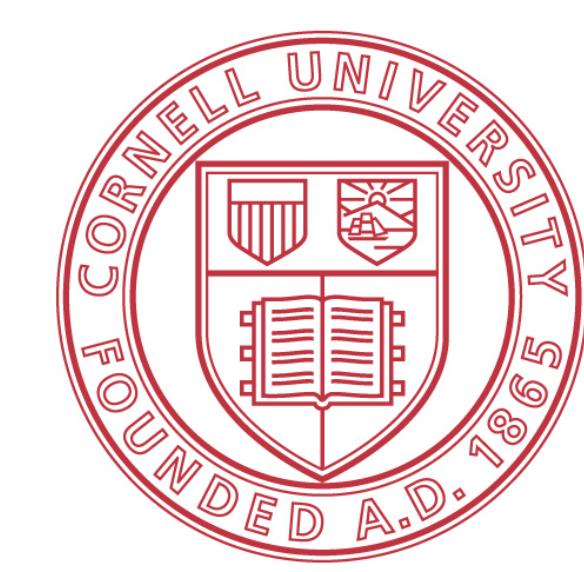


ACCOUNTING FOR FLUCTUATIONS IN STOCHASTIC SIRS MODEL ON NETWORKS



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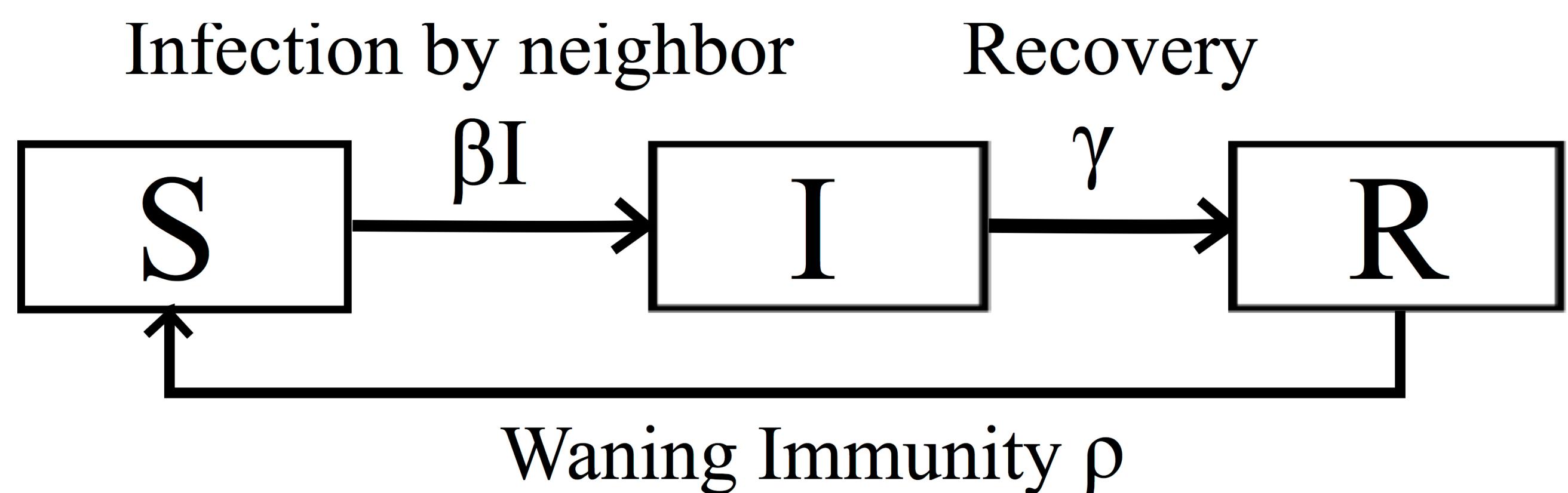
INTRODUCTION

Here we present our analysis of the **stochastic Susceptible-Infected-Recovered-Susceptible (SIRS)** model of infectious disease dynamics on heterogeneous networks. We perform a moment closure analysis to obtain **approximate analytical predictions for the magnitude of fluctuations in the endemic state**. We use the heterogeneous mean field (HMF) to approximate dynamics on a network with degree heterogeneity. This can be used to show how **stochastic fluctuations and spontaneous extinction depend on the size and heterogeneity of the full network**. We present results from our simulations to demonstrate the accuracy and utility of our predictions.

SIRS MODEL OVERVIEW

Waning Immunity

Infection spreads through contact; infected hosts develop immunity; recovered individuals lose immunity over time [1]



Similarity to other models

- SIRS \rightarrow SIR as $\rho \rightarrow 0$ — Disease dies out after initial outbreak
- SIRS \rightarrow SIS as $\rho \rightarrow \infty$ — Persistent endemic disease
- When does endemic disease persist in the SIRS model?

On Networks

- Each node represents a single individual.
- Each node is in one of three states (S, I, R)
- Network defines contacts between individuals
- Network structure accounts for contact heterogeneity [2]

HETEROGENEOUS MEAN FIELD THEORY

- Assume no degree correlations in network
- Partition graph into degree classes, classes couple together
- Account for degree heterogeneity in the mean field

$$\frac{dS_k}{dt} = \underbrace{-\beta S_k k \Theta_k}_{\text{Infection}} + \rho(1 - S_k - I_k), \quad \frac{dI_k}{dt} = \underbrace{\beta S_k k \Theta_k}_{\text{Infection}} - \underbrace{\gamma I_k}_{\text{Recovery}}$$

$$\Theta_k \equiv \sum_{k'} \mathbb{P}(k'|k) I_{k'}, \quad \text{Force of infection between degree classes } k \text{ & } k'$$

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MOMENT CLOSURE

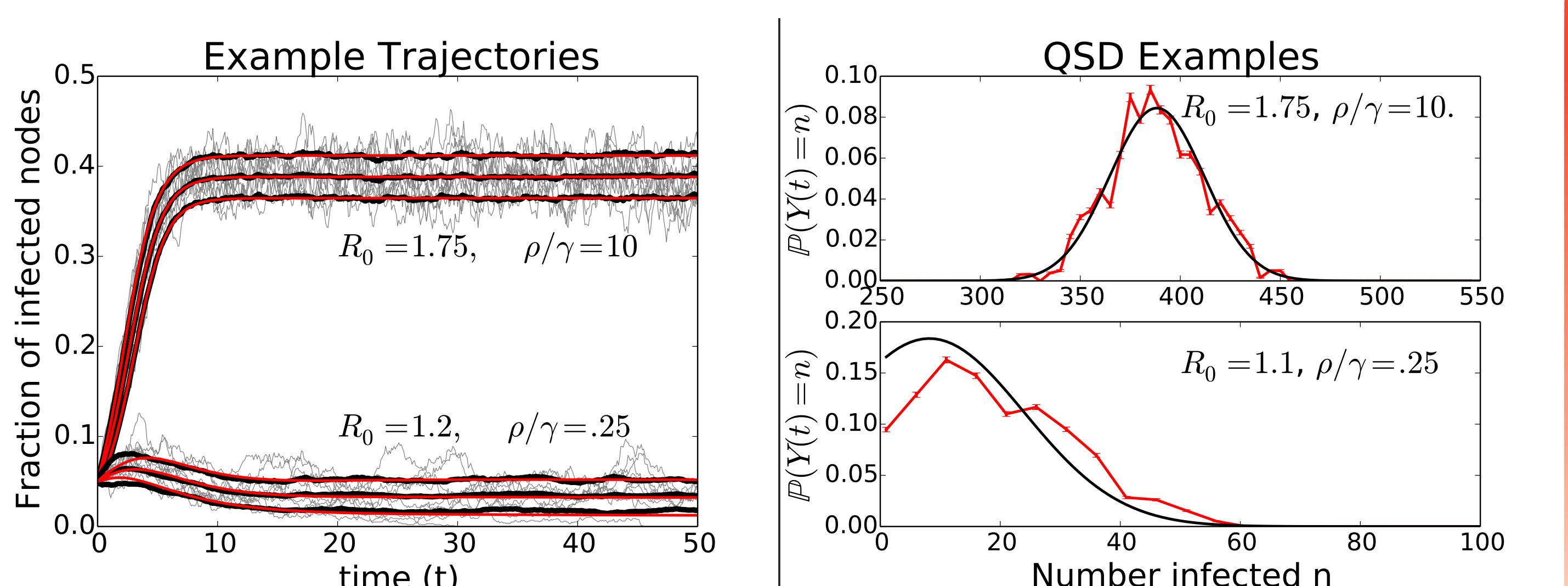
We obtain fluctuation magnitudes by solving for the **quasi-stationary distribution (QSD)** of the number of Infecteds in the population. We use **Gaussian moment-closure** to approximately solve for the QSD.

1. Derive Kolmogorov forward equation (KFE) for the model.
2. Condition on no extinction (> 0 infected individuals, not reaching the absorbing state) to obtain KFE for QSD [3].
3. Obtain PDE for the Probability Generating Function for the QSD.
4. Assume rate of extinction is small, ignore nonlinear sink term [4].
5. Obtain PDE for the Cumulant Generating Function K .
6. Assume Gaussian form of QSD, quadratic K :

$$K(\theta, \phi, t) \approx \mu_x \theta + \mu_y \phi + \sigma_{xy} \theta \phi + \frac{1}{2} \sigma_x^2 \theta^2 + \frac{1}{2} \sigma_y^2 \phi^2 \dots$$

7. Collect terms in series expansion to derive coupled ODEs for all first and second moments of the QSD (means $\{\mu\}$ and variances $\{\sigma^2\}$).
8. Numerically integrate ODEs for $\{\mu\}$ and $\{\sigma^2\}$ as functions of time.
9. Can look for **stationary behavior of QSD, $\{\mu\}$, and $\{\sigma^2\}$** .
10. **Knowing the variances $\{\sigma^2\}$ characterizes the magnitude of stochastic fluctuations.** With this technique, we can account for both finite population size effects and node degree heterogeneity.

QUANTIFYING STOCHASTIC FLUCTUATIONS



Examples of stochastic trajectories in grey, μ_y and $\mu_y \pm \sigma$ in black, moment closure prediction in red. Accurate for small **rate of extinction (y-intercept)**; QSD peaked away from 0 [3, 4].

REFERENCES AND ACKNOWLEDGEMENTS

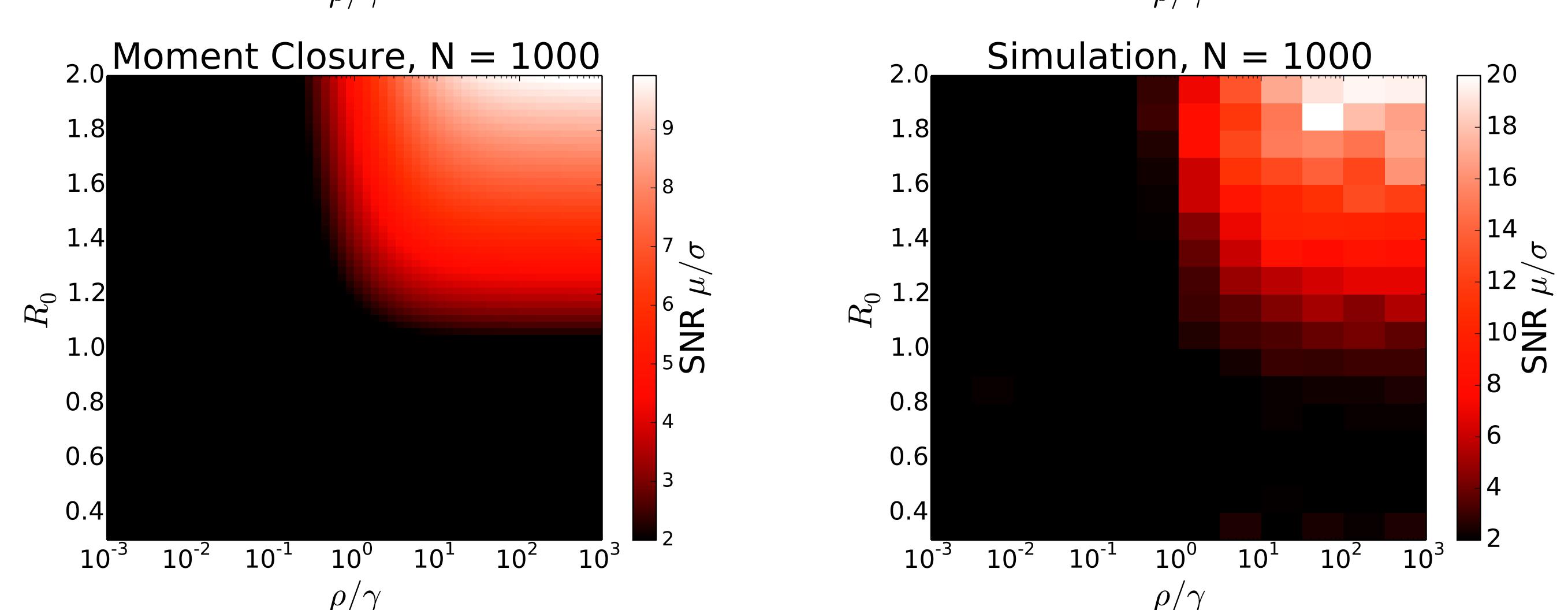
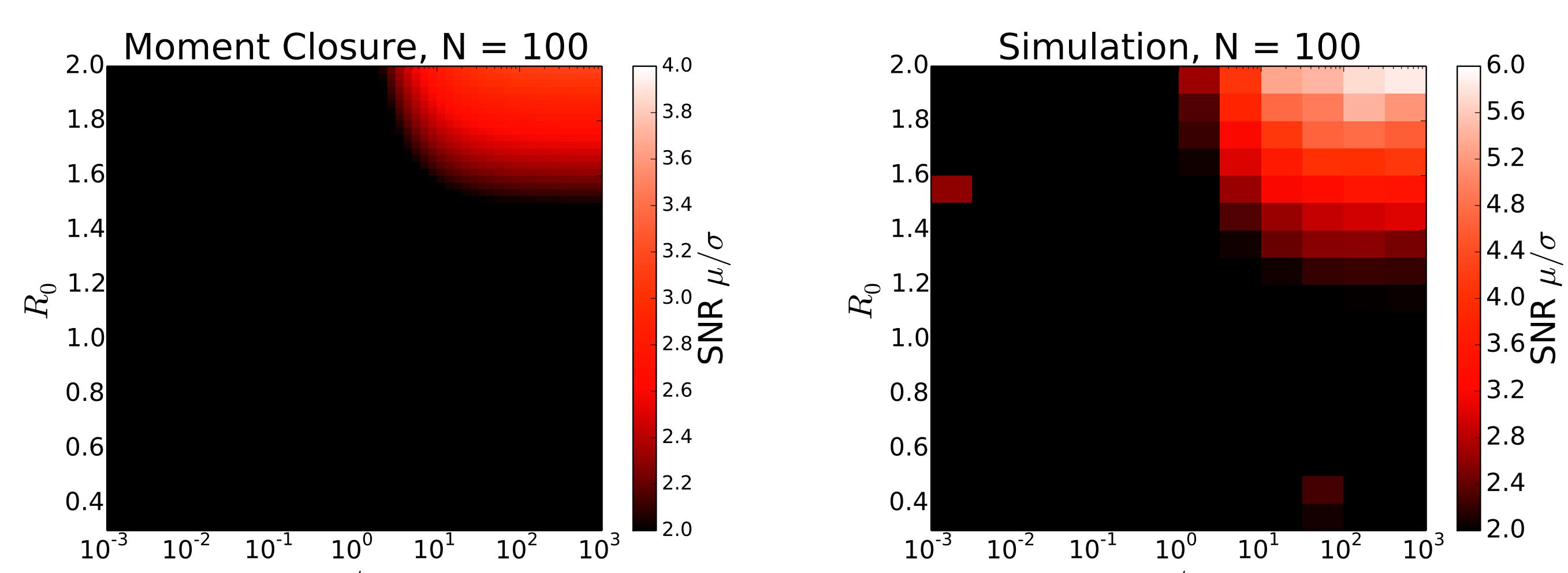
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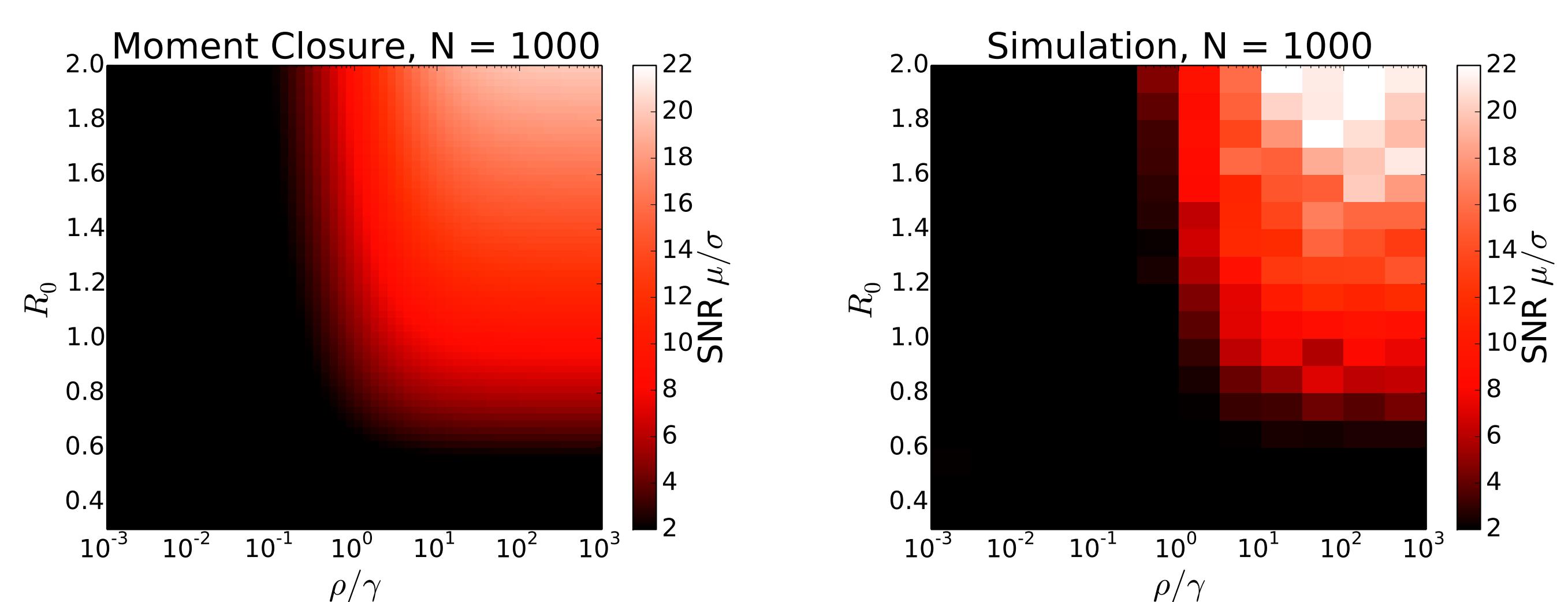
RESULTS

The network can support endemic disease ($\mu_y > 0$) when $R_0 \equiv \beta \langle k \rangle / \gamma$ and ρ/γ are sufficiently large. We plot the **signal to noise ratio (SNR)** of the number of infected individuals (μ_y/σ_y) to show persistence of endemic disease. Network properties also affect endemic disease:

Population Size Predicted and simulated SNR μ_y/σ_y for $G(n, p)$ -type graphs of two different sizes:



Degree Heterogeneity Predicted and simulated SNR μ_y/σ_y for a graph with a skewed degree distribution ($\langle k \rangle^2 / \langle k^2 \rangle = .44$):



FUTURE WORK

- Measure how extinction rates are affected by both finite size and degree heterogeneity in graphs with simulations.
- Examine role of low-degree nodes: moment closure predicts they *decrease* fluctuations in graphs with skewed degree distributions.
- Find a perturbative correction to the initial approximation that accommodates cases where rate of extinction is not small.
- Are two degree classes sufficient to capture network heterogeneity?