Review of Basic Mathematics

An important skill in Chem 260 is the ability to rearrange mathematical expressions to isolate a variable of interest; these questions provide practice in this:

1. Rearrange the equation $(a + b/c)(d - e) = f$ by solving for *a* in terms of the other variables.

   To start, let's rewrite the equation so that the fraction b/c inside the first set of parentheses is displayed in a way that better emphasizes the fraction.

   $$\left(a + \frac{b}{c}\right)(d - e) = f$$

   Next, we divide both sides of the equation by $(d - e)$

   $$\left(a + \frac{b}{c}\right) = \frac{f}{(d - e)}$$

   and then subtract b/c from both sides of the equation to give a final result of

   $$a = \frac{f}{(d - e)} - \frac{b}{c}$$

2. Rearrange the equation $a = b\left(\frac{1}{c} - \frac{1}{d}\right)$ by solving for *c* in terms of the other variables.

   First, we divide both sides of the equation by *b*

   $$\frac{a}{b} = \left(\frac{1}{c} - \frac{1}{d}\right)$$

   and then add $1/d$ to both sides of the equation

   $$\frac{a}{b} + \frac{1}{d} = \left(\frac{1}{c}\right)$$

   This leaves us with $1/c$ on the left side of the equal sign; we want this to be in terms of *c*, so we take the reciprocal of both sides to yield this final result

   $$c = \frac{1}{\frac{a}{b} + \frac{1}{d}}$$

   To clean up the denominator, multiply the right side of the equation by bd/bd to give

   $$c = \frac{bd}{ad + b}$$

To gain comfort with logarithms, determine the value of *x* in the following to two decimal places:

3. $\log(x) = 0.83$

   To determine the value of *x* we take the inverse log of both sides of the equation. Depending on your calculator, you may accomplish this by entering 0.83 and selecting INV LOG or by entering 0.83 and selecting 10^x. In either case, the value of *x* is 6.76.

4. $x = \log(1.35 \times 10^{-5})$

   To determine the value of *x*, enter 1.35×10^{-5} into your calculator and select the LOG key. Be sure that you use the base 10 log, not the base *e* or LN key. The value of *x* is –4.87.

5. $79 = 33^x$

   To solve for x take the log of both sides of the equation to remove x from the exponent

   $$\log(79) = x\log(33)$$

   Taking the log of 79 and of 33 (and keeping an extra decimal point for now), we obtain

   $$1.898 = 1.519x$$

   Finally, solving for x gives its value as 1.25.

Working with quadratic equations is something we will encounter later this semester when solving equilibrium problems; these questions provide practice in working with quadratic equations:

6. Rearrange the equation $\mathbf{0.20} = \frac{x^2}{55-x}$ into the form $ax^2 + bx + c$.

   Begin by multiplying both sides of the equation by $55 - x$

   $$x^2 = 0.20(55 - x)$$

   and then multiplying the 0.20 through the left side of the equation

   $$x^2 = 11 - 0.20x$$

   Next, add $0.20x$ and subtract 11 from both sides of the equation to give

   $$x^2 + 0.20x - 11 = 0$$

7. To three decimal places, what are the roots for the equation $3x^2 + 33x - 6.5 = 0$?

   The roots of a second-order polynomial equation of the form $ax^2 + bx + c = 0$ are given by

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

   Substituting in 3 for a, 33 for b, and –6.5 for c and solving

   $$x = \frac{-33 \pm \sqrt{(33)^2 - (4)(3)(-6.5)}}{(2)(3)} = \frac{-33 \pm \sqrt{1089 + 78}}{6} = \frac{-33 \pm \sqrt{1167}}{6} = \frac{-33 \pm 34.16}{6}$$

   The first root is (–33 + 34.16)/6 or 0.193 and the second root is (–33 – 34.16)/6 or –11.19.

Comfort with scientific notation is important, both in recognizing relative magnitudes and in entering values in your calculator; these questions provide practice with scientific notation:

8. Rank the following numbers from smallest-to-largest in magnitude: 9.0×10^{-6}, 8.1×10^{-6}, 1.6×10^{5}, 4.1×10^{-2}, 5.8×10^{4}

   The order is $8.1\times10^{-6} < 9.0\times10^{-6} < 4.1\times10^{-2} < 5.8\times10^{4} < 1.6\times10^{5}$

9. Convert the following from decimal to scientific notation, or from scientific notation to decimal notation: 0.000139, 452.78, 7.35×10^{-2}, 1.35×10^{5}

   $0.000139 \rightarrow 1.39\times10^{-4}$, $452.78 \rightarrow 4.5278\times10^{2}$, $7.35\times10^{-2} \rightarrow 0.0735$, $1.35\times10^{5} \rightarrow 135{,}000$

10. What is the value of x if $x = \frac{10^{-15}}{3.9\times10^{-7}}$?

    The value of x is 2.56×10^{-9}. If your answer is to the wrong power of 10, then you need to review the proper method for entering scientific notation into your calculator.