Correction to In-Class Exercise

The typo in my notes was the initial concentration of PCl₃, which was intended to be 0.0220 M instead of 0.0120 M. I've corrected the initial concentration in our table and then worked through the solution; much of what we had on the board was okay, as it did not include the typo.

	PCl_3	+	Cl_2	\rightleftharpoons	PCl_5
initial	0.0220		0.00400		0.0400
change	-x		-x		+x
equilibrium	0.0220 - x		0.00400 - x		0.0400 + x

The value of Q is $\frac{0.0400}{(0.0220)(0.00400)} = 454$, which is less than the value for K of 1000; thus, the reaction will shift to the right to reach equilibrium.

To solve for x, we substitute the equilibrium concentrations into the equilibrium constant expression; thus

$$K_{eq} = \frac{[\text{PCl}_5]}{[\text{PCl}_3][\text{Cl}_2]} = \frac{(0.0400 + x)}{(0.00220 - x)(0.00400 - x)} = 1000$$

$$\frac{(0.0400 + x)}{8.8 \times 10^{-5} - 0.026x + x^2} = 1000$$

$$0.0400 + x = 8.8 \times 10^{-5} - 0.026x + x^2$$

$$1000x^2 - 27x + 0.048 = 0$$

$$x = \frac{27 \pm \sqrt{(-27)^2 - 4 \times 1000 \times 0.048}}{(2)(1000)} = \frac{27 \pm 23.173}{2000}$$

$$x = (0.0251, 0.00191)$$

Solving the quadratic equation gives two possible answers, only one of which is correct. In this case, x = 0.0251 leads to a contradiction as it gives a negative concentration for both reactants; thus

$$\begin{split} [PCl_3]_{eq} &= 0.0220 - 0.00191 = 0.0201 \ M \\ [Cl_2]_{eq} &= 0.00400 - 0.00191 = 0.00209 \ M \\ [PCl_5]_{eq} &= 0.0400 + 0.00191 = 0.0419 \ M \end{split}$$

To check our results, we substitute these concentrations into the equilibrium constant expression

$$K_{eq} = \frac{[\text{PCl}_5]}{[\text{PCl}_3][\text{Cl}_2]} = \frac{0.0419}{0.0201 \times 0.00209} = 999$$

and obtain a result that is pretty close to the expected value of 1000 (the difference is due to small round-off errors). Although lengthy, this approach always leads to the correct answer, provided that there are no math (or typographical!) errors.