Lecture 16: Convolution Networks

CS 109B, STAT 121B, AC 209B, CSE 109B

Mark Glickman and Pavlos Protopapas

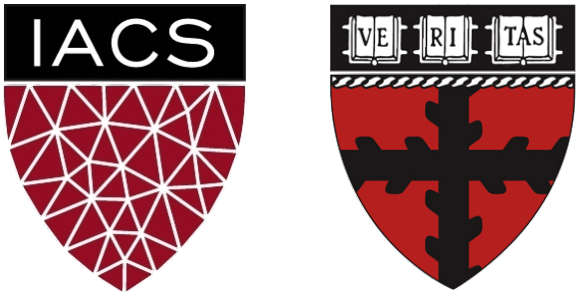


Image Data

- Fully-connected nets infeasible
- E.g. 1K x 1K image (1 million pixels)
  - No. of parameters blow up
- *Convolution Nets*
  - Exploit image structure
  - Use sparse weights between layers

- CNNs don't use convolution
-

Images are Local and Hierarchical

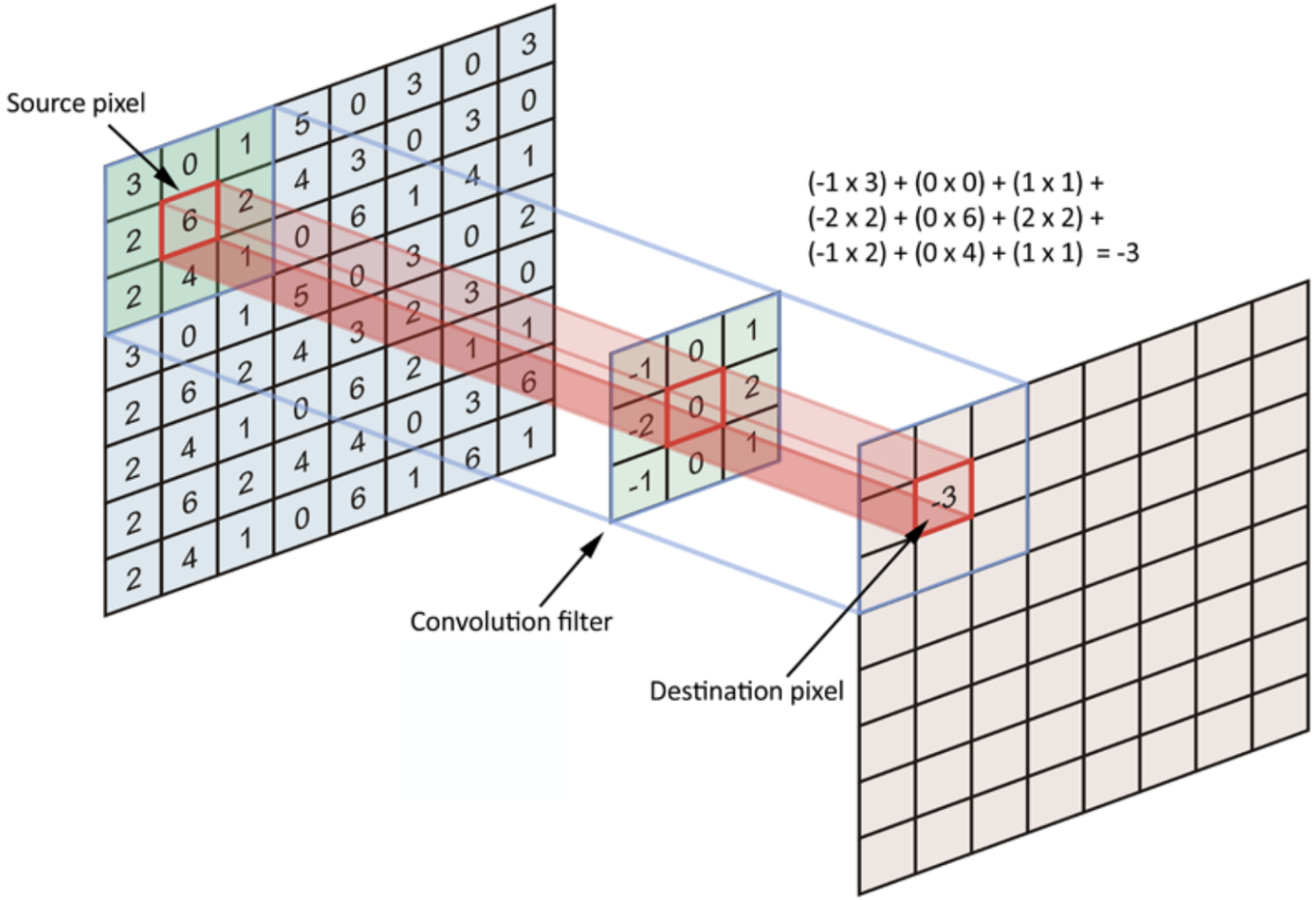
Nearby pixels are more strongly related than distant ones.

Objects are built up out of smaller parts.

Images are Invariant

“Convolution” Operation

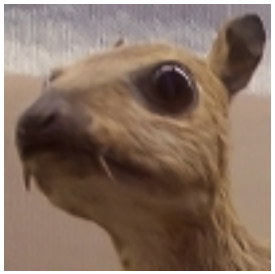
Form a convolution



“Convolution” Operation

Edge detection

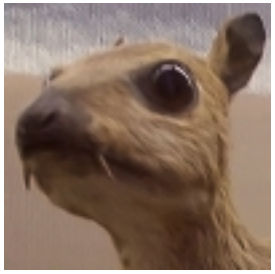
Kernel



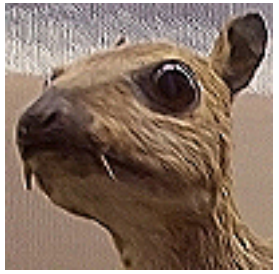
$$* \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} =$$

Text

Sharpen

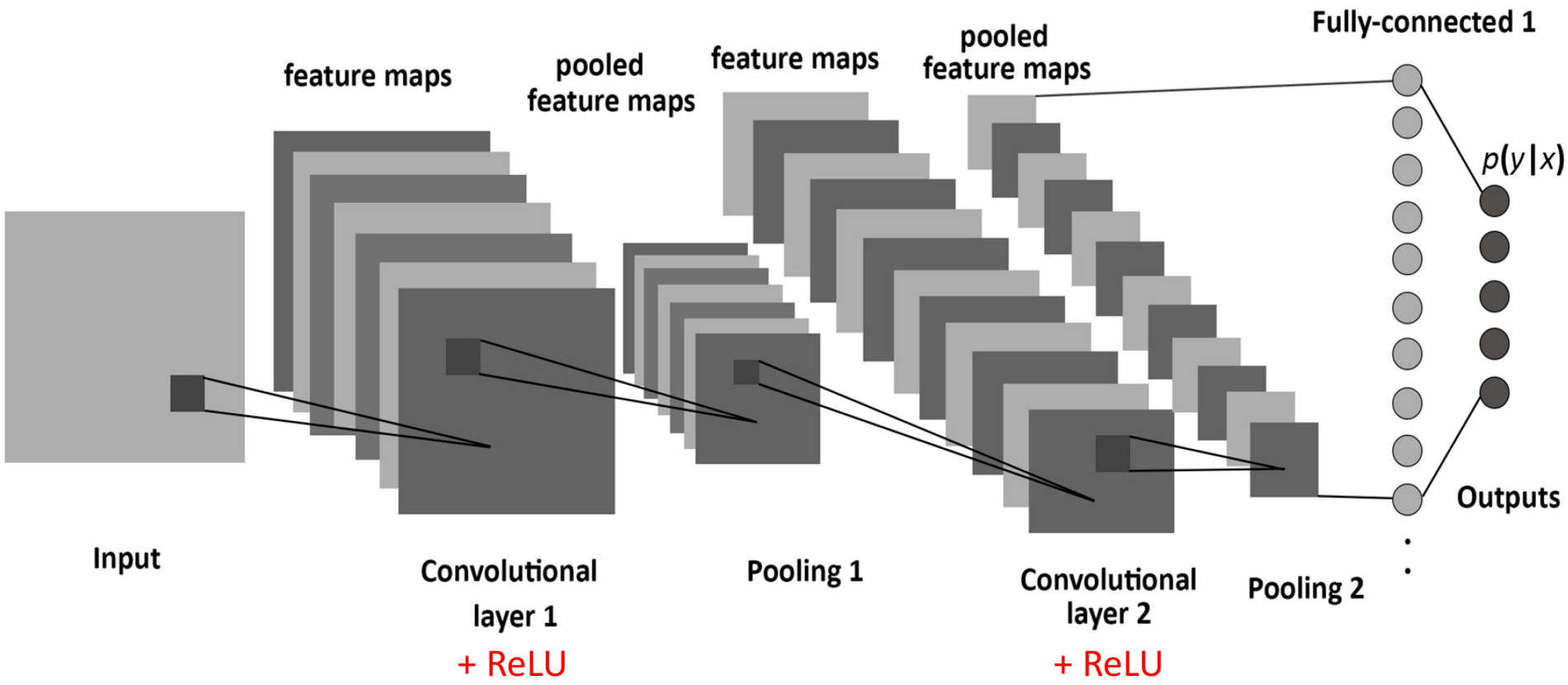


$$* \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} =$$



wikipedia.org

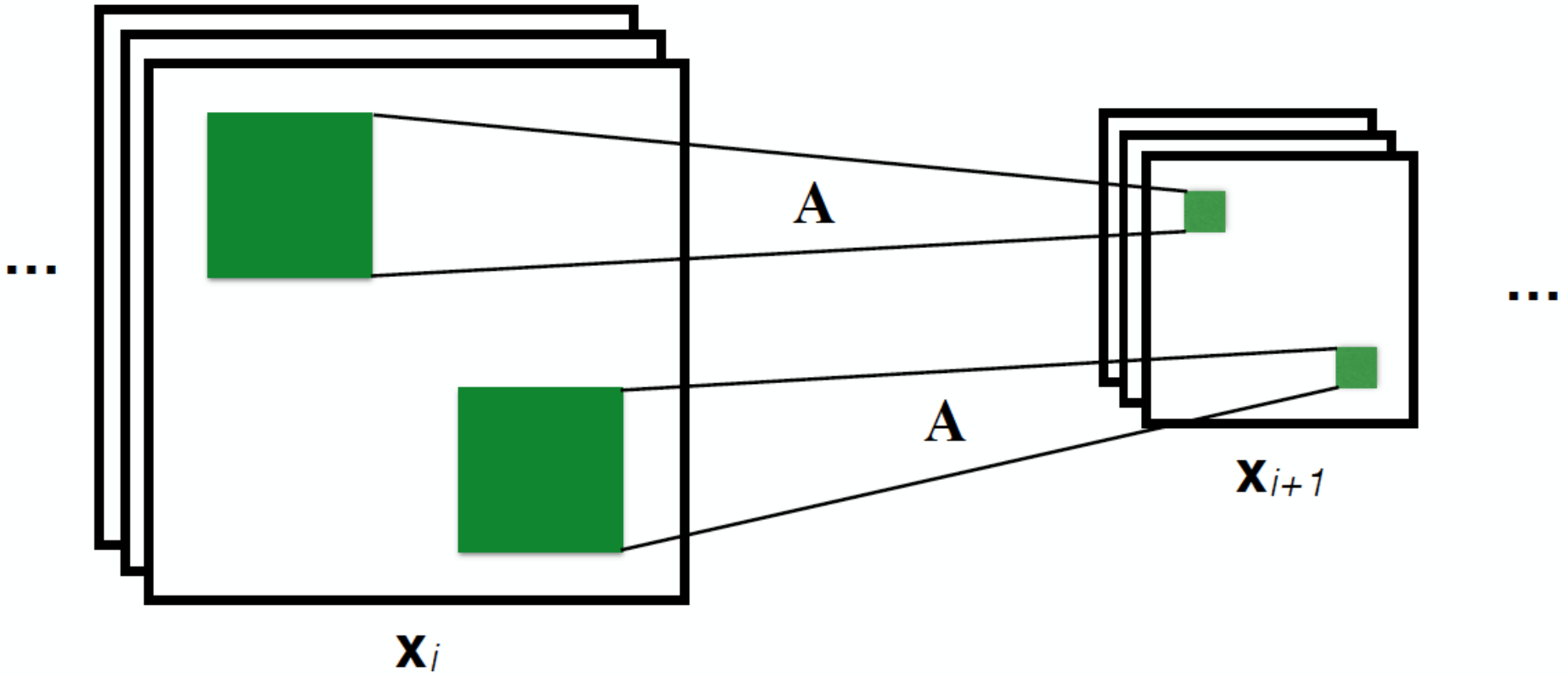
A Convolutional Network



Key Features

- Sparse interaction
- Parameter sharing
- Pooling
- Zero padding

Sparse Interactions



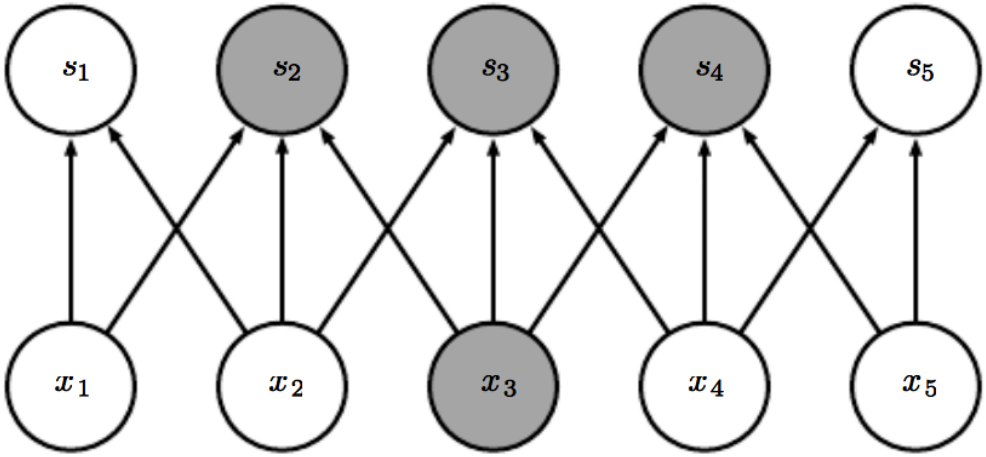
$$\mathbf{x}_{i+1} = \sigma_i(\mathbf{W}_i\mathbf{x}_i + \mathbf{b}_i)$$

Each output interacts to a small region of inputs

Sparse Interactions

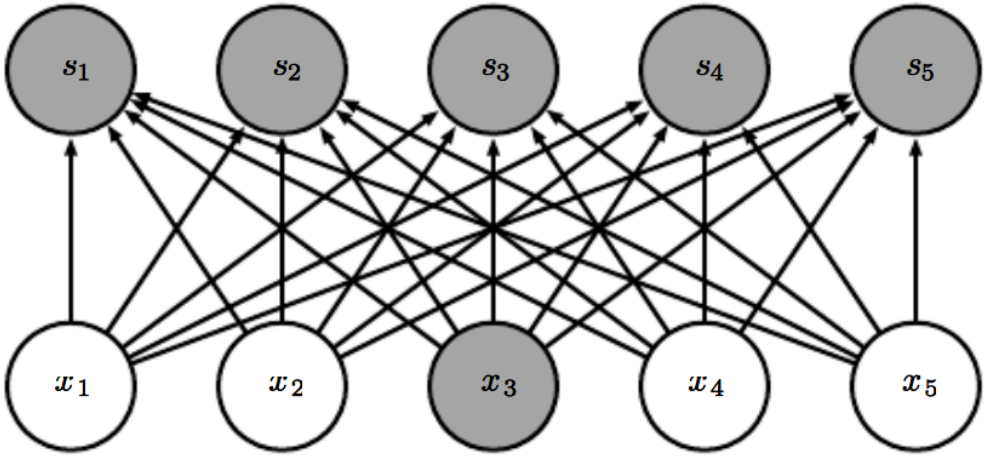
- Kernel size smaller than input
  - Image: usually millions of pixels
  - Kernel: usually 100s or 1000s of parameters
- Reduced memory and run-time, improved statistical efficiency
  - $O(m \times n) \Rightarrow O(k \times n)$ parameters

Sparse Interactions



Convolutional

Much fewer weights in sparse, compared to fully-connected



Fully-connected

Computational Costs

Input size: 320 by 280
Kernel size: 2 by 1
Output size: 319 by 280

	Convolution	Dense matrix	Sparse matrix
Stored floats	2	319*280*320*280 > 8e9	2*319*280 = 178,640
Float muls or adds	319*280*3 = 267,960	> 16e9	Same as convolution (267,960)

Key Features

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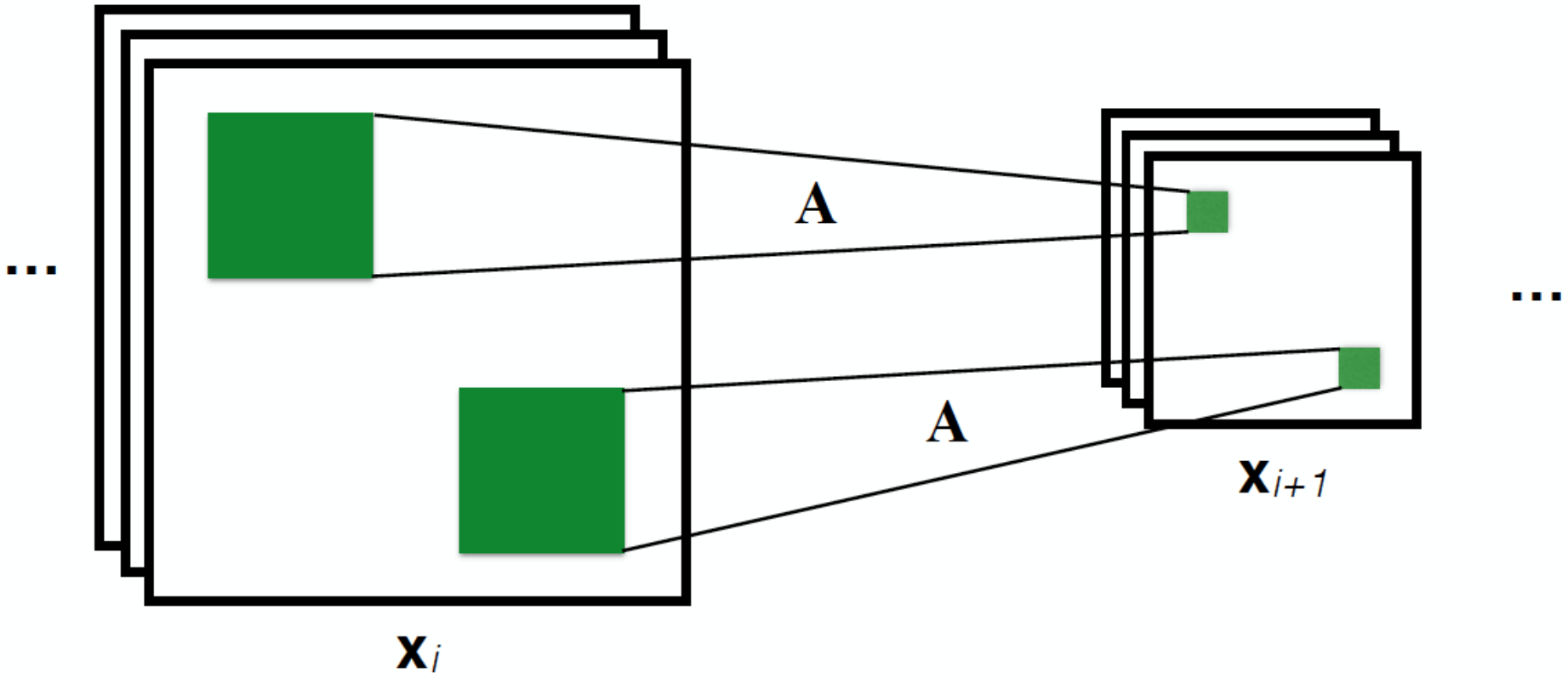
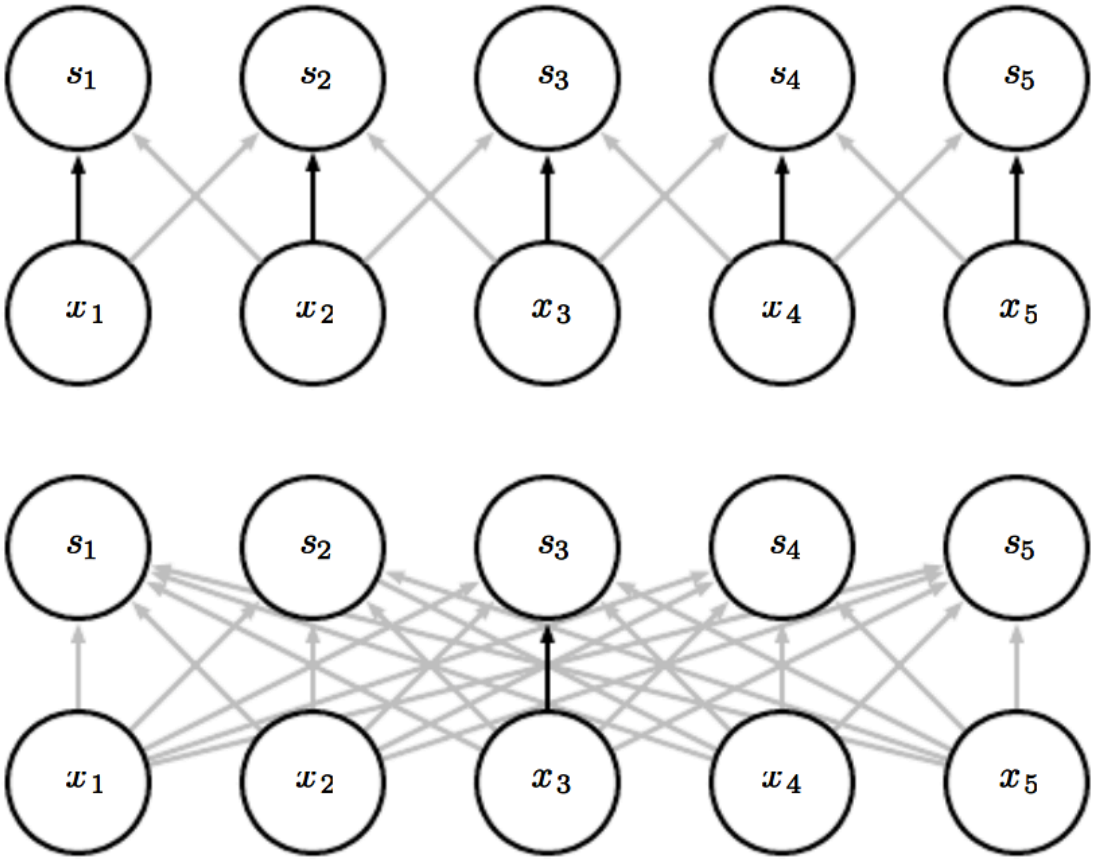
Parameter Sharing



$$\mathbf{x}_{i+1} = \sigma_i(\mathbf{W}_i\mathbf{x}_i + \mathbf{b}_i)$$

Same **A** *in different locations* – only learn **A**

Parameter Sharing

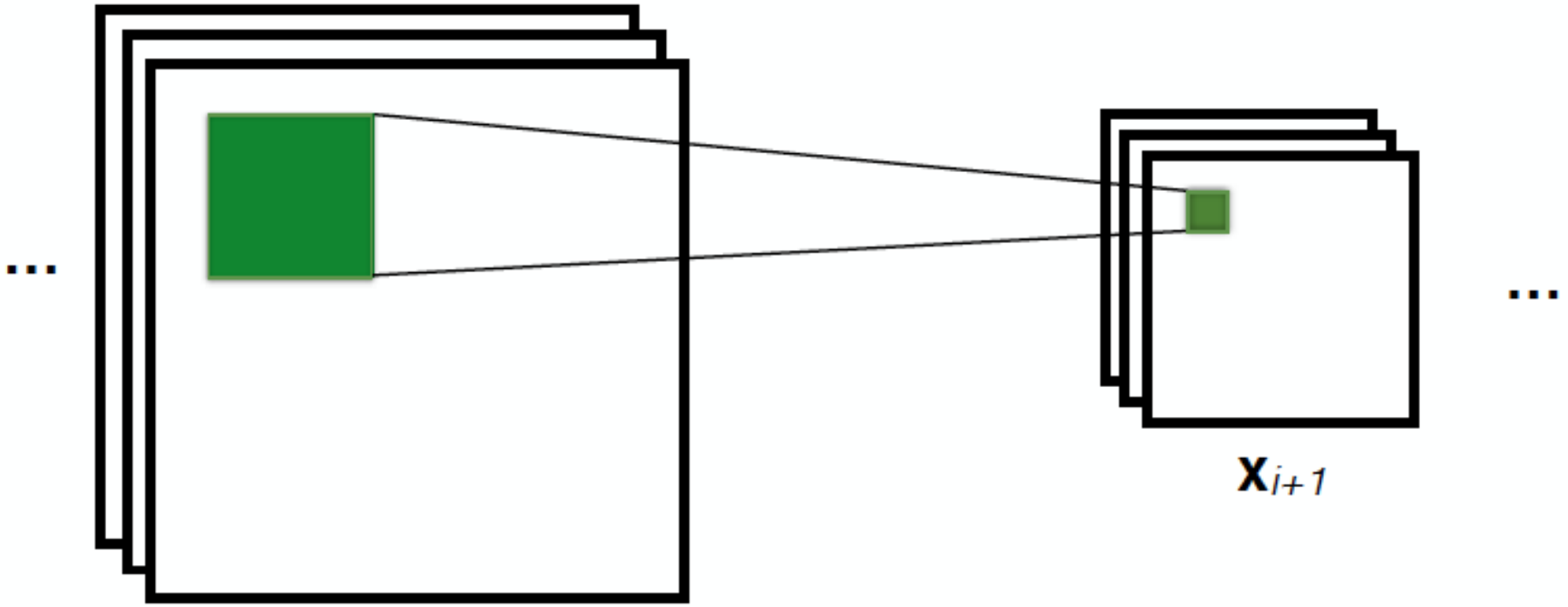


Convolutional

Fully-connected

Parameter Sharing

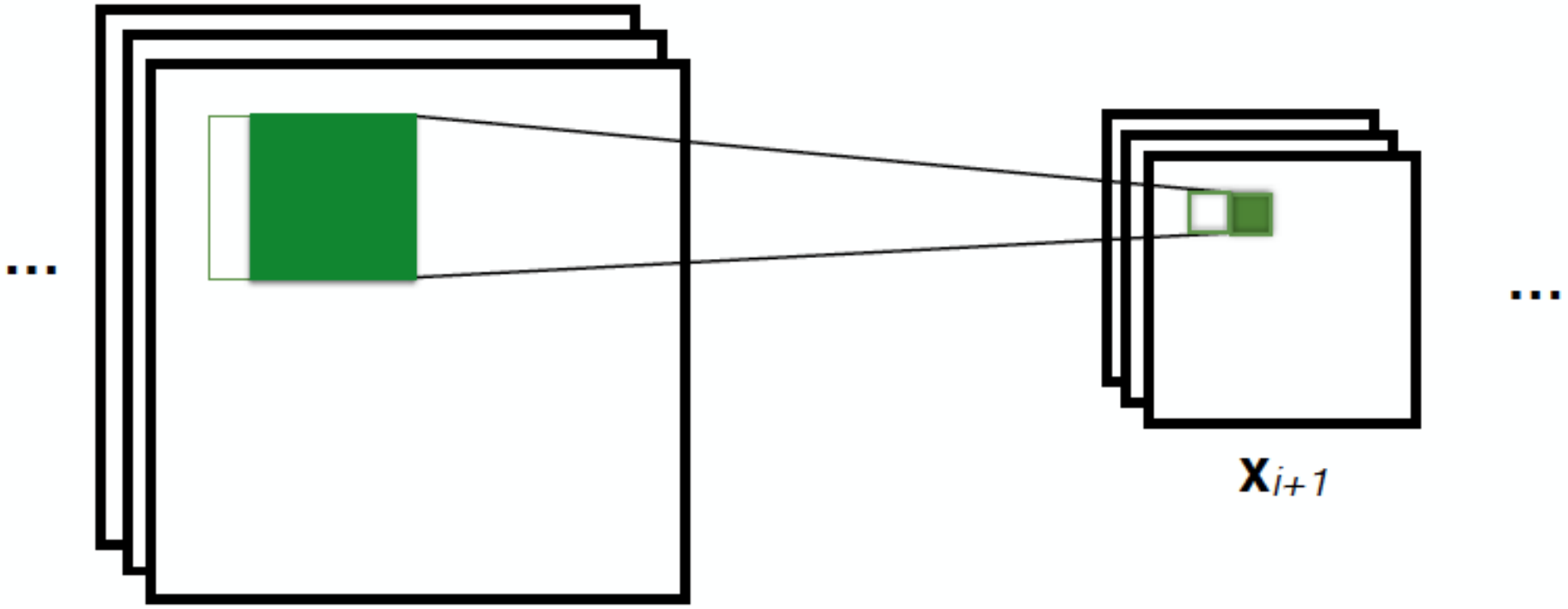
Change in input mapped to change in output



$$\mathbf{x}_{i+1} = \sigma_i(\mathbf{W}_i\mathbf{x}_i + \mathbf{b}_i)$$

Equivariant to translation

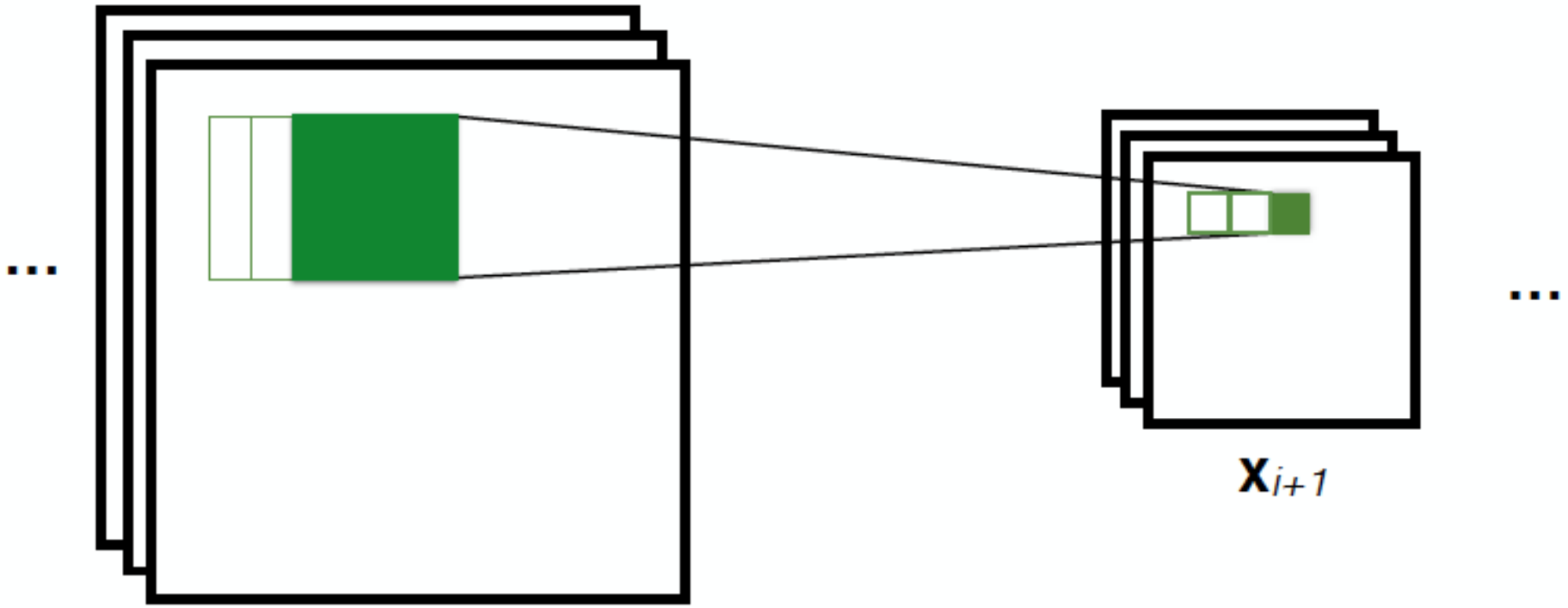
Parameter Sharing



$$\mathbf{x}_{i+1} = \sigma_i(\mathbf{W}_i\mathbf{x}_i + \mathbf{b}_i)$$

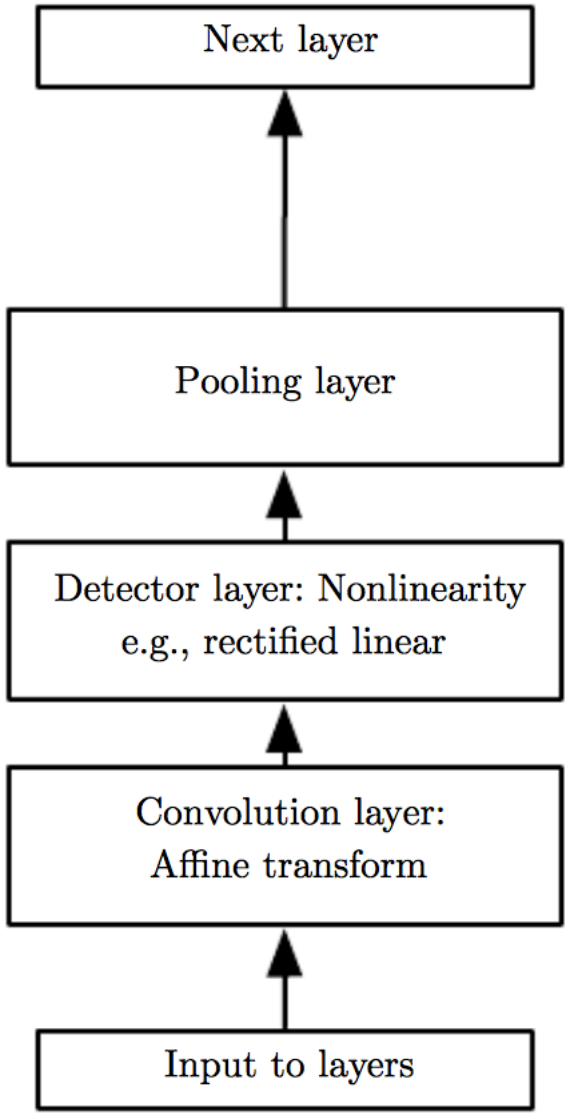
Equivariant to translation

Parameter Sharing



$$\mathbf{x}_{i+1} = \sigma_i(\mathbf{W}_i\mathbf{x}_i + \mathbf{b}_i)$$

Equivariant to translation

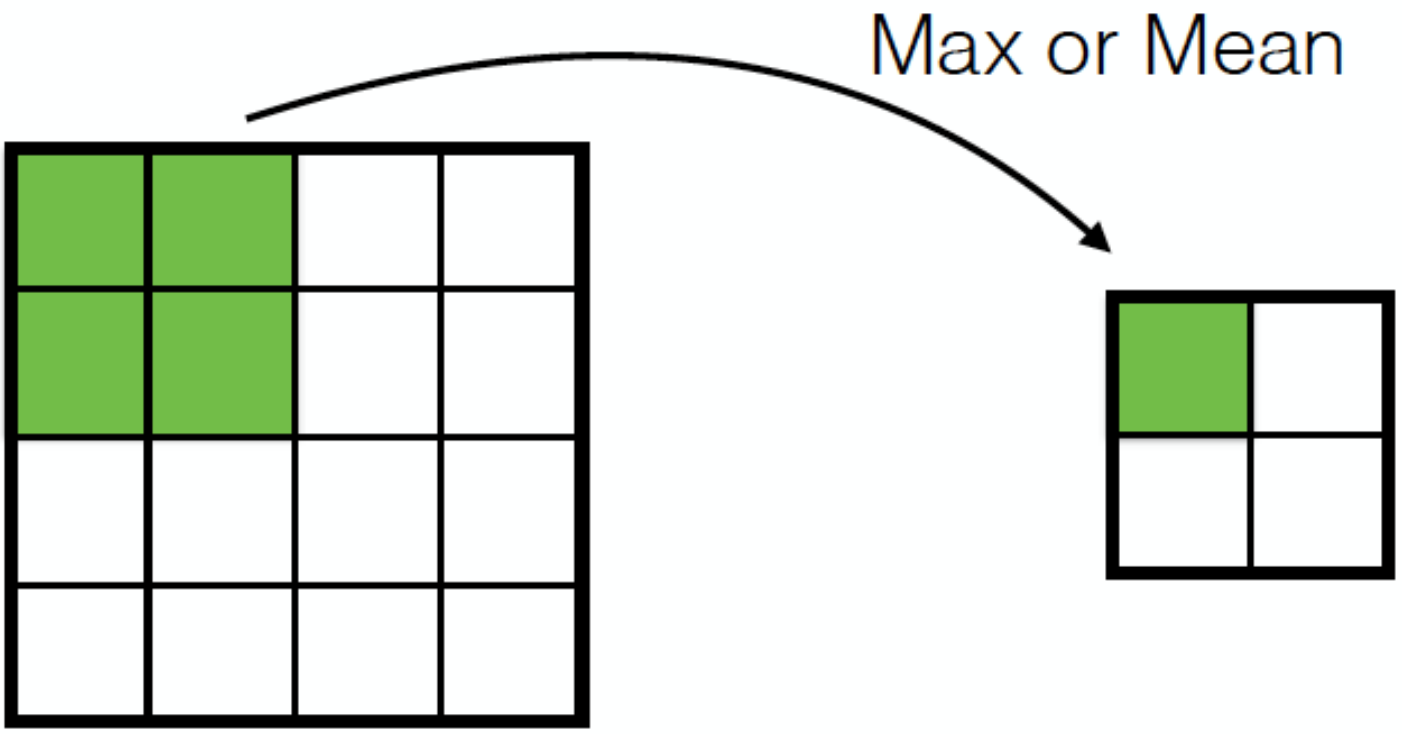


Non-linear activation

Key Features

- Sparse interaction
- Parameter sharing
- Pooling
- Zero padding

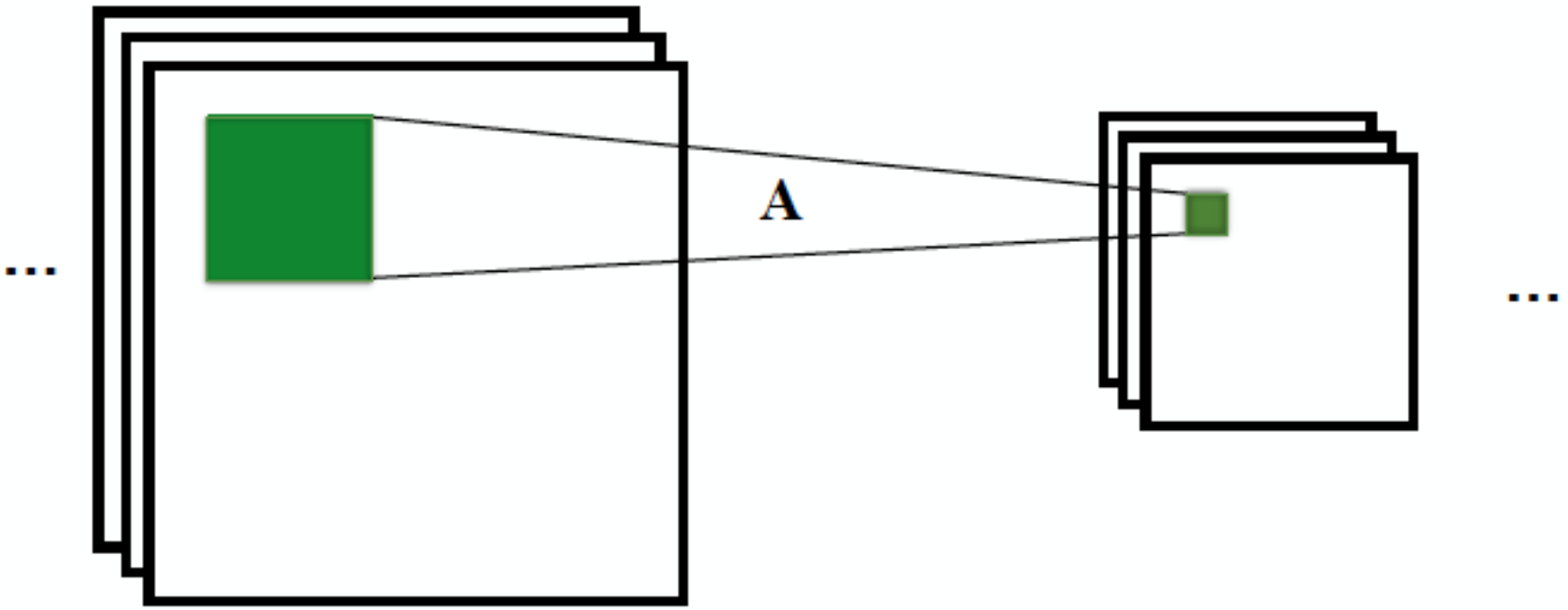
Pooling



Pooling

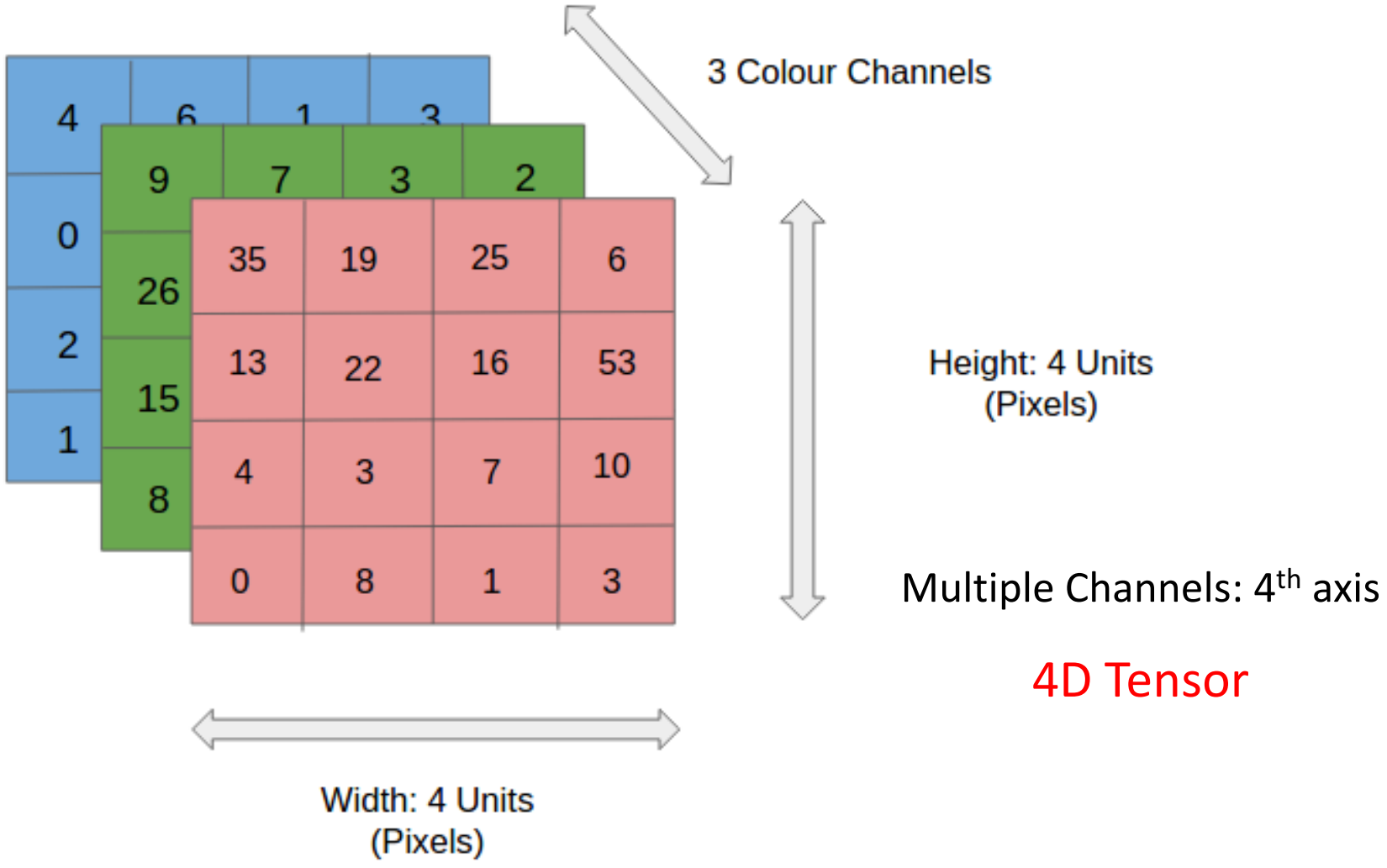
- Invariant to small, “local transitions”
  - Face detection: enough to check the presence of eyes, not their precise location
- Fewer pooling units than detector units
  - Reduces input size to final fully connected layers

Multiple Channels



ConvNets can have multiple channels a.k.a. feature maps, each with separate parameters

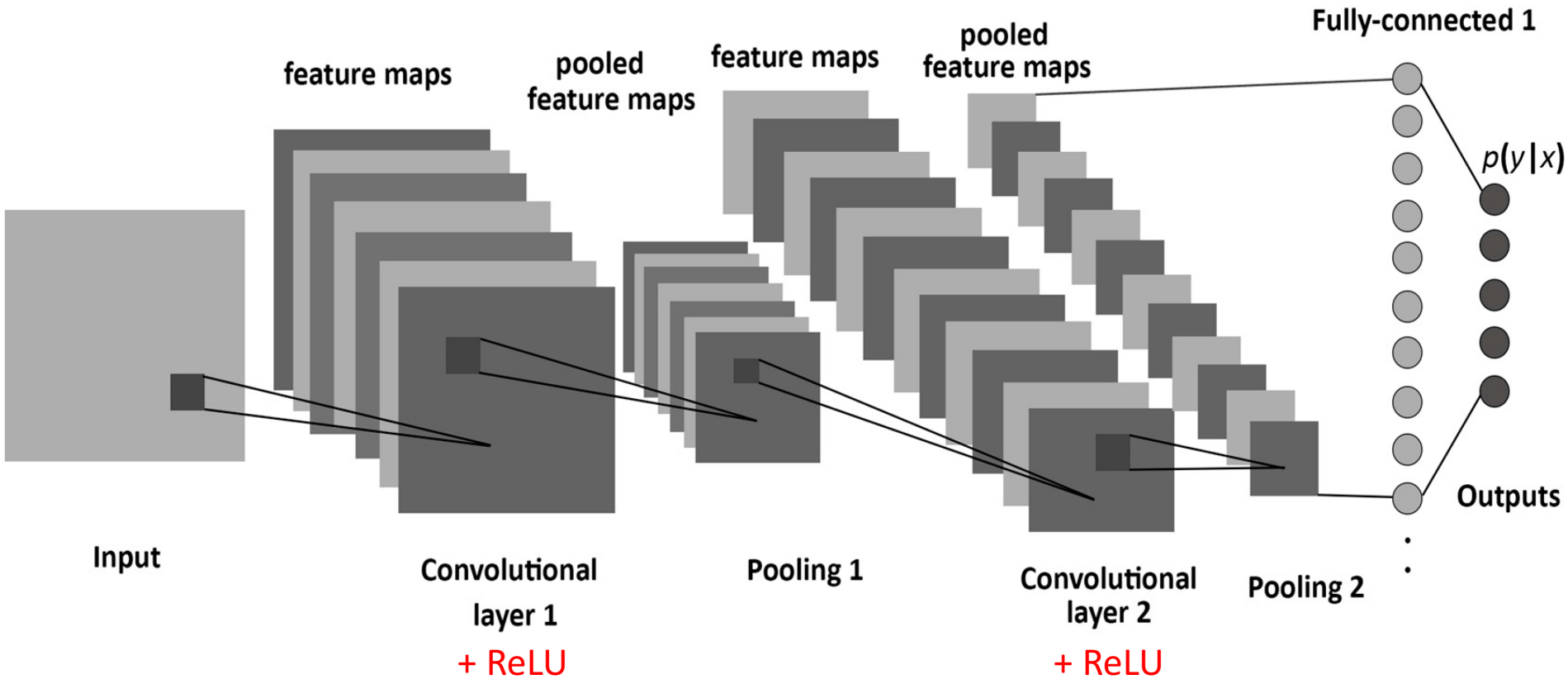
Multiple Dimensions



Multiple Channels: 4th axis

4D Tensor

Overall Architecture



Key Features

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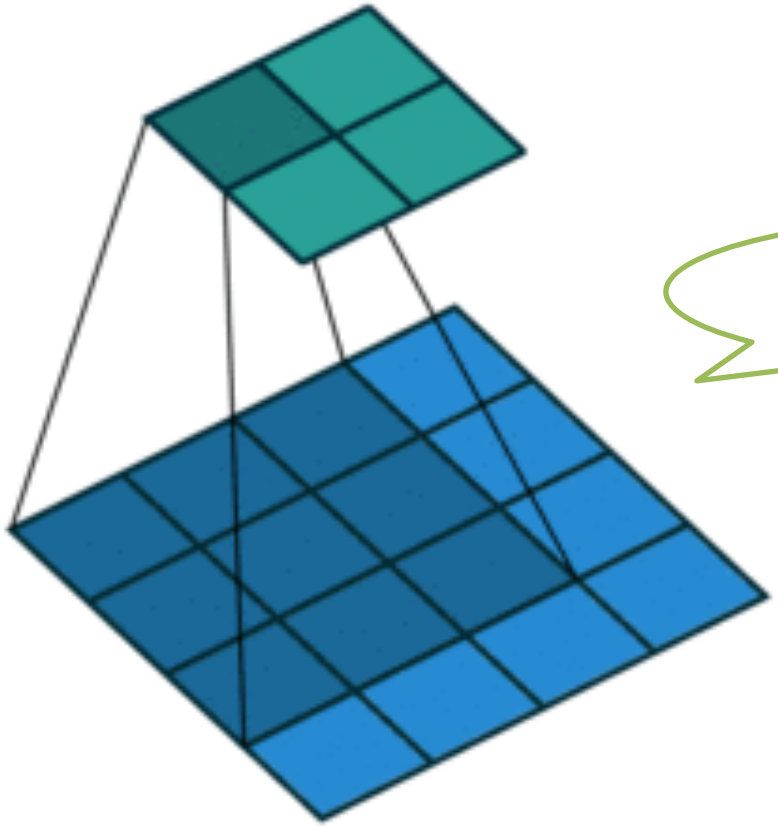
- Zero padding

No Padding

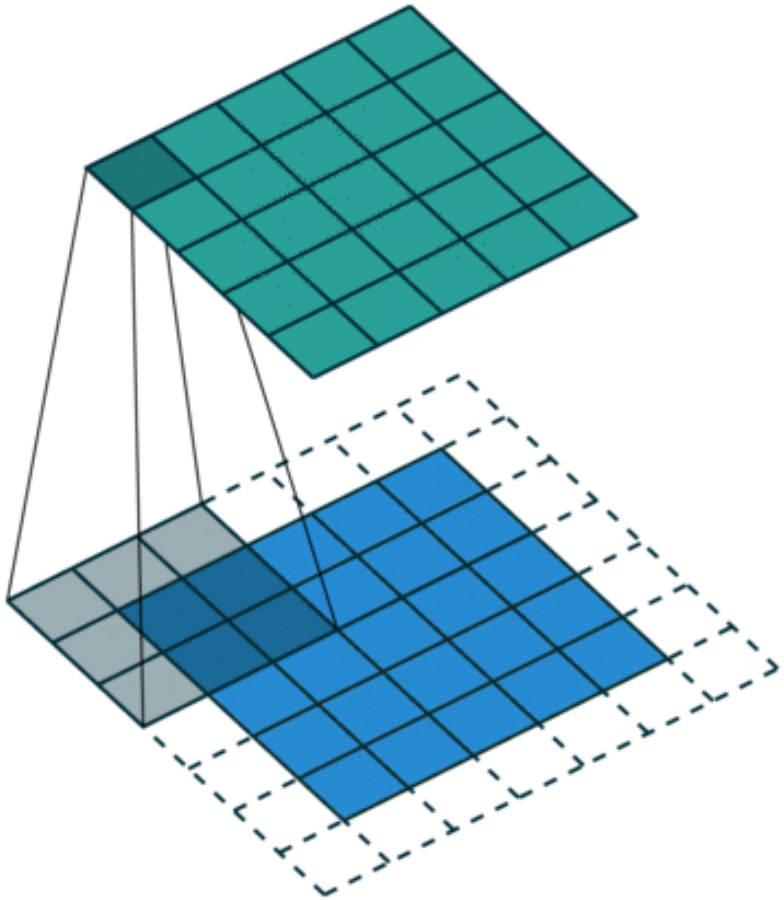
- Representation size shrinks by ‘kernel width – 1’ after each layer



Ignores large portion of input image

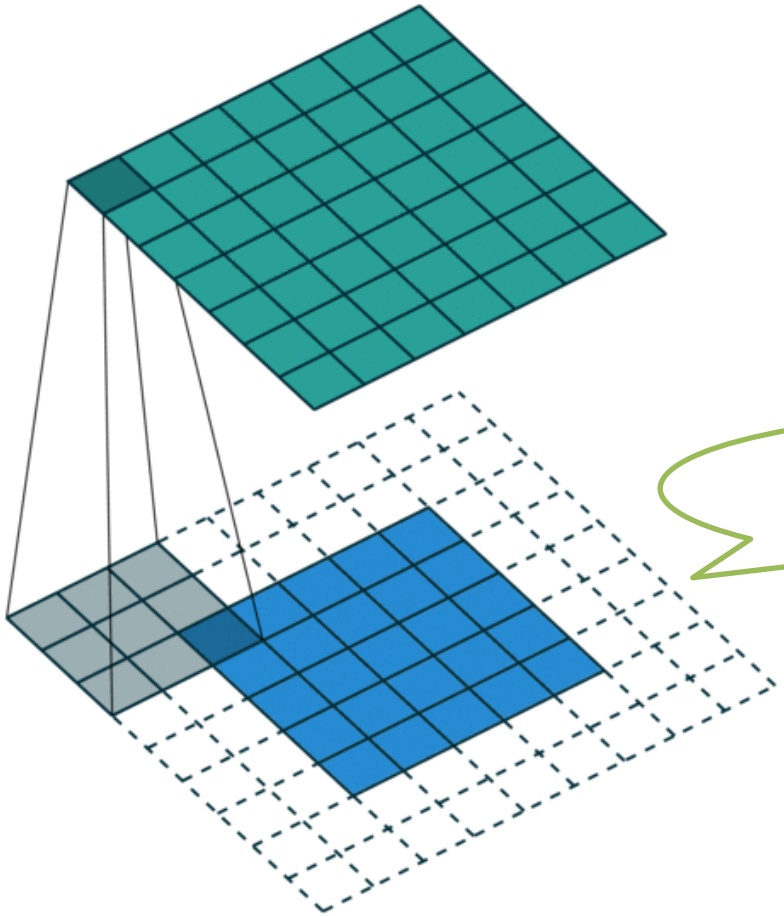
Same Padding

- Pad zeros so that output is same size as input



Full Padding

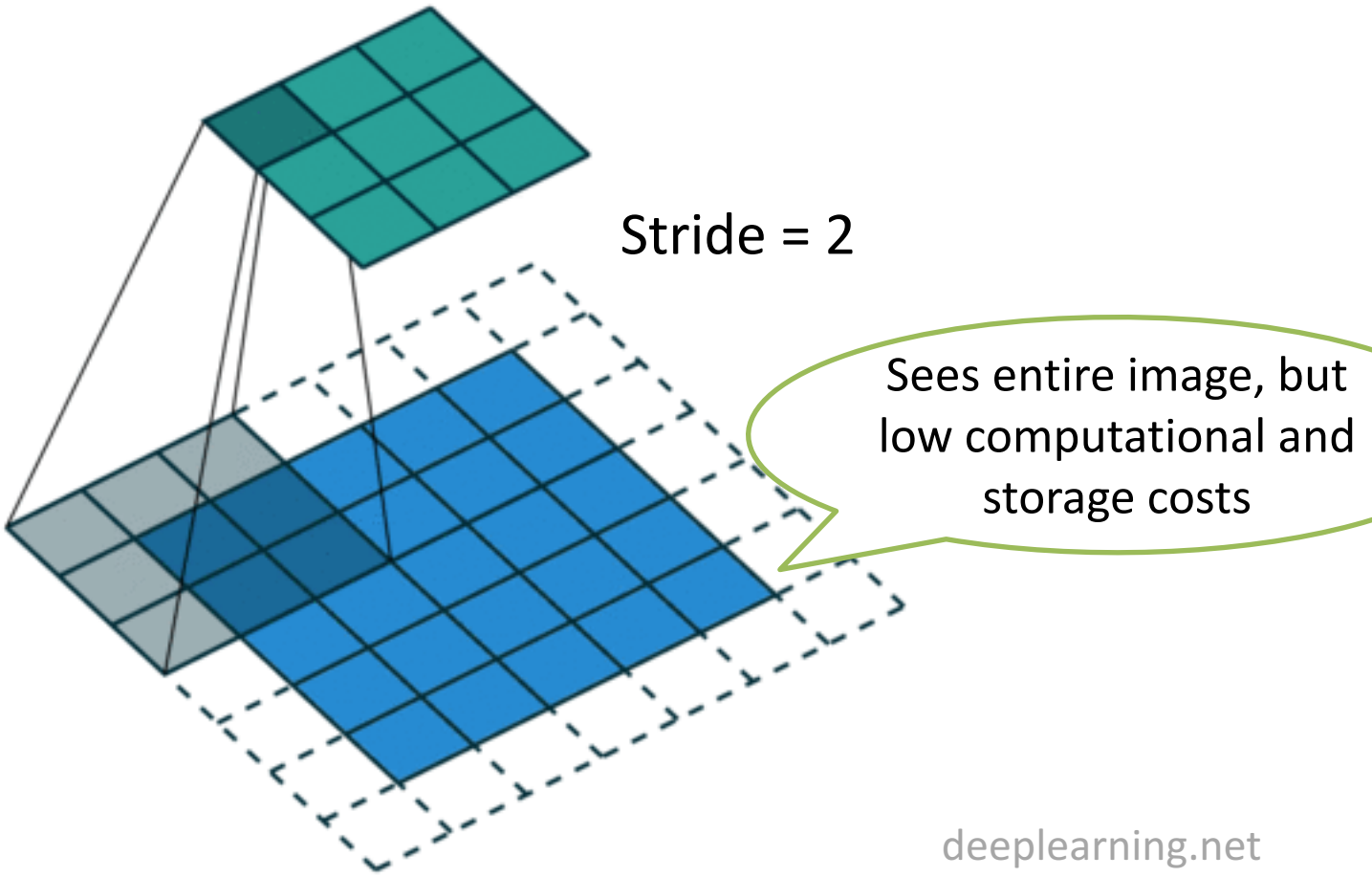
- Pad zeros so that every pixel is visited same number of times



Increases computation and storage cost

Padding with Strides

- Shift the kernel not by 1 pixel, but by k pixels (stride)



Key Features

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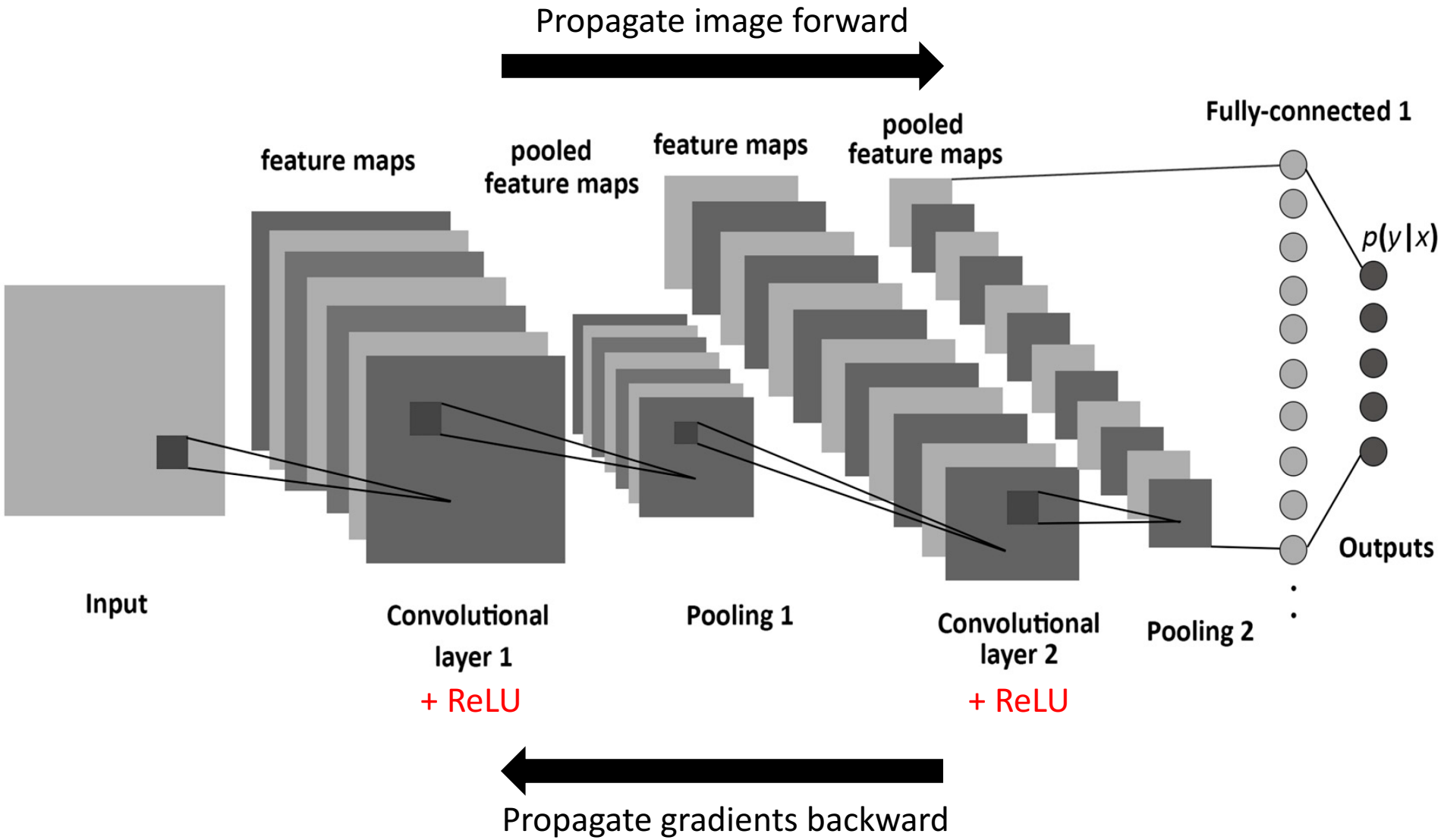
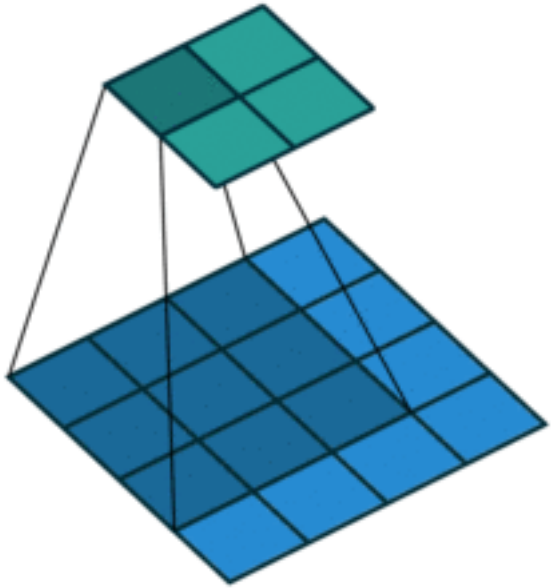
Back-propagation



Back-propagation

- Forward pass through a convolution unit

Kernel: 3 × 3

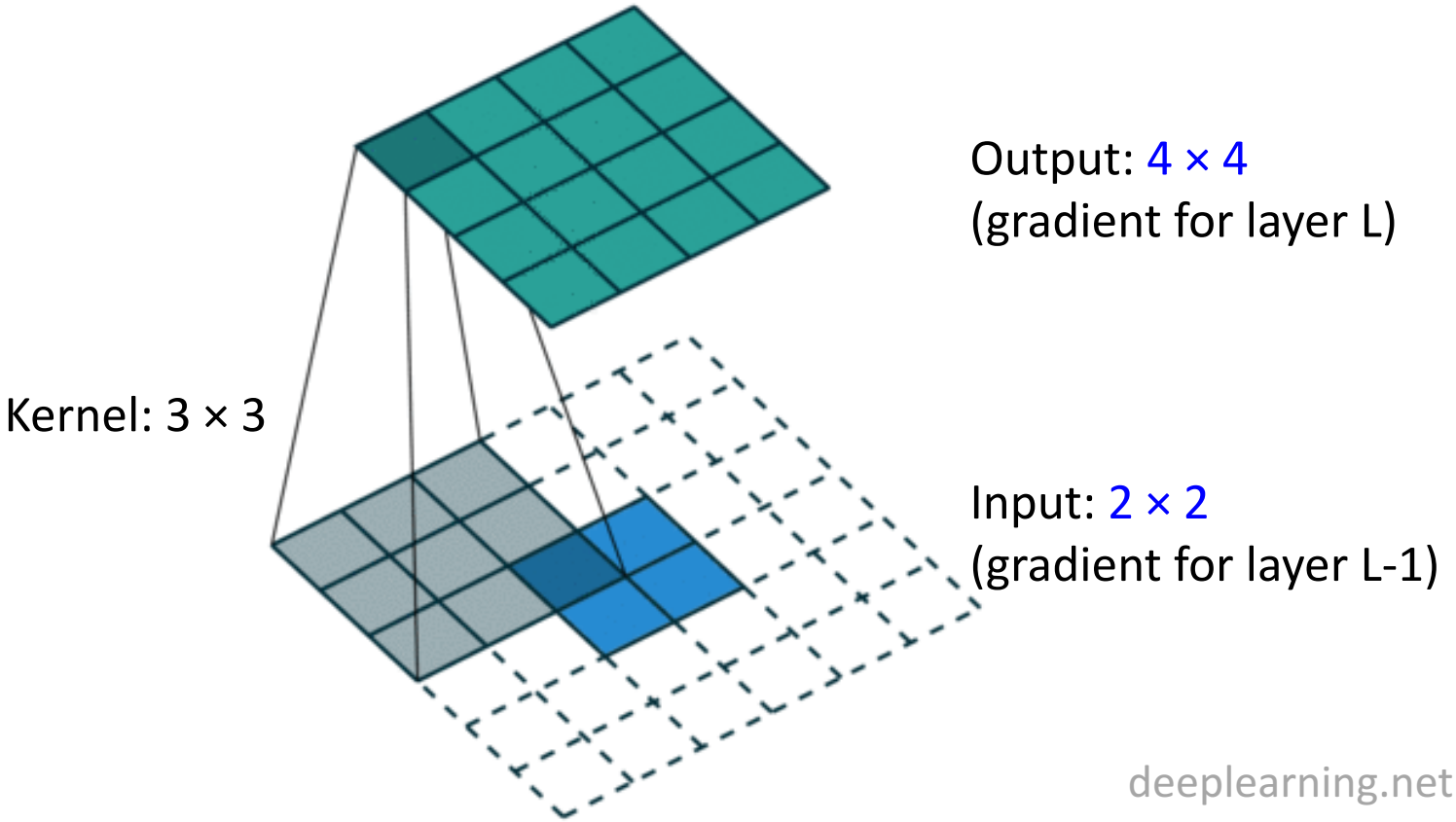


Output: 2 × 2

Input: 4 × 4

Back-propagation

- Backward pass = transpose convolution



Back-propagation

- Parameter matrix **W** contains offset copies of the kernel matrix (Toeplitz matrix)

$$\mathbf{x}^{L} = \mathbf{W}\mathbf{x}^{L-1} \quad \text{(forward)}$$

- Parameter matrix for the transpose convolution is $\mathbf{W}^{\mathrm{T}}$

$$\mathbf{g}^{L-1} = \mathbf{W}^{\mathrm{T}}\mathbf{g}^{L} \quad \text{(backward)}$$

ConvNet Variants

- Locally-connected Layers
  - No parameter sharing, learn separate weights for each spatial location
  - Useful when each output needs to be a function of a small part of the input
- Tiled Convolution
  - Learn set of kernels, rotate them through space
  - Neighboring locations have different filters