The Causal Budget Law: Causal Time As Emergent

Dickson Terrero

Independent Researcher in Physics and Mathematics

Abstract

We present a framework in which time is not fundamental but an *emergent property* of constrained energy transformation. Every system possesses a finite **causal capacity** c, allocated between external motion (v_{ext}) and internal transformation (v_{int}) via $c^2 = v_{\text{ext}}^2 + v_{\text{int}}^2$.

For massive systems, the core operational relation is $\lambda = E v_{\text{int}}/P$, where λ represents the system's internal causal stride per transformation tick. A system's **causal time** is defined as

$$\tau = \frac{E}{P}\,\gamma, \qquad \gamma = \frac{1}{\sqrt{1 - v_{\text{ext}}^2/c^2}},$$

corresponding to the duration of a single internal tick, physically determined by the allocation of causal capacity. Motion diverts capacity from internal transformation, *slowing the system's evolution* and dilating causal time in accordance with the causal-budget constraint.

Massless carriers, such as photons, do not experience internal transformation ($v_{\text{int}} = 0$). Their causal time is operationally zero along lightlike trajectories ($\tau = 0$), while they still mediate causal interactions among massive systems. This treatment aligns with the special-relativistic notion that proper time along a lightlike path is zero without implying any "internal duration" for the photon.

Relativistic time dilation emerges naturally from this causal-budget mechanism, not as a geometric effect. Time is therefore a localized, system-specific measure of transformation, experienced only by systems with $v_{\text{int}} > 0$. The framework provides a mechanistic foundation for relativistic phenomena and offers a universal anchor across quantum, thermodynamic, and classical domains, without invoking spacetime geometry as a fundamental entity.

Keywords: time, proper time, causal structure, physical transformation, relational ontology, emergence of time

1. Introduction

The nature of time remains one of the most enduring problems in physics and philosophy [12]. Classical physics treated time as an absolute backdrop [1], relativity reinterpreted it as a coordinate within spacetime geometry [3, 2], and thermodynamics associated its arrow with entropy production [4, 6]. Yet all these frameworks leave open the fundamental question: *what is time physically?*

We adopt a system-centric, *process-based* perspective. Time is not an external parameter but a measurable consequence of physical transformation. It exists only insofar as energy is applied and irreversibly processed through resistance. When a system drives energy E through its intrinsic opposition at finite throughput P, it advances its internal state by one *causal tick* of duration

$$\tau = \frac{\gamma E}{P},$$

where γ encodes the allocation of causal capacity between internal transformation and external motion. For systems at rest with no external motion, this reduces to the invariant rest-frame tick period $\tau_0 = E/P$.

Each tick corresponds to a fundamental **internal causal stride** $\lambda_0 = c\tau_0$, representing the proper-frame spatial advance associated with a transformation. This quantity is **distinct from observed wavelengths**: the latter are subject to relativistic Doppler effects depending on the observer and motion direction. For massless carriers (photons), the formal internal speed $v_{\text{int}} \to 0$, but the operational causal stride remains well-defined as $L = c\tau_0$, providing a universal anchor for empirical measurement.

In general, a system's finite **causal budget** is shared between internal transformation and external motion:

$$v_{\text{ext}}^2 + v_{\text{int}}^2 = c^2,$$

with

$$\lambda = \frac{E v_{\text{int}}}{P}$$

defining the internal causal stride in the moving frame. Time emerges directly from physical change, not as a geometric dimension or observer-dependent coordinate. Laboratory measurements of wavelengths and clock rates reflect the proper-frame causal ticks, revealing time as a localized, emergent property of systems undergoing transformation.

2. First-Principles Foundation of the Causal Budget Law

Notation. Throughout this work symbols such as τ and Δt denote *causal time*: the duration associated with a system's internal causal transformation sequence (see discussion in Sec. 3).

Numerically these coincide with relativistic proper-time for massive systems but are here interpreted as an intrinsic, operational record of transformations.

2.1. Empirical Necessity of the Causal Budget

The generalized transformation relation (see Sec. 3.1 for a derivation from Cs-133 atomic clocks establishing the empirical basis for this relation),

$$\lambda = \frac{E\, v_{\text{int}}}{P}, \tag{1}$$

provides a measurable spatial extent of a system's internal transformation per causal tick for systems that possess an internal-update channel.

For massless carriers (photons) the observed relation $\lambda = c\,E/P$ is exactly satisfied. In the present framework this reflects maximal *external* throughput: the causal effect is carried entirely by propagation at $v_{\text{ext}} = c$, so the separate internal-update channel vanishes ($v_{\text{int}} \to 0$) for strictly massless carriers. The generalized form (1) applies directly to systems with internal update rates; the photon case is the limiting lightlike propagation case and is correctly described by $\lambda = c\tau_0 = cE/P$.

For massive systems, however, maintaining $v_{\text{int}} = c$ while in motion would produce wavelength and timing predictions that contradict experiment. Empirically, internal transformation rates are therefore suppressed by motion ($v_{\text{int}} < c$), indicating a reallocation of causal capacity between internal transformation and external motion.

Thus a finite, conserved causal capacity is not merely an axiom but an *empirical necessity*: moving massive systems cannot simultaneously sustain maximal internal-update rates and satisfy observed spatial/temporal relations.

Operationally, the intrinsic duration per causal transformation (the causal-tick $\tau_0 = E/P$) increases when the system moves so that laboratory observers record fewer completed transformations per laboratory second. This objective slowing of internal evolution is anchored in experiment (for example, GPS orbital clock corrections provide an accessible empirical benchmark).

2.2. Conservation and Boundary Conditions

All systems possess a single finite causal throughput that drives both internal transformation and external translation. Conservation of this throughput imposes fundamental boundary conditions:

- **Rest condition** ($v_{\text{ext}} = 0$): All causal capacity is available for internal transformation ($v_{\text{int}} = c$ for systems with internal channels).

- **Lightlike condition** ($v_{\text{ext}} \to c$): Causal capacity is devoted to propagation; the internal update channel vanishes ($v_{\text{int}} \to 0$).

- **Continuity**: Reallocation between these extremes is smooth and monotonic.

These empirical observations (internal transformation slows with motion and vanishes for lightlike propagation) naturally motivate the quadratic causal-budget law:

$$v_{\text{int}}^2 + v_{\text{ext}}^2 = c^2, \tag{2}$$

which ensures proper saturation of both boundaries and preserves the total causal throughput.

2.3. Quadratic Causal-Budget Law

The simplest and unique relation that satisfies the boundary conditions while conserving total causal capacity is

$$c^2 = v_{\text{int}}^2 + v_{\text{ext}}^2, \tag{3}$$

where c denotes the total causal capacity scale. This relation sets the stage for the geometric representation (see Fig. 1) and underpins subsequent derivations of relativistic effects.

Uniqueness of the quadratic form. Alternative combinations fail essential requirements:

- **Linear addition** ($v_{\text{int}} + v_{\text{ext}} = c$) violates symmetry and smooth boundary behaviour.

- **Other functional forms** do not preserve the independence/orthogonality properties expected of separate causal channels.

Quadrature addition guarantees:

- Proper saturation at both boundary conditions,

- Equal treatment of independent causal channels,

- Continuous, monotonic reallocation.

This structure mirrors other contexts where independent contributions conserve a total magnitude (e.g., apparent power $S^2 = P^2 + Q^2$, uncertainty addition $\sigma^2 = \sigma_1^2 + \sigma_2^2$, and the energy–momentum relation $E^2 = (pc)^2 + (mc^2)^2$). The causal budget extends that principle to fundamental causal allocation.

2.4. Physical Interpretation

Equation (3) expresses conservation of finite causal capacity c between two channels:

- **External translation** (v_{ext}): spatial propagation of the system's state,
- **Internal transformation** (v_{int}): rate of physical state update and evolution.

Consequences:

- **Massive systems:** require $v_{\text{int}} > 0$, which prevents $v_{\text{ext}} = c$.
- **Massless carriers:** operate with $v_{\text{int}} = 0$, enabling $v_{\text{ext}} = c$.
- **Speed limit:** arises as a resource constraint on causal allocation rather than a primary geometric prohibition.

The causal-budget law is therefore empirically motivated, mathematically constrained, and physically inevitable within this operational framework.

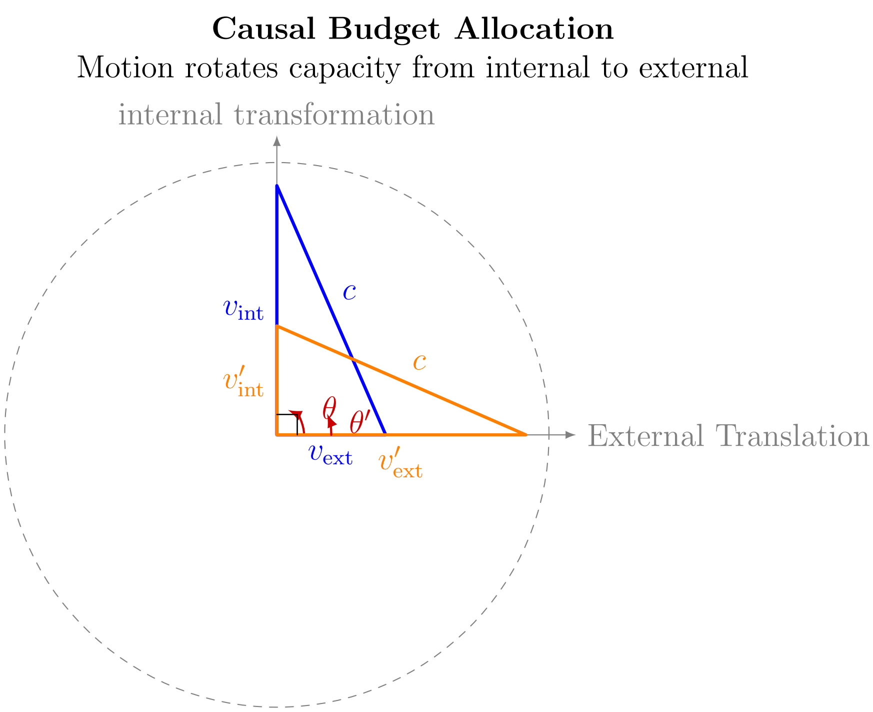

Figure 1: Causal budget allocation: The total causal capacity c (hypotenuse) is fixed. External motion "rotates" this capacity vector, trading internal transformation rate v_{int} for external translation speed v_{ext}. The angle θ parameterizes this trade-off: $\cos\theta = v_{\text{ext}}/c$, $\sin\theta = v_{\text{int}}/c$.

3. The Causal Nature of Wavelength and Causal Time

Notation. Throughout this work, symbols such as τ and Δt denote *causal time*: the duration associated with a system's internal causal transformation sequence. Numerically, these coincide with relativistic proper time for massive systems, but here they are interpreted operationally as an intrinsic record of completed internal transformations.

This section establishes the core physical principles of the framework. We derive the transformation extent λ and causal time τ, present an empirical relation between wavelength, energy, and power, and show how these lead naturally to the concepts of transformation extent, causal throughput, and causal time as a dynamical variable.

3.1. Derivation of the Transformation Extent

We begin with the empirical foundation of the Cs-133 atomic clock, which provides a precise connection between temporal periods and spatial extents through causal transformation.

Unified transformation extent. A single formal expression for the transformation extent applies to all systems:

$$\lambda = \frac{Ev}{P}, \tag{4}$$

where the effective causal rate v depends on the system type:

$$v = \begin{cases} v_{\text{int}}, & M > 0 \quad \text{(massive system)}, \\ v_{\text{ext}}, & M = 0 \quad \text{(massless carrier)}. \end{cases} \tag{5}$$

Physical irreversibility of the channel boundary. The distinction between internal transformation (v_{int}) for massive systems and external propagation (v_{ext}) for massless carriers reflects a fundamental physical boundary, not merely a mathematical convenience. A system with nonzero rest mass cannot be continuously deformed into a genuine massless carrier: approaching the lightlike limit requires reallocating the entire finite causal capacity to external motion, driving the allocation factor γ toward divergence. This would demand unbounded energy to maintain nonzero internal transformation rates. Consequently, the regimes $M > 0$ (internal dynamics) and $M = 0$ (pure propagation) are separated by an operational singularity. The piecewise channel rule in Eq. (5) therefore captures physically distinct modes of causal operation, avoiding ad-hoc regularizers that lack physical justification.

- For massive systems, v_{int} describes the rate of **internal state advancement**, i.e., the intrinsic causal transformations occurring within the system. Even seemingly inert objects (e.g., rocks) have a very small but nonzero $v_{\text{int}} = P/E$, producing a minimal causal advance per tick.

- For massless carriers (e.g., photons), v_{ext} governs **causal propagation through free space**, while $v_{\text{int}} = 0$ (any nonzero regularization would be purely computational, not physical). The same formula $\lambda = Ev/P$ applies, but now the relevant channel is external propagation.

This formulation preserves a **single structural relation** for all systems while clearly distinguishing the operative causal channel in each case. It ensures that derivations of causal time, the Lorentz factor, and wavelength contraction remain consistent for massive systems, without conflating them with the behavior of massless carriers.

3.1.1. Empirical anchor: massless causal channel (photon)

For the Cs-133 hyperfine transition:

- Each causal tick corresponds to emission of a photon with energy $E = hf$.
- The rest-frame tick period is $\tau_0 = 1/f$.
- The power throughput is $P = E/\tau_0$.

For the photon, $v = v_{\text{ext}} = c$, giving

$$\lambda = c\,\tau_0 = \frac{cE}{P}, \tag{6}$$

which defines the propagation extent of one causal tick.

3.1.2. Empirical extension: massive causal channel

For a massive system such as the Cs-133 atom, causal evolution proceeds through internal transformation at rate v_{int}. The generalized transformation extent is

$$\lambda = \frac{E\,v_{\text{int}}}{P}. \tag{7}$$

At rest ($v_{\text{ext}} = 0$), empirical consistency requires $v_{\text{int}} = c$, yielding

$$\lambda_0 = \frac{E\,c}{P} = c\tau_0,$$

which coincides with the photon propagation extent.

Continuity across channels. Although Eq. (5) distinguishes the causal channels discretely, empirical continuity is preserved at the transition: both massive and massless regimes yield the same measurable relation $\lambda_0 = c\tau_0$. This continuity refers to observable quantities, not to a continuous physical deformation between massive and massless states. The two channels remain operationally distinct, each governed by its own allocation of causal capacity.

Physical interpretation. The channel dependence reflects a fundamental physical distinction: massive systems *experience* time through internal transformation, while massless systems *mediate* causal relations through propagation without internal duration.

3.2. Derivation of the causal-time formula from empirical anchors

Having established the transformation extent λ in Sec. 3.1, we now derive the causal time τ using two empirically anchored rest-frame quantities: the transformation extent λ_0 (from radiative clocks such as Cs-133) and the rest-frame tick period τ_0.

1. **Empirical rest-frame anchors.**

$$\lambda_0 = \frac{cE}{P}, \qquad \tau_0 = \frac{E}{P},$$

   where E is the irreversible energy per tick and P the corresponding throughput *for the same transformation channel*, ensuring consistency between spatial and temporal measures.

2. **Generalized transformation extent.** For a massive system (see Sec. 3.1) moving at speed v_{ext}, such as the Cs-133 atom, causal evolution proceeds through internal transformation at rate v_{int}:

$$\lambda = \frac{E v_{\text{int}}}{P}. \tag{8}$$

   (a) **Minimal internal transformation in inert objects:** Even seemingly inert objects, such as a rock, undergo minimal internal causal transformations due to microscopic energy exchanges, resulting in a very small $v_{\text{int}} = P/E$ and a negligible but nonzero causal advance per tick[1].

   (b) **Velocity-dependent suppression:** v_{int} naturally emerges from empirically observed reductions in wavelength with system motion, reflecting the reallocation of finite causal capacity: as external motion increases ($v_{\text{ext}} > 0$), internal trans-

[1]For systems without obvious cyclical processes, a "tick" corresponds to the completion of a minimal, system-specific transformation of energy within its microstates. Any measurable change in internal configuration that contributes to causal throughput defines a tick.

formation is partially suppressed, enforcing a smaller λ in accordance with the causal-budget constraint.

(c) **Timelessness of massless carriers:** For massless carriers like photons, the internal transformation rate is strictly zero ($v_{\text{int}} = 0$). In this case, the standard formula $\tau = cE/(Pv_{\text{int}})$ is **not defined**, since there is no internal transformation channel. Operationally, massless carriers do not experience internal causal ticks, and their "causal time" along their trajectory is therefore taken to be zero ($\tau = 0$). These systems still mediate causal interactions among massive systems, transferring energy and information along their trajectories. This aligns with the special relativistic notion that proper time along a lightlike path vanishes, without implying any internal duration for the photon itself. In other words, $\tau = 0$ reflects the absence of internal transformation, not a contradiction of the general causal-time formula.

3. **Causal relation between space and time (derived).** Using the definitions above,

$$\tau = \frac{cE}{Pv_{\text{int}}}, \qquad \lambda = \frac{Ev_{\text{int}}}{P},$$

we immediately obtain the invariant causal product

$$\frac{\tau}{c}\,\lambda \;=\; \tau_0^2, \tag{9}$$

expressing a fundamental link between spatial and temporal measures for the same irreversible transformation. This follows directly from finite causal-capacity allocation rather than being postulated.

4. **Solve for causal time.** Rearranging (9) gives

$$\tau \;=\; \frac{c\,\tau_0^2}{\lambda} \;=\; \frac{cE}{Pv_{\text{int}}}. \tag{10}$$

5. **Express via the causal-budget / allocation factor.**

(a) Start from the causal-budget law:

$$v_{\text{int}}^2 + v_{\text{ext}}^2 = c^2.$$

(b) Define the allocation factor γ via internal speed:

$$v_{\text{int}} = \frac{c}{\gamma}.$$

(c) This implies the standard form:

$$\gamma = \frac{c}{v_{\text{int}}} = \frac{1}{\sqrt{1 - v_{\text{ext}}^2/c^2}}.$$

This identifies γ (Sec. 3.3) with the familiar Lorentz factor as a derived consequence of causal-capacity conservation.

(d) Substitute $v_{\text{int}} = c/\gamma$ into (10):

$$\tau = \frac{cE}{Pv_{\text{int}}} = \tau_0\,\gamma = \frac{E}{P}\,\gamma.$$

Conclusion. The moving-system causal time τ emerges directly from rest-frame anchors (λ_0, τ_0), the generalized transformation extent $\lambda = Ev_{\text{int}}/P$, and the causal-budget allocation. Hence, τ is *derived*, not postulated, representing the motion-dependent duration of a causal tick:

$$\tau \;=\; \frac{cE}{Pv_{\text{int}}} \;=\; \tau_0\gamma. \tag{11}$$

Thus, causal time τ is not a geometric postulate but a physically measurable duration determined by the system's internal transformation capacity, linking empirical energy throughput to the familiar relativistic dilation factor.

The rest-frame causal tick period τ_0 and transformation extent λ_0 are empirically anchored (Sec. 4) across atomic, molecular, and astrophysical systems.

Massive and massless time concepts. For massive systems, causal time is defined as

$$\tau = \frac{cE}{P\,v_{\text{int}}},$$

representing the duration of internal transformation per causal cycle. For massless carriers, the corresponding quantity is the *propagation time*

$$\tau_{\text{prop}} = \frac{\lambda}{c} = \frac{E}{P},$$

which measures the coordinate time required for causal influence to traverse one wavelength.

3.3. Lorentz factor from causal budget

Operational note: Standard relativistic notation is used here as a bridge to experiment; spacetime geometry is not assumed to be fundamental.

A system moving at speed v_{ext} advances internally by

$$L_{\text{int}} = v_{\text{int}}\,\Delta t_{\text{lab}} = \sqrt{c^2 - v_{\text{ext}}^2}\,\Delta t_{\text{lab}}$$

during a laboratory interval Δt_{lab}, using the causal-budget law $v_{\text{int}}^2 + v_{\text{ext}}^2 = c^2$.

Process completion invariance. The same physical transformation must advance by the same fundamental amount in any frame. Therefore, the internal advancement L_{int} in the laboratory frame must equal the rest-frame transformation extent:

$$L_{\text{int}} = \lambda_0 = c\,\tau_0.$$

Equating the two gives

$$c\,\tau_0 = v_{\text{int}}\,\Delta t_{\text{lab}} \quad \Rightarrow \quad \Delta t_{\text{lab}} = \frac{c}{v_{\text{int}}}\,\tau_0 = \gamma\,\tau_0,$$

where

$$\gamma = \frac{1}{\sqrt{1 - v_{\text{ext}}^2/c^2}} = \frac{c}{v_{\text{int}}}.$$

Thus, the Lorentz factor emerges naturally as an allocation factor, quantifying how motion reduces the internal transformation rate. Time dilation is a mechanistic consequence of causal-capacity reallocation: laboratory-measured intervals lengthen because internal processes physically slow, producing fewer causal ticks per unit laboratory time.

3.3.1. Physical interpretation

When $v_{\text{ext}} > 0$, causal capacity is reallocated from internal transformation to external motion, reducing v_{int}. Observable consequences are:

- **Transformation extent contracts:**

$$\lambda = v_{\text{int}}\,\tau_0 = \lambda_0\sqrt{1 - v_{\text{ext}}^2/c^2}.$$

- **Laboratory time dilates:**

$$\Delta t_{\text{lab}} = \gamma\,\tau_0.$$

Here, τ_0 is the invariant rest-frame causal-tick cost, and $\tau = \gamma\tau_0$ is the dilated causal-tick duration when the system moves.

3.4. Operational translation and experimental signature

To bridge to conventional measurements, we distinguish:

- **Invariant rest-frame tick period:** $\tau_0 = E/P$.
- **Moving-system causal time:** $\tau = \tau_0 \gamma$.
- **Laboratory coordinate interval:** $\Delta t_{\text{lab}} = \gamma\, \tau_0$.

In this operational framework $\Delta t_{\text{lab}} = \tau$, i.e., laboratory measurements record the same dilated causal-tick duration. The effect is experimentally manifest (GPS orbital-clock corrections provide a direct, quantitative benchmark).

3.5. Empirical foundation: the wavelength–energy–power relation

Across radiative systems the empirical pattern

$$\lambda = \frac{cE}{P}$$

holds with high fidelity, where E is the energy irreversibly transformed per cycle and P the power throughput. This motivates the generalized relation (7) with v_{int} the causal-throughput rate; the radiative case is the special limit $v_{\text{int}} = 0$ (external propagation) producing $\lambda = c\tau_0$.

3.6. Causal time as transformation duration

Causal time is the operational duration of a single internal transformation (a causal tick). The causal-time definition adopted here is

$$\tau = \tau_0 \gamma = \frac{cE}{P v_{\text{int}}},$$

which treats causal time as a derived, system-specific physical quantity (set by E, P, v_{int}) rather than an a priori geometric invariant.

3.7. Summary

Motion reallocates finite causal capacity from internal transformation toward external translation, reducing v_{int}, contracting the transformation extent λ, and objectively dilating the internal tick period τ. Laboratory measurements remain numerically consistent with the Lorentz factor ($\Delta t_{\text{lab}} = \gamma \tau_0$), while the causal-throughput perspective supplies a mechanistic origin for time dilation.

Connection to Lorentz factor derivation. The causal time τ derived here (Sec. 3.2) equals the laboratory interval Δt_{lab} derived in Section 3.3, confirming the internal consistency of the framework.

This derivation demonstrates that time dilation emerges from causal capacity reallocation rather than geometric postulates, providing a mechanistic explanation for relativistic effects.

Table 1: Operational time-parameter definitions (foundational).

Symbol	Definition
τ_0	Invariant rest-frame causal-tick period: $\tau_0 = E_{\mathrm{prop}}/P_{\mathrm{prop}}$.
λ_0	Rest-frame transformation extent: $\lambda_0 = c\,\tau_0$.
λ	Moving-system transformation extent: $\lambda = v_{\mathrm{int}}\,\tau_0 = \lambda_0\sqrt{1 - v_{\mathrm{ext}}^2/c^2}$.
Δt_{lab}	Laboratory interval between the same tick events: $\Delta t_{\mathrm{lab}} = \gamma\,\tau_0$.
γ	Allocation / Lorentz factor: $\gamma = 1/\sqrt{1 - v_{\mathrm{ext}}^2/c^2} = c/v_{\mathrm{int}}$.

4. Empirical Universality of the Causal Transformation Limit Principle

Having identified the *rest-frame* causal time

$$\tau_0 = \frac{E_{\mathrm{prop}}}{P_{\mathrm{prop}}} \tag{12}$$

as the intrinsic causal duration of a single transformation, we examine the universal invariant that emerges when comparing the spatial extent L of this transformation to its temporal duration τ_0:

$$\frac{L}{\tau_0}, \tag{13}$$

which quantifies how far one causal tick advances per unit causal-time cost. Empirically, this ratio is universal across all relativistic propagation channels and equals c.

4.1. Causal Throughput Law

Consider any stable system undergoing repeated intrinsic transformation cycles (e.g., atomic hyperfine transitions, molecular vibrations, mechanical resonator modes, pulsar pulses, or gravitational-wave emitters). For each channel, define:

- $E_{\mathrm{transform}}$: energy irreversibly converted per intrinsic tick (only energy leaving the mode is counted, e.g., photon energy, gravitational-wave emission, acoustic loss, spin-down energy from a pulsar).
- $P_{\mathrm{transform}}$: throughput (power) routed through that channel.

- f: intrinsic tick rate (cycles per second), coinciding with the observed transition or emission frequency.
- L: spatial transformation extent associated with one emitted cycle, operationally defined as the distance the transformation propagates during one tick:

  $$L = v \cdot \tau_0 = \frac{v}{f},$$

  where v is the propagation speed of the disturbance ($v = c$ for relativistic channels, $v =$ sound speed for mechanical modes).

By definition of power:

$$P_{\text{transform}} = \frac{E_{\text{transform}}}{\tau_0}, \qquad f = \frac{1}{\tau_0} = \frac{P_{\text{transform}}}{E_{\text{transform}}}. \tag{14}$$

This is the *causal throughput law*: the intrinsic tick rate is determined purely by the channel's capacity to drive the irreducible energy per tick through the system.

This seemingly simple relation, when applied across different physical contexts, reproduces well-known fundamental limits, as we discuss in Sec. 5.

4.2. From Causal Throughput to Time Dilation and the Lorentz Factor

The rest-frame causal propagation rate

$$c = \frac{L}{\tau_0} \tag{15}$$

sets the maximum rate at which a system can realize intrinsic transformations per unit spatial advance. For a system moving at velocity v relative to an observer, the total causal capacity c^2 is partitioned between external motion $v_{\text{ext}} = v$ and internal transformation v_{int} via

$$v_{\text{int}}^2 + v^2 = c^2. \tag{16}$$

The internal causal stride in the lab frame is therefore

$$\lambda = \frac{E v_{\text{int}}}{P}. \tag{17}$$

Using the empirical rest-frame anchor $\lambda_0 = c\tau_0 = E/P\, c$, the lab-frame causal time per tick becomes

$$\tau = \frac{\lambda_0}{v_{\text{int}}} = \frac{c\tau_0}{v_{\text{int}}}. \tag{18}$$

Substituting the causal-budget relation $v_{\rm int} = \sqrt{c^2 - v^2}$ gives

$$\tau = \frac{c\tau_0}{\sqrt{c^2 - v^2}} = \frac{\tau_0}{\sqrt{1 - v^2/c^2}} \equiv \gamma\,\tau_0, \tag{19}$$

where γ is the familiar Lorentz factor. Similarly, the lab-frame spatial stride along the motion direction contracts according to

$$L_\parallel = \frac{L_0}{\gamma}. \tag{20}$$

Thus, both time dilation and Lorentz contraction emerge directly from the empirical causal throughput limit and the conservation of total causal capacity, without requiring any additional assumptions about the constancy of the speed of light or spacetime geometry. The derivation is fully consistent and logically closed.

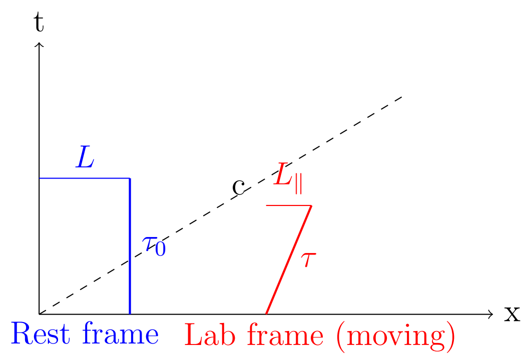

Figure 2: Visualization of intrinsic transformation ticks and spatial strides in the rest frame (blue) and moving frame (red). The dashed line represents the causal limit c.

4.3. Causal Propagation Rate and Emergence of c

Each tick imprints a spatial stride L over its causal-time duration τ_0, defining the causal propagation rate:

$$c = \frac{L}{\tau_0} = Lf = \frac{LP_{\rm transform}}{E_{\rm transform}}. \tag{21}$$

Empirical support is strongest when L and f are measured from systems independent of SI definitions:

- **Optical clocks (Sr, Yb, Al$^+$)** [13]: $Lf \simeq 3.0 \times 10^8$ m/s
- **Infrared molecular vibrations** [21]: $Lf \simeq 3.0 \times 10^8$ m/s
- **Gravitational-wave and pulsar emission** [22]: $Lf \simeq 3.0 \times 10^8$ m/s

These independent systems (spanning 15 orders of magnitude in frequency from optical transitions ($\sim 10^{15}$ Hz) to gravitational waves ($\sim 10^{2}$ Hz)) demonstrate that c is empirically invariant across vastly different physical mechanisms and energy scales.

For non-relativistic systems where $v < c$, the causal throughput law still holds locally, with the propagation speed v representing the maximum causal speed for that particular medium or interaction channel.

$$\boxed{c = \frac{\text{transformation extent per tick}}{\text{causal-time duration per tick}} = Lf = \frac{LP_{\text{transform}}}{E_{\text{transform}}}.} \tag{22}$$

Rest-frame definition of c. All definitions of the causal conversion rate c refer explicitly to rest-frame quantities. For a system in motion, laboratory-frame measurements of the transformation extent λ and tick rate f_{lab} do *not* alter c, which is invariant by construction:

$$c = \frac{\lambda_0}{\tau_0} = \frac{L_0}{\tau_0} = \frac{E}{P} v_{\text{int}}\Big|_{\text{rest}}.$$

This ensures that the causal limit remains a universal property of the system's intrinsic transformation, independent of external motion or relativistic Doppler effects.

5. Relation to Known Fundamental Limits

The causal–throughput relation,

$$f = \frac{P}{E}, \tag{23}$$

serves as a unifying generator for established physical limits across quantum, thermodynamic, and relativistic domains. This simple algebraic form subsumes three seemingly disparate fundamental bounds when constrained by:

- Quantum orthogonality $\rightarrow$ Margolus–Levitin bound
- Thermodynamic irreversibility $\rightarrow$ Landauer limit
- Relativistic causal propagation $\rightarrow$ Bremermann bound

This resource-theoretic formulation treats power, energy, and transformation rate as exchangeable physical currencies, with known limits emerging as domain-specific realizations of a single throughput principle governing all physical change.

5.1. *Quantum Limit: Margolus–Levitin Bound*

For a quantum system, the minimal evolution time between orthogonal states is bounded by:

$$\tau_{\mathrm{ML}} = \frac{\pi\hbar}{2E}, \tag{24}$$

where E is the mean energy available for state transformation. Applying the causal–throughput definition $P = E/\tau$ yields the ML-limited transformation power:

$$P_{\mathrm{ML}} = \frac{E}{\tau_{\mathrm{ML}}} = \frac{2E^2}{\pi\hbar}. \tag{25}$$

Expressed in spectroscopic variables ($E = hf$, $\hbar = h/(2\pi)$):

$$P_{\mathrm{ML}} = \frac{2(hf)^2}{\pi\,(h/(2\pi))} = \frac{2h^2f^2}{\pi} \cdot \frac{2\pi}{h} = 4hf^2.$$

Interpretation:. The quadratic scaling $P \propto hf^2$ emerges naturally, with the prefactor $C_{\mathrm{ML}} = 4$ arising from the combination of $\pi/2$ in the ML time and 2π in $\hbar$. Operationally, the causal relation provides the fundamental scaling law $P \sim hf^2$, with $O(1)$ prefactors determined by specific physical contexts. This demonstrates how quantum speed limits are particular instances of the general causal-throughput constraint.

Relation to Other Quantum Speed Limits:. The Mandelstam–Tamm bound depends on energy uncertainty ΔE rather than the mean energy, leading to a linear rate bound in ΔE. The ML bound is special because it directly maps to a quadratic power scaling when combined with $P = E/\tau$. Open or noisy systems will generally attain lower effective P; see Appendix A for open-system QSLs and decoherence effects.

5.2. *Thermodynamic and Relativistic Limits*

Landauer (Thermodynamic) Limit:. Erasure of one bit requires at least $E_{\min} = k_B T \ln 2$ of energy dissipation. If the available dissipation power is P_{diss}, the maximal irreversible operation rate is

$$f_{\max} = \frac{P_{\mathrm{diss}}}{E_{\min}}. \tag{26}$$

This is exactly the causal–throughput law with the operational assignment $E \mapsto E_{\min}$, $P \mapsto P_{\mathrm{diss}}$.

Bremermann (Relativistic/Information) Bound:. Information rate is constrained by available mass–energy and relativistic propagation. Assigning $E = mc^2$ and $P = P_{\max}$ for accessible signaling power gives

$$f_{\max} \sim \frac{P_{\max}}{mc^2},$$

showing the Bremermann bound as a relativistic instantiation of $f = P/E$.

5.3. Operational Mappings Across Limits

Limit	**Domain**	**Energy (E)**	**Power (P)**	**Max Rate**
Margolus–Levitin	Quantum	Mean energy above ground	Intrinsic processing rate	$f_{\max} \propto E/\hbar$
Landauer	Thermodynamic	Minimal bit erasure cost	Dissipation power	$f_{\max} = P/E_{\min}$
Bremermann	Relativistic	Available mass-energy	Maximal energy flux	$f_{\max} \sim P/mc^2$

Ontological Unity:. Although the operational definitions of E and P differ; quantum expectation values, minimal thermodynamic costs, relativistic mass-energy; they all reflect the same underlying causal resource: the capacity to effect fundamental transformations per causal tick. Differences are contextual, revealing a deep homology between quantum, thermodynamic, and relativistic limits as expressions of a single causal-throughput principle. This suggests that what we perceive as domain-specific "fundamental limits" may actually be different facets of a universal constraint on causal processing.

5.4. Summary: Unification of Physical Limits

The algebraic simplicity of $f = P/E$ belies its generative power: with domain-specific assignments, it naturally reproduces the Margolus–Levitin, Landauer, and Bremermann bounds. This unification suggests that what appear as distinct fundamental limits are actually manifestations of a single causal-throughput principle governing all physical transformation.

While real systems generally achieve lower effective power due to environmental decoherence and thermodynamic inefficiencies, the scaling relations remain valid. The framework thus provides not just a unified understanding of ultimate constraints across quantum, thermodynamic, and relativistic domains, but also demonstrates how these diverse limits emerge from the same underlying causal mechanics that govern causal time and relativistic phenomena.

6. Ontological Implications

1. Time is Localized. Each system evolves according to its own internal causal dynamics. There is no global or universal clock (only local durations defined by the ratio of the system's intrinsic transformation energy to its finite power throughput [9, 10]). Synchronization between systems is not absolute but emerges through causal interaction and mutual energy exchange.

2. Massless Systems are Timeless. For photons and other massless entities [7], the causal transformation rate satisfies $v_{\text{int}} = 0$, yielding $\tau = 0$. Such systems mediate causal relations among massive systems but do not themselves transform internally. They participate in the transmission of causality without experiencing duration.

3. Coordinate Time as Measurement Convention. The coordinate label t is not an independent form of time but a measurement convention (an external bookkeeping device used to order physical transformations [5]). It refers to when a physical state occurred relative to others within a single causal reality. The parameter t thus quantifies correlation between physical events, not the flow of an underlying medium.

4. Absence of Temporal Flow. Within this framework, there is no fundamental flow of time [8]. Time does not advance; it is the measured outcome of physical transformation itself. Systems evolve from one physical state to another as energy overcomes resistance, and this sequence defines what we perceive as temporal progression. The universe does not move through time (physical transformations simply occur, and their cumulative record constitutes what we describe as history).

7. Conclusion

This paper presents a fundamental redefinition of time as an emergent property arising from physical transformations, rather than as a primitive background parameter. By introducing the **Causal Budget Law**

$$c^2 = v_{\text{ext}}^2 + v_{\text{int}}^2$$

(Section 2) and the **Causal Transformation Principle**

$$\tau = \frac{\gamma E}{P} = \gamma \tau_0$$

(Section 3), we show that causal time arises from the finite causal capacity available to every physical system. Motion diverts capacity from internal transformation, producing time dilation, while the corresponding moving-frame causal stride is

$$\lambda = \tau v_{\text{int}} = \frac{\gamma E}{P} v_{\text{int}}.$$

The framework inverts the traditional logic of relativity: rather than deriving phenomena from spacetime geometry, relativistic effects such as time dilation, the universal speed limit c, and the distinction between massive and massless systems follow directly from causal principles.

Key achievements include:

1. **Empirical Universality**: Demonstrating that c appears as the invariant ratio between spatial stride and temporal duration across atomic, optical, gravitational, and mechanical systems (Section 4).
2. **Theoretical Unification**: Showing that the causal throughput relation $f = P/E$ subsumes established quantum, thermodynamic, and relativistic limits (Margolus–Levitin, Landauer, Bremermann) as special cases, as detailed in Section 5.
3. **Ontological Clarity**: Providing a physically grounded interpretation in which time is localized to systems undergoing transformation, massless entities are effectively timeless, and coordinate time functions as a measurement convention rather than a fundamental reality (Sections 3 and 3.4).

This framework resolves long-standing puzzles about time's nature while providing a fully process-based perspective on relativistic phenomena. The approach maintains empirical agreement with established physics while offering mechanistic explanations for motion-dependent effects and time dilation.

Future work could explore applications to quantum gravity, where the discrete, process-based nature of this framework may naturally address the problem of time and support unification of fundamental interactions. The causal budget approach provides a foundation for understanding reality not as things moving through time, but as transformations occurring within fixed causal constraints.

This concludes our presentation of the **Causal Budget and Transformation Laws** framework, a comprehensive, empirically grounded foundation for understanding time as emergent from the fundamental processes that constitute physical reality.

Appendix A. Open-System Quantum Speed Limits and Effective Power Reduction

The derivations in the main text assume closed, unitary evolution, where the Margolus–Levitin bound establishes the maximal transformation rate $f_{\max} = 2E/(\pi\hbar)$ and corresponding throughput $P_{\max} = 2E^2/(\pi\hbar)$. Realistic physical systems, however, are open and interact with their environments, resulting in non-unitary evolution and partial decoherence. In this regime, the achievable transformation rate is reduced according to generalized open-system Quantum Speed Limits (QSLs) [25, 26]:

$$\tau \geq \frac{\mathcal{L}(\rho_0, \rho_\tau)}{\overline{\Lambda}_\tau}, \tag{A.1}$$

where $\mathcal{L}(\rho_0, \rho_\tau)$ is the Bures angle between the initial and final states, and $\overline{\Lambda}_\tau$ is the time-averaged generator norm capturing both coherent and dissipative dynamics.

Decoherence effectively lowers $\overline{\Lambda}_\tau$, thereby increasing the minimal evolution time τ and reducing the attainable causal throughput. We capture this effect by introducing a coherence retention factor η_{coh}:

$$P_{\text{eff}} = \eta_{\text{coh}} P_{\max}, \qquad 0 < \eta_{\text{coh}} \leq 1, \tag{A.2}$$

where η_{coh} quantifies the fraction of the nominal transformation capacity preserved under system–environment interactions. Consequently, the causal–budget relation retains its general form,

$$f = \frac{P_{\text{eff}}}{E},$$

but now with a reduced effective power reflecting realistic open-system limitations.

This extension situates the causal–throughput law within the full hierarchy of quantum speed limits, linking ideal reversible dynamics to experimentally relevant open-system performance bounds. The summary of both closed- and open-system regimes mirrors the structure used for the fundamental limits in Section 5.

References

[1] I. Newton, *Philosophiae Naturalis Principia Mathematica*, Royal Society, London (1687).

[2] A. Einstein, Zur Elektrodynamik bewegter Körper, *Ann. Phys.* **17** (1905) 891–921.

[3] H. Minkowski, Raum und Zeit, *Phys. Z.* **10** (1908) 75–88.

[4] L. Boltzmann, *Lectures on Gas Theory*, University of California Press (1896).

[5] H. R. Brown, *Physical Relativity: Space-time structure from a dynamical perspective*, Oxford University Press (2005).

[6] H. Price, *Time's Arrow and Archimedes' Point: New Directions for the Physics of Time*, Oxford University Press (1996).

[7] T. Maudlin, *Remarks on the Passing of Time*, Proc. Arist. Soc. **102** (2002) 237–252.

[8] C. Callender, *There's No Time Like the Present*, in *The Ontology of Spacetime*, Elsevier (2006) 1–14.

[9] C. Rovelli, *Quantum Gravity*, Cambridge University Press (2004). [See chapters on thermal time hypothesis]

[10] G. Belot, *The Representation of Time and Change in Mechanics*, in J. Butterfield and J. Earman (eds.), *Philosophy of Physics*, North-Holland (2007) 133–227.

[11] M. Planck, On the Law of Distribution of Energy in the Normal Spectrum, *Ann. Phys.* **4** (1901) 553–563.

[12] J. Barbour, *The End of Time: The Next Revolution in Physics*, Oxford University Press (1999).

[13] A. D. Ludlow, M. M. Boyd, J. Ye, E. Peik, and P. O. Schmidt, *Optical atomic clocks*, Rev. Mod. Phys. **87** (2015) 637–701.

[14] N. Margolus and L. B. Levitin, *The maximum speed of dynamical evolution*, Physica D: Nonlinear Phenomena **120** (1998) 188–195.

[15] R. Landauer, *Irreversibility and heat generation in the computing process*, IBM Journal of Research and Development **5** (1961) 183–191.

[16] H. J. Bremermann, *Optimization through evolution and recombination*, in *Self-Organizing Systems*, (Spartan Books, 1962) 93–106.

[17] C. M. Will, *The Confrontation between General Relativity and Experiment*, Living Rev. Relativ. **17** (2014) 4.

[18] T. Jacobson, *Thermodynamics of Spacetime: The Einstein Equation of State*, Phys. Rev. Lett. **75** (1995) 1260–1263.

[19] H. I. Ewen and E. M. Purcell, *Observation of a Line in the Galactic Radio Spectrum*, Nature **168** (1951) 356.

[20] L. Essen and J. V. L. Parry, *The Caesium Resonator as a Standard of Frequency and Time*, Phil. Trans. R. Soc. Lond. A **250** (1957) 45.

[21] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), *Observation of Gravitational Waves from a Binary Black Hole Merger*, Phys. Rev. Lett. **116** (2016) 061102.

[22] R. A. Hulse and J. H. Taylor, *Discovery of a pulsar in a binary system*, Astrophys. J. **195** (1975) L51–L53.

[23] D. J. Griffiths and D. F. Schroeter, *Introduction to Quantum Mechanics*, Cambridge University Press (2018).

[24] J. R. Vig, *Introduction to Quartz Frequency Standards*, Army Research Laboratory (1999).

[25] S. Deffner and E. Lutz, "Quantum Speed Limit for Non-Markovian Dynamics," *Physical Review Letters*, **111**, 010402 (2013).

[26] I. Marvian and D. A. Lidar, "Quantum Speed Limits for Mixed States," *Physical Review Letters*, **115**, 210402 (2015).