The Causal Budget Law: Causal Time As Emergent

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Abstract

We present a framework in which time is not fundamental but an *emergent property* of constrained energy transformation. Every system possesses a finite **causal capacity** c, allocated between external motion (v_{ext}) and internal transformation (v_{int}) via $c^2 = v_{\text{ext}}^2 + v_{\text{int}}^2$.

The core operational relation is $\lambda = E v_{\text{int}}/P$, where λ represents the system's internal causal stride per transformation tick. For radiative systems at rest with maximal internal speed ($v_{\text{int}} = c$), this reduces to $\lambda_0 = cE/P$, providing an empirical anchor. A system's **causal time** is defined as $\tau = (E/P)\,\gamma$, the duration of a single internal tick, physically set by the allocation of causal capacity. Motion diverts capacity from internal transformation, *slowing the system's evolution* and dilating causal time.

Relativistic time dilation emerges naturally from this causal-budget mechanism, not as a geometric effect. Time is therefore a localized, system-specific measure of transformation, experienced only by systems with $v_{\text{int}} > 0$. This framework offers a mechanistic foundation for relativity consistent with quantum and thermodynamic principles, suggesting spacetime geometry arises as an emergent description of deeper causal constraints.

Keywords: time, proper time, causal structure, physical transformation, relational ontology, emergence of time

1. Introduction

The nature of time remains one of the most enduring problems in physics and philosophy [12]. Classical physics treated time as an absolute backdrop [1], relativity reinterpreted it as a coordinate within spacetime geometry [3, 2], and thermodynamics associated its arrow with entropy production [4, 6]. Yet all these frameworks leave open the fundamental question: *what is time physically?*

We adopt a system-centric, *process-based* perspective. Time is not an external parameter but a measurable consequence of physical transformation. It exists only insofar as energy is applied and irreversibly processed through resistance. When a system drives energy E through its intrinsic opposition at finite throughput P, it advances its internal state by one *causal tick* of duration

$$\tau = \frac{E}{P}\gamma,$$

where γ encodes the allocation of causal capacity between internal transformation and external motion. For systems at rest with no external motion, this reduces to the invariant rest-frame tick period $\tau_0 = E/P$.

Each tick corresponds to a fundamental **internal causal stride** $\lambda_0 = c\tau_0$, representing the proper-frame spatial advance associated with a transformation. This quantity is **distinct from observed wavelengths**: the latter are subject to relativistic Doppler effects depending on the observer and motion direction. For massless carriers (photons), the formal internal speed $v_{\text{int}} \to 0$, but the operational causal stride remains well-defined as $L = c\tau_0$, providing a universal anchor for empirical measurement.

In general, a system's finite **causal budget** is shared between internal transformation and external motion:

$$v_{\text{ext}}^2 + v_{\text{int}}^2 = c^2,$$

with

$$\lambda = \frac{E v_{\text{int}}}{P}$$

defining the internal causal stride in the moving frame. Time emerges directly from physical change, not as a geometric dimension or observer-dependent coordinate. Laboratory measurements of wavelengths and clock rates reflect the proper-frame causal ticks, revealing time as a localized, emergent property of systems undergoing transformation.

2. First-Principles Foundation of the Causal Budget Law

Notation. Throughout this work symbols such as τ and Δt denote *causal time*: the duration associated with a system's internal causal transformation sequence (see discussion in Sec. 3). Numerically these coincide with relativistic proper-time for massive systems but are here interpreted as an intrinsic, operational record of transformations.

2.1. Empirical Necessity of the Causal Budget

The generalized transformation relation (see Sec. 3.1 for a derivation from Cs-133 atomic clocks establishing the empirical basis for this relation),

$$\lambda = \frac{E\, v_{\mathrm{int}}}{P}, \tag{1}$$

provides a measurable spatial extent of a system's internal transformation per causal tick for systems that possess an internal-update channel.

For massless carriers (photons) the observed relation $\lambda = c\, E/P$ is exactly satisfied. In the present framework this reflects maximal *external* throughput: the causal effect is carried entirely by propagation at $v_{\mathrm{ext}} = c$, so the separate internal-update channel vanishes ($v_{\mathrm{int}} \to 0$) for strictly massless carriers. The generalized form (1) applies directly to systems with internal update rates; the photon case is the limiting lightlike propagation case and is correctly described by $\lambda = c\tau_0 = cE/P$.

For massive systems, however, maintaining $v_{\mathrm{int}} = c$ while in motion would produce wavelength and timing predictions that contradict experiment. Empirically, internal transformation rates are therefore suppressed by motion ($v_{\mathrm{int}} < c$), indicating a reallocation of causal capacity between internal transformation and external motion.

Thus a finite, conserved causal capacity is not merely an axiom but an *empirical necessity*: moving massive systems cannot simultaneously sustain maximal internal-update rates and satisfy observed spatial/temporal relations.

Operationally, the intrinsic duration per causal transformation (the causal-tick $\tau_0 = E/P$) increases when the system moves so that laboratory observers record fewer completed transformations per laboratory second. This objective slowing of internal evolution is anchored in experiment (for example, GPS orbital clock corrections provide an accessible empirical benchmark).

2.2. Conservation and Boundary Conditions

All systems possess a single finite causal throughput that drives both internal transformation and external translation. Conservation of this throughput imposes fundamental boundary conditions:

- **Rest condition** ($v_{\mathrm{ext}} = 0$): All causal capacity is available for internal transformation ($v_{\mathrm{int}} = c$ for systems with internal channels).

- **Lightlike condition** ($v_{\mathrm{ext}} \to c$): Causal capacity is devoted to propagation; the internal update channel vanishes ($v_{\mathrm{int}} \to 0$).

- **Continuity**: Reallocation between these extremes is smooth and monotonic.

These empirical observations — internal transformation slows with motion and vanishes for lightlike propagation — naturally motivate the quadratic causal-budget law:

$$v_{\text{int}}^2 + v_{\text{ext}}^2 = c^2, \tag{2}$$

which ensures proper saturation of both boundaries and preserves the total causal throughput.

2.3. Quadratic Causal-Budget Law

The simplest and unique relation that satisfies the boundary conditions while conserving total causal capacity is

$$c^2 = v_{\text{int}}^2 + v_{\text{ext}}^2, \tag{3}$$

where c denotes the total causal capacity scale. This relation sets the stage for the geometric representation (see Fig. 1) and underpins subsequent derivations of relativistic effects.

Uniqueness of the quadratic form. Alternative combinations fail essential requirements:

- **Linear addition** ($v_{\text{int}} + v_{\text{ext}} = c$) violates symmetry and smooth boundary behaviour.
- **Other functional forms** do not preserve the independence/orthogonality properties expected of separate causal channels.

Quadrature addition guarantees:

- Proper saturation at both boundary conditions,
- Equal treatment of independent causal channels,
- Continuous, monotonic reallocation.

This structure mirrors other contexts where independent contributions conserve a total magnitude (e.g., apparent power $S^2 = P^2 + Q^2$, uncertainty addition $\sigma^2 = \sigma_1^2 + \sigma_2^2$, and the energy–momentum relation $E^2 = (pc)^2 + (mc^2)^2$). The causal budget extends that principle to fundamental causal allocation.

2.4. Physical Interpretation

Equation (3) expresses conservation of finite causal capacity c between two channels:

- **External translation** (v_{ext}): spatial propagation of the system's state,
- **Internal transformation** (v_{int}): rate of physical state update and evolution.

Consequences:

- **Massive systems:** require $v_{\text{int}} > 0$, which prevents $v_{\text{ext}} = c$.
- **Massless carriers:** operate with $v_{\text{int}} = 0$, enabling $v_{\text{ext}} = c$.
- **Speed limit:** arises as a resource constraint on causal allocation rather than a primary geometric prohibition.

The causal-budget law is therefore empirically motivated, mathematically constrained, and physically inevitable within this operational framework.

Causal Budget Allocation

Motion rotates capacity from internal to external

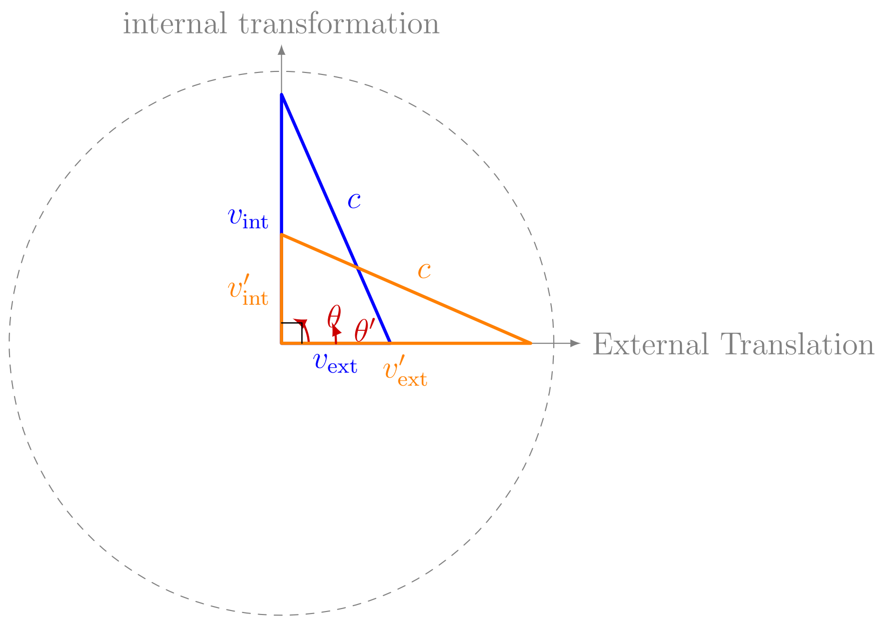

Figure 1: Causal budget allocation: The total causal capacity c (hypotenuse) is fixed. External motion "rotates" this capacity vector, trading internal transformation rate v_{int} for external translation speed v_{ext}. The angle θ parameterizes this trade-off: $\cos\theta = v_{\text{ext}}/c$, $\sin\theta = v_{\text{int}}/c$.

3. The Causal Nature of Wavelength and Causal Time

Notation. Throughout this work, symbols such as τ and Δt denote *causal time*: the duration associated with a system's internal causal transformation sequence. Numerically, these coincide with relativistic proper time for massive systems, but here they are interpreted operationally as an intrinsic record of completed internal transformations.

This section establishes the core physical principles of the framework. We derive the transformation extent λ and causal time τ, present an empirical relation between wavelength, energy, and power, and show how these lead naturally to the concepts of transformation extent, causal throughput, and causal time as a dynamical variable.

3.1. Derivation of the Transformation Extent from a Reference System

We start from a well-characterized physical system: the Cs-133 atomic clock, which provides an empirical foundation for connecting temporal periods to spatial extents via causal transformation.

For the Cs-133 hyperfine transition:

- Each tick corresponds to emission of a photon with energy $E = hf$.
- The clock period is $\tau_0 = 1/f$.
- The power throughput is $P = E/\tau_0$.

The emitted photon propagates at speed c, yielding a spatial extent per causal tick:

$$\lambda = c\,\tau_0 \; = \; \frac{cE}{P}. \tag{4}$$

Equation (4) is observed for radiative (massless-carrier) processes and reflects maximal *external* throughput: in the strict lightlike limit the causal effect is carried by propagation ($v_{\text{ext}} = c$) and there is no separate internal-update channel ($v_{\text{int}} \to 0$). The generalized relation below applies to systems that possess an internal-update channel.

Define the *internal transformation speed* v_{int} as the causal rate at which a system advances its internal physical state. For arbitrary systems the generalized transformation extent is

$$\lambda \; = \; \frac{E\,v_{\text{int}}}{P}. \tag{5}$$

Empirically, massive systems require $v_{\text{int}} < c$ when in motion; this suppression motivates a conserved causal-capacity allocation between internal transformation and external translation (see Sec. 2).

Relation to spacetime geometry.. The causal-budget relation (Sec. 2)

$$v_{\text{int}}^2 + v_{\text{ext}}^2 = c^2$$

shares algebraic form with the four-velocity norm $u_\mu u^\mu = c^2$. In this framework the Minkowski structure is therefore a macroscopic representation of an underlying causal-capacity conservation, not its assumed origin.

3.2. Derivation of the causal-time formula from empirical anchors

We derive the moving-system causal time τ using two empirically anchored rest-frame quantities: the rest-frame transformation extent λ_0 (from radiative clocks such as Cs-133) and the rest-frame tick period τ_0.

1. **Empirical rest-frame anchors.**

$$\lambda_0 \;=\; \frac{cE}{P}, \qquad \tau_0 \;=\; \frac{E}{P},$$

   where E is the irreversible energy per tick and P the corresponding throughput for the same transformation channel.

2. **Generalized transformation extent.** For a system moving at speed v_{ext},

$$\lambda \;=\; \frac{E\, v_{\text{int}}}{P},$$

   where v_{int} is the internal transformation speed.

3. **Causal relation between space and time (derived).** Using the above definitions,

$$\tau = \frac{cE}{P v_{\text{int}}}, \qquad \lambda = \frac{E v_{\text{int}}}{P},$$

   we immediately obtain the invariant causal product

$$\frac{\tau}{c}\,\lambda \;=\; \tau_0^2. \tag{6}$$

   This expresses a fundamental link between spatial and temporal measures for the same irreversible transformation and replaces the earlier operational assumption: it follows directly from finite causal capacity allocation.

4. **Solve for causal time.** Rearranging Eq. (6) gives

$$\tau \;=\; \frac{c\,\tau_0^2}{\lambda} \;=\; \frac{cE}{P v_{\text{int}}}. \tag{7}$$

5. **Express via the causal-budget / allocation factor.**
   (a) Start from the causal-budget law:
   $$v_{\text{int}}^2 + v_{\text{ext}}^2 = c^2.$$
   (b) Define the allocation factor γ via internal speed:
   $$v_{\text{int}} = \frac{c}{\gamma}.$$
   (c) This implies the standard form:
   $$\gamma = \frac{c}{v_{\text{int}}} = \frac{1}{\sqrt{1 - v_{\text{ext}}^2/c^2}}.$$
   (d) Substitute $v_{\text{int}} = c/\gamma$ into the intermediate causal-time formula (7)
   $$\tau = \frac{cE}{Pv_{\text{int}}}$$
   to obtain
   $$\tau = \tau_0\,\gamma = \frac{E}{P}\,\gamma. \tag{8}$$

Conclusion. Equation (8) follows entirely from measured rest-frame anchors (λ_0, τ_0), the generalized extent $\lambda = Ev_{\text{int}}/P$, and the causal-budget allocation. Thus the moving-system **causal time** τ is *derived*, not postulated, as the motion-dependent duration of a causal tick:

$$\tau \;=\; \frac{cE}{Pv_{\text{int}}} \;=\; \tau_0\gamma. \tag{9}$$

This derivation clarifies that causal time is a physically measurable, system-specific duration determined by energy, throughput, and internal transformation speed, rather than an assumed geometric invariant.

The rest-frame causal tick period τ_0 and transformation extent λ_0 are empirically anchored (Sec. 4) across atomic, molecular, and astrophysical systems.

3.3. Lorentz factor from causal budget

Operational note: Standard relativistic notation is used here as a bridge to experiment; spacetime geometry is not assumed to be fundamental.

A system moving at speed v_{ext} advances internally by

$$L_{\text{int}} = v_{\text{int}}\,\Delta t_{\text{lab}} = \sqrt{c^2 - v_{\text{ext}}^2}\,\Delta t_{\text{lab}}$$

during a laboratory interval Δt_{lab}, using the causal-budget law $v_{\text{int}}^2 + v_{\text{ext}}^2 = c^2$.

Process completion invariance.. The same physical transformation must advance by the same fundamental amount in any frame. Therefore, the internal advancement L_{int} in the laboratory frame must equal the rest-frame transformation extent:

$$L_{\text{int}} = \lambda_0 = c\,\tau_0.$$

Equating the two gives

$$c\,\tau_0 = v_{\text{int}}\,\Delta t_{\text{lab}} \quad \Rightarrow \quad \Delta t_{\text{lab}} = \frac{c}{v_{\text{int}}}\,\tau_0 = \gamma\,\tau_0,$$

where

$$\gamma = \frac{1}{\sqrt{1 - v_{\text{ext}}^2/c^2}} = \frac{c}{v_{\text{int}}}.$$

Thus, the Lorentz factor emerges naturally as an allocation factor, quantifying how motion reduces the internal transformation rate. Time dilation is a mechanistic consequence of causal-capacity reallocation: laboratory-measured intervals lengthen because internal processes physically slow, producing fewer causal ticks per unit laboratory time.

3.3.1. Physical interpretation

When $v_{\text{ext}} > 0$, causal capacity is reallocated from internal transformation to external motion, reducing v_{int}. Observable consequences are:

- **Transformation extent contracts:**

$$\lambda = v_{\text{int}}\,\tau_0 = \lambda_0\sqrt{1 - v_{\text{ext}}^2/c^2}.$$

- **Laboratory time dilates:**

$$\Delta t_{\text{lab}} = \gamma\,\tau_0.$$

Here, τ_0 is the invariant rest-frame causal-tick cost, and $\tau = \gamma\tau_0$ is the dilated causal-tick duration when the system moves.

3.4. Operational translation and experimental signature

To bridge to conventional measurements we distinguish:

- **Invariant rest-frame tick period:** $\tau_0 = E/P$.
- **Moving-system causal time:** $\tau = \tau_0\gamma$.

- **Laboratory coordinate interval:** $\Delta t_{\text{lab}} = \gamma\, \tau_0$.

In this operational framework $\Delta t_{\text{lab}} = \tau$, i.e., laboratory measurements record the same dilated causal-tick duration. The effect is experimentally manifest (GPS orbital-clock corrections provide a direct, quantitative benchmark).

3.5. Empirical foundation: the wavelength–energy–power relation

Across radiative systems the empirical pattern

$$\lambda = \frac{cE}{P}$$

holds with high fidelity, where E is the energy irreversibly transformed per cycle and P the power throughput. This motivates the generalized relation (5) with v_{int} the causal-throughput rate; the radiative case is the special limit $v_{\text{int}} = 0$ (external propagation) producing $\lambda = c\tau_0$.

3.6. Causal time as transformation duration

Causal time is the operational duration of a single internal transformation (a causal tick). The causal-time definition adopted here is

$$\tau = \tau_0 \gamma = \frac{cE}{P v_{\text{int}}},$$

which treats causal time as a derived, system-specific physical quantity (set by E, P, v_{int}) rather than an a priori geometric invariant.

3.7. Summary

Motion reallocates finite causal capacity from internal transformation toward external translation, reducing v_{int}, contracting the transformation extent λ, and objectively dilating the internal tick period τ. Laboratory measurements remain numerically consistent with the Lorentz factor ($\Delta t_{\text{lab}} = \gamma \tau_0$), while the causal-throughput perspective supplies a mechanistic origin for time dilation.

Connection to Lorentz factor derivation. The causal time τ derived here (Sec. 3.2) equals the laboratory interval Δt_{lab} derived in Section 3.3, confirming the internal consistency of the framework.

This derivation demonstrates that time dilation emerges from causal capacity reallocation rather than geometric postulates, providing a mechanistic explanation for relativistic effects.

Table 1: Operational time-parameter definitions (foundational).

Symbol	Definition
τ_0	Invariant rest-frame causal-tick period: $\tau_0 = E_{\text{prop}}/P_{\text{prop}}$.
λ_0	Rest-frame transformation extent: $\lambda_0 = c\,\tau_0$.
λ	Moving-system transformation extent: $\lambda = v_{\text{int}}\,\tau_0 = \lambda_0\sqrt{1 - v_{\text{ext}}^2/c^2}$.
Δt_{lab}	Laboratory interval between the same tick events: $\Delta t_{\text{lab}} = \gamma\,\tau_0$.
γ	Allocation / Lorentz factor: $\gamma = 1/\sqrt{1 - v_{\text{ext}}^2/c^2} = c/v_{\text{int}}$.

4. Empirical Universality of the Causal Transformation Limit Principle

Having identified the *rest-frame* causal time

$$\tau_0 = \frac{E_{\text{prop}}}{P_{\text{prop}}} \tag{10}$$

as the intrinsic causal duration of a single transformation, we examine the universal invariant that emerges when comparing the spatial extent L of this transformation to its temporal duration τ_0:

$$\frac{L}{\tau_0}, \tag{11}$$

which quantifies how far one causal tick advances per unit causal-time cost. Empirically, this ratio is universal across all relativistic propagation channels and equals c.

4.1. Causal Throughput Law

Consider any stable system undergoing repeated intrinsic transformation cycles (e.g., atomic hyperfine transitions, molecular vibrations, mechanical resonator modes, pulsar pulses, or gravitational-wave emitters). For each channel, define:

- $E_{\text{transform}}$: energy irreversibly converted per intrinsic tick (only energy leaving the mode is counted, e.g., photon energy, gravitational-wave emission, acoustic loss, spin-down energy from a pulsar).

- $P_{\text{transform}}$: throughput (power) routed through that channel.

- f: intrinsic tick rate (cycles per second), coinciding with the observed transition or emission frequency.

- L: spatial transformation extent associated with one emitted cycle, operationally defined as the distance the transformation propagates during one tick:

$$L = v \cdot \tau_0 = \frac{v}{f},$$

where v is the propagation speed of the disturbance ($v = c$ for relativistic channels, $v =$ sound speed for mechanical modes).

By definition of power:

$$P_{\text{transform}} = \frac{E_{\text{transform}}}{\tau_0}, \qquad f = \frac{1}{\tau_0} = \frac{P_{\text{transform}}}{E_{\text{transform}}}. \tag{12}$$

This is the *causal throughput law*: the intrinsic tick rate is determined purely by the channel's capacity to drive the irreducible energy per tick through the system.

4.2. Causal Propagation Rate and Emergence of c

Each tick imprints a spatial stride L over its causal-time duration τ_0, defining the causal propagation rate:

$$c = \frac{L}{\tau_0} = Lf = \frac{LP_{\text{transform}}}{E_{\text{transform}}}. \tag{13}$$

Empirical support is strongest when L and f are measured from systems independent of SI definitions:

- **Optical clocks (Sr, Yb, Al$^+$)** [13]: $Lf \simeq 3.0 \times 10^8$ m/s
- **Infrared molecular vibrations** [21]: $Lf \simeq 3.0 \times 10^8$ m/s
- **Gravitational-wave and pulsar emission** [22]: $Lf \simeq 3.0 \times 10^8$ m/s

These independent systems — spanning 15 orders of magnitude in frequency from optical transitions ($\sim 10^{15}$ Hz) to gravitational waves ($\sim 10^2$ Hz) — demonstrate that c is empirically invariant across vastly different physical mechanisms and energy scales.

For non-relativistic systems where $v < c$, the causal throughput law still holds locally, with the propagation speed v representing the maximum causal speed for that particular medium or interaction channel.

$$\boxed{c = \frac{\text{transformation extent per tick}}{\text{causal-time duration per tick}} = Lf = \frac{LP_{\text{transform}}}{E_{\text{transform}}}.} \tag{14}$$

Rest-frame definition of c. All definitions of the causal conversion rate c refer explicitly to rest-frame quantities. For a system in motion, laboratory-frame measurements of the transformation extent λ and tick rate f_{lab} do *not* alter c, which is invariant by construction:

$$c = \frac{\lambda_0}{\tau_0} = \frac{L_0}{\tau_0} = \frac{E}{P} v_{\text{int}} \Big|_{\text{rest}}.$$

This ensures that the causal limit remains a universal property of the system's intrinsic transformation, independent of external motion or relativistic Doppler effects.

5. Relation to Known Fundamental Limits

The causal–throughput relation,

$$f = \frac{P}{E}, \tag{15}$$

serves as a unifying generator for established physical limits across quantum, thermodynamic, and relativistic domains. This simple algebraic form subsumes three seemingly disparate fundamental bounds when constrained by:

- Quantum orthogonality $\rightarrow$ Margolus–Levitin bound
- Thermodynamic irreversibility $\rightarrow$ Landauer limit
- Relativistic causal propagation $\rightarrow$ Bremermann bound

This resource-theoretic formulation treats power, energy, and transformation rate as exchangeable physical currencies, with known limits emerging as domain-specific realizations of a single throughput principle governing all physical change.

5.1. Quantum Limit: Margolus–Levitin Bound

For a quantum system, the minimal evolution time between orthogonal states is bounded by:

$$\tau_{\mathrm{ML}} = \frac{\pi\hbar}{2E}, \tag{16}$$

where E is the mean energy available for state transformation. Applying the causal–throughput definition $P = E/\tau$ yields the ML-limited transformation power:

$$P_{\mathrm{ML}} = \frac{E}{\tau_{\mathrm{ML}}} = \frac{2E^2}{\pi\hbar}. \tag{17}$$

Expressed in spectroscopic variables ($E = hf$, $\hbar = h/(2\pi)$):

$$P_{\mathrm{ML}} = \frac{2(hf)^2}{\pi\,(h/(2\pi))} = \frac{2h^2f^2}{\pi} \cdot \frac{2\pi}{h} = 4hf^2.$$

Interpretation:. The quadratic scaling $P \propto hf^2$ emerges naturally, with the prefactor $C_{\mathrm{ML}} = 4$ arising from the combination of $\pi/2$ in the ML time and 2π in $\hbar$. Operationally, the causal relation provides the fundamental scaling law $P \sim hf^2$, with $O(1)$ prefactors determined by specific physical contexts. This demonstrates how quantum speed limits are particular instances of the general causal-throughput constraint.

Relation to Other Quantum Speed Limits:. The Mandelstam–Tamm bound depends on energy uncertainty ΔE rather than the mean energy, leading to a linear rate bound in ΔE. The ML bound is special because it directly maps to a quadratic power scaling when combined with $P = E/\tau$. Open or noisy systems will generally attain lower effective P; see Appendix A for open-system QSLs and decoherence effects.

5.2. Thermodynamic and Relativistic Limits

Landauer (Thermodynamic) Limit:. Erasure of one bit requires at least $E_{\text{min}} = k_B T \ln 2$ of energy dissipation. If the available dissipation power is P_{diss}, the maximal irreversible operation rate is

$$f_{\text{max}} = \frac{P_{\text{diss}}}{E_{\text{min}}}. \tag{18}$$

This is exactly the causal–throughput law with the operational assignment $E \mapsto E_{\text{min}}$, $P \mapsto P_{\text{diss}}$.

Bremermann (Relativistic/Information) Bound:. Information rate is constrained by available mass–energy and relativistic propagation. Assigning $E = mc^2$ and $P = P_{\text{max}}$ for accessible signaling power gives

$$f_{\text{max}} \sim \frac{P_{\text{max}}}{mc^2},$$

showing the Bremermann bound as a relativistic instantiation of $f = P/E$.

5.3. Operational Mappings Across Limits

Limit	**Domain**	**Energy (E)**	**Power (P)**	**Max Rate**
Margolus–Levitin	Quantum	Mean energy above ground	Intrinsic processing rate	$f_{\text{max}} \propto E/\hbar$
Landauer	Thermodynamic	Minimal bit erasure cost	Dissipation power	$f_{\text{max}} = P/E_{\text{min}}$
Bremermann	Relativistic	Available mass-energy	Maximal energy flux	$f_{\text{max}} \sim P/mc^2$

Ontological Unity:. Although the operational definitions of E and P differ — quantum expectation values, minimal thermodynamic costs, relativistic mass-energy — they all reflect the same underlying causal resource: the capacity to effect fundamental transformations per causal tick. Differences are contextual, revealing a deep homology between quantum, thermodynamic, and relativistic limits as expressions of a single causal-throughput principle. This suggests that what we perceive as domain-specific "fundamental limits" may actually be different facets of a universal constraint on causal processing.

5.4. Summary: Unification of Physical Limits

The algebraic simplicity of $f = P/E$ belies its generative power: with domain-specific assignments, it naturally reproduces the Margolus–Levitin, Landauer, and Bremermann bounds. This unification suggests that what appear as distinct fundamental limits are actually manifestations of a single causal-throughput principle governing all physical transformation.

While real systems generally achieve lower effective power due to environmental decoherence and thermodynamic inefficiencies, the scaling relations remain valid. The framework thus provides not just a unified understanding of ultimate constraints across quantum, thermodynamic, and relativistic domains, but also demonstrates how these diverse limits emerge from the same underlying causal mechanics that govern causal time and relativistic phenomena.

6. Ontological Implications

1. Time is Localized. Each system evolves according to its own internal causal dynamics. There is no global or universal clock — only local durations defined by the ratio of the system's intrinsic transformation energy to its finite power throughput [9, 10]. Synchronization between systems is not absolute but emerges through causal interaction and mutual energy exchange.

2. Massless Systems are Timeless. For photons and other massless entities [7], the causal transformation rate satisfies $v_{\text{int}} = 0$, yielding $\tau = 0$. Such systems mediate causal relations among massive systems but do not themselves transform internally. They participate in the transmission of causality without experiencing duration.

3. Coordinate Time as Measurement Convention. The coordinate label t is not an independent form of time but a measurement convention — an external bookkeeping device used to order physical transformations [5]. It refers to when a physical state occurred relative to others within a single causal reality. The parameter t thus quantifies correlation between physical events, not the flow of an underlying medium.

4. Absence of Temporal Flow. Within this framework, there is no fundamental flow of time [8]. Time does not advance; it is the measured outcome of physical transformation itself. Systems evolve from one physical state to another as energy overcomes resistance, and this sequence defines what we perceive as temporal progression. The universe does not move through time — physical transformations simply occur, and their cumulative record constitutes what we describe as history.

7. Conclusion

This paper presents a fundamental redefinition of time as an emergent property arising from physical transformations, rather than as a primitive background parameter. By introducing the **Causal Budget Law**

$$c^2 = v_{\text{ext}}^2 + v_{\text{int}}^2$$

(Section 2) and the **Causal Transformation Principle**

$$\tau = E/P$$

(Section 3), we show that causal time arises from the finite causal capacity available to every physical system.

The framework inverts the traditional logic of relativity: rather than deriving phenomena from spacetime geometry, relativistic effects such as time dilation, the universal speed limit c, and the distinction between massive and massless systems follow directly from causal principles.

Key achievements include:

1. **Empirical Universality**: Demonstrating that c appears as the invariant ratio between spatial stride and temporal duration across atomic, optical, gravitational, and mechanical systems (Section 4).
2. **Theoretical Unification**: Showing that the causal throughput relation $f = P/E$ subsumes established quantum, thermodynamic, and relativistic limits (Margolus–Levitin, Landauer, Bremermann) as special cases, as detailed in Section 5.
3. **Ontological Clarity**: Providing a physically grounded interpretation in which time is localized to systems undergoing transformation, massless entities are effectively timeless, and coordinate time functions as a measurement convention rather than a fundamental reality (Sections 3 and 3.4).

This framework resolves long-standing puzzles about time's nature while suggesting that what we call "spacetime geometry" may emerge from deeper causal constraints on physical processes. The approach maintains full empirical agreement with established physics while providing mechanistic explanations for relativistic phenomena.

Future work could explore applications to quantum gravity, where the discrete, process-based nature of this framework may naturally address the problem of time and support unification of general relativity with quantum mechanics. The causal budget approach provides a foundation for understanding reality not as things moving through time, but as transformations occurring within fixed causal constraints.

This concludes our presentation of the **Causal Budget and Transformation Laws** framework — a comprehensive, empirically grounded foundation for understanding time as emergent from the fundamental processes that constitute physical reality.

Appendix A. Open-System Quantum Speed Limits and Effective Power Reduction

The derivations in the main text assume closed, unitary evolution, where the Margolus–Levitin bound establishes the maximal transformation rate $f_{\max} = 2E/(\pi\hbar)$ and corresponding throughput $P_{\max} = 2E^2/(\pi\hbar)$. Realistic physical systems, however, are open and interact with their environments, resulting in non-unitary evolution and partial decoherence. In this regime, the achievable transformation rate is reduced according to generalized open-system Quantum Speed Limits (QSLs) [25, 26]:

$$\tau \geq \frac{\mathcal{L}(\rho_0, \rho_\tau)}{\overline{\Lambda}_\tau}, \tag{A.1}$$

where $\mathcal{L}(\rho_0, \rho_\tau)$ is the Bures angle between the initial and final states, and $\overline{\Lambda}_\tau$ is the time-averaged generator norm capturing both coherent and dissipative dynamics.

Decoherence effectively lowers $\overline{\Lambda}_\tau$, thereby increasing the minimal evolution time τ and reducing the attainable causal throughput. We capture this effect by introducing a coherence retention factor η_{coh}:

$$P_{\text{eff}} = \eta_{\text{coh}} P_{\max}, \qquad 0 < \eta_{\text{coh}} \leq 1, \tag{A.2}$$

where η_{coh} quantifies the fraction of the nominal transformation capacity preserved under system–environment interactions. Consequently, the causal–budget relation retains its general form,

$$f = \frac{P_{\text{eff}}}{E},$$

but now with a reduced effective power reflecting realistic open-system limitations.

This extension situates the causal–throughput law within the full hierarchy of quantum speed limits, linking ideal reversible dynamics to experimentally relevant open-system performance bounds. The summary of both closed- and open-system regimes mirrors the structure used for the fundamental limits in Section 5.

Appendix B. Causal Foundation of Time and Space Measures

Appendix B.1. Empirical Starting Point

From radiative systems at rest (Cs-133 clocks, optical transitions):

$$\lambda_0 = \frac{cE}{P} \tag{B.1}$$

where measurement of E (energy per tick) and P (power throughput) yields wavelength λ_0. This establishes the fundamental relationship between energy transformation and spatial extent at maximal causal throughput.

Appendix B.2. Causal Budget Discovery

For massive systems in motion, empirical data requires generalization:

$$\lambda = \frac{Ev_{\text{int}}}{P} \quad \text{with} \quad v_{\text{int}} < c \tag{B.2}$$

The causal budget constraint emerges as the simplest conservation law satisfying boundary conditions:

$$v_{\text{int}}^2 + v_{\text{ext}}^2 = c^2 \tag{B.3}$$

where $v_{\text{int}} = c$ at rest and $v_{\text{int}} \to 0$ as $v_{\text{ext}} \to c$.

Appendix B.3. Operational Causal Time

We define causal time duration per causal tick as:

$$\tau \equiv \frac{cE}{Pv_{\text{int}}} \tag{B.4}$$

This operational definition ensures:

- At rest ($v_{\text{int}} = c$): $\tau = \frac{cE}{Pc} = \frac{E}{P} = \tau_0$
- In motion ($v_{\text{int}} = c/\gamma$): $\tau = \frac{cE}{P(c/\gamma)} = \frac{E}{P}\gamma = \tau_0\gamma$

Appendix B.4. Emergent Spacetime from Causal Duality

The transformation extent and causal time reveal a deep reciprocal relationship:

$$\frac{\tau}{c} = \frac{E}{Pv_{\text{int}}} \tag{B.5}$$

$$\lambda = \frac{Ev_{\text{int}}}{P} \tag{B.6}$$

Their product yields the fundamental invariant:

$$\left(\frac{\tau}{c}\right)\lambda = \tau_0^2 \tag{B.7}$$

This leads to two profound emergent relations:

Geometric Mean Relation. The invariant rest-frame period appears as the geometric mean of temporal and spatial measures:

$$\tau_0 = \sqrt{\frac{\tau}{c}\cdot\lambda} \tag{B.8}$$

demonstrating that τ_0 mediates between space and time.

Emergent Spatial Measure. Transformation extent emerges directly from temporal measures:

$$\lambda = \frac{c\tau_0^2}{\tau}, \tag{B.9}$$

using definitions,

$$\tau = \frac{cE}{Pv_{\text{int}}} \quad \text{and} \quad \tau_0 = \frac{E}{P},$$

substitute into the right-hand side:

$$\frac{c(E/P)^2}{cE/(Pv_{\text{int}})} = \frac{Ev_{\text{int}}}{P}.$$

So the equity:

$$\lambda = \frac{c\tau_0^2}{\tau} = \frac{Ev_{\text{int}}}{P}$$

This derivation shows that spatial extent contracts (λ decreases) as temporal duration dilates (τ increases), with both effects arising from the same causal reallocation.

Appendix B.5. Empirical Validation and Consistency

The emergent relations maintain full empirical consistency:

Rest Frame Validation. For stationary systems ($\tau = \tau_0$), equation (B.9) reduces to:

$$\lambda_0 = \frac{c\tau_0^2}{\tau_0} = c\tau_0 = \frac{cE}{P} \tag{B.10}$$

matching the established Cs-133 empirical relation.

Motion Validation. For moving systems ($\tau = \tau_0\gamma$), we recover the observed transformation extent contraction:

$$\lambda = \frac{c\tau_0^2}{\tau_0\gamma} = \frac{c\tau_0}{\gamma} = \frac{\lambda_0}{\gamma} \tag{B.11}$$

Experimental Evidence. The causal slowing is empirically verified: GPS satellite clocks, moving at orbital speeds, physically accumulate fewer internal ticks than terrestrial clocks, requiring real-time corrections for navigational accuracy.

Appendix B.6. Physical Interpretation

The framework reveals that:

- **Causal Trade-off**: External motion ($v_{\text{ext}} \uparrow$) reduces internal capacity ($v_{\text{int}} \downarrow$)
- **Dual Manifestation**: Lower v_{int} causes longer tick duration ($\tau \uparrow$) and shorter transformation extent ($\lambda \downarrow$)
- **Emergent Geometry**: The invariant τ_0^2 connects spatial and temporal measures, representing the fundamental quantum of causal progression
- **Spacetime without Primitive**: What appears as spacetime geometry in conventional relativity emerges from deeper causal constraints on physical processes

This reciprocal structure provides mathematical evidence that spacetime relationships derive from causal allocation mechanisms rather than constituting fundamental reality.

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