NEWTON - RAPHSON

$$0 = f_1(x_1, \ldots, x_N)$$

$$\vdots$$

$$0 = f_N(x_1, \ldots, x_N)$$

TAYLOR SERIES

$$f_i(x_1, \ldots x_N) = f_i(x_1^*, \ldots x_N^*)$$

$$+ (x_1 - x_1^*) \left.\frac{\partial f_i}{\partial x_1}\right|_{\vec{x}^*}$$

$$+ (x_2 - x_2^*) \left.\frac{\partial f_i}{\partial x_2}\right|_{\vec{x}^*}$$

$$\vdots$$

$$+ (x_N - x_N^*) \left.\frac{\partial f_i}{\partial x_N}\right|_{\vec{x}^*}$$

$$\underbrace{f_i(x_1 \cdots x_N)}_{②} = f_i(x_1^* \cdots x_N^*) + \underbrace{\sum_{j=1}^{N} (x_j - x_j^*) \frac{\partial f_i}{\partial x_j}}_{①}\Bigg|_{\vec{x}^*} + \cdots$$

①
$$\sum_{j=1}^{N} \frac{\partial f_i}{\partial x_j}(x_j - x_j^*) = \underbrace{\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_N} \\ \vdots & & & \\ \frac{\partial f_N}{\partial x_1} & \cdots & & \frac{\partial f_N}{\partial x_N} \end{bmatrix}}_{\bar{J} \text{ JACOBIAN MATRIX}} \begin{bmatrix} x_1 - x_1^* \\ \vdots \\ x_N - x_N^* \end{bmatrix}$$

②
$$\begin{pmatrix} f_1(\vec{x}) \\ f_2(\vec{x}) \\ \vdots \\ f_N(\vec{x}) \end{pmatrix}$$

$$\vec{f}(\vec{x}) = \vec{f}(\vec{x}^*) + \bar{J}\cdot(\vec{x} - \vec{x}^*)$$

WHERE $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix}$ $\qquad J_{ij} = \left.\dfrac{df_i}{dx_j}\right|_{\vec{x}^*}$

$$\Delta x = \frac{-f(x_0)}{f'(x_0)}$$

- IF $\vec{\Delta x} = (\vec{x}^* - \vec{x})$ IS STEP TOWARD SOLUTION

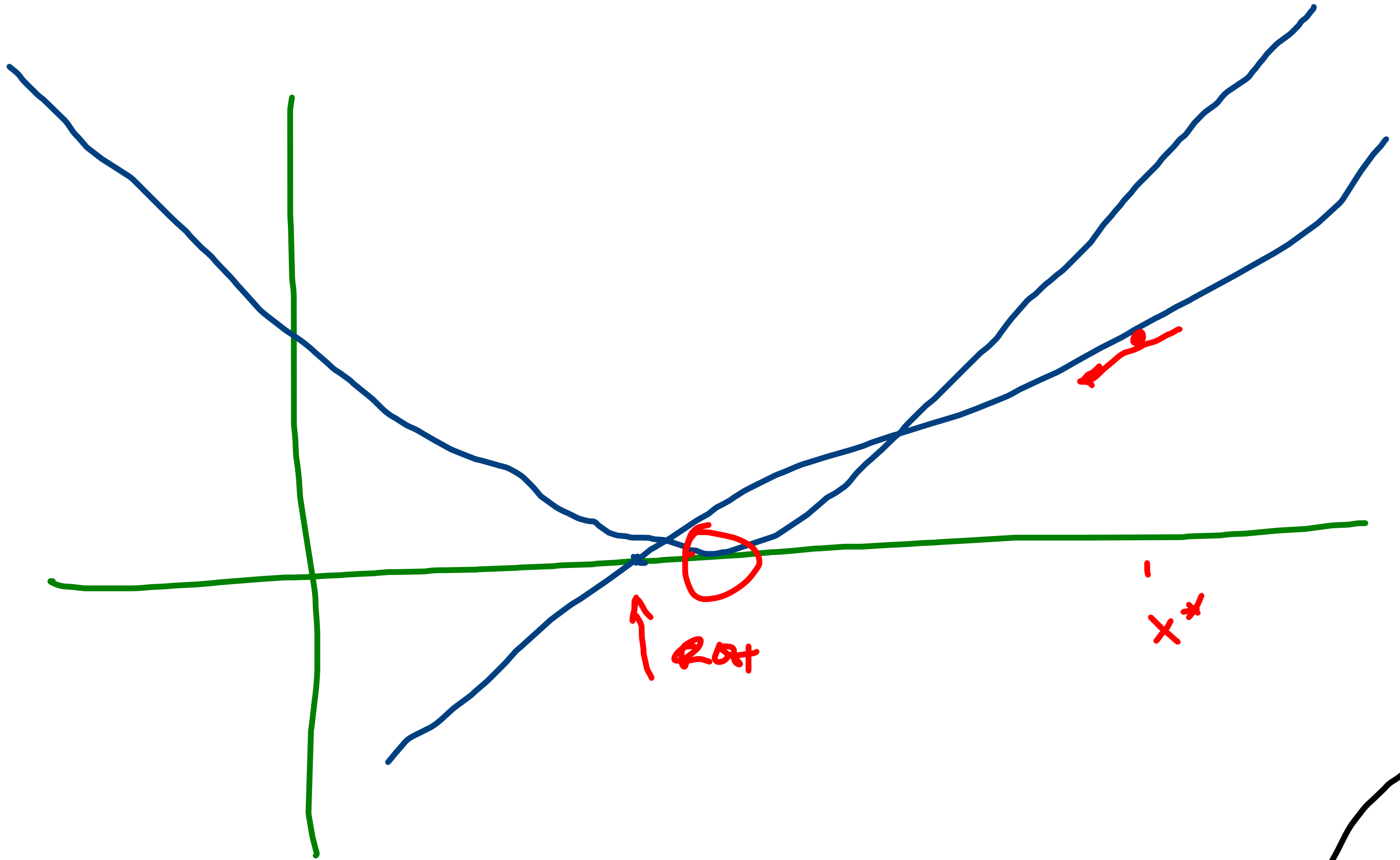
- IF $\vec{x}$ IS SOLUTION $\therefore$ LHS $= 0$

$$\boxed{\vec{f}(\vec{x}^*) = \bar{J}\cdot\vec{\Delta x}}$$

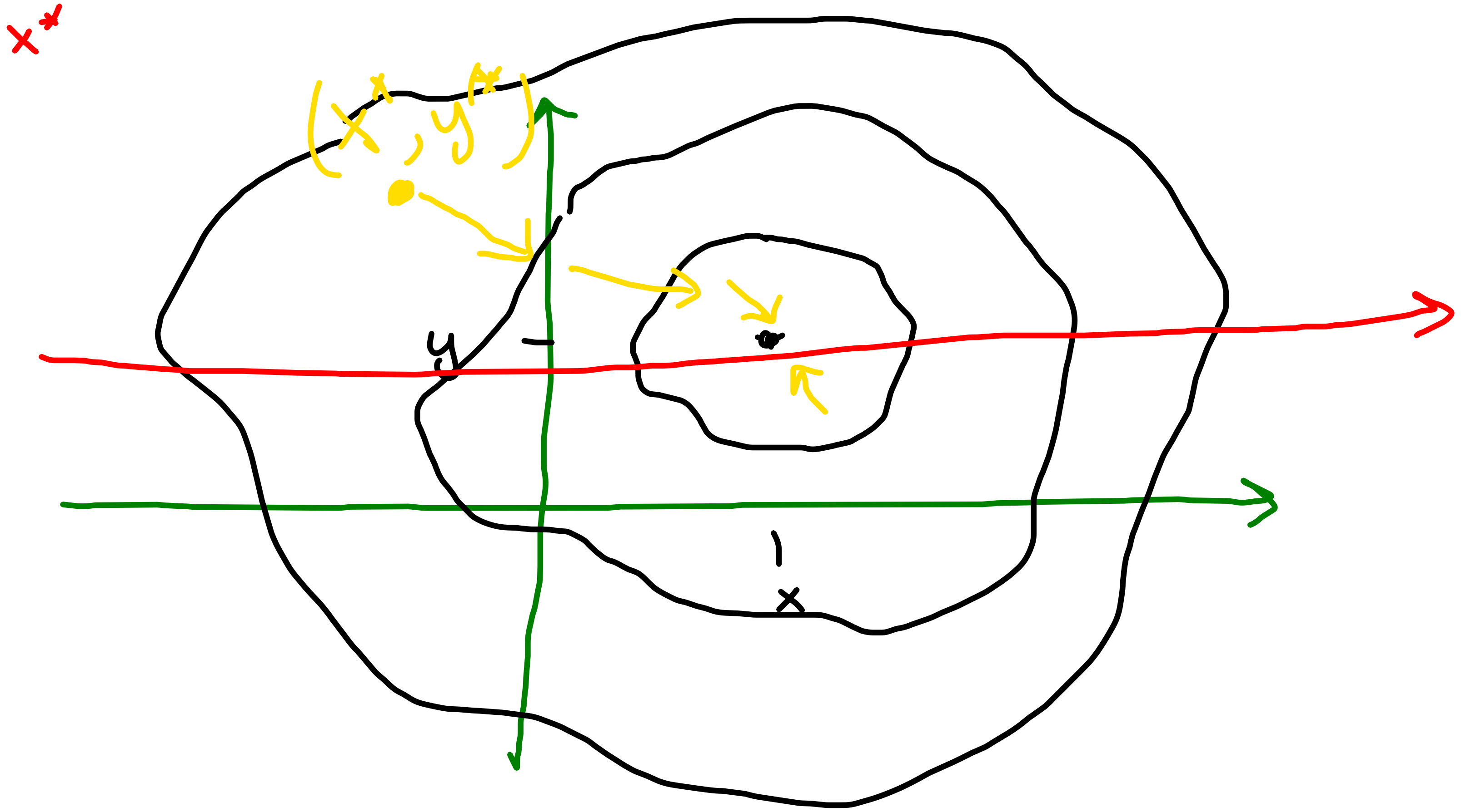
$$\vec{b} = \bar{A}\,\vec{x}$$

$$\vec{\Delta x} = ? = \boxed{\bar{J}^{-1}\cdot\vec{f}(\vec{x}^*) = \Delta x}$$

0. INITIAL GUESS $\vec{x}^*$
1. CALCULATE $\vec{f}(\vec{x}^*)$ AND $\bar{J}\big|_{\vec{x}^*}$
2. SOLVE $\bar{J}\,\vec{\Delta x} = \vec{f}$ FOR $\vec{\Delta x}$
3. $\vec{x}' = \vec{x}^* - \vec{\Delta x}$



$$\Delta x = -\frac{f(x^*)}{f'(x^*)}$$

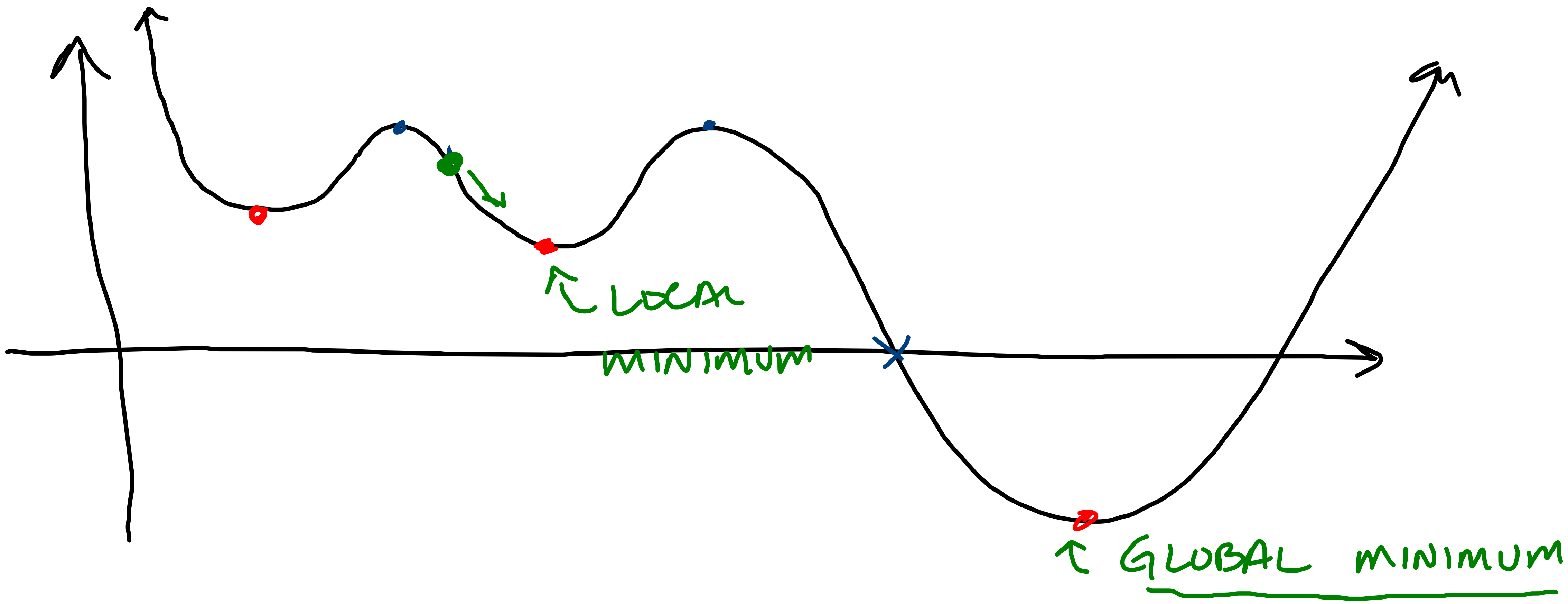


EXTREMUM OF FUNCTION

(OPTIMIZATION)

IF $\vec{x}$ SATISFIES $\vec{f}(\vec{x}) = 0$

$\vec{x}$ IS AN EXTREMUM OF $\vec{g}(\vec{x})$ IF $\vec{\nabla}_{\vec{x}} \vec{g} = \vec{f}$



① GOOD GUESS REQUIRED