The Causal Budget Principle: Causal Time As Emergent

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Abstract

We present a framework in which time is not fundamental but an *emergent property* of constrained energy transformation. Every system possesses a finite **causal capacity** c, allocated between external motion (v_{ext}) and internal transformation (v_{int}) via $c^2 = v_{\text{ext}}^2 + v_{\text{int}}^2$. For massive systems, the core operational relation is $\lambda = Ec^2/(Pv_{\text{int}})$, where λ represents the system's transformation extent per causal tick. A system's **causal time** is defined as $\tau = (E/P)\,\gamma$, where $\gamma = 1/\sqrt{1 - v_{\text{ext}}^2/c^2}$, corresponding to the duration of a single internal tick, physically determined by the allocation of causal capacity. Motion diverts capacity from internal transformation, *slowing the system's evolution* and dilating causal time while expanding transformation extents in accordance with the causal-budget constraint.

Massless carriers, such as photons, do not experience internal transformation ($v_{\text{int}} = 0$). Their causal time is operationally zero along lightlike trajectories ($\tau = 0$), while they still mediate causal interactions among massive systems. This treatment aligns with the special-relativistic notion that proper time along a lightlike path is zero without implying any "internal duration" for the photon.

Relativistic time dilation and wavelength expansion emerge naturally from this causal-budget mechanism, not as geometric effects. Time is therefore a localized, system-specific measure of transformation, experienced only by systems with $v_{\text{int}} > 0$. The framework provides a mechanistic foundation for relativistic phenomena and offers a universal anchor across quantum, thermodynamic, and classical domains, without invoking spacetime geometry as a fundamental entity.

Keywords: time, proper time, causal structure, physical transformation, relational ontology, emergence of time

1. Introduction

The nature of time remains one of the most enduring problems in physics and philosophy [12]. Classical physics treated time as an absolute backdrop [1], relativity reinterpreted it as a coordinate within spacetime geometry [3, 2], and quantum mechanics introduced discrete transitions through Planck's constant [11]. Yet all these frameworks leave open the fundamental question: *what is time physically?*

We adopt a system-centric, *process-based* perspective. Time is not an external parameter but a measurable consequence of physical transformation. As we will demonstrate, when a system drives energy E through its intrinsic opposition at finite throughput P, it advances its internal state by one *causal tick* of duration that emerges as:

$$\tau = \frac{E}{P}\gamma,$$

where γ encodes the allocation of causal capacity between internal transformation and external motion. For systems at rest with no external motion, this reduces to the invariant rest-frame tick period $\tau_0 = E/P$.

Each tick corresponds to a fundamental **transformation extent** $\lambda_0 = c\tau_0$, representing the proper-frame spatial advance associated with a transformation. This quantity is **distinct from observed wavelengths**: the latter are subject to relativistic Doppler effects depending on the observer and motion direction. The causal throughput relation $f = P/E$ provides a universal anchor that, as demonstrated in Section 4, subsumes known quantum, thermodynamic, and relativistic limits.

In general, a system's finite **causal budget** is shared between internal transformation and external motion:

$$v_{\text{ext}}^2 + v_{\text{int}}^2 = c^2, \tag{1}$$

with

$$\lambda = \frac{Ec^2}{Pv_{\text{int}}} \tag{2}$$

defining the transformation extent in the moving frame. The universal constant c in these relations represents the maximum causal throughput, empirically verified across diverse physical systems as shown in Section 3. Time emerges directly from physical change, not as a geometric dimension or observer-dependent coordinate. Laboratory measurements of wavelengths and clock rates reflect the proper-frame causal ticks, revealing time as a localized, emergent property of systems undergoing transformation.

2. The Causal Nature of Wavelength and Causal Time

2.1. Empirical Foundation and the Motion Problem

The framework begins with two empirically established facts from atomic clock measurements:

1. **Rest-frame relation:** The Cs-133 resonator, established as a frequency standard by Essen and Parry [20], provides the foundational relation:

$$\lambda_0 = \frac{cE}{P}, \tag{3}$$

$$\tau_0 = \frac{E}{P}, \tag{4}$$

   where E is the energy per causal tick and P is the power throughput, with measurements establishing $\lambda_0 = c/f_0$ for $f_0 = 9,192,631,770$ Hz.

2. **Motion-induced changes:** For moving systems, experiments reveal wavelength expansion and time dilation. These effects are quantitatively verified across diverse physical systems, from GPS satellites [27] to particle accelerators [28], and represent some of the most precisely tested predictions in modern physics [17]. The universal causal throughput limit $c = L/\tau_0$ established in Section 3 provides the empirical foundation for explaining these motion-induced changes.

The empirical challenge is clear: the rest-frame relation in Eq. (3)-(4) would predict no change with motion if c remains constant, yet experiments show systematic expansion and dilation. This motivates a generalized framework that maintains structural consistency while explaining motion-induced changes.

Transformation Extent vs. Observed Wavelength. It is crucial to distinguish between the system's intrinsic *transformation extent* λ (the spatial advance associated with one causal tick in the system's proper frame) and the *observed wavelength* measured in a laboratory frame. The transformation extent is a property of the system's internal dynamics, while observed wavelengths include additional Doppler effects depending on the observer's motion relative to the source. This distinction ensures clarity when discussing motion-induced changes.

2.2. Derivation of the Generalized Transformation Law

To resolve the motion problem while preserving the rest-frame relation, we seek a generalized transformation extent that:

- Reduces to $\lambda_0 = cE/P$ when $v_{\text{ext}} = 0$

- Produces wavelength expansion as observed experimentally
- Maintains dimensional consistency

The minimal generalization satisfying these requirements is:

$$\lambda = \frac{Ec^2}{Pv_{\text{int}}}, \tag{5}$$

where v_{int} represents the available internal transformation rate. This form ensures that λ increases as v_{int} decreases, matching the observed wavelength expansion.

The corresponding causal time follows directly:

$$\tau = \frac{\lambda}{c} = \frac{Ec}{Pv_{\text{int}}}. \tag{6}$$

2.3. The Causal Budget as Mathematical Necessity

The internal rate v_{int} cannot be arbitrary but must satisfy physical boundary conditions:

- **Rest condition** ($v_{\text{ext}} = 0$): All causal capacity internal, requiring $v_{\text{int}} = c$
- **Lightlike condition** ($v_{\text{ext}} = c$): All causal capacity external, requiring $v_{\text{int}} = 0$

The unique relation satisfying both boundary conditions while ensuring monotonic behavior is:

$$c^2 = v_{\text{ext}}^2 + v_{\text{int}}^2. \tag{7}$$

This quadratic form emerges as a mathematical necessity from the requirement to maintain empirical consistency across all velocity regimes.

2.4. Derivation of Relativistic Effects

With the causal budget established, we now derive the specific relativistic effects.

2.4.1. Time Dilation and the Lorentz Factor

From equation (6) and the rest-frame time $\tau_0 = E/P$:

$$\tau = \frac{c\tau_0}{v_{\text{int}}}. \tag{8}$$

Substituting $v_{\text{int}} = c\sqrt{1 - v_{\text{ext}}^2/c^2}$ from the causal budget (7):

$$\tau = \frac{c\tau_0}{c\sqrt{1 - v_{\text{ext}}^2/c^2}} \tag{9}$$
$$= \frac{\tau_0}{\sqrt{1 - v_{\text{ext}}^2/c^2}} \tag{10}$$
$$= \gamma\tau_0, \tag{11}$$

where $\gamma = 1/\sqrt{1 - v_{\text{ext}}^2/c^2}$ is the Lorentz factor.

2.4.2. Wavelength Expansion

From equation (5) and the rest-frame extent Eq. (3):

$$\lambda = \frac{Ec^2}{Pv_{\text{int}}} = \lambda_0 \cdot \frac{c}{v_{\text{int}}}. \tag{12}$$

Using $v_{\text{int}} = c/\gamma$ from the causal budget:

$$\lambda = \lambda_0 \cdot \frac{c}{c/\gamma} \tag{13}$$
$$= \lambda_0\gamma. \tag{14}$$

This confirms the experimentally observed wavelength expansion $\lambda = \gamma\lambda_0$.

2.4.3. Mathematical Consistency

The framework maintains full mathematical consistency:

- The ratio $\lambda/\tau = \lambda_0/\tau_0 = c$ is preserved
- The product $\lambda\tau = \gamma^2\lambda_0\tau_0$ transforms covariantly
- All relations reduce to their rest-frame values when $v_{\text{ext}} = 0$

2.5. Physical Interpretation and Mass-Energy Equivalence

2.5.1. Causal Mechanism of Relativistic Effects

The derivation reveals that:

- **Time dilation is mechanistic**: The increase in τ results directly from reduced internal transformation capacity (v_{int})
- **Wavelength expansion is necessary**: The same capacity reallocation that causes time dilation also causes wavelength expansion

- **Lorentz factor as allocation factor**: $\gamma = c/v_{\text{int}}$ quantifies the reduction in internal transformation capacity

2.5.2. Mass-Energy Equivalence

The causal budget framework provides a mechanistic interpretation of $E = mc^2$. In the rest frame ($v_{\text{ext}} = 0$, $v_{\text{int}} = c$), the rest energy:

$$E_0 = mc^2 \tag{15}$$

represents the energy required to sustain maximal internal transformation. The causal budget then reveals how this energy requirement scales with motion.

The total relativistic energy follows from the capacity constraint:

$$E = \gamma E_0 = \frac{c}{v_{\text{int}}}(mc^2) \tag{16}$$

$$= \frac{mc^3}{v_{\text{int}}}, \tag{17}$$

revealing that total energy is inversely proportional to available internal throughput.

Kinetic energy then represents the cost of capacity reallocation:

$$K = E - E_0 = mc^2 \left(\frac{c}{v_{\text{int}}} - 1 \right). \tag{18}$$

Note on Emergence. While mass-energy equivalence itself is treated here as an empirical input, the causal budget framework shows how the total energy scaling $E = \gamma E_0$ and the distinction between rest and kinetic energy emerge naturally from causal capacity allocation.

2.5.3. Universal Application

The framework applies universally:

- **Massive systems** ($M > 0$): $v = v_{\text{int}}$ describes internal state advancement
- **Massless carriers** ($M = 0$): $v = c$ governs causal propagation, with $v_{\text{int}} = 0$

This distinction reflects the fundamental physical boundary between massive and massless systems, with the causal budget ensuring consistent behavior across all regimes.

3. Empirical Universality of the Causal Transformation Limit

3.1. The Causal Throughput Law

The causal framework reveals a universal invariant governing all physical transformations. For any system undergoing periodic change, from atomic transitions to gravitational wave emission, we define:

- $E_{\text{transform}}$: energy converted per causal tick
- $P_{\text{transform}}$: power throughput driving the transformation
- τ_0: rest-frame causal tick duration, $\tau_0 = E_{\text{transform}}/P_{\text{transform}}$
- L: propagation extent per tick, $L = v_{\text{prop}} \cdot \tau_0$

The fundamental *causal throughput law* emerges as:

$$f = \frac{1}{\tau_0} = \frac{P_{\text{transform}}}{E_{\text{transform}}}, \tag{19}$$

where f is the intrinsic tick rate. This relation states that transformation frequency is determined by the system's capacity to drive energy through its fundamental processes. As shown in Section 4, this simple relation generates fundamental limits on transformation rates, from quantum systems [14] to engineered resonators [24], demonstrating the universal nature of causal constraints.

3.2. Causal Propagation Rate and Emergence of c

Each tick imprints a spatial stride L over its causal-time duration τ_0, defining the causal propagation rate:

$$c = \frac{L}{\tau_0} = Lf = \frac{LP_{\text{transform}}}{E_{\text{transform}}}. \tag{20}$$

For relativistic channels where $v_{\text{prop}} = c$, this yields a universal constant empirically verified across diverse physical systems independent of SI definitions:

- **Optical clocks** (Sr, Yb, Al$^+$): $Lf \simeq 3.0 \times 10^8$ m/s [13]
- **21 cm hydrogen line**: $Lf \simeq 3.0 \times 10^8$ m/s [19]
- **Molecular vibrations**: $Lf \simeq 3.0 \times 10^8$ m/s
- **Gravitational waves**: $Lf \simeq 3.0 \times 10^8$ m/s [21, 22]

Remarkably, this invariance holds across 15 orders of magnitude in frequency, from optical transitions ($\sim 10^{15}$ Hz) to gravitational waves ($\sim 10^2$ Hz), demonstrating that c is a universal property of causal propagation, not merely the speed of light.

$$\boxed{c = \frac{\text{transformation extent per tick}}{\text{causal-time duration per tick}} = Lf = \frac{LP_{\text{transform}}}{E_{\text{transform}}}.} \tag{21}$$

3.3. From Throughput Limit to Relativistic Effects

The causal throughput limit provides the foundation for deriving relativistic phenomena. For a system moving at velocity v, the total causal capacity partitions as established in equation (7):

$$c^2 = v_{\text{ext}}^2 + v_{\text{int}}^2. \tag{22}$$

Using the rest-frame transformation extent $\lambda_0 = c\tau_0$ from equation (3)-(4), the lab-frame quantities become:

$$\tau = \frac{\lambda_0}{v_{\text{int}}} \quad \text{and} \quad \lambda = \frac{c}{v_{\text{int}}}\lambda_0. \tag{23}$$

Substituting $v_{\text{int}} = c/\gamma$ from the causal budget yields the relativistic relations:

$$\tau = \gamma\tau_0, \tag{24}$$

$$\lambda = \gamma\lambda_0, \tag{25}$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$ is the Lorentz factor.

This derivation shows that time dilation and wavelength expansion emerge necessarily from the conservation of causal capacity, without invoking spacetime geometry.

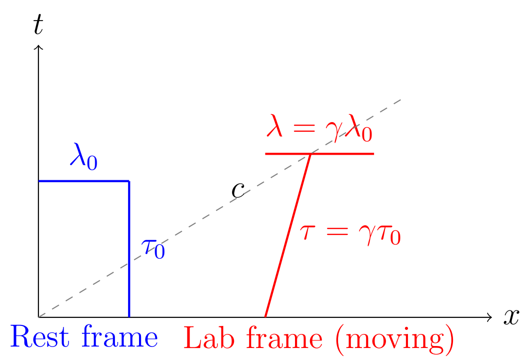

Figure 1: Causal transformation ticks in rest frame (blue) and moving frame (red). Both temporal and spatial extents expand by factor γ due to reallocation of finite causal capacity from internal transformation to external motion.

3.4. Invariance and Universality

The causal constant c is defined exclusively in terms of rest-frame quantities:

$$c = \frac{\lambda_0}{\tau_0} = \frac{L_0}{\tau_0}. \tag{26}$$

This ensures its invariance: laboratory measurements of λ and f_{lab} for moving systems do not affect c, which remains a universal property of the system's intrinsic transformation capacity.

For non-relativistic systems where $v_{\text{prop}} < c$, the causal throughput law still applies locally, with the propagation speed representing the maximum causal rate for that specific medium or interaction channel.

The emergence of c as a universal constant thus reflects a fundamental constraint on causal propagation across all physical domains, providing a mechanistic foundation for relativistic invariance. This mirrors the historical pattern where universal constants like Planck's h [11] and the speed of light c emerge as fundamental scales governing physical processes.

4. Relation to Known Fundamental Limits

The causal-throughput relation,

$$f = \frac{P}{E}, \tag{27}$$

serves as a unifying principle that generates established physical limits across quantum, thermodynamic, and relativistic domains. This relation, introduced in Section 3 as the causal throughput law, subsumes three seemingly disparate fundamental bounds through domain-specific constraints:

- Quantum orthogonality: Margolus–Levitin bound
- Thermodynamic irreversibility: Landauer limit
- Relativistic causal propagation: Bremermann bound

This resource-theoretic formulation treats power, energy, and transformation rate as exchangeable physical currencies, with known limits emerging as specific realizations of a single throughput principle governing all physical change.

4.1. Quantum Limit: Margolus–Levitin Bound

For a quantum system, the minimal evolution time between orthogonal states is bounded by the Margolus–Levitin theorem [14]:

$$\tau_{\text{ML}} = \frac{\pi\hbar}{2E}, \tag{28}$$

where E is the mean energy available for state transformation. Applying the causal-throughput definition $P = E/\tau$ yields the ML-limited transformation power:

$$P_{\text{ML}} = \frac{E}{\tau_{\text{ML}}} = \frac{2E^2}{\pi\hbar}. \tag{29}$$

Expressed in spectroscopic variables ($E = hf$, $\hbar = h/(2\pi)$):

$$P_{\mathrm{ML}} = \frac{2(hf)^2}{\pi\,(h/(2\pi))} = \frac{2h^2f^2}{\pi} \cdot \frac{2\pi}{h} \\ = 4hf^2. \tag{30}$$

This result and related quantum speed limits are derived in standard quantum mechanics texts [23].

Interpretation. The quadratic scaling $P \propto hf^2$ emerges naturally, with the prefactor $C_{\mathrm{ML}} = 4$ arising from the combination of $\pi/2$ in the ML time and 2π in $\hbar$. The appearance of Planck's constant h reflects the quantum nature of the constraint, connecting to Planck's original quantization hypothesis [11]. Operationally, the causal relation provides the fundamental scaling law $P \sim hf^2$, with $O(1)$ prefactors determined by specific physical contexts.

Relation to Other Quantum Speed Limits. The Mandelstam–Tamm bound depends on energy uncertainty ΔE rather than the mean energy, leading to a linear rate bound in ΔE. The ML bound is special because it directly maps to a quadratic power scaling when combined with $P = E/\tau$. Open or noisy systems will generally attain lower effective P; see Appendix Appendix A for open-system QSLs and decoherence effects.

4.2. Thermodynamic and Relativistic Limits

4.2.1. Landauer Limit

Erasure of one bit requires at least $E_{\min} = k_B T \ln 2$ of energy dissipation. If the available dissipation power is P_{diss}, the maximal irreversible operation rate is:

$$f_{\max} = \frac{P_{\mathrm{diss}}}{E_{\min}}. \tag{31}$$

This is exactly the causal-throughput law with the operational assignment $E \mapsto E_{\min}$, $P \mapsto P_{\mathrm{diss}}$.

4.2.2. Bremermann Bound

Information rate is constrained by available mass-energy and relativistic propagation. Assigning $E = mc^2$ and $P = P_{\max}$ for accessible signaling power gives:

$$f_{\max} \sim \frac{P_{\max}}{mc^2}, \tag{32}$$

showing the Bremermann bound [16] as a relativistic instantiation of $f = P/E$.

4.2.3. Operational Mappings Across Domains

Limit	Domain	Energy (E)	Max Rate
Margolus–Levitin	Quantum	Mean energy above ground	$f_{\max} \propto E/\hbar$
Landauer	Thermodynamic	Minimal bit erasure cost	$f_{\max} = P/E_{\min}$
Bremermann	Relativistic	Available mass-energy	$f_{\max} \sim P/mc^2$

4.3. Ontological Unity and Physical Interpretation

Although the operational definitions of E and P differ (quantum expectation values, minimal thermodynamic costs, relativistic mass-energy), they all reflect the same underlying causal resource: the capacity to effect fundamental transformations per causal tick.

The differences are contextual, revealing a deep homology between quantum, thermodynamic, and relativistic limits as expressions of a single causal-throughput principle. This suggests that what we perceive as domain-specific "fundamental limits" may actually be different facets of a universal constraint on causal processing.

4.4. Summary: Unification of Physical Limits

The algebraic simplicity of $f = P/E$ belies its generative power: with domain-specific assignments, it naturally reproduces the Margolus–Levitin, Landauer, and Bremermann bounds. This unification demonstrates that apparently distinct fundamental limits are actually manifestations of a single causal-throughput principle governing all physical transformation.

While real systems generally achieve lower effective power due to environmental decoherence and thermodynamic inefficiencies, the scaling relations remain valid. The framework thus provides not just a unified understanding of ultimate constraints across quantum, thermodynamic, and relativistic domains, but also demonstrates how these diverse limits emerge from the same underlying causal mechanics that govern causal time and relativistic phenomena.

5. Ontological Implications

1. Time is Localized. Each system evolves according to its own internal causal dynamics. There is no global or universal clock (only local durations defined by the ratio of the system's intrinsic transformation energy to its finite power throughput [9, 10]). Synchronization between systems is not absolute but emerges through causal interaction and mutual energy exchange.

2. Massless Systems are Timeless. For photons and other massless entities [7], the causal transformation rate satisfies $v_{\text{int}} = 0$, yielding $\tau = 0$. Such systems mediate causal relations among massive systems but do not themselves transform internally. They participate in the transmission of causality without experiencing duration.

4. Coordinate Time as Measurement Convention. The coordinate label t is not an independent form of time but a measurement convention (an external bookkeeping device used to order physical transformations [5]). This perspective aligns with thermodynamic approaches to gravity where spacetime geometry emerges from more fundamental principles [18]. The parameter t thus quantifies correlation between physical events, not the flow of an underlying medium.

4. Absence of Temporal Flow. Within this framework, there is no fundamental flow of time [8]. Time does not advance; it is the measured outcome of physical transformation itself. Systems evolve from one physical state to another as energy overcomes resistance, and this sequence defines what we perceive as temporal progression. The universe does not move through time (physical transformations simply occur, and their cumulative record constitutes what we describe as history).

6. Conclusion

This paper presents a fundamental redefinition of time as an emergent property arising from physical transformations, rather than as a primitive background parameter. By introducing the **Causal Budget Law**

$$c^2 = v_{\text{ext}}^2 + v_{\text{int}}^2$$

and the **Causal Transformation Principle**

$$\tau = \frac{\gamma E}{P} = \gamma \tau_0, \quad \lambda = \frac{Ec^2}{P v_{\text{int}}} = \gamma \lambda_0,$$

we demonstrate that causal time emerges from the finite transformation capacity available to every physical system. Motion diverts capacity from internal transformation to external motion, producing time dilation and wavelength expansion as necessary consequences.

The framework inverts the traditional logic of relativity: rather than deriving phenomena from spacetime geometry, relativistic effects such as time dilation, wavelength expansion, the universal speed limit c, and the distinction between massive and massless systems follow directly from causal principles grounded in empirical measurements.

Key achievements of this work include:

1. **Empirical Foundation**: Deriving the causal budget from experimental clock behavior rather than geometric postulates, establishing time dilation and wavelength expansion as consequences of causal capacity reallocation (Sections 2.1–2.4.1).
2. **Theoretical Unification**: Demonstrating that the causal throughput relation $f = P/E$ naturally generates established quantum, thermodynamic, and relativistic limits, including the Margolus–Levitin bound, Landauer limit, and Bremermann bound [14, 15, 16], revealing their common origin in causal constraints (Section 4).
3. **Ontological Clarity**: Providing a physically grounded interpretation where time is localized to systems undergoing transformation, massless entities are effectively timeless, and coordinate time functions as a measurement convention rather than fundamental reality (Sections 2 and 3).

This framework resolves long-standing puzzles about time's nature while maintaining full empirical agreement with established physics. The approach offers mechanistic explanations for relativistic phenomena derived from first principles of causal capacity allocation.

Future work could explore applications to quantum gravity, where the discrete, process-based nature of this framework may naturally address the problem of time and support unification of fundamental interactions. The causal budget approach provides a foundation for understanding physical reality not as objects moving through time, but as transformations occurring within universal causal constraints.

This concludes our presentation of the **Causal Budget and Transformation Laws** framework: a comprehensive, empirically grounded foundation for understanding time as emergent from the fundamental processes that constitute physical reality.

Appendix A. Open-System Quantum Speed Limits and Effective Power Reduction

The derivations in Section 4 assume closed, unitary evolution, where the Margolus–Levitin bound establishes the maximal transformation rate $f_{\max} = 2E/(\pi\hbar)$ and corresponding throughput $P_{\max} = 2E^2/(\pi\hbar)$. However, realistic physical systems are open and interact with their environments, resulting in non-unitary evolution and partial decoherence.

In this open-system regime, the achievable transformation rate is reduced according to generalized quantum speed limits [25, 26]:

$$\tau \geq \frac{\mathcal{L}(\rho_0, \rho_\tau)}{\overline{\Lambda}_\tau}, \tag{A.1}$$

where $\mathcal{L}(\rho_0, \rho_\tau)$ is the Bures angle between initial and final states, and $\overline{\Lambda}_\tau$ is the time-averaged generator norm capturing both coherent and dissipative dynamics.

Decoherence effectively lowers $\overline{\Lambda}_\tau$, increasing the minimal evolution time τ and reducing the attainable causal throughput. We capture this environmental effect by introducing a coherence retention factor:

$$P_{\text{eff}} = \eta_{\text{coh}} P_{\max}, \qquad 0 < \eta_{\text{coh}} \leq 1, \tag{A.2}$$

where η_{coh} quantifies the fraction of nominal transformation capacity preserved under system–environment interactions.

The causal-throughput relation consequently maintains its general form,

$$f = \frac{P_{\text{eff}}}{E}, \tag{A.3}$$

but with reduced effective power reflecting realistic open-system limitations.

This extension situates the causal-throughput law within the complete hierarchy of quantum speed limits, linking ideal reversible dynamics to experimentally relevant open-system performance bounds. The consistent treatment of both closed and open-system regimes demonstrates the robustness of the causal framework across different physical contexts, complementing the unification of fundamental limits presented in Section 4.

Appendix B. Symbols Definition and Physical Units

Table B.1: Operational time-parameter definitions (foundational).

Symbol	Definition
τ_0	Invariant rest-frame causal-tick period: $\tau_0 = E/P$.
λ_0	Rest-frame transformation extent: $\lambda_0 = c\tau_0$.
λ	Moving-system transformation extent: $\lambda = \dfrac{Ec^2}{Pv_{\text{int}}} = \gamma\lambda_0$.
Δt_{lab}	Laboratory interval between the same tick events: $\Delta t_{\text{lab}} = \gamma\tau_0$.
γ	Allocation / Lorentz factor: $\gamma = 1/\sqrt{1 - v_{\text{ext}}^2/c^2} = c/v_{\text{int}}$.

Table B.2: Symbols and their physical units.

Symbol	**Unit / Meaning**
E	[J] Energy per causal tick
P	[W] Power throughput
τ_0	[s] Rest-frame causal tick period
f_0	[Hz] Rest-frame causal frequency ($f_0 = 1/\tau_0$)
λ	[m] Transformation extent per causal tick
v_{int}	[m·s^{-1}] Internal transformation speed
f_{int}	[Hz] Internal transformation rate

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