

Linear Algebra

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May 26, 2016

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Chapter 1

Groups, Rings and Fields

The concept of mathematical sets is important in all fields of mathematics, and is especially true in linear algebra given the ubiquity of vector spaces and subspaces. As such, the idea of fields, a close analog to vector spaces, is quite important to mention in a proper reading of the topic, and thus, we begin by introducing groups, upon which we build rings, which we consequently use to build a field.

1.1 Groups

1.2 Rings

1.3 Fields

Chapter 2

Vector Spaces, Subspaces and Quotient Spaces

Chapter 3

Spans, Linear Independence and Bases

Chapter 4

Linear Transformations and the Isomorphism Theorems

4.1 Nilpotent Transformations

4.2 Projection Transformations

Chapter 5

Matrices and Linear Systems

Chapter 6

Applications

At this point, we bring up some interesting applications that require only the knowledge of solving linear systems using basic row reduction operations.

6.1 Discrete Dynamics

6.2 Markov Chains

6.3 Stochastic Matrices

Chapter 7

Determinants, Invertibility, and Eigen-theory

In this chapter, we'll introduce the determinant function, which is a special function (in its alternating and multilinear characteristic) that allows us to introduce another perspective of linear transformations. More specifically, we'll look at how transformations can be inverted (i.e. when they are bijective), and see how this may be useful in developing the idea of similar transformations.

7.1 Determinants

7.2 Invertibility

7.3 Eigenvalues and Eigenvectors

7.4 Diagonalization and Similarity

7.5 Spectral Value Decomposition

Chapter 8

Inner Products

Chapter 9

Adjoint, Spectral Theorem, Principal Axis Theorem

Chapter 10

Jordan and Rational Canonical Forms

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10.1 Invariant Subspaces

10.2 Jordan Canonical Forms

10.3 Rational Canonical Forms

10.4 Applications

Chapter 11

Application to Differential Equations

Chapter 12

The Similarity Problem