

I. PIVOT

PROBLEM DESCRIPTION

Given a “pivot’d” array (the one used in quicksort). Determine how many elements in the array could have been the chosen pivot.

SOLUTION TECHNIQUES

Dynamic Programming

SOLUTION SKETCHES

An element $a[i]$ could have been the pivot, if all the elements on the left side $\leq a[i]$ and $a[i] <$ all the elements on the right side.

To determine whether this is true, we only need to know whether $a[i]$ is greater than or equal to the greatest element of the left side and whether $a[i]$ is less than the smallest element on the right side. This can be efficiently with Dynamic Programming using a prefix-sum-like approach.

Let $f[i]$ be $\max_{0 \leq j \leq i} a_j$ and $b[i]$ be $\min_{i \leq j \leq n-1} a_j$

$$f[i] = \begin{cases} a_0, & i = 0 \\ \max(f[i-1], a_i), & i > 0 \end{cases}$$

And likewise for $b[i]$.

An element $a[i]$ could have been the pivot if $f[i-1] \leq a[i] < b[i+1]$.

TIME COMPLEXITY

$O(n)$

SOLUTION PROGRAM FOR REFERENCE

```
1. #include <iostream>
2. #include <stdio>
3. #include <cstring>
4. #include <algorithm>
5.
6. using namespace std;
7.
8. const int N = 1e5 + 5;
9.
10. int a[N];
11. int f[N], b[N];
12.
13. int main()
14. {
15.     int i, n, ans = 0;
16.     scanf("%d", &n);
17.     for (i = 0; i < n; i++) scanf("%d", &a[i]);
18.     f[0] = a[0];
19.     for (i = 1; i < n; i++)
20.         f[i] = max(f[i - 1], a[i]);
21.     b[n - 1] = a[n - 1];
22.     for (i = n - 2; i >= 0; i--)
23.         b[i] = min(b[i + 1], a[i]);
24.     if (a[0] < b[1]) ans++;
25.     if (f[n - 2] <= a[n - 1]) ans++;
26.     for (i = 1; i <= n - 2; i++)
27.         if (f[i - 1] <= a[i] && a[i] < b[i + 1])
28.             ans++;
29.     printf("%d\n", ans);
30.     return 0;
31. }
```