More Buffer Problems

1. A biochemist must prepare a solution as a medium in an experiment involving acid-producing bacteria. The pH of the medium must not change by more than 0.05 pH units for every 0.0010 mol H₃O⁺ generated by the organisms per liter of medium. A medium consisting of 0.10 M HA and 0.10 M A⁻ is prepared with a total volume of 1.0 L. Will the buffer capacity be sufficient for this experiment?

With a volume of 1.0 L and concentrations of 0.10 M HA and 0.10 M A⁻ we know that the initial amount of HA and A⁻ are each 0.10 moles. Adding a strong acid to the buffer causes the following reaction

$$A^-(aq) + H_3O^+(aq) \rightarrow H_2O(l) + HA(aq)$$

to go to completion; thus, we know that when equilibrium is reached

moles
$$A^- = (\text{moles A}^-)_0 - \text{moles H}_3 O^+$$

moles $HA = (\text{moles HA})_0 + \text{moles H}_3 O^+$

From the Henderson-Hasselbalch equation

$$pH = pK_a + log \frac{[A^-]}{[HA]} = pK_a + log \frac{(moles A^-)}{(moles HA)}$$

we define ΔpH as

$$\Delta pH = pH_{final} - pH_{initial} = \left(\log \frac{(\text{moles A}^{-})}{(\text{moles HA})}\right)_{final} - \left(\log \frac{(\text{moles A}^{-})}{(\text{moles HA})}\right)_{initial}$$

Adding a strong acid makes the pH smaller; thus Δ pH is -0.05 and

$$-0.05 = \left(\log \frac{0.10 - \text{moles H}_3\text{O}^+}{0.10 + \text{moles H}_3\text{O}^+}\right)_{\text{final}} - \left(\log \frac{0.10}{0.10}\right)_{\text{intiial}}$$

$$\frac{0.10 - \text{moles H}_3\text{O}^+}{0.10 + \text{moles H}_3\text{O}^+} = 0.891$$

$$0.10 - \text{moles H}_3\text{O}^+ = 0.0891 + 0.891 \times \text{moles H}_3\text{O}^+$$

$$\text{moles H}_3\text{O}^+ = 5.76 \times 10^{-3} \text{ mole}$$

Thus, we can add as much as 5.76×10^{-3} moles of strong acid and maintain a ΔpH of 0.05; as this is better than the stated requirements, the buffer is acceptable.

2. A 1.00-L buffer is prepared that is 0.2000 M in the weak acid, HA, and 0.1500 M in the weak base NaA. The buffer has a pH of 3.35. What is the pK_a for the weak acid?

Using the Henderson-Hasselbalch equation

$$3.35 = pK_a + \log \frac{0.1500}{0.2000}$$

Gives the p K_a as 3.47.

Is this buffer better at neutralizing strong acid or strong base?

The buffer has more of its conjugate weak acid, HA, than its conjugate weak base, A⁻; therefore, the buffer is better at neutralizing strong base.

What is the buffer's capacity to neutralize strong acid?

We can add strong acid until the ratio [A⁻]/[HA] is 0.1000. Adding a strong acid converts A⁻ to HA; thus

$$\frac{\text{moles A}^{-} - \text{moles H}_{3}\text{O}^{+}}{\text{moles HA} + \text{moles H}_{3}\text{O}^{+}} = 0.1000$$

Since we have 1.00-L of the buffer, we have 0.1500 moles of A⁻ and 0.2000 moles of HA; thus

$$\frac{0.1500 - \text{moles H}_3\text{O}^+}{0.2000 + \text{moles H}_3\text{O}^+} = 0.1000$$

$$0.1500 - \text{moles H}_3\text{O}^+ = 0.02000 + 0.1000 \times \text{moles H}_3\text{O}^+$$

$$\text{moles H}_3\text{O}^+ = 0.1182$$

A total of 0.1182 moles of strong acid can be neutralized.

What is the buffer's capacity to neutralize strong base?

We can add strong base until the ratio $[A^-]/[HA]$ is 10.00. Adding a strong base converts HA to A^- ; thus

$$\frac{\text{moles A}^{-} + \text{moles OH}^{-}}{\text{moles HA} - \text{moles OH}^{-}} = 10.00$$

Since we have 1.00-L of the buffer, we have 0.1500 moles of A⁻ and 0.2000 moles of HA; thus

$$\frac{0.1500 + \text{moles OH}^{-}}{0.2000 - \text{moles OH}^{-}} = 10.00$$

$$0.1500 + \text{moles OH}^{-} = 2.000 - 10.00 \times \text{moles OH}^{-}$$

$$\text{moles OH}^{-} = 0.1682$$

A total of 0.1682 moles of strong acid can be neutralized. Note that this is consistent with the expectation that the buffer can neutralize more strong base.

What is the buffer's pH of the buffer if 0.0015 mol NaOH is added to 0.5000 L of the buffer?

Since we are working with only $\frac{1}{2}$ of the buffer, the amount of HA and A⁻ are 0.1000 moles HA and 0.0750 moles A⁻; thus

pH =
$$3.47 + \log \frac{0.0750 + 0.0015}{0.1000 - 0.0015} = 3.36$$

3. An environmental chemist needs a carbonate buffer of pH 10.00 to study the effects of the acidification of limestone-rich soils. How many grams of Na₂CO₃ must she add to 1.5 L of freshly prepared 0.20 M NaHCO₃ to prepare this buffer?

This buffer is based on HCO_3^- and CO_3^{2-} for which the p K_a is 10.33. The total moles of HCO_3^- at the start is

$$0.20 \text{ M NaHCO}_3 \times 1.5 \text{ L} = 0.30 \text{ moles HCO}_3^-$$

Using the Henderson-Hasselbalch equation

$$10.00 = 10.33 + \log \frac{\text{moles CO}_3^{2-}}{0.30 \text{ moles HCO}_3^{-}}$$

Solving for the moles of CO₃²⁻ gives 0.140, which is equivalent to 14.9 g of Na₂CO₃.