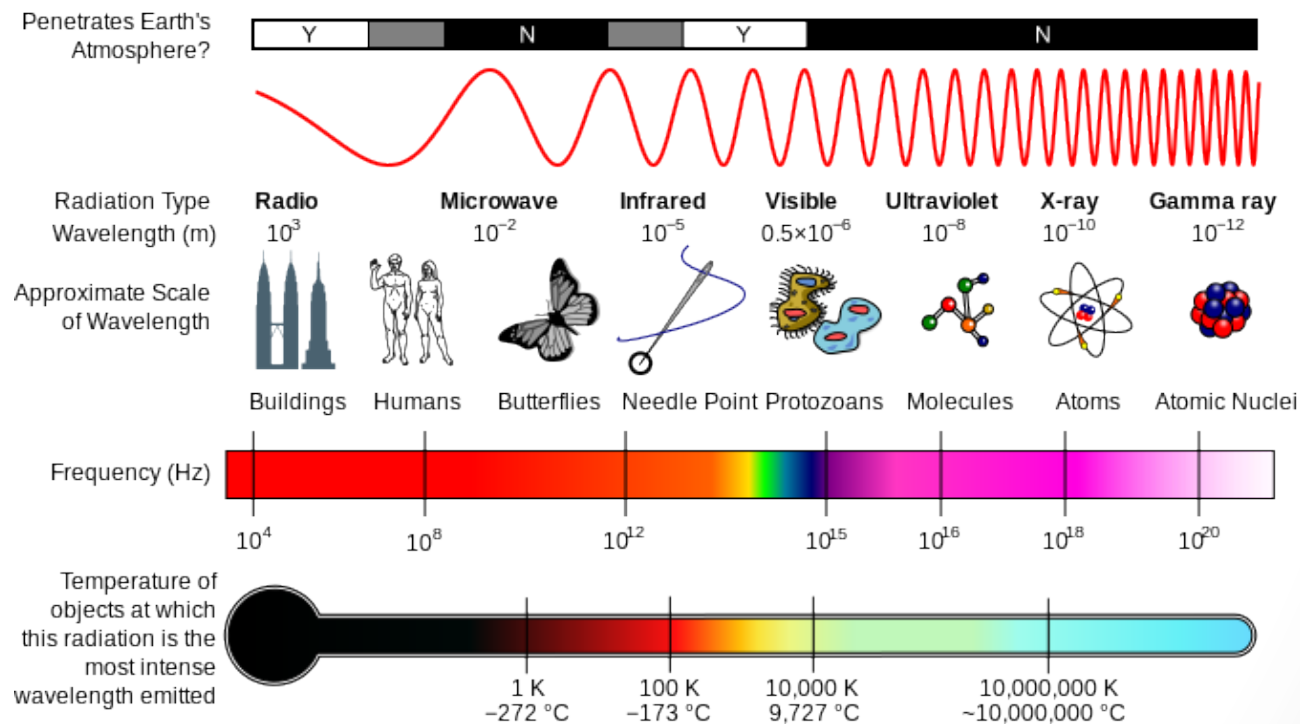
15. If the five lenses shown below are made of the same material, which lens has the shortest positive focal length?



# Wave Model of Light: Interference and Diffraction

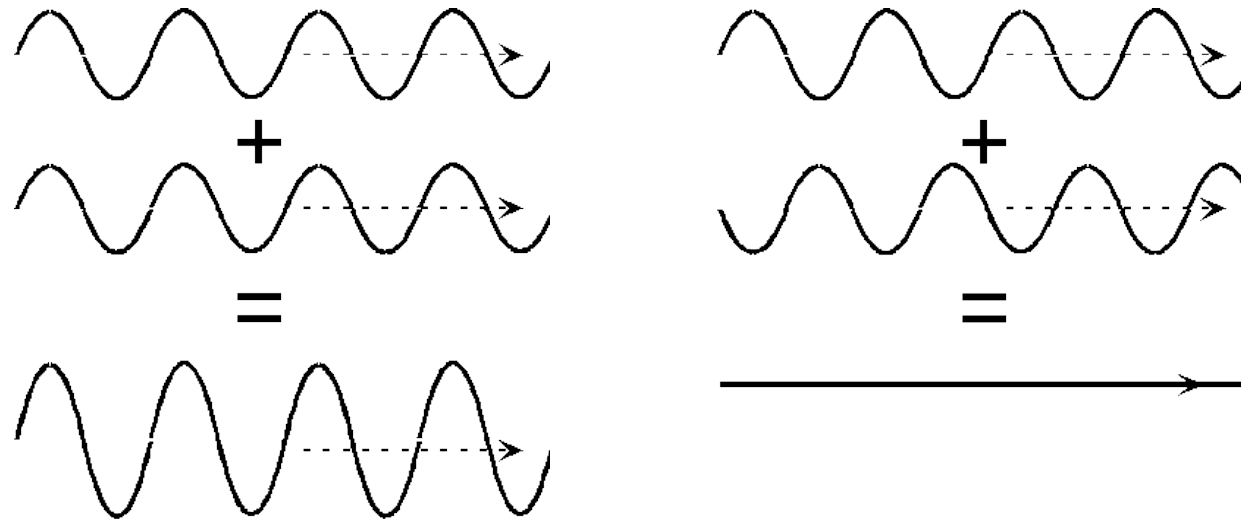
# Wave Model of Light

- At smaller length scales, can treat light as a wave
- From classical E&M, light is an electromagnetic wave
  - Intensity  $I \propto |E|^2$
- (Good to know some wavelengths of parts of the E-M spectrum: visible light, X-rays, etc)



# Wave Model of Light: Interference

- Waves can add constructively or destructively:

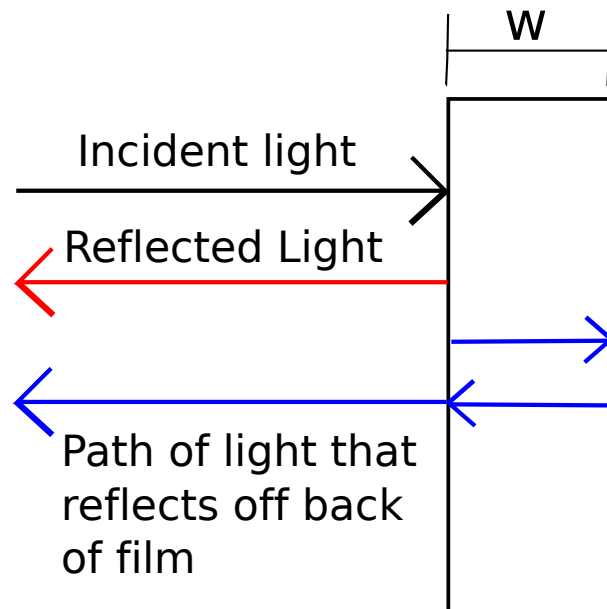


- All interference and diffraction problems:
  - Locate the two waves that are interfering
  - Determine path difference  $d$
  - Determine phase change  $\phi$ 
    - Constructive interference for  $\phi = \text{even multiples of } \pi$
    - Destructive interference for  $\phi = \text{odd multiples of } \pi$

$$\frac{\phi}{2\pi} = \frac{d}{\lambda}$$

# Interference: Thin Films

- Beam splits at first interface
- **Impedance mismatch at first interface  $\Rightarrow$  adds  $\pi$  phase shift**
- Path difference is twice film thickness
  - $d = 2w$
  - $(\phi + \pi) / 2\pi = 2w / \lambda$

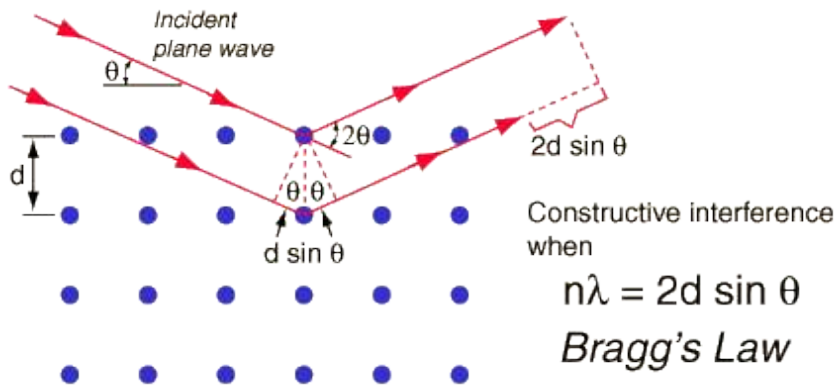


82. Consider two horizontal glass plates with a thin film of air between them. For what values of the thickness of the film of air will the film, as seen by reflected light, appear bright if it is illuminated normally from above by blue light of wavelength 488 nanometers?

- (A) 0, 122 nm, 244 nm
- (B) 0, 122 nm, 366 nm
- (C) 0, 244 nm, 488 nm
- (D) 122 nm, 244 nm, 366 nm
- (E) 122 nm, 366 nm, 610 nm

# Diffraction patterns – Bragg's Law

- Incident light enters crystal and rebounds at an angle, forming distinct diffraction points
- Treat lattice planes of crystal as diffraction slits (or as thin film, at an angle)



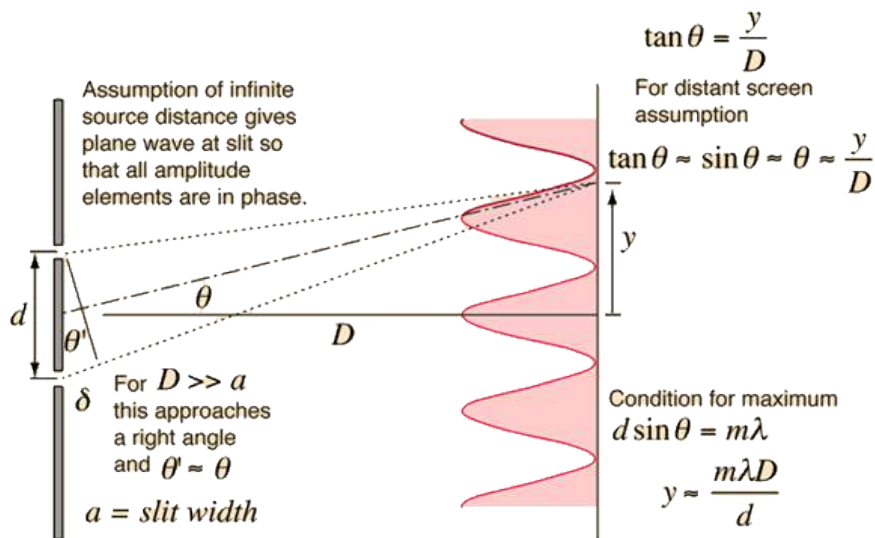
91. When a narrow beam of monoenergetic electrons impinges on the surface of a single metal crystal at an angle of 30 degrees with the plane of the surface, first-order reflection is observed. If the spacing of the reflecting crystal planes is known from x-ray measurements to be 3 ångstroms, the speed of the electrons is most nearly

- (A)  $1.4 \times 10^{-4}$  m/s
- (B) 2.4 m/s
- (C)  $5.0 \times 10^3$  m/s
- (D)  $2.4 \times 10^6$  m/s
- (E)  $4.5 \times 10^9$  m/s

- (In practice, this becomes a 3D problem)

# Diffraction patterns – Double Slit

- Path difference:
  - Look at difference between light passing through two slits
  - Condition on maxima becomes  $d \sin \theta = n\lambda$



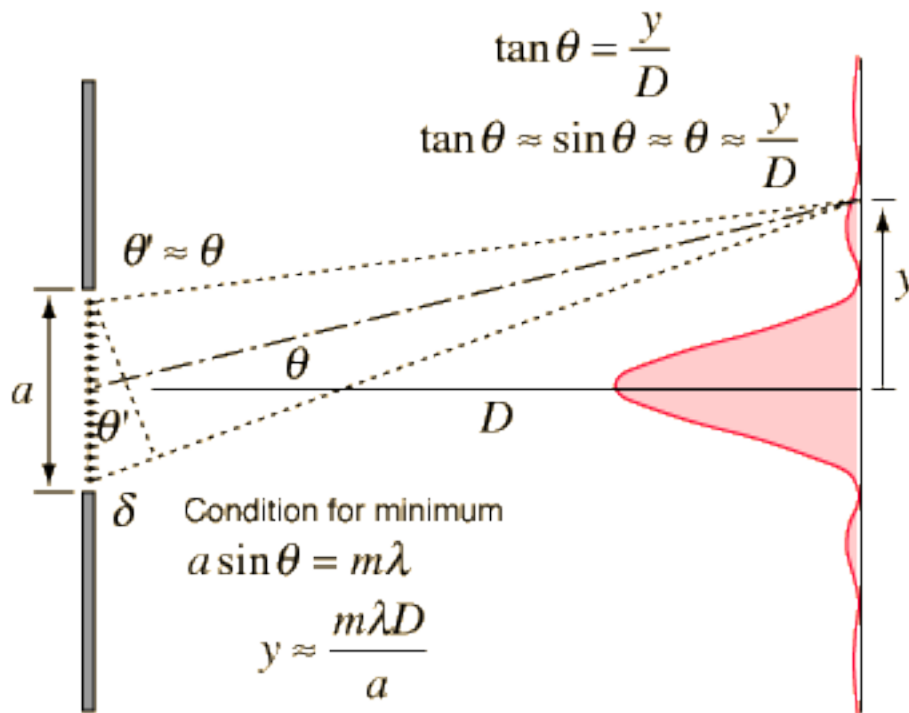
70. Light from a laser falls on a pair of very narrow slits separated by 0.5 micrometer, and bright fringes separated by 1.0 millimeter are observed on a distant screen. If the frequency of the laser light is doubled, what will be the separation of the bright fringes?

- (A) 0.25 mm
- (B) 0.5 mm
- (C) 1.0 mm
- (D) 2.0 mm
- (E) 2.5 mm

- Note: Actual diffraction pattern intensity will be bounded by single-slit interference pattern (see next slide)

# Interference: Single Slit Diffraction

- Derivation is a bit trickier, involves dividing wide slit into  $N$  very narrow slits and taking  $N$  to infinity
- Diffraction pattern goes as  $\sin(\theta)/(\theta)$  with large peak in center
  - Condition on minima becomes:  $a \sin \theta = m\lambda$
  - Similar to double slit condition for maxima (easy to get confused)

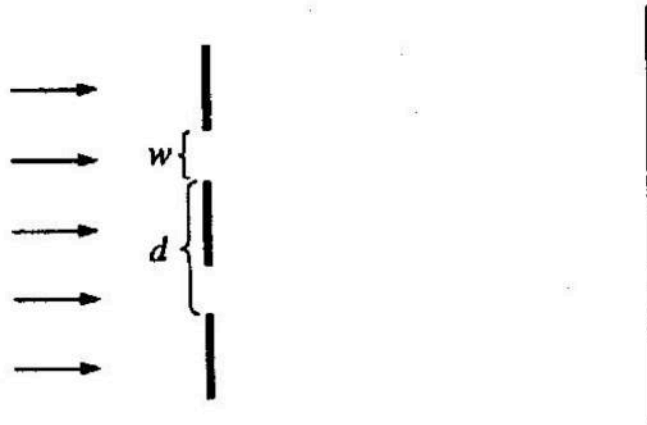


57. Consider a single-slit diffraction pattern for a slit of width  $d$ . It is observed that for light of wavelength 400 nanometers, the angle between the first minimum and the central maximum is  $4 \times 10^{-3}$  radians. The value of  $d$  is

- (A)  $1 \times 10^{-5}$  m
- (B)  $5 \times 10^{-5}$  m
- (C)  $1 \times 10^{-4}$  m
- (D)  $2 \times 10^{-4}$  m
- (E)  $1 \times 10^{-3}$  m



# Diffraction patterns: example



20. In a double-slit interference experiment,  $d$  is the distance between the centers of the slits and  $w$  is the width of each slit, as shown in the figure above. For incident plane waves, an interference maximum on a distant screen will be “missing” when

- (A)  $d = \sqrt{2}w$
- (B)  $d = \sqrt{3}w$
- (C)  $2d = w$
- (D)  $2d = 3w$
- (E)  $3d = 2w$

- When does a minimum from the single slit cancel out a maximum from the double slit?
- Single-slit condition for minimum is same as double-slit condition for maximum

$$\frac{\sin(\theta)}{\lambda} = \frac{m}{w} = \frac{n}{d}$$

- Which answers are plausible
- Which are not?

# Interference and Diffraction: Summary

- Wave model of light
- **Finding path differences**
- Thin film interference
- Bragg scattering condition
  - Crystallography
- Double-slit diffraction
- Single-slit interference
  - How single-slit differs from double-slit
- Additional concepts and equations:
  - Interferometry

# Properties of the Wave Equation

# Wave Equation

- Used to describe:
  - Sound waves in fluids
  - Vibrations in solids
  - Electromagnetic waves (solving Maxwell's equations in vacuum)

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

- Solutions in one dimension for a single Fourier mode:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \Rightarrow x(t) = A e^{i(kx - \omega t)}, \quad c = \omega/k$$

- A depends on boundary conditions
  - In practice, full solutions are superpositions of many modes
- In higher dimensions, solutions can be longitudinal (sound) or transverse (light) or both (earthquakes)

# Sound Wave Velocity and Power

- Rule of thumb for velocities

$$v \propto \sqrt{\text{Restoring force} / \text{Inertia of medium}}$$

- Examples:
  - Sound: Restoring force is pressure, Inertia is density
  - On string: Restoring force is tension, Inertia is linear density
- Derivation of wave power derivation

$$P = F_{rest.} \cdot v_{trans} = T \frac{\partial z}{\partial x} \frac{\partial z}{\partial t} \propto \mu v \omega^2 A^2$$

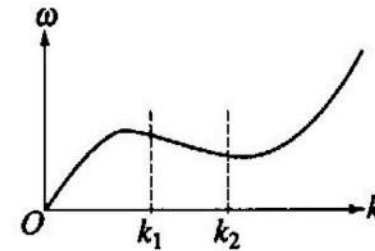
(Given power P, tension T, density  $\mu$ )

- Linear energy density  $u = P/v$
- Potential energy from stressing the medium
- Kinetic energy from motion of medium

# Group Velocity vs. Phase Velocity

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \Rightarrow x(t) = Ae^{i(kx - \omega t)}, \quad c = \omega/k$$

- $c = \omega/k$  is speed of the wave in the medium  $\Rightarrow$  Phase Velocity
  - Speed at which wave packet travels
- $v = d\omega/dk$  is called the Group Velocity
  - Given a superposition of waves, how does the phase change
  - Can be negative in some regimes
- $\omega(k)$  is the dispersion relation, depends on the medium



79. The dispersion curve shown above relates the angular frequency  $\omega$  to the wave number  $k$ . For waves with wave numbers lying in the range  $k_1 < k < k_2$ , which of the following is true of the phase velocity and the group velocity?

- (A) They are in opposite directions.
- (B) They are in the same direction and the phase velocity is larger.
- (C) They are in the same direction and the group velocity is larger.
- (D) The phase velocity is infinite and the group velocity is finite.
- (E) They are the same in direction and magnitude.

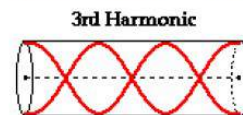
Excellent demo:

[http://en.wikipedia.org/wiki/Phase\\_velocity](http://en.wikipedia.org/wiki/Phase_velocity)

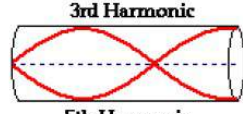
# Standing wave boundary conditions

- Mechanical wave solutions depend on response of medium
- At a hard boundary, amplitude must go to zero
- At an open boundary, amplitude is allowed to be a maximum

## Open at Both Ends



## Closed at One End



Harmonic	Wavelength $\lambda$	Frequency $f$
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1 <sup>st</sup>	$2L$	$f_1$
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2 <sup>nd</sup>	$L$	$2f_1$
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3 <sup>rd</sup>	$2L/3$	$3f_1$
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Odd and Even Harmonics

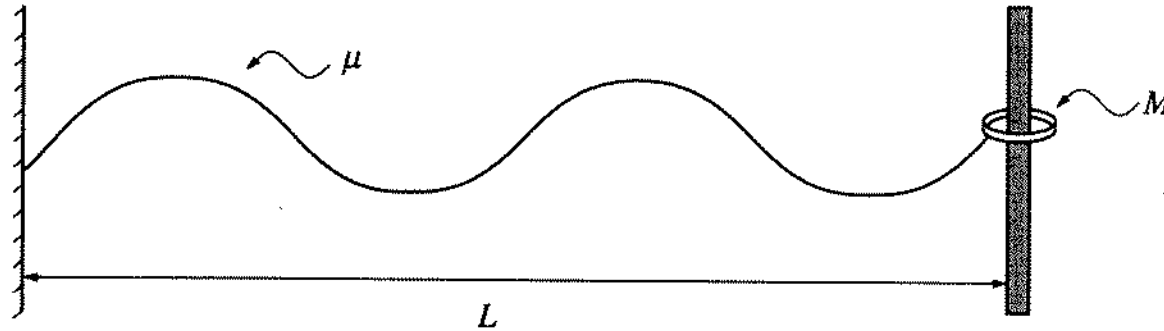
1 <sup>st</sup>	$4L$	$f_1$
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3 <sup>rd</sup>	$4L/3$	$3f_1$
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5 <sup>th</sup>	$4L/5$	$5f_1$
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Odd Harmonics

# Waves – Example problem



85. Small-amplitude standing waves of wavelength  $\lambda$  occur on a string with tension  $T$ , mass per unit length  $\mu$ , and length  $L$ . One end of the string is fixed and the other end is attached to a ring of mass  $M$  that slides on a frictionless rod, as shown in the figure above. When gravity is neglected, which of the following conditions correctly determines the wavelength? (You might want to consider the limiting cases  $M \rightarrow 0$  and  $M \rightarrow \infty$ .)

(A)  $\mu/M = \frac{2\pi}{\lambda} \cot \frac{2\pi L}{\lambda}$

(B)  $\mu/M = \frac{2\pi}{\lambda} \tan \frac{2\pi L}{\lambda}$

(C)  $\mu/M = \frac{2\pi}{\lambda} \sin \frac{2\pi L}{\lambda}$

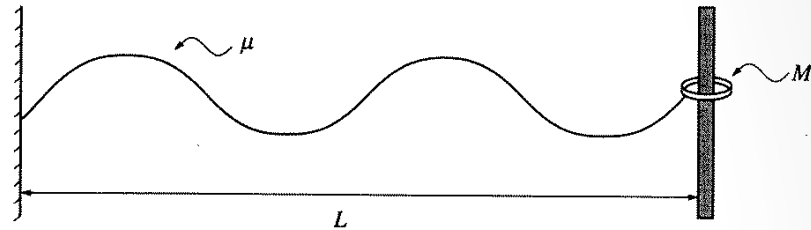
(D)  $\lambda = 2L/n$ ,  $n = 1, 2, 3, \dots$

(E)  $\lambda = 2L/(n + \frac{1}{2})$ ,  $n = 1, 2, 3, \dots$



# Waves – Example problem

- Treat waves as standing wave
- Take limits!
- How does the boundary change when...
  - $M \rightarrow \infty$
  - $M \rightarrow 0$
- So, what wavelengths are and are not allowed?



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(D)  $\lambda = 2L/n, \quad n = 1, 2, 3, \dots$

(E)  $\lambda = 2L/(n + \frac{1}{2}), \quad n = 1, 2, 3, \dots$

# Waves – Interfaces

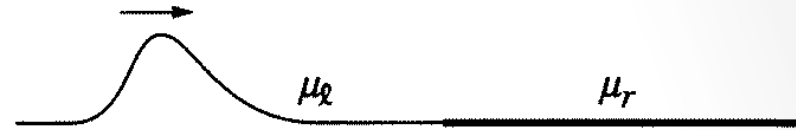
- What does a wave do when it hits a boundary between media?
  - Incoming wave from medium 1 enters medium 2
  - See also Quantum mechanical scattering from a potential barrier
- Part is reflected, part is transmitted
- Given Incident, Transmitted, and Reflected waves, match the conditions at the boundary
- Use  $n$ , the index of refraction where  $v = c/n$ 
  - Large  $n$  means stiff medium, slow wave velocity
- Obtain equations for Amplitudes ( $A$ ) for the three waves:

$$A_R = \frac{n_1 - n_2}{n_1 + n_2} A_I, \quad A_T = \frac{2n_2}{n_1 + n_2} A_I$$

- Observe:
  - $A_R < 0$  if  $n_1 < n_2$ ,  $\pi$  phase difference added if medium 2 is stiffer
  - $A_R = 0$  if  $n_1 = n_2$ , media are identical
  - $A_T > 0$  always
  - $A_T \rightarrow 0$  for  $n_1 \gg n_2$
  - The more similar the medium, the more is transferred

# Waves at Interfaces Example

- Answer should depend on the properties of the media
- What is the sign of the amplitude of the transmitted wave?
- Take limits: what happens when  $\mu_L = \mu_R$ , or  $\mu_L \ll \mu_R$  ?



80. A string consists of two parts attached at  $x = 0$ . The right part of the string ( $x > 0$ ) has mass  $\mu_r$  per unit length and the left part of the string ( $x < 0$ ) has mass  $\mu_l$  per unit length. The string tension is  $T$ . If a wave of unit amplitude travels along the left part of the string, as shown in the figure above, what is the amplitude of the wave that is transmitted to the right part of the string?

(A) 1

(B)  $\frac{2}{1 + \sqrt{\mu_l/\mu_r}}$

(C)  $\frac{2\sqrt{\mu_l/\mu_r}}{1 + \sqrt{\mu_l/\mu_r}}$

(D)  $\frac{\sqrt{\mu_l/\mu_r} - 1}{\sqrt{\mu_l/\mu_r} + 1}$

(E) 0

# Wave Equation: Summary

- The wave equation and its solutions
  - Group velocity vs. phase velocity
- Boundaries and interfaces
  - Important to keep in mind medium properties
- Other concepts:
  - Doppler shift (sound waves and relativistic shift)

$$f' = \frac{v - v_{\text{object}}}{v - v_{\text{source}}} f \quad \text{-or-} \quad \sqrt{\frac{1 + \beta}{1 - \beta}} f$$

- Beats

$$f_{\text{beats}} = \frac{1}{2} |f_1 - f_2|$$

- Aperture formula

$$\theta \cong 1.22 \lambda / d$$