Sol Key

Q2 ① (i) $(X_c^T X_c)^T = X_c^T (X_c^T)^T = X_c^T X_c$. Hence $X_c^T X_c$ is symmetric

① (ii) $X_c^T X_c = (U \Sigma V^T)^T U \Sigma V^T$

$= V \Sigma^T U^T U \Sigma V^T$ ($\because U^T U = I$ and $\Sigma^T = \Sigma$, Σ is diagonal matrix)

$= V \Sigma^2 V^T$

Hence V contains eigen vector of $X_c^T X_c$

② As per calculation in part (ii) Σ^2 contains eigen values of $X_c^T X_c$

Q2 (i) $K(x, x') = K_1(x, x') + K_2(x, x') \geq 0$ because each term is bigger than 0

(ii) symmetric $K(x, x') = K_1(x, x') + K_2(x, x')$

$= K_1(x', x) + K_2(x', x)$ $\because K_1, K_2$ are symmetric

$= K(x', x)$

(iii) let $z \neq 0 \in \mathbb{R}^d$

let $K_{N\times N}$, $K_{1\,N\times N}$, $K_{2\,N\times N}$ be the Gram matrix of K, K_1, K_2 for any N.

clearly $K_{N\times N} = K_{1\,N\times N} + K_{2\,N\times N}$

because any entry in matrix $K_{N\times N}$

$K(x_i, x_j) = K_1(x_i, x_j) + K_2(x_i, x_j)$

We need to show $K_{N\times N}$ is Positive definite

Then $z^T (K) z = z^T (K_1 + K_2) z$

$= \underbrace{z^T K_1 z}_{>0} + \underbrace{z^T K_2 z}_{>0}$

> 0 for any $z \neq 0$.

Q3 RBF Kernel is a mercer Kernel, Hence there exists a mapping ϕ such that

For $x, x' \in \mathcal{X}$

$$\phi^T(x)\,\phi(x') = \exp\left(-\frac{1}{2}\frac{\|x-x'\|_2^2}{6}\right)$$

then For point x

$$\phi^T(x)\,\phi^T(x) = \exp\left(-\frac{1}{2}\frac{\|x-x\|_2^2}{6}\right) = \exp(0) = 1$$

$$\|\phi^T(x)\|_2^2 = 1$$

Hence points are mapped to unit hyper sphere.