Homework 10: Nonlinear Programs

Due: Wednesday 26 April 2023, 11:59pm
See the course website for instructions and submission details.

Note: for this Homework set, I suggest using the `Ipopt` solver in JuMP. Be sure to make use of the macros `@NLconstraint` and `@NLobjective` when specifying nonlinear constraints or objectives respectively.

1. **Hexagon construction.** The goal is to figure out the optimal geometry of a hexagon such that (1) the hexagon's area is maximized and (2) the hexagon's diameter does not exceed 1. The *diameter* of the hexagon is the largest distance between any pair of points belonging to the hexagon. There are many different ways to solve this problem. A few suggestions/hints:

   - Place one of the vertices at the origin so that you don't need as many decision variables.
   - A simple way of computing the area of a hexagon is to split it into four triangles that share a common vertex (e.g. the one at the origin). The area of the hexagon is then the sum of the areas of the four triangles. (You need to find the formula for calculating area of triangles!)
   - The optimal solution will look like a legitimate hexagon. It will not be degenerate (no coincident vertices), nor will it be perfectly regular (all sidelengths equal).
   - I suggest to run it several times from random starting points to verify that you find the best solution. (Because this is a nonconvex problem, it could have multiple local minima.)

   Output the **area** of the optimal hexagon and **plot a picture** of it.

2. **Fertilizer influence model.** A series of experiments is conducted to determine the effect of a particular fertilizer on wheat crop yield. Here are the results: Such situations typically involve some sort

fertilizer rate (x)	crop yield (y)
-5	127
-3	151
-1	379
1	421
3	460
5	426

   of "diminishing returns". So there is a limit to how much the fertilizer can boost the yield. A popular model for this relationship is the following:

   $$y = k_1 + k_2 \exp(k_3 x)$$

   Use nonlinear least squares to determine the values of (k_1, k_2, k_3) that provide the best fit to the data. We are expecting coefficients in the neighborhood of $(500, -200, -1)$.

   To display your result, produce a plot of the original data points and the best-fit curve.