

Key for Review of Basic Mathematics

An important skill in Chem 260 is the ability to rearrange mathematical expressions to isolate a variable of interest; these two questions provide practice in this:

1. Rearrange the following equation by solving for a in terms of the other variables.

$$\left(a + \frac{b}{c}\right) \times (d - e) = f$$

First, let's divide both sides of the equation by $(d - e)$

$$\left(a + \frac{b}{c}\right) = \frac{f}{d - e}$$

and then subtract $\frac{b}{c}$ from both sides of the equation to arrive at the final answer

$$a = \frac{f}{d - e} - \frac{b}{c}$$

2. Rearrange the following equation by solving for c in terms of the other variables.

$$a = b \left(\frac{1}{c} - \frac{1}{d} \right)$$

First, let's divide both sides of the equation by b

$$\frac{a}{b} = \left(\frac{1}{c} - \frac{1}{d} \right)$$

and then add $\frac{1}{d}$ to both sides of the equation

$$\frac{a}{b} + \frac{1}{d} = \frac{1}{c}$$

This leaves us with $\frac{1}{c}$ on the left side of the equal sign; we want to write this in terms of c , so we take the reciprocal of both sides, arriving at

$$c = \frac{1}{\frac{a}{b} + \frac{1}{d}}$$

which we clean up by multiplying the right side of the equation by bd/bd

$$c = \frac{bd}{ad + b}$$

To gain comfort with natural and base 10 logarithms, determine the value of x to two decimal places for each of the following four problems.

3. $\log(x) = 0.83$

To determine the value of x we take the inverse log of both sides of the equation. Depending on your calculator, you may accomplish this by entering 0.83 and selecting INV LOG or by selecting 10^x ; in either case, the value of x is 6.76.

4. $x = \log(0.0135)$

To determine the value of x , enter 0.0135 into your calculator and select the LOG key (not the LN key, which is for base e). The value of x is -1.87 .

5. $\ln(x) = 0.122$

To solve for x , we take the inverse natural log of both sides of the equation. Depending on your calculator, you may accomplish this by entering 0.122 and selecting INV LN or by selecting e^x ; in either case, the value of x is 1.130.

6. $x = \ln(1.83)$

To determine the value of x , enter 1.83 into your calculator and select the LN key (not the LOG key, which is for base 10). The value of x is 0.604.

We will work with quadratic equations later this semester when solving equilibrium problems; these two questions provide practice in working with quadratic equations:

7. Rearrange the following equation into the form $ax^2 + bx + c = 0$.

$$0.20 = \frac{x^2}{55 - x}$$

We begin by multiplying both sides of the equation by $55 - x$

$$x^2 = 0.20(55 - x)$$

and then multiply the 0.20 through the right side of the equation

$$x^2 = 11 - 0.20x$$

Next, we add $0.20x$ and subtract 11 from both sides of the equation to give

$$x^2 + 0.20x - 11 = 0$$

8. Determine the roots for the equation $2x^2 - x - 15 = 0$ by factorization.

First, we need to rewrite this equation as the product of two terms. The first term of the polynomial equation, $2x^2$, tells us that factorization is of the form $(2x \pm a)$ and $(x \pm b)$. The last term of the polynomial equation, -15 , means that a and b must be $+5$ and -3 , or -5 and $+3$; the first of these combinations is the one that leads to the polynomial equation's second term of $-x$, which makes the results $2x^2 - x - 15 = 0 = (2x + 5)(x - 3)$. To find the roots, we set each of the two terms equal to zero and solve; thus, $2x + 5 = 0$ gives $x = -2.5$ and $x - 3 = 0$ gives $x = +3$.

9. Using the quadratic formula, what are the roots for the equation $3x^2 + 33x - 65 = 0$ to three decimal places?

The roots of a polynomial equation of the form $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substituting in 3 for a , 33 for b , and -6.5 for c and solving gives

$$x = \frac{-33 \pm \sqrt{33^2 - (4)(3)(-6.5)}}{(2)(3)} = \frac{-33 \pm \sqrt{1089 + 78}}{6} = \frac{-33 \pm \sqrt{1167}}{6} = \frac{-33 \pm 34.16}{6}$$

The first root is $(-33 + 34.16)/6$ or 0.193 and the second root is $(-33 - 34.16)/6$ or -11.193 .

Comfort with scientific notation is important, both in recognizing relative magnitudes and when you enter values in your calculator; these three problems provide practice with scientific notation.

10. Rank the following numbers from smallest-to-largest in magnitude: 9.0×10^{-6} , 8.1×10^{-6} , 1.6×10^5 , 4.1×10^{-2} , 5.8×10^4 .

The order is $8.1 \times 10^{-6} < 9.0 \times 10^{-6} < 4.1 \times 10^{-2} < 5.8 \times 10^4 < 1.6 \times 10^5$

11. Convert the following from decimal notation to scientific notation, or from scientific notation to decimal notation: 0.000139, 452.78, 7.35×10^{-2} , 1.35×10^5 .

$0.000139 = 1.39 \times 10^{-4}$, $452.78 = 4.5278 \times 10^2$, $7.35 \times 10^{-2} = 0.0735$, $1.35 \times 10^5 = 135,000$

12. What is the value of x if

$$x = \frac{10^{-15}}{3.9 \times 10^{-7}}$$

The value of x is 2.56×10^{-9} . If your answer is to the wrong power of 10, then you need to review the proper method for entering scientific notation in your calculator.