Two-User Interference Channels With Local Views: On Capacity Regions of TDM-Dominating Policies

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***Abstract*—We study the limits of reliable communication in two-user interference channels where each of the two transmitters knows a different subset of the four channel gains characterizing the network state. In order to systematically analyze this problem, we introduce two concepts. First, we define a local view model for network state information at each transmitter, wherein each transmitter knows only a subset of the four channel gains. This subset may be mismatched from that of the other transmitter, limiting the ability of the two transmitters to coordinate transmission decisions. Second, we define a notion of maximal rate regions achievable by transmission policies that dominate time-division multiplexing (TDM). Specifically, these "TDM-dominating capacity regions" characterize rates achievable when transmission schemes must, for every possible realization of network state, achieve rates at least as good as what can be achieved through TDM. We consider a set of seven symmetric local views based on an assumption that each transmitter uses the same mechanism to gather its local view. For five out of the seven local views, we show that TDM is sufficient to achieve the full TDM-dominating capacity region for the linear deterministic interference channel. For these five local views, our result implies that no single policy can achieve a rate point outside the TDM region without inducing sub-TDM performance in another network state. The common traits shared by the two remaining local views (those with better performance than TDM) are: first, each transmitter knows its outgoing interference channel gain, and second, there exists at least one channel gain known to both transmitters. For these two local views, transmitters can use their knowledge to achieve opportunistic rate gains beyond TDM. Using the relationship between the linear deterministic channel and the Gaussian channel, we extend our conclusions to bounded gap characterizations of the TDM-dominating capacity region for the Gaussian interference channel with local views.**

***Index Terms*—Compound channel, interference channel (IC), local views, wireless networks.**

I. Introduction

INTERFERENCE occurs in wireless networks when multiple transmitters simultaneously attempt to convey messages to distinct receivers using a shared medium. The mixing of both desired and undesired signals at each receiver poses a challenge to reliable communication. An interference channel (IC) is a specific mathematical model of interference in multiuser networks, wherein for each transmitter, there is exactly one unique intended receiver and salient effects of the physical medium are captured in a probabilistic input–output relationship. The capacity region of the IC in general remains unsolved. However, the capacity of two-user Gaussian IC, a canonical model for wireless networks depicted in Fig. 1, is characterized for some specific regimes [1]–[5]. Moreover, it has been shown in [6] that a simple version of the Han–Kobayashi (HK) coding scheme [7], which was introduced in the 1980s, achieves to within one bit the full capacity region of the two-user Gaussian IC for all regimes.

Despite these results, there exists a gap between transmission schemes that are theoretically near-optimal, and schemes that are used in practice. Modern wireless networks do not use HK codes when dealing with interference, and almost all commercial approaches to dealing with interference can be categorized as one of two simpler methods: treating interference as noise or orthogonalization. Rationales for the use of suboptimal schemes often cite the challenge or high cost of gathering perfect knowledge about the network state [8] and difficulty in coordinating numerous independent transmitter–receiver pairs [9]. In this paper, we incorporate these challenges into the network model and address analytically this gap between theory and practice within the framework of the two-user IC. Our contributions are fourfold.

1) Local View Knowledge Model: First, we model the distributed nature of networks by introducing the notion of a *local view* of global network state at each transmitter, which the transmitter uses as the basis for transmission decisions. The network state or global view is the four-tuple of all the channels in the two-user IC, and the local view of a transmitter has two properties.

1) Transmitters' local view of network may be *incomplete*: Each transmitter only has knowledge of a subset of the channels from the four-tuple network state.
2) Transmitters' local view of network may be *mismatched*: The subset of channel gains known to one transmitter may be different from the subset known to the other transmitter. This feature is notably missing in previously studied channel state knowledge models.

As an example, consider the IC in Fig. 1. Transmitter a's local view may consist of knowledge about h_{aa} and h_{ab}, while Transmitter b local view may consist only of h_{bb} and h_{ba}. Thus, their local views are incomplete and mismatched to each other's.

In a wireless network, each node often employs the same channel state learning mechanism to arrive at their local views.

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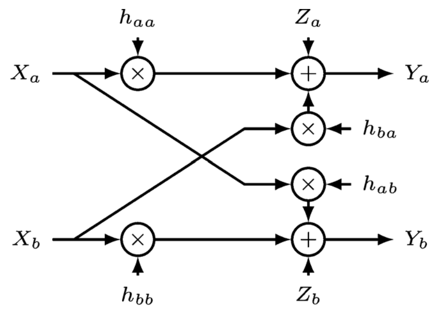


Fig. 1. Gaussian interference channel.

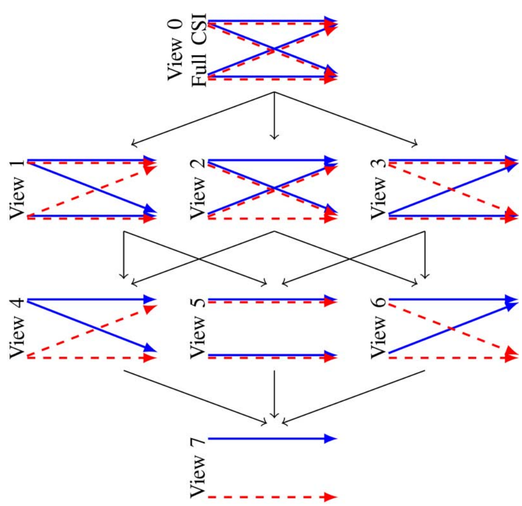


Fig. 2. Local Views: solid blue edges are known to Transmitter a and dashed red edges are known to Transmitter b. Digraph also depicts relationships between preferences between local views. An arrow from one view to another signifies that the first has a strictly more complete understanding of channel state, and its TDM-dominating capacity region contains the capacity region of the other.

That is, Transmitters a and b may use the same channel state acquisition and sharing method to acquire their local views. Hence, we limit our attention to *symmetric* local views, the case where from each transmitter's point of view, the same subset of channels are acquired. The above example where Transmitter a knows (h_{aa}, h_{ab}) and Transmitter b knows (h_{bb}, h_{ba}) is a case of symmetric local view.[1] For the two-user IC, the constraint of symmetric local views leads to the seven pairs of local views depicted in Fig. 2.

2) Framework for Analysis: Second, we introduce a framework for studying performance with local views. Our framework consists of a notion of distributed *policies* and a new measure of performance called a TDM-dominating capacity region. A policy is a formalization of the idea that the action of a node, specifically the encoding and signaling scheme of each transmitter, is a response to knowledge about the network state. In our formulation, the policy is a deterministic mapping from local view to transmission scheme, and this mapping is globally known, i.e., we model a network where the protocol used by each device is known. As such, a key distinction is made between the traditional notion of a scheme, which may be optimized for a single network state, and a policy, which dictates the scheme *for all possible* network states.

Our performance measure, TDM-dominating capacity regions, captures the fundamental limits of error free communication of a policy with a constraint that the policy must be preferable to time-division multiplexing (TDM) for all channel states. TDM can be employed with the minimal local view where each transmitter knows just the channel gain to its intended receiver, and thus is a natural baseline for performance.[2] With the requirement of performance universally "at least as good as TDM," we are able to infer how local views with more knowledge facilitate performance gains over orthogonalization approaches used in current networks.

3) Analysis of a Linear Deterministic Channel: Third, we first analyze a two-user linear deterministic interference channel (LDIC) that approximates the Gaussian interference channel (GIC). We derive the exact TDM-dominating capacity regions for a two-user LDIC for the local views in Fig. 2 and find that the seven symmetric local views can be classified into two categories: those where opportunistic HK codes can exceed TDM region and those where TDM is an optimal approach.

The first category, where opportunistic HK codes can exceed the TDM region, includes only two out of seven possible symmetric views, and both require transmitter knowledge of three out of four channels including the channel coefficient of outgoing interference. Transmitters may use knowledge of the outgoing channel gains to utilize HK codes and opportunistically increase the rates of private and common codebooks, resulting in rates above those achievable through TDM.

The second category includes the remaining five views, for which we show that no transmission policy exists that can universally outperform TDM. This category surprisingly includes the case of View 3 from Fig. 2 where each transmitter knows all of the channel gains except the outgoing interference, emphasizing that mechanisms to learn network state, if poorly designed, can provide a nearly complete characterization of the network and yet still provide no additional benefit.

4) Extension to the Gaussian Channel: Finally, we use the similarities between the linear deterministic and Gaussian channel models to analyze the gap between the TDM-dominating capacity regions of the LDIC and the TDM-dominating capacity regions of the Gaussian IC. We introduce a method utilizing carefully structured genie signals that allows us to extend the construction of the outer bounds in the LDIC to the GIC.

Using this extension, for four local views, the gap is constant irrespective of the network state. For the remaining three local views, the gap is dependent only on the relative channel gain values, which allows us to characterize the generalized degrees of freedom (GDoF) regions of the Gaussian IC with TDM-dominating policies.

Our results provide key intuitions into the development of wireless networks. First, although we analyze local views

[1]An example of asymmetric local view, in the context of Fig. 1, is when Transmitter a knows (h_{aa}, h_{ab}) and Transmitter b knows h_{bb}.

[2]We have implicitly assumed that the whole network is temporally synchronized irrespective of the amount of knowledge at any node.

without assuming a learning mechanism (e.g., training, feedback, message passing, etc.), our results can provide intuition into the value of learning each aspect of the network state. For instance, we showed that in both Views 3 and 7, TDM is an equally good transmission policy, despite the fact that in View 3, each transmitter has three times as much knowledge as View 7. On the other hand, if we desire spectral efficiency exceeding that can be provided by TDM, we find that transmitters must acquire knowledge about their outgoing IC gain; a learning mechanism should result in View 1 or 2. Second, for currently deployed wireless networks, wherein nodes typically have very little knowledge about the current network state, our results for Views 3–7 suggest that commonly used orthogonalization-based approaches, such as random access, high-frequency reuse factors, and CDMA, are appropriate design choices.

The remainder of this paper is structured as follows. Section II details related work and discusses how our contributions relate to previous findings. In Section III, we formalize our analysis: we review Gaussian and LDIC models, describe the local view model of distributed network state knowledge, discuss the relative importance of the local views studied, and provide mathematical preliminaries for the statement of main results. Within this section, we also clarify our notion of TDM-dominating capacity regions and, through analysis of a local view multiple-access channel, demonstrate the ambiguity of traditional notions of capacity resulting from a local view model and how our definition of TDM-dominating capacity regions applies. Section IV describes the techniques used in defining inner and outer bounds for both channel models, and in Section V, we present our main results: TDM-dominating capacity regions for the linear deterministic IC for each of the seven views studied. In Section VI, we examine the TDM-dominating capacity regions of the Gaussian IC and extend intuitions drawn in the LDIC to the Gaussian domain.

II. Related Work

Information-theoretic study of interference began in the 1960s [10]. In particular, the issue of the capacity region of the GIC was the impetus for development of many new achievable schemes and outer bounding techniques [1]–[7], [11]–[17].

Among these, [15] was instrumental in studying the LDIC model, a subclass of the channels studied in [18]. The linear deterministic signal model, first introduced in [19], is used within this study and, as shown in [15] and [19], is an excellent approximation of the two-user GIC. Two desirable consequences of using the linear deterministic model will be discussed in the context of our paper.

1) Approximately optimal layered approaches like the HK code of [6] and the lattice approach of [16] are revealed naturally in the linear deterministic model.
2) The relationship between the capacity regions of the linear deterministic channel and Gaussian channel and near-equivalence in the high-SNR regime (GDoF), provides intuition into solving real systems.

In [20], the authors introduced interference alignment (IA), a transmission scheme that is degrees of freedom optimal. While recent IA work also studies limited knowledge conditions, e.g., no-CSIT [21] and “blind” [22] cases, we study a case without knowledge of network state statistics and our metric (capacity and approximate capacity *regions*) reveal more about the performance capabilities of the network than degrees of freedom which is not specific to network state.

Another way to model channel uncertainty is the approach taken in compound channels [23] which specify a set of possible network states, and the objective is to define a scheme that will maximize rate regardless of the actual network state. Within this domain, the work in [24], where the authors provide the capacity of the compound IC to within a constant gap, is closest to our own. The network state uncertainty in the compound IC is synchronized among the nodes. In contrast, our contribution emphasizes transmitters having different uncertainty regarding the network state.[3]

Local views are a newer field of study, and so far the emphasis has been on notions of sum-rate [25], [26] or feasibility of linear IA [27]. We instead derive a full capacity region, thereby taking a first step towards considering alternative notions of optimal rate allocation, such as fairness metrics which require knowledge of the full feasible set of rates [28]. Additionally, the uncertainty model in [25] and [26] quantifies the locality of the view by counting the number of “hops” of information. Our approach is flexible in the sense that we do not assume a mechanism for acquiring the view, and instead examine a comprehensive subset of views, each of which may result from a different knowledge acquisition mechanism.

Commodity WiFi network architecture provides a common example of how network design results in the most limited of local views we study: mobiles only measure the channel gain to the intended access point, and therefore, mobiles have no knowledge of how much interference they may inflict on a flow associated with a neighboring access point. Most 3G cellular networks are similar in this respect, since no spectral resources are dedicated to training for channel gains characterizing intercell interference. On the other hand, protocols using inverse power tones or echoes [29], [30] have also been proposed, which provide transmitters with an estimate of the amount of interference they inflict on neighboring nodes but not the quality of the other user’s direct link.

III. Preliminaries

This section defines our problem formulation. In Section III-A, two IC models are defined: the two-user GIC used in modeling wireless networks, and the two-user LDIC which is a discrete abstraction of the Gaussian model. We define the *local view* formalization of distributed knowledge in Section III-B. Section III-C reviews theoretical preliminaries relevant to our study, and in Sections III-D and III-E, we define notions of policies, achievability, and TDM-dominating capacity specific to the local view model. Finally, in Section III-F, we provide an example of a local view multiple-access channel which clarifies our model and reinforces the importance of each aspect of our problem formulation.

[3]We note that View 5 represents the single exception, the two transmitters have the same knowledge, and thus, our results for View 5 may also be derived from by extending those of [24].

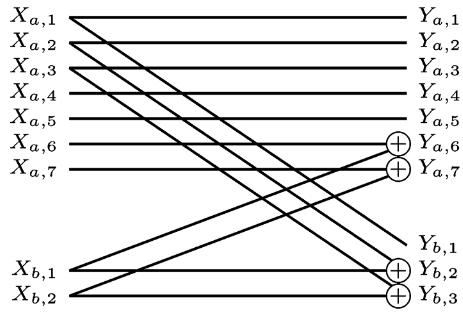


Fig. 3. LDIC where $g_{aa} = 7$, $g_{ab} = 3$, $g_{ba} = 2$, and $g_{bb} = 2$.

A. Channel Models

We study both Gaussian and linear deterministic two-user IC models.

1) Gaussian IC: The GIC (shown in Fig. 1) consists of two transmitter–receiver pairs, labeled a and b, and four point-to-point *links*: two direct and two interfering. The signal strengths of the four point-to-point links are represented by the four complex channel gains, h_{aa}, h_{ab}, h_{ba}, and h_{bb}.

Definition 1 (Network State): The network state is defined as the collective four-tuple, $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$.

The relationship between complex channel inputs X_a and X_b and the received channel outputs Y_a and Y_b is given by

$$\begin{aligned} Y_a &= h_{aa} X_a + h_{ba} X_b + Z_a, \\ Y_b &= h_{bb} X_b + h_{ab} X_a + Z_b, \end{aligned}$$

where we have a power constraint $\frac{1}{n}\sum_{t=1}^{n} |X_i|^2 \leq 1$ for $i \in \{a, b\}$, and Z_a and Z_b are single samples of a zero-mean, circularly symmetric, unit-variance white Gaussian random process.

2) Linear Deterministic IC: The linear deterministic signal model [19] captures the broadcast and superposition aspects of the wireless channel while abstracting the receiver noise into a signal level "floor" at each receiver. In doing so, noise becomes a deterministic effect, facilitating the analysis of the impact of interference. Fig. 3 depicts an example of an LDIC, which is used repeatedly within this document to demonstrate new concepts.

For the LDIC, we denote the binary vectors representing single-channel-use input (output) of Transmitter i (Receiver j) as $\mathbf{X}_i$ ($\mathbf{Y}_j$), and to length-n sequences of channel inputs (outputs) as $\mathbf{X}_i^n$ ($\mathbf{Y}_j^n$), respectively. Such sequences represent the uses of the channel n times. We also refer to the collection of all four channel gains as the *network state* $G = (g_{aa}, g_{ab}, g_{ba}, g_{bb})$ and express the input–output relationship of the LDIC in the form of shift matrix operations and elementwise modulo-2 addition. Notice that we differentiate LDIC variables from those of the GIC with bold weight typeface for channel input and output variables and use of G instead of H for network state. The input–output relationship is given by

$$\begin{aligned} \mathbf{Y}_a &= S_{q_a}^{q_a - g_{aa}} \mathbf{X}_a \oplus S_{q_a}^{q_a - g_{ba}} \mathbf{X}_b, \\ \mathbf{Y}_b &= S_{q_b}^{q_b - g_{bb}} \mathbf{X}_b \oplus S_{q_b}^{q_b - g_{ab}} \mathbf{X}_a, \end{aligned}$$

where $q_a = \max\{g_{aa}, g_{ba}\}$, $q_b = \max\{g_{bb}, g_{ab}\}$, $\oplus$ represents elementwise modulo-2 sum operation, and S_q is the $q \times q$ shift matrix

$$S_q = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix}.$$

By properties of matrix exponentiation, S_q^0 is a $q \times q$ identity matrix.

Each set of GIC channel gains, H, is associated with an analogous set of LDIC channel gains, G, by the following relationship:

$$g_{aa} = \lfloor \log\left(|h_{aa}|^2\right) \rfloor^+, \tag{1}$$
$$g_{ab} = \lfloor \log\left(|h_{ab}|^2\right) \rfloor^+, \tag{2}$$
$$g_{ba} = \lfloor \log\left(|h_{ba}|^2\right) \rfloor^+, \tag{3}$$
$$g_{bb} = \lfloor \log\left(|h_{bb}|^2\right) \rfloor^+. \tag{4}$$

This linear deterministic approximation was proposed in [19] and its usefulness for analyzing the GIC was demonstrated in [15], where it was shown that the capacity of a two-user GIC is within a constant number of bits of the associated LDIC. Moreover, the intuition provided by segmentation of signal space into discrete levels provided an intuition on construction of nearly optimal codes.

B. Local Views: A Model for Distributed Knowledge

In our model, Transmitter a's knowledge is composed of only a subset of the four channel gains in the network. When a channel gain is known, it is known without error. When it is unknown, Transmitter a has no knowledge of its value besides its support, which is the complex field for the GIC and all nonnegative integers for the LDIC. We refer to this model of incomplete and mismatched knowledge about the network state as a *local view* defined here:

Definition 2 (Local View): Transmitter i's local view is the subset of channel gains known to Transmitter i. We denote it as $\widehat{H}_a$ for the GIC and $\widehat{G}_i$ for the LDIC.

When specifying the view, the symbol $\varnothing$ is used to represent the value of unknown channel gains. For example, if Transmitter a knew all of the gains besides that of the direct link between Transmitter b and Receiver b, we would express this view as

$$\widehat{G}_a = (g_{aa}, g_{ab}, g_{ba}, \varnothing).$$

We clarify that both transmitters are aware of the size of the network, as well as the structure of the local view of the other transmitter (which channel gains are known and unknown). Receivers are assumed to have adequate knowledge to accommodate transmitter decisions and decode messages coherently. Ostensibly, it is sufficient to assume that receivers have a full view of the entire network state; however, we note that to implement some schemes proposed in this paper, receivers need only know their incoming channel gains and the transmitter decisions on codebook and rate may be embedded in a low-rate transmission header.

Rather than analyzing all 256 possible pairs of transmitter local views, this paper is restricted to cases where each Transmitter knows the channel gain to its intended receiver (i.e., Transmitter a knows h_{aa}) and where the two views are *symmetric* relative to perspective of the viewer: e.g., if h_{ij} is (un)known to Transmitter a, then h_{ji} is (un)known to Transmitter b. This restriction helps to bound the scope of the problem but also models, for the two-user IC, cases where the same method is used by each transmitter to learn about the network state. All eight symmetric views are depicted in Fig. 2, and notice that while the structure of local views exhibit relative symmetry, the exact sets of channel gains known to each node for all but Views 0 and 5 are not equal.

C. Mathematical Preliminaries

We now review the standard information-theoretic definitions for reliable communication. In order to communicate, each transmitter uses an encoding function $c_{i,n}$ to encode a message m_i drawn independently from a uniformly distributed set $M_i = \{1, \ldots, 2^{nr_i}\}$ into a codeword of n symbols, $\mathbf{X}_i^n = (\mathbf{X}_i[1], \ldots, \mathbf{X}_i[n])$. For the LDIC, each symbol is a binary vector. For the GIC, inputs are subject to a unit power constraint $\frac{1}{n}\sum_{t=1}^{n}|X_i[t]|^2 \leq 1$.

Each receiver observes its channel outputs, $\mathbf{Y}_i^n = (\mathbf{Y}_i[1], \ldots, \mathbf{Y}_i[n])$, and uses a decoding function $f_{i,n}$ to arrive at an estimate $\widehat{m}_i \in M_i$ of the encoded message m_i. An error occurs whenever $\widehat{m}_i \neq m_i$. The average probability of error for User i is given by

$$\epsilon_{i,n} = E[\Pr(\widehat{m}_i \neq m_i)],$$

where the expectation is taken with respect to the distributions of messages m_a and m_b.

A rate pair (r_a, r_b) is achievable if there exists a family of codebook pairs $\{c_{a,n}, c_{b,n}\}_{n\in\mathbb{N}}$ indexed by the block length n with codewords satisfying input constraints, and decoding functions $\{f_{a,n}(\cdot), f_{b,n}(\cdot)\}_{n\in\mathbb{N}}$, such that the average decoding error probabilities $\epsilon_{a,n}$ and $\epsilon_{b,n}$ vanish as block length n goes to infinity. By applying Shannon's coding theorem for the point-to-point channel, the set of achievable rate points can be determined by the following.

Lemma 1 [15, Lemma 1]: The rate point (r_a, r_b) is achievable if and only if for every $\epsilon > 0$, there exists a block length n and distributions $p(\mathbf{X}_a^n)$ and $p(\mathbf{X}_b^n)$ such that

$$nr_a - \epsilon \leq I(\mathbf{X}_a^n; \mathbf{Y}_a^n), \tag{5}$$
$$nr_b - \epsilon \leq I(\mathbf{X}_b^n; \mathbf{Y}_b^n). \tag{6}$$

The capacity region $\mathcal{C}$ of the IC is the closure of the set of all achievable rate pairs.

D. Distributed Policies

In a centralized network, encoding functions may be designed jointly to adapt to the network state. On the other hand, in our model, each transmitter selects its encoding function based on its respective local view, which consists of only a subset of channel gains. We refer to the mapping from view to transmission scheme as the *policy* employed by a transmitter and define it here.

Definition 3 (Policy): A policy is the mapping from view, $\widehat{G}_i$, to encoding function, $c_{i,n}(m_i; \widehat{G}_i)$. This implies that the rate of the transmission $(r_i(\widehat{G}_i))$ and distribution of channel inputs $(p(\mathbf{X}_i; \widehat{G}_i))$ are also dependent on view, and thus characteristics of the policy.

We assume each transmitter's policy is both *deterministic* and *globally known*; i.e., although Transmitter a may not know Transmitter b's exact choice of codebook and rate due to mismatched views of a and b, Transmitter a knows how b would respond to a particular network state. The globally known policies emulate predetermined protocols used by the two transmitters or specified by the network architect.

A policy couples the uncertainty of the interferer's choice of encoding function to the uncertainty in network state. Accordingly, the ability to coordinate selection of encoding functions and the resulting performance is still dependent on what knowledge is available to each transmitter. In our formulation, we define achievability of a pair of policy-defined rates, $r_a(\widehat{G}_a)$ and $r_b(\widehat{G}_b)$, by extending Lemma 1 to apply to local view-dependent encoding functions, and requiring achievability of the respective encoding functions for all network states consistent with the local view. Mathematically, achievability requires the existence of local view-dependent input distributions $p(\mathbf{X}_a^n; \widehat{G}_a)$ and $p(\mathbf{X}_b^n; \widehat{G}_b)$ such that the following two expressions hold for all G:

$$nr_a(\widehat{G}_a) - \epsilon \leq I(\mathbf{X}_a^n; \mathbf{Y}_a^n), \tag{7}$$
$$nr_b(\widehat{G}_b) - \epsilon \leq I(\mathbf{X}_b^n; \mathbf{Y}_b^n). \tag{8}$$

E. TDM-Dominating Capacity Region

The inequalities (7) and (8) provide necessary conditions for policies to achieve their target rates. However, (7) and (8) do not capture how policies may prescribe an aggressive, capacity-achieving scheme for one network state at the cost of low rate in another network state.

Policies based on TDM are unique in this regard because using a fixed time-division results in a rate pair that is the same across all values of IC gains g_{ab} and g_{ba}. TDM is also a policy that is achievable for all local views shown in Fig. 2 and, therefore, provides a natural baseline for comparison of performance across local views. Therefore, we restrict analysis to only those policies that guarantee performance equal to or better than a TDM policy regardless of network state.

We now formalize a notion of capacity qualified by a TDM minimum performance criterion with the following two definitions.

Definition 4 (TDM-Dominating Policies): Let $\overline{\mathcal{R}}^{\mathsf{TDM}}$ be defined as the set of Pareto optimal rates achieved using TDM. Specifically,

$$\overline{\mathcal{R}}^{\mathsf{TDM}} \triangleq \{(r_a, r_b) : r_a = (1-\tau)C_a, r_b = \tau C_b, \quad \tau \in [0,1]\}, \tag{9}$$

where $C_a = g_{aa}$, $C_b = g_{bb}$ for the LDIC, and $C_a = \log\left(1+|h_{aa}|^2\right)$, $C_b = \log\left(1+|h_{bb}|^2\right)$ for the GIC. We refer to a pair of policies as *TDM-dominating* if, for every network state G and views $\widehat{G}_a = V_a(G)$ and $\widehat{G}_b = V_b(G)$, there exists some $(r_a^{\mathsf{TDM}}, r_b^{\mathsf{TDM}}) \in \overline{\mathcal{R}}^{\mathsf{TDM}}$ such that

$$\begin{aligned} r_a(\widehat{G}_a) &\geq r_a^{\mathsf{TDM}}, \\ r_b(\widehat{G}_b) &\geq r_b^{\mathsf{TDM}}. \end{aligned}$$

Equivalently, policies are TDM-dominating if they satisfy for every possible network state, G, the minimum performance criterion

$$\frac{r_a(\widehat{G}_a)}{C_a} + \frac{r_b(\widehat{G}_b)}{C_b} \geq 1. \tag{10}$$

We say that the policy strictly dominates TDM if (10) is satisfied for all network states, and for at least one network state, (10) is strict. The analogous definition of TDM-dominating policies applies to the GIC.

Definition 5 (TDM-Dominating Capacity Region): A TDM-dominating capacity region for a given network state G is the set of all rate pairs achievable by TDM-dominating policies.

If the TDM-dominating capacity region is strictly larger than the TDM rate region, then there exists a policy which strictly dominates TDM, or any other orthogonalized scheme, across all network states. Without the criterion (10), the standard definition of a capacity region for network state G is misleading with respect to distributed protocol design. For a given network state, a policy that prescribes a full-view capacity-achieving scheme *always* exists. However, as we demonstrate in the next section, use of such a policy limits what is achievable in other realizations of network state. By analyzing TDM-dominating policies and the resulting TDM-dominating capacity region, we present rate pairs that are achievable for one network state without inducing performance poorer than TDM in another network state.

We note that our problem formulation is an analysis of worst-case or robust communication within a network with distributed uncertainty. When available and valid, knowledge of statistics of the network state can provide a method of improving the average performance of the network. On the other hand, our formulation is readily applicable to scenarios where the wireless fading statistics are poorly understood, difficult to model, or difficult to measure. Additionally, when the measure of system performance cannot be measured ergodically averaging over many network state realizations, e.g., systems sensitive to delay like streaming media, first responder systems, and system critical control, our results can provide more insight into what guarantees can be made regarding a network's ability to communicate.

For the sake of brevity, we occasionally shorten the term "TDM-dominating capacity" to the term "capacity." When a distinction must be made, we refer to the notion of capacity as defined by Shannon as the "full-view" capacity and our definition as TDM-dominating capacity.

F. Example: A Local View Multiple-Access Channel

Consider the following two-user local view linear deterministic multiple-access channel (LV-MAC). The full-view capacity region for any particular network state is [19] the set of rates (r_a, r_b) satisfying

$$r_a \leq g_a, \tag{11}$$
$$r_b \leq g_b, \tag{12}$$
$$r_a + r_b \leq \max\{g_a, g_b\}. \tag{13}$$

The local views are such that each transmitter only knows its direct link channel gain, i.e., $\widehat{G}_a = (g_a, \varnothing)$ and $\widehat{G}_b = (\varnothing, g_b)$. As in the local view IC, transmitters must select a codebook and rate given incomplete, mismatched knowledge of the network state. There exists a policy that for $g_a = 2$ and $g_b = 1$ achieves, using random codebooks[4] and joint decoding, the rate point $(1, 1)$ which is a corner point on the full-view capacity region, i.e.,

$$r_a(\widehat{G}_a)|_{g_a=2} = 1, \tag{14}$$
$$r_b(\widehat{G}_b)|_{g_b=1} = 1. \tag{15}$$

In order to satisfy (13) when $G = (1,1)$ or $G = (2,2)$, and assuming the policy chosen is such that (14) and (15) hold, we find the following constraints on policy responses:

$$\begin{aligned} r_a(\widehat{G}_a)|_{g_a=1} &\leq \max\{1,1\} - r_b(\widehat{G}_b)|_{g_b=1} = 0, \\ r_b(\widehat{G}_b)|_{g_b=2} &\leq \max\{2,2\} - r_a(\widehat{G}_a)|_{g_a=2} = 1. \end{aligned}$$

For the network states $G = (1,1)$, $G = (2,1)$, and $G = (2,2)$, the policy results in rate points on the boundary of the respective ideal capacity regions. However, for the case where $G = (1,2)$, the resulting extremal rate point, $r = (0,1)$, is not only an interior point, but also less efficient than TDM. Therefore, although the policy designed thus far strictly dominates TDM for some network states, the performance increase comes at the expense of what may occur in the state $G = (1,2)$.

We now show that no policy strictly dominating TDM exists. In each of the two network states $G = (1,1)$ and $G = (2,2)$, TDM is capacity achieving even with a full view. Let the time-division parameters in state $G = (1,1)$ be defined as $\tau_a(1)$ and $\tau_b(1)$, where $\tau_a(1) + \tau_b(1) = 1$. Similarly, we define $\tau_a(2)$ and $\tau_b(2)$, where $\tau_a(2) + \tau_b(2) = 1$. The rates resulting from this policy are $r_a(\widehat{G}_a)|_{g_a=s} = s\tau_a(s)$ and $r_b(\widehat{G}_b)|_{g_b=t} = t\tau_b(t)$.

Assume $\tau_a(1) \leq \tau_a(2)$, which implies $\tau_b(2) \leq \tau_b(1)$. When the network state is $G = (1,2)$, we find

$$\frac{r_a(\widehat{G}_a)}{g_a} + \frac{r_b(\widehat{G}_b)}{g_b} = \tau_a(1) + \tau_b(2) \leq 1,$$

with equality if and only if $\tau_a(1) = \tau_a(2)$ and $\tau_b(2) = \tau_b(1)$, i.e., if the rate achieved in each network state dominates TDM, then not only are the capacity regions of all four possible network states the TDM region, but also all four states are tied to the same time division.

In fact, the restriction to a single time division regardless of network state holds true for a more general case as well: the TDM-capacity of the K-user linear deterministic multiple-access channel where each transmitter only knows its direct

[4]Assume block codes of length n where each entry of each $g_i \times n$ matrix codeword is drawn from a Bernoulli distribution with $p = \frac{1}{2}$.

channel gain cannot strictly dominate TDM. The proof can be found in Appendix A.

Theorem 2 (K-User LDMAC TDM-Dominating Capacity Region): Let an LDMAC be defined as a K-user linear deterministic multiple-access channel where for each transmitter, $\widehat{G}_k = (\varnothing, \ldots, \varnothing, g_k, \varnothing, \ldots)$. The TDM-dominating capacity region, $\mathcal{C}_{MAC}$, is equal to the set of rate tuples achievable by TDM, $\mathcal{R}^{\mathsf{TDM}}_{\mathrm{MAC}}$, i.e.,

$$\mathcal{C}_{\mathrm{MAC}} = \mathcal{R}^{\mathsf{TDM}}_{\mathrm{MAC}}.$$

IV. Bounding Techniques

Though the precise analysis for each local view varies, the basic techniques employed to establish inner and outer bounds on the TDM-dominating capacity region are summarized in this section.

A. Inner Bounds

In this study, we reference only two types of achievable schemes: TDM and the simple HK scheme from [6] whose achievable region was shown (in the same reference) to be within one bit per user of the full-view two-user GIC capacity region.

1) Time-Division Multiplexing: The rate pairs achievable through TDM (the boundary of which was specified in Section III-E) are achieved through time orthogonalization. In the case of the GIC, our TDM-based rate region assumes no power scaling; however, the gap between the two regions can be verified to be less than 2 bits per user.

2) Simple HK Codes: In general HK schemes, each transmitter splits the contents of its message into a common message and a private message. The simple HK codes of [6] use random Gaussian codebooks for both the common and private encoding functions, with a division in power between the two chosen such that the private component of the message is received at the unintended receiver "in the noise floor"; i.e., the private and common codebooks are drawn from independent zero-mean Gaussian distributions with variances $P_{i,p} = \min\left\{\frac{1}{|h_{ij}|^2}, 1\right\}$ $(i \neq j)$ and $P_{i,c} = 1 - P_{i,p}$, respectively.

At the receiver, the private message of the undesired signal is treated as noise, thereby at most doubling the power of the interference-noise floor. The receiver jointly decodes both common messages and the desired private message, forming a virtual three user multiple-access channel. The resulting rate region $\mathcal{R}^{\mathsf{HK}}_{G}$ was shown to be approximately capacity achieving and is given by all nonnegative rate pairs (r_a, r_b) such that $r_a \leq r_{a,p} + r_{a,c}$ and $r_b \leq r_{b,p} + r_{b,c}$ satisfying

$$r_{a,p} \leq \log\left(1 + \frac{\min\left\{\frac{|h_{aa}|^2}{|h_{ab}|^2}, |h_{aa}|^2\right\}}{1 + \min\{|h_{ba}|^2, 1\}}\right),$$

$$r_{a,c} \leq \min\left\{ \log\left(1 + \frac{|h_{aa}|^2 - \min\left\{\frac{|h_{aa}|^2}{|h_{ab}|^2}, |h_{aa}|^2\right\}}{1 + \min\{|h_{ba}|^2, 1\}}\right), \log\left(1 + \frac{|h_{ab}|^2 - \min\{1, |h_{ab}|^2\}}{1 + \min\{|h_{ab}|^2, 1\}}\right)\right\},$$

$$r_{a,p} + r_{a,c} \leq \log\left(1 + \frac{|h_{aa}|^2}{1 + \min\{|h_{ba}|^2, 1\}}\right),$$

$$r_{a,p} + r_{b,c} \leq \log\left(1 + \frac{\min\left\{\frac{|h_{aa}|^2}{|h_{ab}|^2}, |h_{aa}|^2\right\} + \left(|h_{ba}|^2 - 1\right)^+}{1 + \min\{|h_{ba}|^2, 1\}}\right),$$

$$r_{a,p} + r_{a,c} + r_{b,c} \leq \log\left(1 + \frac{|h_{aa}|^2 + \left(|h_{ba}|^2 - 1\right)^+}{1 + \min\{|h_{ba}|^2, 1\}}\right),$$

$$r_{b,p} \leq \log\left(1 + \frac{\min\left\{\frac{|h_{bb}|^2}{|h_{ba}|^2}, |h_{bb}|^2\right\}}{1 + \min\{|h_{ab}|^2, 1\}}\right),$$

$$r_{b,c} \leq \min\left\{ \log\left(1 + \frac{|h_{bb}|^2 - \min\left\{\frac{|h_{bb}|^2}{|h_{ba}|^2}, |h_{bb}|^2\right\}}{1 + \min\{|h_{ab}|^2, 1\}}\right), \log\left(1 + \frac{|h_{ba}|^2 - \min\{1, |h_{ba}|^2\}}{1 + \min\{|h_{ba}|^2, 1\}}\right)\right\},$$

$$r_{b,p} + r_{b,c} \leq \log\left(1 + \frac{|h_{bb}|^2}{1 + \min\{|h_{ab}|^2, 1\}}\right),$$

$$r_{b,p} + r_{a,c} \leq \log\left(1 + \frac{\min\left\{\frac{|h_{bb}|^2}{|h_{ba}|^2}, |h_{bb}|^2\right\} + \left(|h_{ab}|^2 - 1\right)^+}{1 + \min\{|h_{ab}|^2, 1\}}\right),$$

$$r_{b,p} + r_{b,c} + r_{a,c} \leq \log\left(1 + \frac{|h_{bb}|^2 + \left(|h_{ab}|^2 - 1\right)^+}{1 + \min\{|h_{ab}|^2, 1\}}\right),$$

$$r_{a,c} + r_{b,c} \leq \log\left(1 + \frac{|h_{aa}|^2\left(\frac{|h_{ab}|^2 - 1}{|h_{ab}|^2}\right)^+ + \left(|h_{ba}|^2 - 1\right)^+}{1 + \min\{|h_{ba}|^2, 1\}}\right),$$

$$r_{a,c} + r_{b,c} \leq \log\left(1 + \frac{|h_{bb}|^2\left(\frac{|h_{ba}|^2 - 1}{|h_{ba}|^2}\right)^+ + \left(|h_{ab}|^2 - 1\right)^+}{1 + \min\{|h_{ab}|^2, 1\}}\right).$$

The analogous approach in the LDIC is to similarly split each user's message into common and private parts, where the private message is carried by the signal levels that are not seen at the unintended receiver (shown in Fig. 4). As with simple HK codes for the GIC, receivers decode the two common messages and desired private message jointly, resulting in the capacity-achieving rate region, $\mathcal{R}^{\mathsf{HK}}_{D}$, given by all nonnegative rate pairs (r_a, r_b) such that $r_a \leq r_{a,p} + r_{a,c}$ and $r_b \leq r_{b,p} + r_{b,c}$ satisfying

$$r_{a,p} \leq (g_{aa} - g_{ab})^+, \quad (16)$$

$$r_{a,c} \leq \min\{g_{aa}, g_{ab}\}, \quad (17)$$

$$r_{a,p} + r_{a,c} \leq g_{aa}, \quad (18)$$

$$r_{a,p} + r_{b,c} \leq \max\{g_{aa} - g_{ab}, g_{ba}\}, \quad (19)$$

$$r_{a,p} + r_{a,c} + r_{b,c} \leq \max\{g_{aa}, g_{ba}\}, \quad (20)$$

$$r_{b,p} \leq (g_{bb} - g_{ba})^+, \quad (21)$$

$$r_{b,c} \leq \min\{g_{bb}, g_{ba}\}, \quad (22)$$

$$r_{b,p} + r_{b,c} \leq g_{bb}, \quad (23)$$

$$r_{b,p} + r_{a,c} \leq \max\{g_{bb} - g_{ba}, g_{ab}\}, \quad (24)$$

$$r_{b,p} + r_{b,c} + r_{a,c} \leq \max\{g_{bb}, g_{ab}\}, \quad (25)$$

$$r_{a,c} + r_{b,c} \leq \max\{g_{aa}, g_{ba}\}, \quad (26)$$

$$r_{a,c} + r_{b,c} \leq \max\{g_{bb}, g_{ab}\}. \quad (27)$$

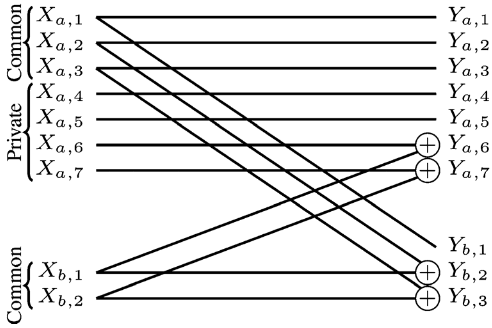


Fig. 4. Separation of usable linear deterministic channel levels into common and private components.

These expressions can also be adapted from [15] by removing redundant constraints, but are presented in a form that emphasizes similarities to the virtual three-user MAC seen at each receiver in the GIC and facilitates comparison between achievable component rates. By maximizing the difference between the right-hand side of each pair of analogous expressions, we have the following.

Lemma 3: Let a GIC network state, H, and analogous LDIC network state, G, defined by (1)–(4) be given. If the tuple $(r_a^p, r_a^c, r_b^p, r_b^c)$ achievable by the Gaussian simple HK scheme, the tuple $(r_a^p - 2, r_a^c - 2, r_b^p - 2, r_b^c - 2)$ is achievable in the associated linear deterministic scheme. Conversely, if the tuple $(r_a^p, r_a^c, r_b^p, r_b^c)$ achievable by the linear deterministic HK scheme, the tuple $(r_a^p - 2, r_a^c - 2, r_b^p - 2, r_b^c - 2)$ is achievable in the associated Gaussian scheme.

Consequently, if (r_a, r_b) is achievable for either channel model using the described HK scheme, then $(r_a - 4, r_b - 4)$ is achievable in the alternate model.

Although a more concise set of inequalities from [15] and more generally from [18] is shown in (28)–(34) below for the LDIC (the same region approximates the GIC), the component-separated expressions (16)–(25) reveals where opportunities for increased rate over orthogonalized schemes exist. We will show that transmitters may only capitalize on an available opportunity if that opportunity is revealed by a local view

$$r_a \le g_{aa}, \tag{28}$$

$$r_b \le g_{aa}, \tag{29}$$

$$r_a + r_b \le (g_{aa} - g_{ba})^+ + \max\{g_{bb}, g_{ba}\}, \tag{30}$$

$$r_a + r_b \le (g_{bb} - g_{ab})^+ + \max\{g_{aa}, g_{ab}\}, \tag{31}$$

$$\begin{aligned} r_a + r_b \le{}& \max\{g_{ab}, (g_{aa} - g_{ba})^+\} \\ &+ \max\{g_{ba}, (g_{bb} - g_{ab})^+\}, \end{aligned} \tag{32}$$

$$\begin{aligned} 2r_a + r_b \le{}& \max\{g_{aa}, g_{ab}\} + (g_{aa} - g_{ba})^+ \\ &+ \max\{g_{ba}, (g_{bb} - g_{ab})^+\}, \end{aligned} \tag{33}$$

$$\begin{aligned} r_a + 2r_b \le{}& \max\{g_{bb}, g_{ba}\} + (g_{bb} - g_{ab})^+ \\ &+ \max\{g_{ab}, (g_{aa} - g_{ba})^+\}. \end{aligned} \tag{34}$$

B. Outer Bounds

We build our outer bounds on the capacity region of local view policies on two key techniques.

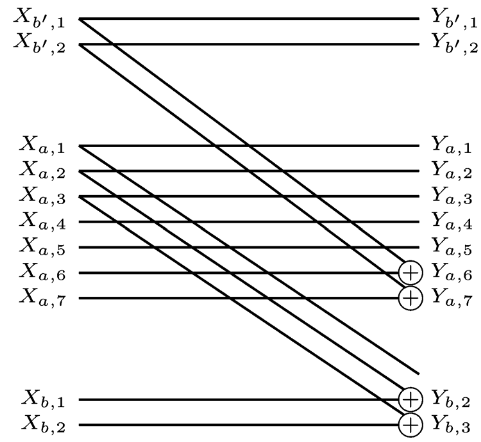


Fig. 5. LDIC of Fig. 3 unwrapped into a double Z-channel.

1) Expanded Virtual Z-Channel: In an IC, construction of each transmitter's encoding scheme is faced with two objectives:

a) On the direct link, a transmitter seeks to adapt its signal to increase its rate (increase entropy) in the presence of an interference signal.
b) On the outgoing interference link, a transmitter seeks to minimize its impact (reduce entropy).

A special case of IC, known as the Z-channel, often provides clarity regarding these competing objectives by considering the effect of only one of the two interference links. The relationship between the Z-channel and IC has been noted previously, e.g., in derivation of outer bounds [13].

However, instead of analyzing a single Z-channel, we go one step further by "unwrapping" the two-user local view IC into a series of Z-channels, so as to simultaneously focus on effects of both outgoing and incoming interference on achievable user policy responses (see Fig. 5).

In order to use the unwrapped IC as an outer bound, we impose constraints on the inputs of users in the unwrapped channel. In the case shown in Fig. 5, if we assume full views of network state, the constraint that the exact message and channel inputs of b and b' are the same is enough to specify a double Z-channel whose capacity region characterizes (and thus bounds) the original IC. Notice that the restriction of b and b' to the same messages and inputs results in $r_b = r_{b'}$ as long as both receivers can decode.

For full view, the resulting bound does not supply any intuition beyond that given by known genie-aided bounds. On the other hand, because a transmitter with a local view is uncertain of at least one channel gain in the unwrapped channel, it must account for every possibility of unknown channel gains and resulting policy responses of the other transmitter. Moreover, from the perspective of User a, the input $\mathbf{X}_{b'}(\widehat{G}_{b'})$ most detrimental at Receiver a may result from one local view at Transmitter b, $\widehat{G}_{b'}$, while the input most sensitive to interference at Receiver b may result from a different local view, $\widehat{G}_b$. Therefore, the constraints we impose on inputs $\mathbf{X}_{b'}(\widehat{G}_{b'})$ and $\mathbf{X}_b(\widehat{G}_b)$ are that the policy mapping view $\widehat{G}_b$ to scheme must be consistent.

We visualize design of local view policies with a series of virtual users arranged in a larger Z-channel with the following properties:

1) Each transmitter–receiver pair in an unwrapped channel is a *virtual user* corresponding to a policy response of either User a or User b in the original local view IC.
2) A virtual User a always interferes with a virtual User b (and vice-versa), or does not interfere at all (terminates the Z-channel). The Z-channel may be cyclic.
3) Each virtual transmitter uses the policy governing channel inputs corresponding to its local view.
4) Any channel gain known to both transmitters must be consistent throughout the virtual channel.

By visualizing the design of local view policies in sets of expanded Z-channels, we can analyze possibly many worst case network states, each of which corresponds to a sequence of dependences between policy responses.

2) Genies: In the following, we show how a particular level-by-level application of the chain rule in LDIC analysis can be emulated for the GIC using a well-designed genie. The genie we describe here is implicit in analysis of the LDIC, owing to the fact that the channel is noiseless, but construction of the GIC genie is motivated by intuitions drawn from how, in the LDIC, entropies of signals may be decomposed level-by-level through application of the chain rule and the entropy of each level may be examined conditioned on higher levels.

Consider an LDIC where $g_{aa} \geq g_{ba} > 0$ (i.e., the direct link between Transmitter a and Receiver a has a higher channel gain than the impinging interference from Transmitter b). In this case, the mutual information of User a can be expressed as

$$\begin{aligned} I(\mathbf{X}_a^n;\mathbf{Y}_a^n) &= H(\mathbf{Y}_a^n) - H(\mathbf{Y}_a^n|\mathbf{X}_a^n) \\ &\overset{(a)}{=} H(Y_{a,1}^n,\ldots,Y_{a,g_{aa}-g_{ba}}^n) - H(X_{b,1}^n,\ldots,X_{b,g_{ba}}^n) \\ &\quad + H(Y_{a,g_{aa}-g_{ba}+1}^n,\ldots,Y_{a,g_{aa}}^N|Y_{a,1}^n,\ldots,Y_{a,g_{aa}-g_{ba}}^n) \\ &\overset{(b)}{=} H(X_{a,1}^n,\ldots,X_{a,g_{aa}-g_{ba}}^n) - H(X_{b,1}^n,\ldots,X_{b,g_{ba}}^n) \\ &\quad + H(Y_{a,g_{aa}-g_{ba}+1}^n,\ldots,Y_{a,g_{aa}}^n|X_{a,1}^n,\ldots,X_{a,g_{aa}-g_{ba}}^n). \end{aligned} \tag{35}$$

Step (a) results from an application of the chain rule, and (b) notes that the chain rule was applied at the boundary between interfered and uninterfered receive signal levels. If on the other hand $g_{ba} > g_{aa}$, we have

$$\begin{aligned} I(\mathbf{X}_a^n;\mathbf{Y}_a^n) &= H(X_{b,1}^n,\ldots,X_{b,g_{ba}-g_{aa}}^n) - H(X_{b,1}^n,\ldots,X_{b,g_{ba}}^n) \\ &\quad + H(Y_{a,g_{ba}-g_{aa}+1}^n,\ldots,Y_{a,g_{ba}}^n|X_{b,1}^n,\ldots,X_{b,g_{ba}-g_{aa}}^n). \end{aligned} \tag{36}$$

Given the following definitions:

$$\begin{aligned} L_{a,i} &\triangleq H(X_{a,i}^n|X_{a,1}^n\ldots,X_{a,i-1}^n), \\ L_{b,j} &\triangleq H(X_{b,j}^n|X_{b,1}^n\ldots,X_{b,j-1}^n), \\ u_a^+ &\triangleq (g_{aa}-g_{ba})^+, \\ u_a^- &\triangleq (g_{ba}-g_{aa})^+, \\ u_b^+ &\triangleq (g_{bb}-g_{ab})^+, \\ u_b^- &\triangleq (g_{ab}-g_{bb})^+, \end{aligned}$$

the relations (35) and (36) for User a can be more generally stated in (37) as

$$\begin{aligned} I(\mathbf{X}_a^n;\mathbf{Y}_a^n) \leq &\left(n\min\{g_{aa},g_{ba}\} - \sum_{k=1}^{g_{ba}} L_{b,k}\right) \\ &+ \left(\sum_{i=1}^{u_a^+} L_{a,i} + \sum_{j=1}^{u_a^-} L_{b,j}\right), \end{aligned} \tag{37}$$

and similarly, for User b

$$\begin{aligned} I(\mathbf{X}_b^n;\mathbf{Y}_b^n) \leq &\left(n\min\{g_{bb},g_{ab}\} - \sum_{k=1}^{g_{ab}} L_{a,k}\right) \\ &+ \left(\sum_{j=1}^{u_b^+} L_{b,j} + \sum_{i=1}^{u_b^-} L_{a,i}\right). \end{aligned} \tag{38}$$

The first two quantities in both decompositions (37) and (38) emphasize that if the strengths of incoming signals are not equal, the most significant bits of the combined received signal are easy to decode, and the contention occurs in those levels where the two signals overlap.

For the GIC, it is not apparent how to "decode the most significant bits" without restricting analysis to layered coding schemes. However, if the upper levels of the signal (those modeled as noninterfered bits in the LDIC) are "easy to decode," then there should be little benefit in supplying these levels of the signal separately to the receiver. This provides the intuition behind our genie, which provides a set of signals intended to emulate the layering of message content that is implicit in the LDIC model.

Our approach for constructing the genie is similar to [24] in the sense that each signal is derived from a series of degraded signals, each representing a component of a received signal that may be received in a potential network state. Furthermore, like [24], our genie signals are conditionally independent from the original received signal.

Assume, for ease of explanation of the genie, that GIC channel gains result in integer LDIC channel gains without the use of the floor function:

$$g_{ij} = \log\left(|h_{ij}|^2\right)^+,\ i,j \in \{a,b\}.$$

Let $Z_{a,\ell}^n \sim N(0,\mathbf{I})$ for $\ell \in \mathbb{N}$ be a series of independent (across ℓ) length-n i.i.d. zero-mean complex Gaussian random vectors. We define the maximum number of signals derived from Transmitter a's input as

$$\ell_a^\star \triangleq \max\{g_{aa},g_{ab}\},$$

and the signal $U_{a,\ell_a^\star}^n$ be given as

$$U_{a,\ell_a^\star}^n \triangleq \begin{cases} |h_{aa}|X_a^n + Z_{a,\ell_a^\star}^n, & \text{if } |h_{aa}| \geq |h_{ab}| \\ |h_{ab}|X_a^n + Z_{a,\ell_a^\star}^n, & \text{if } |h_{aa}| < |h_{ab}|. \end{cases}$$

From $U^n_{a,\ell^\star_a}$, we define our a collection of serially degraded signals $\{W_{a,\ell}\}_{\ell\in\mathbb{N}}$

$$\begin{aligned} U^n_{a,\ell^\star_a-1} &\triangleq U^n_{a,\ell^\star_a} + Z^n_{a,\ell^\star_a-1},\\ U^n_{a,\ell^\star_a-2} &\triangleq U^n_{a,\ell^\star_a-1} + \sqrt{2}Z^n_{a,\ell^\star_a-2},\\ &\vdots\\ U^n_{a,\ell} &\triangleq U^n_{a,\ell-1} + \sqrt{2^{\ell^\star_a-\ell-1}}Z^n_{a,\ell},\\ &\vdots\\ U^n_{a,1} &\triangleq \frac{1}{\sqrt{2}}U^n_{a,2} + \sqrt{2^{\ell^\star_a-2}}Z^n_{a,1}. \end{aligned}$$

In each successive signal $U^n_{a,\ell}$, the power in the total noise term doubles. Additionally, the following Markov relationship is formed:

$$U^n_{a,1} — U^n_{a,2} — \cdots — U^n_{a,\ell^\star_a} — X^n_a — Y^n_j,$$

where Y^n_j is either received signal. A similar collection of signals, $\{U_{b,\ell}\}_{\ell\in\mathbb{N}}$, is defined for Transmitter b's input as well.

To these signals, we add a phase correction term of the form

$$\begin{aligned} W^n_{aa,\ell} &\triangleq \Phi_{aa}U^n_{a,\ell},\\ W^n_{ab,\ell} &\triangleq \Phi_{ab}U^n_{a,\ell},\\ W^n_{ba,\ell} &\triangleq \Phi_{ba}U^n_{b,\ell},\\ W^n_{bb,\ell} &\triangleq \Phi_{bb}U^n_{b,\ell}, \end{aligned}$$

where $\Phi_{ij} = e^{j\angle h_{ij}}$ incorporates the appropriate phase into the genie signals.

At each receiver, the genie provides those signals that represent the "easy to decode" large scale variations. For instance, if $|h_{aa}| > |h_{ba}|$, then signals $\{W_{aa,\ell}\}_{\ell\in\{1,\ldots,g_{aa}-g_{ba}\}}$ are provided to Receiver a, as shown in Fig. 6.

Assuming $|h_{aa}| > |h_{ba}|$, our newly defined genie allows us to arrive at

$$\begin{aligned} I(X^n_a;Y^n_a) &\le I(X^n_a;Y^n_a, W_{aa,1}, W_{aa,2},\ldots,W_{aa,u^+_a})\\ &= h(Y^n_a, W_{aa,1}, W_{aa,2},\ldots,W_{aa,u^+_a})\\ &\quad - h(Y^n_a, W_{aa,1}, W_{aa,2},\ldots,W_{aa,u^+_a}|X^n_a)\\ &= h(Y^n_a|W_{aa,u^+_a}) - h(Y^n_a|W_{aa,u^+_a},X^n_a)\\ &\quad + \sum_{\ell=1}^{u^+_a} h(W_{aa,\ell}|W_{aa,1},\ldots,W_{aa,\ell-1})\\ &\quad - h(W_{aa,\ell}|W_{aa,1},\ldots,W_{aa,\ell-1},X^n_a)\\ &= h(Y^n_a|W_{aa,u^+_a}) - h(Y^n_a|W_{aa,u^+_a},X^n_a)\\ &\quad + \sum_{\ell=1}^{u^+_a} I(X^n_a;W_{aa,\ell}|W_{aa,1},\ldots,W_{aa,\ell-1}). \end{aligned} \tag{39}$$

As desired, expression (39) mimics (37) in its isolation of larger signal variations (more significant bits) from the variations which are contested by both direct and interference signals. Moreover, we notice that using a substitution of statistically equivalent noise terms $Z^{n\prime} \sim N(0,\mathbf{I})$ and $Z^{n\prime\prime} \sim N(0,\mathbf{I})$, it is clear that, as in the LDIC, the payload of each genie signal level except for $\ell = 1$[5] is constrained to at most 1 bit:

$$\Lambda_{a,\ell} \triangleq I(X^n_a; W_{ai,\ell}|W_{ai,1},\ldots,W_{ai,\ell-1}) \le 1.$$

Additionally, we note that

$$\begin{aligned} \sum_{\ell=k}^{K}\Lambda_{a,\ell} &= \sum_{\ell=k}^{K} I(X^n_a;W_{ai,\ell}|W_{ai,1},\ldots,W_{ai,\ell-1})\\ &= I(X^n_a;W_{ai,K}|W_{ai,1},\ldots,W_{ai,k}), \end{aligned}$$

is the point-to-point rate of the kth through Kth levels. Using the interference-noise term of (39), we have

$$\begin{aligned} h(Y^n_a|W_{aa,u^+_a},X^n_a) &= h(h_{ba}X^n_b + Z^n_a)\\ &= I(X^n_b;W_{ba,g_{ba}}) + h(Z^n_a)\\ &= I(X^n_b;W_{ba,g_{ba}}) + n\log(2\pi e). \end{aligned}$$

If $g_{ba} > g_{aa}$ and the genie supplies Receiver a with levels from Transmitter b, we can also say

$$\begin{aligned} h(Y^n_a|W_{ba,u^-_a},X^n_a) &= h(h_{ba}X^n_b + Z^n_a|W_{ba,u^-_a})\\ &= I(X^n_b;W_{ba,g_{ba}}|W_{ba,u^-_a}) + n\log(2\pi e). \end{aligned}$$

Consequently, the genie-aided decomposition of mutual information at each receiver can be bounded by

$$\begin{aligned} I(X^n_a;Y^n_a) &\le h(Y^n_a|W_{aa,u^+_a},W_{ba,u^-_a}) - n\log(2\pi e)\\ &\quad - \sum_{\ell=1}^{g_{ba}}\Lambda_{b,\ell} + \sum_{\ell=1}^{u^+_a}\Lambda_{a,\ell} + \sum_{\ell=1}^{u^-_a}\Lambda_{b,\ell}, \end{aligned} \tag{40}$$

$$\begin{aligned} I(X^n_b;Y^n_b) &\le h(Y^n_b|W_{bb,u^+_b},W_{ab,u^-_b}) - n\log(2\pi e)\\ &\quad - \sum_{\ell=1}^{g_{ab}}\Lambda_{a,\ell} + \sum_{\ell=1}^{u^+_b}\Lambda_{b,\ell} + \sum_{\ell=1}^{u^-_b}\Lambda_{a,\ell}, \end{aligned} \tag{41}$$

where $W_{ij,0}$ for $i,j\in\{a,b\}$ exist only as dummy (independent of the system or constant) signals. Expressions (40) and (41) will later be used in Section VI to extend results for the LDIC to results for associated GICs. The similarities between the decoupling of signal levels in the LDIC and genie-aided decomposition for the GIC will permit similar analysis for both while clarifying how to account for the gap between the two models.

V. Results for the LDIC

In this section, we state results for the LDIC. The TDM-dominating capacity regions for each of the seven views shown in Fig. 2 fall in one of two categories. In the first category, we have Views 1 and 2 which enable opportunistic HK codes, thereby achieving rates dominating $\mathcal{R}_D^{\mathsf{TDM}}$. In the second category, containing Views 3–7, to achieve any point in the capacity region, a TDM scheme is sufficient.

Before stating the results, we draw the reader's attention to relationships between views shown in Fig. 2. The set of all local views may be interpreted as a power set of the set of channel

[5]The topmost level of the signal $\ell = 1$ is not subject to conditioning and thus is bounded by $\log(3)$.

gains and the chart depicts a partial ordering based on the inclusion of a channel gain in the view. Ordered relationships are depicted by directed edges (or paths along multiple directed edges) with the view at the tail of the edge preferable to the view at the head of an edge: The transmitters lose knowledge of one channel gain for each edge on a path.

Since a less preferable view has only a subset of the knowledge available to more preferable views, intuitively one might assume that, for a given network state, the capacity region of a particular local view IC may be bounded by the capacity region of any more preferable view. This is indeed the case and simplifies the analysis of the many local views. Consequently, we need only analyze Views 1–3 in full detail and, subsequently, apply the results to Views 4–7.

Our TDM-dominating capacity regions are expressed as parameterizations based on potential policies, and highlight coupling of policy responses in different network states. When appropriate, we also include a more concise set of inequalities stemming from the union over all such policies. Proofs are relegated to the appendixes.

To further provide intuition as to why each view either enables or inhibits opportunities for advanced transmission schemes, for Views 1–3 we use the LDIC shown in Fig. 3 and either define a policy that strictly dominates TDM, or demonstrate why strictly dominating TDM is impossible.

A. Opportunity-Enabling Local Views

1) View 1: The TDM-dominating capacity region for View 1 may strictly dominate TDM. However, in order for a policy to achieve a rate pair on the Pareto boundary of the TDM-dominating capacity region for one network state realization, the rates achieved in other potential network states must be constrained to exactly those achievable by TDM. Consequently, regions of achievable rates are coupled across different network state realizations.

Theorem 4 (View 1 TDM-Dominating Capacity Region): Let $\widehat{G}_a = (g_{aa}, g_{ab}, \emptyset, g_{bb})$ and $\widehat{G}_b = (g_{aa}, \emptyset, g_{ba}, g_{bb})$. Additionally, WLOG let $g_{aa} \geq g_{bb}$, and define for a specific network state $G = (g_{aa}, g_{ab}, g_{ba}, g_{bb})$ the value

$$\delta \triangleq g_{aa} - g_{bb}.$$

The TDM-dominating capacity region for View 1 with network state G is the union of all regions indexed by $\tau_b(g_{aa}, g_{bb}) \in [0,1]$, each region containing all tuples $(r_a(\widehat{G}_a), r_b(\widehat{G}_b))$ satisfying

$$r_a(\widehat{G}_a) \leq r_a^c(\widehat{G}_a) + r_a^p(\widehat{G}_a),$$
$$r_b(\widehat{G}_b) \leq r_b^c(\widehat{G}_b) + r_b^p(\widehat{G}_b),$$

where $r_a^c(\widehat{G}_a)$, $r_a^p(\widehat{G}_a)$, $r_b^c(\widehat{G}_b)$, and $r_b^p(\widehat{G}_b)$ are nonnegative and satisfy (42)–(55), shown at the bottom of the page.

Though the parametrized characterization of the region is somewhat unwieldy, each expression in (42)–(55) results from a particular class of virtual Z-channels. Moreover, minimization over ℓ_i in an expression actually describes at most two "worst-cases." Which of the two cases is truly worst depends on the value of $\tau_b(g_{aa}, g_{bb})$ and the network state G.

$$r_a(\widehat{G}_a) \leq g_{aa} - g_{bb}\tau_b(g_{aa}, g_{bb}), \tag{42}$$
$$r_b(\widehat{G}_b) \leq g_{aa}\tau_b(g_{aa}, g_{bb}), \tag{43}$$
$$r_a^c(\widehat{G}_a) \leq \min_{\ell \geq 0}[\max\{g_{bb} - \ell\delta, g_{ab}\} + \ell\delta\tau_b(g_{aa}, g_{bb}) - g_{bb}\tau_b(g_{aa}, g_{bb})], \tag{44}$$
$$r_a^c(\widehat{G}_a) \leq g_{ab}, \tag{45}$$
$$r_b^c(\widehat{G}_b) \leq g_{aa}\tau_b(g_{aa}, g_{bb}), \tag{46}$$
$$r_b^c(\widehat{G}_b) \leq \min_{\ell \geq 0}[\max\{g_{ba} - \ell\delta, (g_{ba} - g_{aa})^+\} + \ell\delta\tau_b(g_{aa}, g_{bb})], \tag{47}$$
$$r_a^c(\widehat{G}_a) + r_b(\widehat{G}_b) \leq \min_{\ell \geq 0}[\max\{g_{bb} - \ell\delta, g_{ab}\} + \ell\delta\tau_b(g_{aa}, g_{bb})], \tag{48}$$
$$r_b^c(\widehat{G}_b) + r_a(\widehat{G}_a) \leq \max\{g_{ba}, g_{aa}\}, \tag{49}$$
$$r_b^c(\widehat{G}_b) + r_b(\widehat{G}_b) \leq \min_{\ell \geq 0}[\max\{g_{ba} - (\ell+1)\delta, (g_{ba} - g_{aa})^+\} + (g_{aa} + \ell\delta)\tau_b(g_{aa}, g_{bb})], \tag{50}$$
$$r_a^c(\widehat{G}_a) + r_b^p(\widehat{G}_b) \leq \min_{\ell \geq 0}[\max\{g_{ab}, g_{bb} - g_{ba} - \ell\delta\} + \ell\delta\tau_b(g_{aa}, g_{bb})], \tag{51}$$
$$r_b^c(\widehat{G}_b) + r_a^p(\widehat{G}_a) \leq \min_{\ell \geq 0}[\max\{g_{ba} - \ell\delta, (g_{aa} - g_{ab})^+, g_{ba} - g_{ab}, g_{ba} - g_{aa}\} + \ell\delta\tau_b(g_{aa}, g_{bb})], \tag{52}$$
$$r_a^p(\widehat{G}_a) \leq \min_{\ell \geq 0}[\max\{(g_{aa} - g_{ab})^+, g_{bb} - \ell\delta\} - g_{bb}\tau_b(g_{aa}, g_{bb}) + \ell\delta\tau_b(g_{aa}, g_{bb})], \tag{53}$$
$$r_a^p(\widehat{G}_a) \leq (g_{aa} - g_{ab})^+, \tag{54}$$
$$r_b^p(\widehat{G}_b) \leq \min_{\ell \geq 0}[(g_{bb} - g_{ba} - \ell\delta)^+ + \ell\delta\tau_b(g_{aa}, g_{bb})]. \tag{55}$$

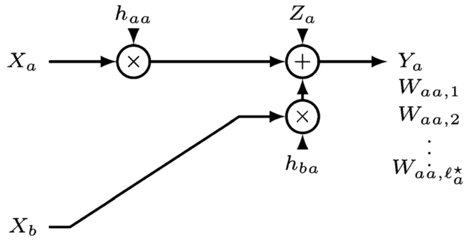


Fig. 6. Genie for receiver a in the GIC.

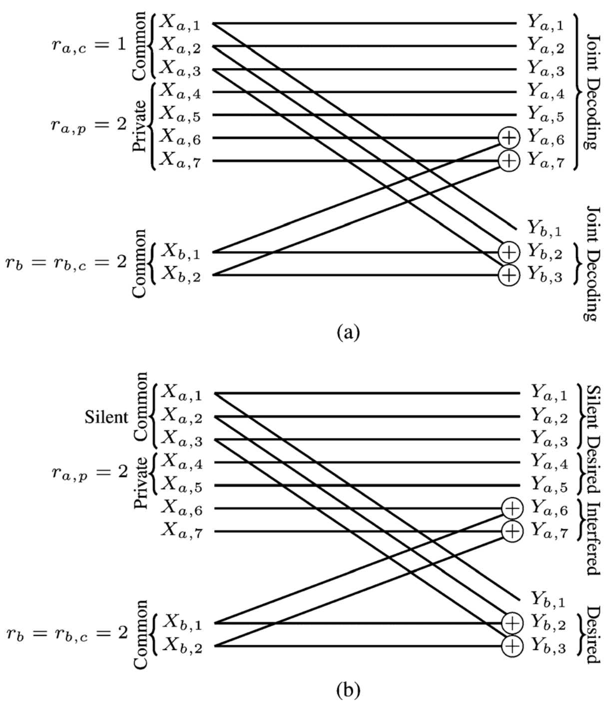


Fig. 7. Policy dictated schemes responding to (a) View 1 and (b) View 2. In each, the view provides enough information to achieve a rate point strictly dominating TDM. (a) View 1 Example. (b) View 2 Example.

Though it is possible to state the capacity region in a nonparametric form (through Fourier–Motzkin elimination applied categorically for each of many different regimes), such a presentation is extremely unwieldy and does little to illuminate the form of the region. We find it more illustrative to provide an example of a policy strictly dominating TDM in the example channel [see Fig. 7(a)]. Transmitter b transmits at full rate $(r_b(\widehat{G}_b) = g_{bb} = 2)$ while Transmitter a uses an HK code where the common message has rate $r_{a,c} = g_{ab} - g_{bb} = 1$. All three interfering signal levels are used in a codebook drawn from a random distribution, which can be interpreted as the most significant bit carrying a 1-bit message, and the next two signal levels providing parity. A private message is encoded over all $g_{aa} - g_{ab} = 4$ private signal levels at rate $r_{a,p} = 2$. Regardless of the value of g_{ba} (which is unknown to Transmitter a) the component rates at Receiver a satisfy (16)–(20), so the rate point $(r_{a,c} + r_{a,p}, r_{b,c}) = (3, 2)$ is achievable. We see that

$$\frac{r_a(\widehat{G}_a)}{g_{aa}} + \frac{r_b(\widehat{G}_b)}{g_{bb}} = \frac{3}{7} + \frac{2}{2} = \frac{10}{7} > 1.$$

2) View 2: For View 2, each transmitter is aware of which of its signal levels may be causing interference, and which may be interfered with. Thus, each transmitter may opportunistically align bits to appropriate signal levels. Note that in the parametrized characterization, the constraint on rate for each transmitter is independent of the unknown channel gain. This is because the worst case(s), which results in TDM even in the full-view scenario, has already been addressed, and additional bits are gained through opportunism.

Theorem 5 (View 2 TDM-Dominating Capacity Region): If $\widehat{G}_a = (g_{aa}, g_{ab}, g_{ba}, \varnothing)$ and $\widehat{G}_b = (\varnothing, g_{ab}, g_{ba}, g_{bb})$, then the TDM-dominating capacity region is given by the union of all regions indexed by values $\tau_a(g_{ab}, g_{ba})$ and $\tau_b(g_{ab}, g_{ba})$, with $\tau_a(g_{ab}, g_{ba}) + \tau_b(g_{ab}, g_{ba}) = 1$, such that $(r_a(\widehat{G}_a), r_b(\widehat{G}_b))$ is nonnegative and

$$r_a(\widehat{G}_a) \leq g_{aa}, \tag{56}$$
$$r_a(\widehat{G}_a) \leq (g_{aa} - g_{ab})^+ + g_{ab}\tau_a(g_{ab}, g_{ba}), \tag{57}$$
$$r_a(\widehat{G}_a) \leq (g_{aa} - g_{ba})^+ + g_{ba}\tau_a(g_{ab}, g_{ba}), \tag{58}$$
$$r_a(\widehat{G}_a) \leq (g_{aa} - g_{ab} - g_{ba})^+ + (g_{ab} + g_{ba})\tau_a(g_{ab}, g_{ba}), \tag{59}$$
$$r_b(\widehat{G}_b) \leq g_{bb}, \tag{60}$$
$$r_b(\widehat{G}_b) \leq (g_{bb} - g_{ba})^+ + g_{ba}\tau_b(g_{ab}, g_{ba}), \tag{61}$$
$$r_b(\widehat{G}_b) \leq (g_{bb} - g_{ab})^+ + g_{ab}\tau_b(g_{ab}, g_{ba}), \tag{62}$$
$$r_b(\widehat{G}_b) \leq (g_{bb} - g_{ab} - g_{ba})^+ + (g_{ab} + g_{ba})\tau_b(g_{ab}, g_{ba}). \tag{63}$$

In general, this region cannot be achieved with a simple orthogonalized scheme. The details of the scheme are more rigorously explained within the proof, but it is essentially a linear deterministic analogue of the approach used in [6]. Transmitter knowledge of the channel gain to unintended receiver enables each transmitter to split its message into a common and private component, where the common message is coded using channel inputs that interfere with the other transmission. The private message is sent on the remaining inputs of the direct link, essentially "hidden in the noise floor" at the other receiver. Each receiver treats the desired common and private messages and the common message of the other user as a virtual MAC, and jointly decodes the components.

The similarity to the result of [6] also extends to the nonparametric characterization of the region.

Corollary 6: The View 2 TDM-dominating capacity region consists of all nonnegative rate points satisfying

$$r_a(\widehat{G}_a) \leq g_{aa}, \tag{64}$$
$$r_b(\widehat{G}_b) \leq g_{bb}, \tag{65}$$
$$r_a(\widehat{G}_a) + r_b(\widehat{G}_b) \leq (g_{aa} - g_{ba})^+ + \max\{g_{bb}, g_{ba}\}, \tag{66}$$
$$r_a(\widehat{G}_a) + r_b(\widehat{G}_b) \leq (g_{bb} - g_{ab})^+ + \max\{g_{aa}, g_{ab}\}, \tag{67}$$
$$r_a(\widehat{G}_a) + r_b(\widehat{G}_b) \leq \max\{g_{ab}, g_{aa} - g_{ba}\} + \max\{g_{ba}, g_{bb} - g_{ab}\}, \tag{68}$$

$$\frac{g_{ab}+g_{ba}}{g_{ba}} r_a(\widehat{G}_a) + r_b(\widehat{G}_b) \le \frac{g_{ab}}{g_{ba}} \max\{g_{aa}, g_{ba}\} + (g_{aa}-g_{ba})^+ + \max\{g_{ba}, g_{bb}-g_{ab}\}, \tag{69}$$

$$r_a(\widehat{G}_a) + \frac{g_{ab}+g_{ba}}{g_{ba}} r_b(\widehat{G}_b) \le \frac{g_{ab}}{g_{ba}} \max\{g_{bb}, g_{ba}\} + (g_{bb}-g_{ba})^+ + \max\{g_{ba}, g_{aa}-g_{ab}\}, \tag{70}$$

$$\frac{g_{ab}+g_{ba}}{g_{ab}} r_a(\widehat{G}_a) + r_b(\widehat{G}_b) \le \frac{g_{ba}}{g_{ab}} \max\{g_{aa}, g_{ab}\} + (g_{aa}-g_{ab})^+ + \max\{g_{ab}, g_{bb}-g_{ba}\}, \tag{71}$$

$$r_a(\widehat{G}_a) + \frac{g_{ab}+g_{ba}}{g_{ab}} r_b(\widehat{G}_b) \le \frac{g_{ba}}{g_{ab}} \max\{g_{bb}, g_{ab}\} + (g_{bb}-g_{ab})^+ + \max\{g_{ab}, g_{aa}-g_{ba}\}. \tag{72}$$

The region specified in (64)–(72) results from Fourier–Motzkin elimination [31] of the time-sharing parameters τ_a and τ_b and is not included for brevity.

Notice that the Fourier–Motzkin eliminated region for View 2, (64)–(72), is similar to the full-view capacity region, (28)–(34). Specifically, (64)–(68) match (28)–(32) exactly. Of the remaining inequalities, if $g_{ab} = g_{ba}$, then (69) and (70) are equivalent to (71) and (72) as well as the inequalities (33) and (34) of the full-view case. These inequalities characterize the only loss from the full-view capacity region when transmitters are provided with View 2, and if either $g_{ab} = g_{ba}$ or bounds (69)–(72) are dominated by the sum-rate bounds, then the View 2 region and the full-view region coincide. However, this does not imply a lack of operational loss between a full view and View 2. The parametrized characterization of Theorem 5 highlights the lack of system flexibility needed to achieve points within this region.

For an example of a policy outperforming TDM in the example channel from Fig. 3, allow Transmitter b to always transmit at full rate ($r_b(\widehat{G}_b) = g_{bb} = 2$). Transmitter a uses an HK code; however, the common message is constrained to rate $r_{a,c} = 0$. The private message is encoded over all the top two of the noninterfering signal levels ($X_{a,4}$ and $X_{a,5}$) at rate $r_{a,p} = 2$. Receiver a treats the interference as noise and decodes the private message. Consequently, we have

$$\frac{r_a(\widehat{G}_a)}{g_{aa}} + \frac{r_b(\widehat{G}_b)}{g_{bb}} = \frac{2}{7} + \frac{2}{2} = \frac{9}{7} > 1,$$

as desired. The coding scheme described is depicted below in Fig. 7(b).

B. TDM-Optimal Local Views

In each of the remaining five views, performance strictly dominating TDM is not possible. However, we separate our statements of the remaining results to emphasize the distinction between views that still facilitate some degree of transmission coordination.

1) Views 3 and 5: Although Views 3 and 5 do not permit use of strictly TDM-dominating policies, both may capitalize on common knowledge of g_{aa} and g_{bb} to adjust which point on the TDM region boundary is used.

Theorem 7 (Views 3 and 5 TDM-Dominating Capacity Region): If either $\widehat{G}_a = (g_{aa}, \varnothing, g_{ba}, g_{bb})$ and $\widehat{G}_b = (g_{aa}, g_{ab}, \varnothing, g_{bb})$ or $\widehat{G}_a = (g_{aa}, \varnothing, \varnothing, g_{bb})$ and $\widehat{G}_b = (g_{aa}, \varnothing, \varnothing, g_{bb})$, then the TDM-dominating capacity region consists of all nonnegative tuples $(r_a(\widehat{G}_a), r_b(\widehat{G}_b))$ such that

$$r_a(\widehat{G}_a) \le g_{aa}\tau_a(g_{aa}, g_{bb}),$$
$$r_b(\widehat{G}_b) \le g_{bb}\tau_b(g_{aa}, g_{bb}),$$

with $\tau_a(g_{aa}, g_{bb}) + \tau_b(g_{aa}, g_{bb}) = 1$.

The capacity region for View 3 is the most negative finding of this work. Despite *almost* complete knowledge of the network, transmitters are unable to strictly dominate TDM, which suggests that the costs associated with acquiring knowledge of the incoming IC gain and the other direct link channel gain were wasted.

Although detailed construction of the outer bound is left to the Appendix, we demonstrate below that the range of channel gains needed to construct the converse is bounded. Therefore, while our problem formulation specifies unbounded support for unknown channel gains, even a bounded set may result in limited performance.

For an example of how the converse may be constructed, we again refer to the channel in Fig. 3. In order for a policy to strictly dominate TDM, there must exist some $\tau_a^{\min}$ and $\tau_b^{\min}$, where $\tau_a^{\min} + \tau_b^{\min} = 1$, such that $r_a(\widehat{G}_a) \ge \tau_a^{\min} g_{aa}$ and $r_b(\widehat{G}_b) \ge \tau_b^{\min} g_{bb}$.

Under View 3, each transmitter does not know its outgoing IC gain, but does know the channel gain between the other transmitter and receiver. Consider the point of view of Transmitter a under the possibility $g'_{ab} = 1$ ($G' = (g_{aa}, g'_{ab} = 1, g_{ba}, g_{bb})$). From (38), we have

$$\begin{aligned} n\tau_b^{\min} g_{bb} &\le r_b(\widehat{G}_b^{(1)}) \\ &\le I(\mathbf{X}_b^n; \mathbf{Y}_b^n) \\ &\le n \min\{g_{bb}, g_{ab}\} - \sum_{k=1}^{g_{ab}} L_{a,k}(\widehat{G}_a) \\ &\quad + \sum_{j=1}^{u_b^+} L_{b,j}(\widehat{G}_b^{(1)}) + \sum_{i=1}^{u_b^-} L_{a,i}(\widehat{G}_a) \\ &\le n - L_{a,1}(\widehat{G}_a) + L_{b,1}(\widehat{G}'_b). \end{aligned}$$

Sweeping across a range of possible (from the perspective of Transmitter a) channel gains, we have

$$2\tau_b^{\min} \le \frac{1}{n}\left(L_{b,1}(\widehat{G}'_b) + n - L_{a,1}(\widehat{G}_a)\right), \tag{73}$$

$$2\tau_b^{\min} \le \frac{1}{n}\left(2n - L_{a,\ell}(\widehat{G}_a) - L_{a,\ell+1}(\widehat{G}_a)\right), \tag{74}$$

for any integer $\ell \in \{1,2,3,4,5,6\}$. Expressions (73)–(74) already suggest a microcosm of an inability to strictly dominate TDM; every pair of signal levels consecutive in significance already simulates an orthogonalized scheme. Combining (73) and (74) for $\ell \in \{2,4,6\}$ gives us

$$\frac{1}{n}\sum_{i=1}^{7} L_{a,i}(\widehat{G}_a) \le \frac{1}{n}L_{b,1}(\widehat{G}_b') - \tau_b^{\min} + 7\tau_a^{\min}, \tag{75}$$

and combining (74) for $\ell \in \{1,3,5\}$ yields

$$\frac{1}{n}\sum_{i=1}^{6} L_{a,i}(\widehat{G}_a) \le 6\tau_a^{\min}. \tag{76}$$

If we recall that $n\tau_a^{\min} g_{aa} \le n r_a(\widehat{G}_a) \le \sum_i L_{a,i}$, expression (76) becomes

$$\frac{1}{n}L_{b,1}(\widehat{G}_b') \ge \tau_b^{\min}. \tag{77}$$

Since we have not used Transmitter a's knowledge of g_{ba}, (75)–(77) also hold for any other values of g_{ba} as well including $g''_{ba} = 1$. Let $G'' = (g_{aa}, g'_{ab}, g''_{ba}, g_{bb}) = (7,1,1,2)$. Then, in another network state, to decode reliably at Receiver a,

$$\begin{aligned} r_a(\widehat{G}''_a) &\le I(\mathbf{X}_a;\mathbf{Y}_a) \\ &\le \frac{1}{n}\sum_{i=1}^{6} L_{a,i}(\widehat{G}''_a) + 1 - \frac{1}{n}L_{b,1}(\widehat{G}'_b) \\ &\le 6\tau_a^{\min} + 1 - \frac{1}{n}L_{b,1}(\widehat{G}'_b). \end{aligned} \tag{78}$$

Combining the fact $r_a(\widehat{G}''_a) \ge 7\tau_a^{\min}$ and (77) implies $\frac{1}{n}L_{b,1}(\widehat{G}'_b) = \tau_b^{\min}$, which with (78) proves

$$r_a(\widehat{G}_a) = 7\tau_a^{\min} = \tau_a^{\min} g_{aa}.$$

This example demonstrates how Transmitter a's inability to effectively align its interference signal prevents performance strictly dominating TDM. A similar series of arguments confirms that Transmitter b also is limited to the TDM rate; however, we omit this in lieu of the more general proof in the Appendix.

In View 5, transmitters have even less knowledge than in View 3 and thus is bounded by the same level of performance. However, it is interesting to note that the knowledge common to both transmitters is all the knowledge available to each transmitter. This not only allows them to synchronize their decision, but also suggests that View 5 models a centralized compound IC. It is, therefore, worth mentioning that the extension to multiple network states discussed in [24] results in the same conclusion for the View 5 LDIC.

2) Views 4, 6, and 7: As the three local views with the smallest local views of network state and no channel gain commonly known to both transmitters, the following result confirms the intuition that TDM is a good policy.

Theorem 8 (Views 4, 6, and 7 TDM-Dominating Capacity Regions): If the views are any of the following three cases

1) $\widehat{G}_a = (g_{aa}, \varnothing, g_{ba}, \varnothing)$ and $\widehat{G}_b = (\varnothing, g_{ab}, \varnothing, g_{bb})$
2) $\widehat{G}_a = (g_{aa}, g_{ab}, \varnothing, \varnothing)$ and $\widehat{G}_b = (\varnothing, \varnothing, g_{ba}, g_{bb})$
3) $\widehat{G}_a = (g_{aa}, \varnothing, \varnothing, \varnothing)$ and $\widehat{G}_b = (\varnothing, \varnothing, \varnothing, g_{bb})$,

then the TDM-dominating capacity region consists of all nonnegative tuples $(r_a(\widehat{G}_a), r_b(\widehat{G}_b))$ such that

$$\begin{aligned} r_a(\widehat{G}_a) &\le g_{aa}\tau_a, \\ r_b(\widehat{G}_b) &\le g_{bb}\tau_b, \end{aligned}$$

with $\tau_a + \tau_b = 1$.

As described while referencing Fig. 2, policies relying on Views 4–7 can perform no better than views containing more information. Therefore, it is not surprising that no policy can strictly dominate TDM in any of these views.

VI. Results for the GIC

In this section, we extend our results to the GIC and characterize TDM-dominating capacity regions to within a bounded gap, and comment upon the GDoF of the GIC with TDM-dominating policies.

A. Main Result

Theorem 9 (Approximate TDM-Dominating Capacity Regions of Local View Gaussian ICs): Given the relationship between LDIC and GIC channel gains presented in (1)–(4), and assuming WLOG $g_{aa} \ge g_{bb}$, for local view k, the per-user gap between the GIC TDM-dominating capacity region and the TDM-dominating capacity region of its LDIC analogue is less than Δ_k bits where Δ_k is given in Table I.

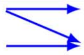
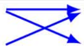
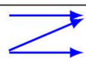
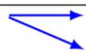
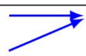
Note that for Views 1, 3, and 5, the gap is dependent on network state and not universal in the sense of prior work on approximate capacity. This stems from the outer bound based on Z-channel expansion. However, the gap is dependent on only ratios between channel gains, and under an appropriate scaling of the magnitudes of channel gains discussed in Section VI-C, the gap remains bounded by the same constant as SNR goes to infinity.

Detailed proofs for each view are located in Appendix F. In the remainder of this section, we comment on the sources of gaps between models and intuitions that can be drawn from our result. Prior to explaining calculation of gaps between the regions given in Theorems 4–8 and their GIC counterparts, we first recall from Section IV-A that each scheme prescribed for every local view LDIC (either TDM or a simple HK-based code) approximates an analogous achievable scheme for the GIC. At the same time, the rates achieved using each scheme dictated by LDIC policies may be approximately achieved using the analogous GIC scheme. Therefore, we do not explicitly define the policy for each local view GIC, but instead rely on results of the LDIC to characterize the capacity approximation gaps.

B. Approximate TDM-Dominating Capacity

To account for the gaps shown in Table I, we use the results of the LDIC as an intermediate step in determining the gap from GIC achievable policy to GIC outer bound. Note that if TDM is the prescribed policy for the LDIC case (Views 3–7), the GIC TDM fully contains the analogous LDIC TDM region, and thus, the rate prescribed by the LDIC policy is achievable. If the prescribed policy is based on an HK scheme (Views 1 and 2), we

TABLE I
PER-USER GAP BETWEEN GIC AND LDIC TDM-DOMINATING CAPACITY REGIONS

View (k)	**View Diagram**	Δ_k	Common Knowledge
1		$\begin{cases} \log(6)+4 & \text{if } g_{aa}=g_{bb} \\ \log(9)+2\max\left\{2\left\lceil\frac{g_{bb}}{g_{aa}-g_{bb}}\right\rceil+1, \left\lceil\frac{g_{ba}}{g_{aa}-g_{bb}}\right\rceil+\left\lceil\frac{(g_{bb}-g_{ba})^+}{g_{aa}-g_{bb}}\right\rceil\right\}+4 & \text{else} \end{cases}$	g_{aa}, g_{bb}
2		$2\log(6)+\log(3)+4$	g_{ab}, g_{ba}
3		$\left(\frac{\mathrm{LCM}(g_{aa},g_{bb})}{g_{aa}}+\frac{\mathrm{LCM}(g_{aa},g_{bb})}{g_{bb}}-1\right)\log(6)$	g_{aa}, g_{bb}
4		$\log(6)$	$\emptyset$
5		$\left(\frac{\mathrm{LCM}(g_{aa},g_{bb})}{g_{aa}}+\frac{\mathrm{LCM}(g_{aa},g_{bb})}{g_{bb}}-1\right)\log(6)$	g_{aa}, g_{bb}
6		$\log(6)$	$\emptyset$
7		$\log(6)$	$\emptyset$

apply Lemma 3 to bound the gap between rates achievable by analogous schemes in GIC and LDIC models to 4 bits per user.

Since the LDIC characterization is tight, we proceed to bound the gap between LDIC and GIC outer bounds. We illustrate the impact of each of the two main features of the linear deterministic approximation of the Gaussian channel (quantization of a complex channel gain h into an integer value g and representation of a superposition of signals with modulo addition) by comparing the level-by-level decompositions of mutual information bounds for the LDIC and GIC.

Consider the first term in both (37) and (40). In (37), the maximum entropy of the signal levels affected by both the desired and interfering signals is no higher than if maximizing the entropy of just one of the two signals. On the other hand, the analogous term in the Gaussian IC is not so easily bounded. If $g_{aa} > g_{ba} > 0$, we have

$$\begin{aligned}
&h\big(Y_a^n \big| W_{aa,u_a^+}, W_{ba,u_a^-}\big) - n\log(2\pi e) \\
&\quad = h\left(Y_a^n \middle| W_{aa,u_a^+}\right) - n\log(2\pi e) \\
&\quad = h\left(h_{aa}X_a^n + h_{ba}X_b^n + Z_a^n \middle| W_{aa,u_a^+}\right) \\
&\qquad - n\log(2\pi e) \\
&\quad \overset{(a)}{\leq} h\left(\Omega\Phi_{aa}X_a^n + h_{ba}X_b^n + Z_a^n - \sqrt{2^{g_{ba}}}Z^{n\prime}\right) \\
&\qquad - n\log(2\pi e) \\
&\quad \overset{(b)}{\leq} n\log\left(|\Omega|^2 + |h_{ba}|^2 + 1 + 2^{g_{ba}}\right) \\
&\quad \overset{(c)}{\leq} n\log\left(2 + |h_{ba}|^2 + 1 + 2^{g_{ba}}\right) \\
&\quad \leq n\log\left(3(2^{g_{ba}}) + 3\right),
\end{aligned}$$

where

$$\Omega \triangleq |h_{aa}| - \sqrt{2^{g_{aa}}}\frac{\max\{|h_{aa}|, |h_{ab}|\}}{\sqrt{2^{\max\{g_{aa},g_{ab}\}}}}.$$

In (a), we subtract the genie signal scaled by $\frac{\sqrt{2^{g_{aa}}}}{\sqrt{2^{\max\{g_{aa},g_{ab}\}}}}$; in (b), we invoke a maximum entropy argument; and in (c), we bound the variance of the uncanceled part of X_a^n. If $g_{aa} < g_{ba}$, then through an analogous series of steps, we find

$$\begin{aligned}
&h\big(Y_a^n \big| W_{aa,u_a^+}, W_{ba,u_a^-}\big) - n\log(2\pi e) \\
&\quad = h\left(Y_a^n \middle| W_{ba,u_a^-}\right) - n\log(2\pi e) \\
&\quad \leq n\log\left(3(2^{g_{aa}}) + 3\right).
\end{aligned}$$

Finally, if $g_{aa} = g_{ba}$, then

$$\begin{aligned}
&h\big(Y_a^n \big| W_{aa,u_a^+}, W_{ba,u_a^-}\big) - n\log(2\pi e) \\
&\quad = h(Y_a^n) - n\log(2\pi e) \\
&\quad = h\left(h_{aa}X_a^n + h_{ba}X_b^n + Z_a^n\right) - n\log(2\pi e) \\
&\quad \leq n\log\left(|h_{aa}|^2 + |h_{ba}|^2 + 1\right) \\
&\quad \leq n\log\left(4(2^{g_{aa}}) + 1\right).
\end{aligned}$$

Defining $\tilde{g} \triangleq \min\{g_{aa}, g_{ba}\}$, we upper bound the gap between this term and its LDIC counterpart over all channel gains:

$$\begin{aligned}
&h\big(Y_a^n \big| W_{aa,u_a^+}, W_{ba,u_a^-}\big) - n\log(2\pi e) - n\tilde{g} \\
&\quad \leq n\max\left\{\log\left(3\left(2^{\tilde{g}}\right) + 3\right), \log\left(4\left(2^{\tilde{g}}\right) + 1\right)\right\} - n\tilde{g} \\
&\quad \leq n\log(6), \qquad (79)
\end{aligned}$$

which implies that in each interference scenario, there may be up to $\log(6)$ bits per channel use that is not utilized by rates prescribed by the LDIC region.

This extra headroom is partially the result of aggregation of power that occurs when summing signals in the Gaussian model. Additionally, the quantized channel magnitudes in the linear deterministic model also incur a reduction in represented signal strength of both desired and interference signal components.

In the context of TDM-dominating capacity analysis, this gap between the (tight) linear deterministic bound and our Gaussian outer bound exists for each interference scenario. Therefore, the larger the number of network states that jointly constrain the rate of a transmitter (i.e., the number of virtual Z-channel transmitter–receiver pairs used to establish an outer bound), the larger the gap between LDIC and GIC TDM-dominating capacity region boundaries. This is reflected in Views 4, 6, and

7, where due to extremely limited local view, only a single network state can be established as a worst case, and our gap is relatively small.

Our bound is admittedly not tight, and for certain cases (Views 2 and 5), application of existing techniques ([6] and [24], respectively) may result in smaller gaps. However, our analysis better parallels intuition imparted by the linear deterministic model applied in the local view setting.

C. GDoF of TDM-Dominating Policies

Our approximation gaps for GIC TDM-dominating capacity depend both on local view and network state and thus are not universal. However, the notion of GDoF characterizes the high-SNR behavior of each combination of local view and network state, thereby skirting the nonuniversality of gaps and presenting a clearer comparison between local view GICs at high SNR.

Let the parameter $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ be defined as

$$\alpha_1 \triangleq \frac{\log |h_{bb}|^2}{\log |h_{aa}|^2}, \quad \alpha_2 \triangleq \frac{\log |h_{ba}|^2}{\log |h_{aa}|^2}, \quad \alpha_3 \triangleq \frac{\log |h_{ab}|^2}{\log |h_{aa}|^2}$$

and $\mathcal{C}(H)$ be the capacity region of the complex Gaussian IC given by network state $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$. The GDoF region was defined in [6] as

$$\mathcal{D}(\alpha) = \lim_{|h| \to \infty} \left\{ \left(\frac{r_a}{\log |h_{aa}|^2}, \frac{r_b}{\log |h_{bb}|^2} \right) : (r_a, r_b) \in \mathcal{C}(H) \right\},$$

where the limit is taken with α and $\angle h_{ij}$ held constant for all i, j. Although the exact impact of phase on the capacity region is unknown, because the one bit characterization of [6] provided a phase-independent approximation of capacity the high-SNR characterization of GDoF can be tight without considering phase.

In order to comment on GDoF of local view GICs, we note the following property regarding integer multiples of LDIC network states. The claim is due to the linearity of the model and can be verified by examination of expressions defining each region:

Property 1 (Integer Multiples of Network States): If all the channel gains of an LDIC network state G' can be expressed as an integer multiple of another network state, $G' = cG$, where c is a positive integer, then the capacity region given G' is the integer multiple of the capacity region of G:

$$\mathcal{C}(G') = c\,\mathcal{C}(G).$$

With respect to the Gaussian channel, this property coupled with the gap analysis results allows us to comment on the GDoF region subject to restriction to TDM-dominating policies for each view:

Corollary 10 (GDoF Regions of TDM-Dominating Policies): Let α_1, α_2, and α_3, be positive rational values. The GDoF region of TDM-dominating policies with View k is given by

$$\mathcal{D}_k(\alpha) = \left\{ \left(\frac{r_a}{g_{aa}}, \frac{r_b}{g_{bb}} \right) : (r_a, r_b) \in \mathcal{C}_{D,k}(G) \right\},$$

where G is such that $\frac{g_{bb}}{g_{aa}} = \alpha_1$, $\frac{g_{ba}}{g_{aa}} = \alpha_2$, $\frac{g_{ab}}{g_{aa}} = \alpha_3$.

While this does not define a GDoF region for all values of α, because the rationals are dense in the reals, the GDoF region of TDM-dominating policies can be found for an arbitrarily precise approximation of the parameter α.

VII. Summary

We studied the impact of incomplete, mismatched views of network state at each transmitter on performance in the two-user IC. We proposed a new formalization of maximally achievable rate regions for local view ICs called a TDM-dominating capacity region, and presented exact characterizations of the TDM-dominating capacity region of the linear deterministic IC with local views. One of our major conclusions shows the critical importance of each transmitter knowing the channel gain to its unintended receiver to achieve performance strictly dominating TDM. Finally, we extended our analysis to the two-user Gaussian IC, where we bounded gaps between the Gaussian IC TDM-dominating capacity region and its linear deterministic analogue, and used these results to comment on the GDoF region of TDM-dominating policies for the Gaussian IC with local views.

Appendix

A) Proof for LV-MAC: From the inequalities defining the boundary of the MAC capacity region, we have

$$\sum_{k=1}^{K} r_k(\widehat{G}_k)|_{g_k=d} \le d, \tag{80}$$

where h is any potential channel gain. Applying the minimum performance criterion yields

$$1 \le \sum_{k=1}^{K} \frac{r_k(\widehat{G}_k)|_{g_k=d}}{d} \le 1, \tag{81}$$

or

$$\sum_{k=1}^{K} \frac{r_k(\widehat{G}_k)|_{g_k=d}}{d} = 1. \tag{82}$$

Select K nonnegative integer values, $d_1, \ldots, d_K$, and notice

$$\sum_{k=1}^{K} \frac{r_k(\widehat{G}_k)|_{g_k=d_k}}{d_k} = \sum_{k=1}^{K} \frac{r_k(\widehat{G}_k)|_{g_k=d_k}}{d_k} + K - \sum_{\ell=1}^{K} \left(\sum_{k=1}^{K} \frac{r_k(\widehat{G}_k)|_{g_k=d_\ell}}{d_\ell} \right)$$

$$= K - \sum_{\ell=1}^{K-1}\left(\left(\sum_{k=1}^{\ell-1}\frac{r_k(\widehat{G}_k)|_{g_k=d_\ell}}{d_\ell}\right)+\frac{r_\ell(\widehat{G}_\ell)|_{g_\ell=d_K}}{d_K}\right.$$
$$\left.+\left(\sum_{k=\ell+1}^{K}\frac{r_k(\widehat{G}_k)|_{g_k=d_\ell}}{d_\ell}\right)\right). \tag{83}$$

Applying the minimum performance criterion to the left-hand side, as well as to each of the terms indexed by ℓ on the right-hand side in (83), we have

$$1 \le \sum_{k=1}^{K}\frac{r_k(\widehat{G}_k)|_{g_k=d_k}}{d_k} \le K-(K-1)=1. \tag{84}$$

Since this holds for any nonnegative values of $d_1,\ldots,d_K$ the theorem holds.

B) Proof for View 1:

Outer Bound: For any policy satisfying the minimum performance criteria, by definition there must exist nonnegative τ_a and τ_b such that for all G,

$$\tau_a(g_{aa},g_{bb})+\tau_b(g_{aa},g_{bb})=1, \tag{85}$$

and

$$r_a(\widehat{G}_a) \ge g_{aa}\tau_a(g_{aa},g_{bb}), \tag{86}$$

$$r_b(\widehat{G}_b) \ge g_{bb}\tau_b(g_{aa},g_{bb}), \tag{87}$$

where system-wide parameters are allowed to depend on the common knowledge (g_{aa} and g_{bb}).

Two virtual single Z-channels immediately result in bounds (42) and (44). At Receiver a, if $g_{ba}=g_{bb}$ we apply (37) and find

$$\begin{aligned}
&nr_a(\widehat{G}_a)\\
&\le \left.\left(ng_{ba}-\sum_{j=1}^{g_{ba}}L_{b,j}(\widehat{G}_b)+\sum_{i=1}^{g_{aa}-g_{ba}}L_{a,i}(\widehat{G}_a)\right)\right|_{g_{ba}=g_{bb}}\\
&\le n\left(g_{bb}-r_b(\widehat{G}_b)|_{g_{ba}=g_{bb}}\right)+\sum_{i=1}^{g_{aa}-g_{bb}}L_{a,i}(\widehat{G}_a)\\
&\le n\left[g_{aa}-\tau_b(g_{aa},g_{bb})g_{bb}\right].
\end{aligned} \tag{88}$$

Similarly, at Receiver b if $g_{ab}=g_{aa}$ we apply (38)

$$\begin{aligned}
&nr_b(\widehat{G}_b)\\
&\le \left.\left(ng_{bb}-\sum_{j=1}^{g_{ab}}L_{a,j}(\widehat{G}_a)+\sum_{i=1}^{g_{ab}-g_{bb}}L_{a,i}(\widehat{G}_a)\right)\right|_{g_{ab}=g_{aa}}\\
&\le n\left(g_{bb}-r_a(\widehat{G}_a)|_{g_{ab}=g_{aa}}\right)+\sum_{i=1}^{g_{aa}-g_{bb}}L_{a,i}(\widehat{G}_a)\\
&\le n\left(g_{aa}-r_a(\widehat{G}_a)|_{g_{ab}=g_{aa}}\right)\\
&\le n\left(g_{aa}-g_{aa}\tau_a(g_{aa},g_{bb})\right)\\
&\le ng_{aa}\tau_b(g_{aa},g_{bb}).
\end{aligned} \tag{89}$$

To arrive at the other bounds, we note in (37) that regardless of the incoming interference gain, g_{ba}, the following statement is necessary for achievability:

$$\begin{aligned}
nr_a(\widehat{G}_a) \le& \sum_{i=1}^{g_{ab}}L_{a,i}(\widehat{G}_a)+n\max(g_{aa}-g_{ab},g_{ba})\\
&-\sum_{j=1}^{g_{ba}}L_{b,j}(\widehat{G}_b).
\end{aligned} \tag{90}$$

In this expression, we draw a distinction between the entropy of the interference component and the entropy of the noninterfering component of the signal. To clarify analysis, we define the average entropies of the interference components of each transmitter's input as

$$\overline{r}_a^c(\widehat{G}_a) \triangleq \frac{1}{n}\sum_{i=1}^{g_{ab}}L_{a,i}(\widehat{G}_a), \tag{91}$$

$$\overline{r}_b^c(\widehat{G}_b) \triangleq \frac{1}{n}\sum_{j=1}^{g_{ba}}L_{b,j}(\widehat{G}_b). \tag{92}$$

Each transmitter's interference component and the noninterference component separately by constructing Z-channels both in the forward (adding virtual users that receive interference) and backward (adding virtual users that may induce interference) directions, as described in Section IV-B1. Examples of the virtual Z-channels and their relation to specific bounds are shown in Fig. 8.

First, consider Transmitter a's interference component. If the Z-channel terminates at the next signal, the following are two necessary conditions to guarantee achievability of rate $r_b(\widehat{G}_b')$:

$$\overline{r}_a^c(\widehat{G}_a) \le \max\{g_{bb},g_{ab}\}-r_b(\widehat{G}_b'), \tag{93}$$

$$\begin{aligned}
\overline{r}_a^c(\widehat{G}_a) &\le \frac{1}{n}\sum_{j=1}^{g_{ba}'}L_{b,j}(\widehat{G}_b')+\max\{g_{bb}-g_{ba}',g_{ab}\}-r_b(\widehat{G}_b')\\
&=\overline{r}_b^c(\widehat{G}_b')+\max\{g_{bb}-g_{ba}',g_{ab}\}-r_b(\widehat{G}_b').
\end{aligned} \tag{94}$$

Similarly,

$$\overline{r}_b^c(\widehat{G}_b) \le \max\{g_{aa},g_{ba}\}-r_a(\widehat{G}_a'), \tag{95}$$
$$\overline{r}_b^c(\widehat{G}_b) \le \overline{r}_a^c(\widehat{G}_a')+\max\{g_{aa}-g_{ab}',g_{ba}\}-r_a(\widehat{G}_a'). \tag{96}$$

We may expand the virtual Z-channel in the forward direction by substitution of (95) or (96) into (94), or (93) or (94) into (96). This expansion may be repeated indefinitely resulting in a virtual Z-channel that incorporates the possibility of many different channel states $G^{(1)},G^{(2)},\ldots$ and local views $\widehat{G}_a^{(1)},\widehat{G}_a^{(2)},\ldots$ and $\widehat{G}_b^{(1)},\widehat{G}_b^{(2)},\ldots$ seen by the respective virtual users. Expansion of (94) in the forward direction, and indexing

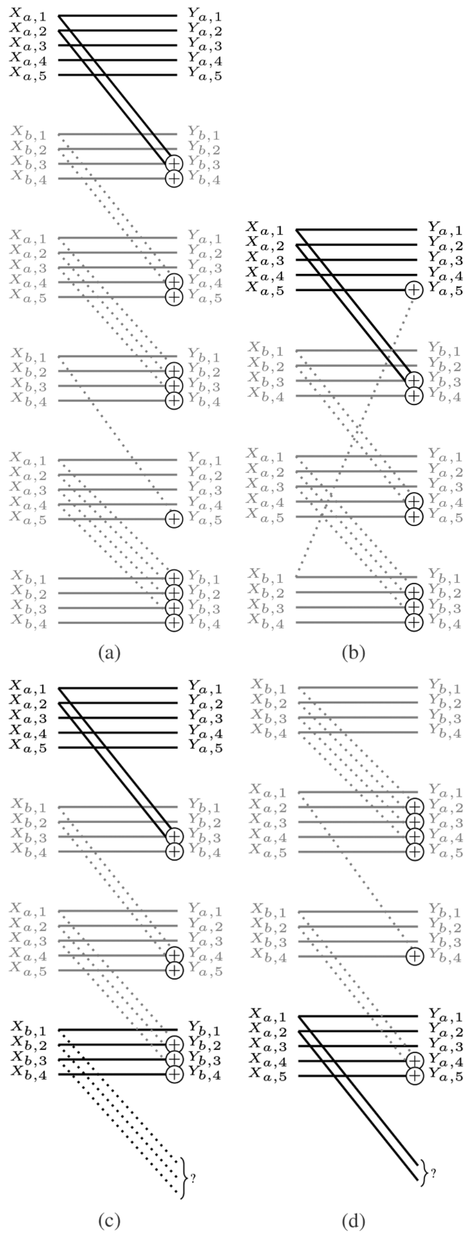


Fig. 8. Some virtual Z-channels used in deriving outer bound for View 1. Optimized channel state is $g_{aa} = 5, g_{ab} = 2, g_{ba} = 3, g_{bb} = 4$, and the expansion is from Transmitter a's POV. Black lines depict optimized channel states, solid lines reflect "known" link gains. Virtual Z-channel forward expansion to arrive at (a) bound of type (106), (b) bound of type (112), (c) bound of type (107) or (109), and (d) bound of type (119).

with θ each of Θ considered channel states, yields the following four families of bounds where $M \in \mathbb{Z}^+$:

$$\begin{aligned}\overline{r}_a^c(\widehat{G}_a) \leq\ & \max\{g_{bb} - g_{ba}^{(1)}, g_{ab}\} - r_b(\widehat{G}_b^{(1)}) \\ & + \sum_{\theta=1}^{\Theta-2} \Big[\max\{g_{aa} - g_{ab}^{(\theta)}, g_{ba}^{(\theta)}\} - \overline{r}_a(\widehat{G}_a^{(\theta)}) \\ & + \max\{g_{bb} - g_{ba}^{(\theta+1)}, g_{ab}^{(\theta)}\} - r_b(\widehat{G}_b^{(\theta+1)})\Big] \\ & + \max\{g_{aa} - g_{ab}^{(\Theta-1)}, g_{ba}^{(\Theta-1)}\} - \overline{r}_a(\widehat{G}_a^{(\Theta-1)}) \\ & + \max\{g_{bb}, g_{ab}^{(\Theta-1)}\} - r_b(\widehat{G}_b^{(\Theta)}),\end{aligned} \tag{97}$$

$$\begin{aligned}\overline{r}_a^c(\widehat{G}_a) \leq\ & \max\{g_{bb} - g_{ba}^{(1)}, g_{ab}\} - r_b(\widehat{G}_b^{(1)}) \\ & + \sum_{\theta=1}^{\Theta-2} \Big[\max\{g_{aa} - g_{ab}^{(\theta)}, g_{ba}^{(\theta)}\} - \overline{r}_a(\widehat{G}_a^{(\theta)}) \\ & + \max\{g_{bb} - g_{ba}^{(\theta+1)}, g_{ab}^{(\theta)}\} - r_b(\widehat{G}_b^{(\theta+1)})\Big] \\ & + \max\{g_{aa} - g_{ab}^{(M-1)}, g_{ba}^{(\Theta-1)}\} - \overline{r}_a(\widehat{G}_a^{(\Theta-1)}) \\ & + \max\{g_{bb} - g_{ba}^{(\Theta)}, g_{ab}^{(\Theta-1)}\} \\ & - r_b(\widehat{G}_b^{(\Theta)}) + \overline{r}_b^c(\widehat{G}_b^{(\Theta)}),\end{aligned} \tag{98}$$

$$\begin{aligned}\overline{r}_a^c(\widehat{G}_a) \leq\ & \max\{g_{bb} - g_{ba}^{(1)}, g_{ab}\} - r_b(\widehat{G}_b^{(1)}) \\ & + \sum_{\theta=1}^{\Theta-2} \Big[\max\{g_{aa} - g_{ab}^{(\theta)}, g_{ba}^{(\theta)}\} - \overline{r}_a(\widehat{G}_a^{(\theta)}) \\ & + \max\{g_{bb} - g_{ba}^{(\theta+1)}, g_{ab}^{(\theta)}\} - r_b(\widehat{G}_b^{(\theta+1)})\Big] \\ & + \max\{g_{aa}, g_{ba}^{(\Theta)}\} - \overline{r}_a(\widehat{G}_a^{(\Theta)}),\end{aligned} \tag{99}$$

$$\begin{aligned}\overline{r}_a^c(\widehat{G}_a) \leq\ & \max\{g_{bb} - g_{ba}^{(1)}, g_{ab}\} - r_b(\widehat{G}_b^{(1)}) \\ & + \sum_{\theta=1}^{\Theta-2} \Big[\max\{g_{aa} - g_{ab}^{(\theta)}, g_{ba}^{(\theta)}\} - \overline{r}_a(\widehat{G}_a^{(\theta)}) \\ & + \max\{g_{bb} - g_{ba}^{(\theta+1)}, g_{ab}^{(\theta)}\} - r_b(\widehat{G}_b^{(\theta+1)})\Big] \\ & + \max\{g_{aa} - g_{ab}^{(\Theta)}, g_{ba}^{(\Theta)}\} \\ & - \overline{r}_a(\widehat{G}_a^{(\Theta)}) + \overline{r}_a^c(\widehat{G}_a^{(\Theta)}),\end{aligned} \tag{100}$$

which hold for any $\Theta \in \mathbb{Z}^+$, and arbitrary values of $G^{(\theta)}$. We tighten the bounds, by applying (86) and (87) and selecting the values of Θ and $G^{(\theta)}$ that minimize the right-hand sides of (97)–(100).

Applying (86) and (87), expression (97) becomes

$$\begin{aligned}\overline{r}_a^c(\widehat{G}_a) \leq\ & \max\{g_{bb} - g_{ba}^{(1)}, g_{ab}\} - g_{bb}\tau_b(g_{aa}, g_{bb}) \\ & + \sum_{\theta=1}^{\Theta-2} \big[\max\{g_{aa} - g_{ab}^{(\theta)}, g_{ba}^{(\theta)}\} \\ & + \max\{g_{bb} - g_{ba}^{(\theta+1)}, g_{ab}^{(\theta)}\} - (g_{bb} + \delta\tau_a(g_{aa}, g_{bb}))\big] \\ & + \max\{g_{aa} - g_{ab}^{(\Theta-1)}, g_{ba}^{(\Theta-1)}\} - g_{aa}\tau_a(g_{aa}, g_{bb}) \\ & + \max\{g_{bb}, g_{ab}^{(\Theta-1)}\} - g_{bb}\tau_b(g_{aa}, g_{bb}) \qquad (101) \\ =\ & \max\{g_{bb} - g_{ba}^{(1)}, g_{ab}\} \\ & - (\Theta - 1)(g_{bb} + \delta\tau_a(g_{aa}, g_{bb})) - g_{bb}\tau_b(g_{aa}, g_{bb}) \\ & + \sum_{\theta=1}^{\Theta-2} \big[\max\{g_{aa} - g_{ab}^{(\theta)}, g_{ba}^{(\theta)}\} \\ & + \max\{g_{bb} - g_{ba}^{(\theta+1)}, g_{ab}^{(\theta)}\}\big] \\ & + \max\{g_{aa} - g_{ab}^{(\Theta-1)}, g_{ba}^{(\Theta-1)}\} \\ & + \max\{g_{bb}, g_{ab}^{(\Theta-1)}\}.\end{aligned} \tag{102}$$

In order to minimize this expression for a given Θ, we assign the values of $g_{ab}^{(\theta)}$ and $g_{ab}^{(\theta)}$ as

$$g_{ab}^{(\Theta-1)} = g_{bb}, \tag{103}$$

$$g_{ba}^{(\theta)} = g_{aa} - g_{ab}^{(\theta)}, \tag{104}$$

$$g_{ab}^{(\theta-1)} = \Big(g_{bb} = g_{ba}^{(\theta)}\Big)^+. \tag{105}$$

Substituting $\ell = \Theta - 1$ and noting $g_{ab} \geq 0$,we arrive at the first bound on interference component entropy

$$\overline{r}_a^c(\widehat{G}_a) \leq \max\{g_{bb} - \ell\delta, g_{ab}\} + \ell\delta\tau_b(g_{aa}, g_{bb}) - g_{bb}\tau_b(g_{aa}, g_{bb}). \quad (106)$$

We can also study (97) and (99) where the terminal link represents the response to the optimized channel state (i.e., $g_{ba}^{(\Theta)} = g_{ba}$), and derive from (97) the bound

$$\overline{r}_a^c(\widehat{G}_a) + r_b(\widehat{G}_b) \leq \max\{g_{bb} - \ell\delta, g_{ab}\} + \ell\delta\tau_b(g_{aa}, g_{bb}). \quad (107)$$

For (98), selecting $g_{ba}^{(\Theta)} = g_{ba}$ yields

$$\begin{aligned}
&\overline{r}_a^c(\widehat{G}_a) + r_b(\widehat{G}_b) - \overline{r}_b^c(\widehat{G}_b) \\
&\quad \leq \max\{g_{bb} - g_{ba}^{(1)}, g_{ab}\} - (\Theta - 1)\left(g_{bb} + \delta\tau_a(g_{aa}, g_{bb})\right) \\
&\qquad + \sum_{\theta=1}^{\Theta-1}\big[\max\{g_{aa} - g_{ab}^{(\theta)}, g_{ba}^{(\theta)}\} \\
&\qquad + \max\{g_{bb} - g_{ba}^{(\theta+1)}, g_{ab}^{(\theta)}\}\big] \\
&\qquad + \max\{g_{aa} - g_{ab}^{(\Theta-1)}, g_{ba}^{(\Theta-1)}\} \\
&\qquad + \max\{g_{bb} - g_{ba}, g_{ab}^{(\Theta-1)}\}. \qquad (108)
\end{aligned}$$

Selecting possible interference gains as,

$$\begin{aligned}
g_{ab}^{(\Theta-1)} &= (g_{bb} - g_{ba})^+, \\
g_{ba}^{(\theta)} &= g_{aa} - g_{ab}^{(\theta)}, \\
g_{ab}^{(\theta-1)} &= (g_{bb} - g_{ba}^{(\theta)})^+,
\end{aligned}$$

results in the bound (given $\ell \geq 0$)

$$\overline{r}_a^c(\widehat{G}_a) + r_b(\widehat{G}_b) - \overline{r}_b^c(\widehat{G}_b) \leq \max\{g_{bb} - g_{ba} - \ell\delta, g_{ab}\} + \ell\delta\tau_b(g_{aa}, g_{bb}). \quad (109)$$

Similar analysis and choices for free parameters from the expressions (99) and (100) yield the following bounds (given $\ell \geq 0$):

$$\overline{r}_a^c(\widehat{G}_a) \leq g_{ab} + \ell\delta\tau_b(g_{aa}, g_{bb}), \quad (110)$$

$$\overline{r}_a^c(\widehat{G}_a) + r_a(\widehat{G}_a) \leq g_{ab} + (\ell+1)\delta\tau_b(g_{aa}, g_{bb}) + g_{aa}\tau_a(g_{aa}, g_{bb}), \quad (111)$$

$$r_a(\widehat{G}_a) \leq \max\{g_{aa}, g_{ab}\} + \ell\delta\tau_b(g_{aa}, g_{bb}) - g_{bb}\tau_b(g_{aa}, g_{bb}). \quad (112)$$

Analogously, from expansion from the interference component of b, we have (given $\ell \geq 0$)

$$\overline{r}_b^c(\widehat{G}_b) \leq \max\{g_{ba}, g_{aa}\} - g_{aa}\tau_a(g_{aa}, g_{bb}) + \ell\delta\tau_b(g_{aa}, g_{bb}), \quad (113)$$

$$\overline{r}_b^c(\widehat{G}_b) + r_a(\widehat{G}_a) \leq \max\{g_{ba}, g_{aa}\} + \ell\delta\tau_b(g_{aa}, g_{bb}), \quad (114)$$

$$\begin{aligned}
&\overline{r}_b^c(\widehat{G}_a) + r_a(\widehat{G}_b) - \overline{r}_a^c(\widehat{G}_b) \\
&\quad \leq \max\{g_{ba} - \ell\delta, (g_{aa} - g_{ab}), (g_{ba} - g_{ab}), (g_{ba} - g_{aa}), 0\} \\
&\qquad + \ell\delta\tau_b(g_{aa}, g_{bb}), \qquad (115)
\end{aligned}$$

$$\overline{r}_b^c(\widehat{G}_b) \leq \max\{g_{ba} - \ell\delta, (g_{ba} - g_{aa}), 0\} + \ell\delta\tau_b(g_{aa}, g_{bb}), \quad (116)$$

$$\overline{r}_b^c(\widehat{G}_a) + r_b(\widehat{G}_b) \leq \max\{g_{ba} - (\ell+1)\delta, (g_{ba} - g_{aa}), 0\} + (g_{bb} + (\ell+1)\delta)\tau_b(g_{aa}, g_{bb}), \quad (117)$$

$$r_b(\widehat{G}_b) \leq \max\{g_{ba}, g_{aa}\} - g_{aa}\tau_a(g_{aa}, g_{bb}) + \ell\delta\tau_b(g_{aa}, g_{bb}). \quad (118)$$

Although these bounds were derived by expanding a Z-channel forward from each transmitter's interference component, expressions (109) and (115) also account for the message component not contained in the interference signal: $r_b(\widehat{G}_b) - \overline{r}_b^c(\widehat{G}_b)$ and $r_a(\widehat{G}_a) - \overline{r}_a^c(\widehat{G}_a)$, respectively.

We now establish an additional pair of bounds for the component representative of the "private" message component in HK coding, derived from extending a Z-channel in the reverse direction [see Fig. 8(d)]. To terminate each chain, we assume the interference gain of the final link is equal to the direct link gain. Consequently, given $\ell \geq 0$

$$r_a(\widehat{G}_a) - \overline{r}_a^c(\widehat{G}_a) \leq \max\{(g_{aa} - g_{ab})^+, g_{bb} - \ell\delta\} - g_{bb}\tau_b(g_{aa}, g_{bb}) + \ell\delta\tau_b(g_{aa}, g_{bb}), \quad (119)$$

$$r_a(\widehat{G}_a) - \overline{r}_a^c(\widehat{G}_a) \leq (g_{aa} - g_{ab})^+ + \ell\delta\tau_b(g_{aa}, g_{bb}), \quad (120)$$

$$r_b(\widehat{G}_b) - \overline{r}_b^c(\widehat{G}_b) \leq (g_{aa} + \ell\delta)\tau_b(g_{aa}, g_{bb}), \quad (121)$$

$$r_b(\widehat{G}_b) - \overline{r}_b^c(\widehat{G}_b) \leq (g_{bb} - g_{ba} - \ell\delta)^+ + \ell\delta\tau_b(g_{aa}, g_{bb}). \quad (122)$$

We remove redundant bounds from the signal component bounds derived thus far. For instance, (112), (118), and (121) are undeniably looser bounds than (88), (89), and (122), respectively. Additionally, (111) is the sum of (88) and (106). In addition to redundancies, the inequality (113) can be tightened by observing its relationship to (89): as a bound on a "common" component of the signal, the entropy bounded in (113) must be less than the entropies of the full transmitted signals.

In summary, we have the set of bounds detailed in (123)–(136), as shown at the bottom of the next page.

Achievable Scheme: To complete the proof, we have three tasks:

1) Define a policy that specifies the transmission scheme not only for the channel state at hand, but for all states with the same direct link gains.
2) Show that any rate point on the outer bound can be achieved by such a scheme.
3) Confirm that the rates prescribed for other channel states are achievable and satisfy the minimum specified rate.

As in View 2 and [6], the achievable scheme relies on each message being divided into common and a private components. For each channel state, the common component of Sender a's codebook is generated by randomly selecting nr_a^c codewords from the set of all $n \times \max\{g_{ab}, g_{aa}\}$ binary matrices. The private message codebook is generated by randomly selecting nr_a^p codewords from the set of $n \times (g_{aa} - g_{ab})^+$ matrices. On outgoing links that interfere with Link b, the g_{ab} most significant levels of the common message are sent. If $g_{aa} - g_{ab} > 0$, then the modulo addition of the private message and lower levels of the common message is transmitted. Similarly, for Sender b, r_b^c and r_b^p govern the number of codewords (randomly drawn) in the common and private codebooks of Link b.

The size of component codebooks varies for different channel states, and is a function of each sender's local view. For the channel state being considered, the number of codewords in each component codebook are chosen such that $r_a^c(\widehat{G}_a)$, $r_a^p(\widehat{G}_a)$, $r_a(\widehat{G}_a)$, $r_b^c(\widehat{G}_b)$, $r_b^p(\widehat{G}_b)$, and $r_b(\widehat{G}_b)$ obey (42)–(55).

We assume joint decoding of all received components (i.e., each receiver perceives a virtual three-user MAC) which implies that the proposed policy is achievable if nonnegative r_a^c, r_a^p, r_b^c, r_b^p satisfy (16)–(27). Noting that the restrictions imposed in (42)–(55) are actually stricter than (16)–(27), the policy proposed thus far is achievable, thus completing Step 2.

For the responses to other channel states with local views $\widehat{G}'_a \neq \widehat{G}_a$ and $\widehat{G}'_b \neq \widehat{G}_b$, the common and private codebook sizes must also conform to a similar set of bounds such that the remain consistent with the responses of the considered channel state. Applying a similar virtual Z-channel expansion to arbitrary channel states, and assuming

$$r_a(\widehat{G}'_a) = r_a^c(\widehat{G}'_a) + r_a^p(\widehat{G}'_a) \geq g_{aa}\tau_a(g_{aa}, g_{bb}), \quad (137)$$

$$r_b(\widehat{G}'_b) = r_b^c(\widehat{G}'_b) + r_b^p(\widehat{G}'_b) \geq g_{bb}\tau_b(g_{aa}, g_{bb}), \quad (138)$$

we find for local views $\widehat{G}''_a \neq \widehat{G}_a$ and $\widehat{G}''_b \neq \widehat{G}_b$ and $\ell \geq 0$ that the component rates must satisfy (139)–(160), shown at the bottom of the next page.

Using these expressions along with (42)–(55), we define the rates (and by proxy size) of codebooks in each policy response. Moreover, through substitution of (42)–(55) into (139)–(160) we see that the rate satisfies the minimum performance criterion as desired.

C) Proof for View 2:

Outer Bound: We first analyze two limiting cases. If $g_{aa} = g_{bb} = g_{ab}$, from (31), we have

$$r_a(\widehat{G}_a)|_{g_{aa}=g_{ab}} + r_b(\widehat{G}_b)|_{g_{bb}=g_{ab}} \leq g_{ab} = g_{ab},$$

where the equality is enforced in order to satisfy the minimum performance criterion. Similarly, if $g_{aa} = g_{bb} = g_{ba}$, we have

$$r_a(\widehat{G}_a)|_{g_{aa}=g_{ba}} + r_b(\widehat{G}_b)|_{g_{bb}=g_{ba}} = g_{ba}.$$

These two cases can be restated as

$$\frac{r_a(\widehat{G}_a)|_{g_{aa}=g_{ab}}}{g_{ab}} + \frac{r_b(\widehat{G}_b)|_{g_{bb}=g_{ab}}}{g_{ab}} = 1,$$
$$\frac{r_a(\widehat{G}_a)|_{g_{aa}=g_{ba}}}{g_{ba}} + \frac{r_b(\widehat{G}_b)|_{g_{bb}=g_{ba}}}{g_{ba}} = 1$$

and when summed we have

$$\frac{r_a(\widehat{G}_a)|_{g_{aa}=g_{ab}}}{g_{ab}} + \frac{r_b(\widehat{G}_b)|_{g_{bb}=g_{ba}}}{g_{ba}} = 2 - \frac{r_a(\widehat{G}_a)|_{g_{aa}=g_{ba}}}{g_{ba}} - \frac{r_b(\widehat{G}_b)|_{g_{bb}=g_{ab}}}{g_{ab}} \leq 1, \quad (161)$$

where (161) is due to the minimum performance criterion. By applying the same constraint on the other side, we have

$$1 \leq \frac{r_a(\widehat{G}_a)|_{g_{aa}=g_{ab}}}{g_{ab}} + \frac{r_b(\widehat{G}_b)|_{g_{bb}=g_{ba}}}{g_{ba}} \leq 1, \quad (162)$$

$$r_a(\widehat{G}_a) \leq g_{aa} - g_{bb}\tau_b(g_{aa}, g_{bb}), \quad (123)$$
$$r_b(\widehat{G}_b) \leq g_{aa}\tau_b(g_{axa}, g_{bb}), \quad (124)$$
$$\overline{r}_a^c(\widehat{G}_a) \leq \min_{\ell\geq 0}[\max\{g_{bb} - \ell\delta, g_{ab}\} + \ell\delta\tau_b(g_{aa}, g_{bb}) - g_{bb}\tau_b(g_{aa}, g_{bb})], \quad (125)$$
$$\overline{r}_a^c(\widehat{G}_a) \leq g_{ab}, \quad (126)$$
$$\overline{r}_b^c(\widehat{G}_b) \leq g_{aa}\tau_b(g_{aa}, g_{bb}), \quad (127)$$
$$\overline{r}_b^c(\widehat{G}_b) \leq \min_{\ell\geq 0}[\max\{g_{ba} - \ell\delta, (g_{ba} - g_{aa})^+\} + \ell\delta\tau_b(g_{aa}, g_{bb})], \quad (128)$$
$$\overline{r}_a^c(\widehat{G}_a) + r_b(\widehat{G}_b) \leq \min_{\ell\geq 0}[\max\{g_{bb} - \ell\delta, g_{ab}\} + \ell\delta\tau_b(g_{aa}, g_{bb})], \quad (129)$$
$$\overline{r}_b^c(\widehat{G}_b) + r_a(\widehat{G}_a) \leq \max\{g_{ba}, g_{aa}\}, \quad (130)$$
$$\overline{r}_b^c(\widehat{G}_b) + r_b(\widehat{G}_b) \leq \min_{\ell\geq 0}[\max\{g_{ba} - (\ell+1)\delta, (g_{ba} - g_{aa})^+\} + (g_{aa} + \ell\delta)\tau_b(g_{aa}, g_{bb})], \quad (131)$$
$$\overline{r}_a^c(\widehat{G}_a) + r_b(\widehat{G}_b) - \overline{r}_b^c(\widehat{G}_b) \leq \min_{\ell\geq 0}[\max\{g_{ab}, g_{bb} - g_{ba} - \ell\delta\} + \ell\delta\tau_b(g_{aa}, g_{bb})], \quad (132)$$
$$\overline{r}_b^c(\widehat{G}_b) + r_a(\widehat{G}_a) - \overline{r}_a^c(\widehat{G}_a) \leq \min_{\ell\geq 0}[\max\{g_{ba} - \ell\delta, (g_{aa} - g_{ab})^+, g_{ba} - g_{ab}, g_{ba} - g_{aa}\} + \ell\delta\tau_b(g_{aa}, g_{bb})], \quad (133)$$
$$r_a(\widehat{G}_a) - \overline{r}_a^c(\widehat{G}_a) \leq \min_{\ell\geq 0}[\max\{(g_{aa} - g_{ab})^+, g_{bb} - \ell\delta\} - g_{bb}\tau_b(g_{aa}, g_{bb}) + \ell\delta\tau_b(g_{aa}, g_{bb})], \quad (134)$$
$$r_a(\widehat{G}_a) - \overline{r}_a^c(\widehat{G}_a) \leq (g_{aa} - g_{ab})^+, \quad (135)$$
$$r_b(\widehat{G}_b) - \overline{r}_b^c(\widehat{G}_b) \leq \min_{\ell\geq 0}[(g_{bb} - g_{ba} - \ell\delta)^+ + \ell\delta\tau_b(g_{aa}, g_{bb})]. \quad (136)$$

which implies that the two cases discussed are not only both constrained to a region where TDM is sufficient, but the operating points must be consistent, i.e.,

$$\begin{aligned} \frac{r_a(\widehat{G}_a)|_{g_{aa}=g_{ab}}}{g_{ab}} &= \frac{r_a(\widehat{G}_a)|_{g_{aa}=g_{ba}}}{g_{ba}} = \tau_a\left(g_{ab}, g_{ba}\right), \\ \frac{r_b(\widehat{G}_b)|_{g_{bb}=g_{ab}}}{g_{ab}} &= \frac{r_b(\widehat{G}_b)|_{g_{bb}=g_{ba}}}{g_{ba}} = \tau_b\left(g_{ab}, g_{ba}\right), \\ \tau_a\left(g_{ab}, g_{ba}\right) &+ \tau_b\left(g_{ab}, g_{ba}\right) = 1. \end{aligned}$$

For other cases of direct link gain, we assume the viewpoint of Transmitter a and examine its policy options. As we have shown, there must exist $\tau_a(g_{ab}, g_{ba})$ and $\tau_a(g_{ab}, g_{ba})$ summing to one, such that

$$\begin{aligned} r_a(\widehat{G}_a) &\geq g_{aa}\tau_a\left(g_{ab}, g_{ba}\right), \\ r_b(\widehat{G}_b) &\geq g_{bb}\tau_b\left(g_{ab}, g_{ba}\right). \end{aligned}$$

When $g_{aa} \notin \{g_{ab}, g_{ba}\}$, there still exists a possibility of the other direct link being fully interfering/interfered, $g_{bb} \in \{g_{ab}, g_{ba}\}$. Therefore, regardless of the known direct link, the channel input and decoding process must both accommodate the constraints imposed by these two limiting possibilities, resulting in the virtual Z-channel shown in Fig. 9 for Transmitter a's view.

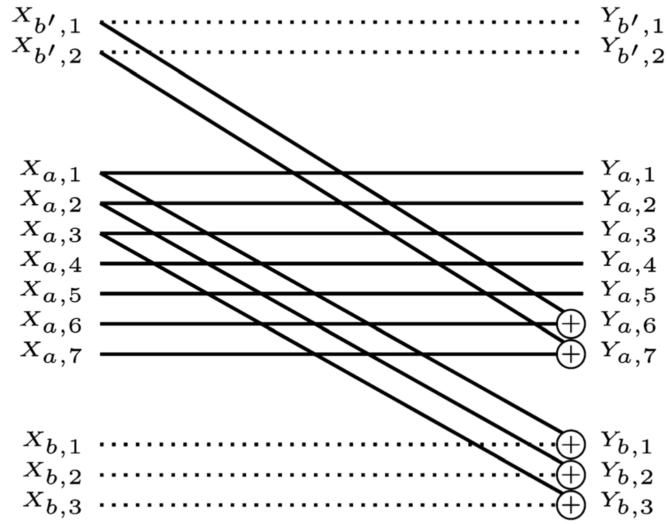


Fig. 9. Virtual double Z-channel for Transmitter a's in View 2. Dotted segments represent unknown link gains.

This provides us with three constraints related to the limiting cases, and one which is essentially the point-to-point capacity:

$$\begin{aligned} r_a(\widehat{G}_a) &\leq \frac{1}{n}\sum_{i=1}^{g_{aa}} L_{a,i}(\widehat{G}_a) \\ &\leq g_{aa}. \end{aligned} \tag{163}$$

$$\begin{aligned}
r_a^c(\widehat{G}'_a) &\leq g'_{ab}, && (139)\\
r_a^c(\widehat{G}'_a) &\leq \max\{g'_{ab}, g_{bb}-g_{ba}-\ell\delta\} + \ell\delta\tau_b(g_{aa}, g_{bb}) - r_b^p(\widehat{G}_b), && (140)\\
r_a^c(\widehat{G}'_a) &\leq \max\{g'_{ab}, g_{bb}+g_{ab}-g_{aa}-\ell\delta\} + \ell\delta\tau_b(g_{aa}, g_{bb}) + (g_{aa}-g_{ab})^+ - g_{bb}\tau_b(g_{aa}, g_{bb}) - r_a^p(\widehat{G}_a), && (141)\\
r_a^c(\widehat{G}'_a) &\leq \max\{g'_{ab}, g_{bb}-\ell\delta\} + \ell\delta\tau_b(g_{aa}, g_{bb}) - r_b(\widehat{G}_b), && (142)\\
r_a^c(\widehat{G}'_a) &\leq \max\{g'_{ab}, g_{bb}-\ell\delta\} + \ell\delta\tau_b(g_{aa}, g_{bb}) + g_{aa} - g_{bb}\tau_b(g_{aa}, g_{bb}) - r_a(\widehat{G}_a), && (143)\\
r_a^p(\widehat{G}'_a) &\leq (g_{aa}-g'_{ab})^+, && (144)\\
r_a^p(\widehat{G}'_a) &\leq \max\{g_{ba}-\ell\delta, g_{aa}-g'_{ab}\} + \ell\delta\tau_b(g_{aa}, g_{bb}) - r_b^c(\widehat{G}_b), && (145)\\
r_a^p(\widehat{G}'_a) &\leq (g_{aa}-g'_{ab})^+ - g_{bb}\tau_b(g_{aa}, g_{bb}) + \max\{g_{ab}, g_{bb}-(g_{aa}-g'_{ab}\}-\ell\delta) + \ell\delta\tau_b(g_{aa}, g_{bb}) - r_a^c(\widehat{G}_a), && (146)\\
r_a^p(\widehat{G}'_a) + r_a^c(\widehat{G}'_a) &\leq g_{aa}, && (147)\\
r_a^p(\widehat{G}'_a) + r_a^c(\widehat{G}'_a) &\leq \max\{g_{ba}-\ell\delta, g_{aa}\} + \ell\delta\tau_b(g_{aa}, g_{bb}) - r_b^c(\widehat{G}_b), && (148)\\
r_a^p(\widehat{G}'_a) + r_a^c(\widehat{G}'_a) &\leq c g_{ab} + g_{aa} - r_a^c(\widehat{G}_a) - g_{bb}\tau_b(g_{aa}, g_{bb}), && (149)\\
r_b^c(\widehat{G}'_a) &\leq g'_{ba}, && (150)\\
r_b^c(\widehat{G}'_a) &\leq \max\{g'_{ba}-\ell\delta, g_{aa}-g_{ab}\} + \ell\delta\tau_b(g_{aa}, g_{bb}) - r_a^p(\widehat{G}_a), && (151)\\
r_b^c(\widehat{G}'_a) &\leq \max\{g'_{ba}-\ell\delta, g_{aa}-(g_{bb}-g_{ba})\} + \ell\delta\tau_b(g_{aa}, g_{bb}) - g_{aa}\tau_a(g_{aa}, g_{bb}) + (g_{bb}-g_{ba})^+ - r_b^p(\widehat{G}_b), && (152)\\
r_b^c(\widehat{G}'_a) &\leq \max\{g'_{ba}-\ell\delta, g_{aa}\} + \ell\delta\tau_b(g_{aa}, g_{bb}) - r_a(\widehat{G}_a), && (153)\\
r_b^c(\widehat{G}'_a) &\leq \max\{g'_{ba}-\ell\delta, \delta\} + \ell\delta\tau_b(g_{aa}, g_{bb}) - g_{aa}\tau_a(g_{aa}, g_{bb}) + g_{bb} - r_b(\widehat{G}_b), && (154)\\
r_b^p(\widehat{G}'_a) &\leq (g_{bb}-g'_{ba})^+, && (155)\\
r_b^p(\widehat{G}'_b) &\leq \max\{g_{ab}, g_{bb}-g'_{ba}-\ell\delta\} + \ell\delta\tau_b(g_{aa}, g_{bb}) - r_a^c(\widehat{G}_a), && (156)\\
r_b^p(\widehat{G}'_b) &\leq \max\{g_{aa}-g_{ba}, g_{bb}-g'_{ba}\} - g_{aa}\tau_a(g_{aa}, g_{bb}) + g_{ba} - r_b^c(\widehat{G}_b), && (157)\\
r_b^p(\widehat{G}'_b) + r_b^c(\widehat{G}'_b) &\leq g_{bb}, && (158)\\
r_b^p(\widehat{G}'_b) + r_b^c(\widehat{G}'_b) &\leq \max\{g_{ab}, g_{bb}-\ell\delta\} + \ell\delta\tau_b(g_{aa}, g_{bb}) - r_a^c(\widehat{G}_a), && (159)\\
r_b^p(\widehat{G}'_b) + r_b^c(\widehat{G}'_b) &\leq \max\{g_{ba}+g_{bb}-\ell\delta, g_{aa}\} + \ell\delta\tau_b(g_{aa}, g_{bb}) - r_b^c(\widehat{G}_b) - g_{aa}\tau_a(g_{aa}, g_{bb}). && (160)
\end{aligned}$$

Of the remaining three bounds, we begin with the first with one resulting from an adaptation of (38) and isolating the lower of the two Z-channels depicted in the three-user Z-channel,

$$\begin{aligned}\sum_{i=1}^{g_{ab}} L_{a,i}(\widehat{G}_a) &\leq \sum_{j=1}^{g_{bb}-g_{ab}} L_{b,j}(\widehat{G}_b) + ng_{ab} - nr_b(\widehat{G}_b)\\ &\leq ng_{ab} - nr_b(\widehat{G}_b)|_{g_{bb}=g_{ab}}\\ &\leq ng_{ab}\tau_a\left(g_{ab}, g_{ba}\right),\end{aligned}$$

which we apply in

$$\begin{aligned}r_a(\widehat{G}_a) &\leq \frac{1}{n}\sum_{i=1}^{g_{aa}} L_{a,i}(\widehat{G}_a)\\ &= \frac{1}{n}\left(\sum_{i=1}^{g_{ab}} L_{a,i}(\widehat{G}_a) + \sum_{i=g_{ab}+1}^{g_{aa}} L_{a,i}(\widehat{G}_a)\right)\\ &\leq g_{ab}\tau_a\left(g_{ab}, g_{ba}\right) + \left(g_{aa} - g_{ab}\right)^+. \qquad (164)\end{aligned}$$

For the third bound on $r_a(\widehat{G}_a)$, we recall (7) and isolate only the upper of the two Z-channels:

$$\begin{aligned}&r_a(\widehat{G}_a)\\ &\leq \frac{1}{n}\left(\sum_{i=1}^{g_{aa}-g_{ba}} L_{a,i}(\widehat{G}_a) - \sum_{j=1}^{g_{ba}} L_{b,j}(\widehat{G}_b)\right) + g_{ba}\\ &\leq \frac{1}{n}\left(\sum_{i=1}^{g_{aa}-g_{ba}} l_{a,i}(\widehat{G}_a) + ng_{ba} - \sum_{j=1}^{g_{ba}} L_{b,j}(\widehat{G}_b)|_{g_{bb}=g_{ba}}\right)\\ &\leq \frac{1}{n}\left(\sum_{i=1}^{g_{aa}-g_{ba}} L_{a,i}(\widehat{G}_a) + ng_{ba} - nr_b(\widehat{G}_b)|_{g_{bb}=g_{ba}}\right)\\ &= \frac{1}{n}\left(\sum_{i=1}^{g_{aa}-g_{ba}} L_{a,i} + ng_{ba} - ng_{ba}\tau_b\left(g_{ab}, g_{ba}\right)\right) \qquad (165)\\ &\leq \left(g_{aa} - g_{ba}\right)^+ + g_{ba}\tau_a\left(g_{ab}, g_{ba}\right). \qquad (166)\end{aligned}$$

For the fourth bound, we also apply (7) but deviate at (165):

$$\begin{aligned}&r_a(\widehat{G}_a)\\ &\leq \frac{1}{n}\left(\sum_{i=1}^{g_{aa}-g_{ba}} L_{a,i} + ng_{ba} - ng_{ba}\tau_b\left(g_{ab}, g_{ba}\right)\right)\\ &\leq \frac{1}{n}\sum_{i=1}^{g_{aa}-g_{ba}} L_{a,i}(\widehat{G}_a) + g_{ba}\tau_a\left(g_{ab}, g_{ba}\right)\\ &\leq \frac{1}{n}\sum_{i=1}^{g_{ab}} L_{a,i} + \frac{1}{n}\sum_{i=g_{ab}+1}^{g_{aa}-g_{ba}} L_{a,i} + g_{ba}\tau_a(g_{ab}, g_{ba})\\ &\leq g_{ab}\tau_a\left(g_{ab}, g_{ba}\right) + \left(g_{aa} - g_{ab} - g_{ba}\right)^+ + g_{ba}\tau_a\left(g_{ab}, g_{ba}\right). \qquad (167)\end{aligned}$$

In summary, we have for User a:

$$r_a(\widehat{G}_a) \leq g_{aa}, \qquad (168)$$

$$r_a(\widehat{G}_a) \leq \left(g_{aa} - g_{ab}\right)^+ + g_{ab}\tau_a\left(g_{ab}, g_{ba}\right), \qquad (169)$$

$$r_a(\widehat{G}_a) \leq \left(g_{aa} - g_{ba}\right)^+ + g_{ba}\tau_a\left(g_{ab}, g_{ba}\right), \qquad (170)$$

$$r_a(\widehat{G}_a) \leq \left(g_{aa} - g_{ab} - g_{ba}\right)^+ + \left(g_{ab} + g_{ba}\right)\tau_a\left(g_{ab}, g_{ba}\right). \qquad (171)$$

The analogous bounds on the rate chosen by Sender b follow the same process.

Achievable Scheme: The scheme used in this scenario is the linear deterministic model version of the simple HK scheme proposed in [6], and described in Section IV-A2.

We generate common and private codebooks using random codes. For the common message of Sender a, let the $\nu_a = \min\{g_{aa}, g_{ab}\}$. We choose $n\nu_a r_a^c(\widehat{G}_a)$ codewords randomly from the set of all $n \times \nu_a$ binary vectors using a uniform distribution over the set. If $g_{ab} < g_{aa}$, we also choose $n\nu_a r_a^p(\widehat{G}_a)$ codewords randomly from the set of all $n \times (g_{aa} - g_{ab})$ binary vectors again using a uniform distribution. At Sender b, we do the same for $n\nu_b = \min\{g_{bb}, g_{ba}\}$, $r_b^c(\widehat{G}_b)$ and $r_b^c(\widehat{G}_b)$.

The set of decodable rates $r_a^c(\widehat{G}_a)$, $r_a^p(\widehat{G}_a)$, $r_b^c(\widehat{G}_b)$, and $r_b^c(\widehat{G}_b)$ at Receiver a is given by (16)–(27). Since it is necessary for Sender a to know the rate of Sender b's common message in order to determine limits on its own common and private rates, we impose the constraints

$$r_a^c(\widehat{G}_a) \leq g_{ab}\tau_a\left(g_{ab}, g_{ba}\right), \qquad (172)$$

$$r_b^c(\widehat{G}_b) \leq g_{ba}\tau_b\left(g_{ab}, g_{ba}\right), \qquad (173)$$

chosen based on our understanding of the two limiting cases in the outer bound. Furthermore, we note that in order to satisfy the minimum performance criterion

$$r_a^c(\widehat{G}_a)|_{g_{aa}=g_{ab}} = r_a(\widehat{G}_a)|_{g_{aa}=g_{ab}} = g_{ab}\tau_a\left(g_{ab}, g_{ba}\right), \qquad (174)$$

$$r_b^c(\widehat{G}_b)|_{g_{bb}=g_{ba}} = r_b(\widehat{G}_a)|_{g_{bb}=g_{ba}} = g_{ba}\tau_b\left(g_{ab}, g_{ba}\right). \qquad (175)$$

The resulting region of rates achievable for the common and private messages of a are

$$r_a^c(\widehat{G}_a) \leq \min\left\{g_{aa}, g_{ab}\right\}, \qquad (176)$$

$$r_a^p(\widehat{G}_a) \leq \left(g_{aa} - g_{ab}\right)^+, \qquad (177)$$

$$r_a^c(\widehat{G}_a) + r_a^p(\widehat{G}_a) \leq g_{aa}, \qquad (178)$$

$$\begin{aligned}r_a^c(\widehat{G}_a) &\leq \max\left\{g_{aa}, g_{ba}\right\} - r_b^c(\widehat{G}_b)\\ &\leq \max\left\{g_{aa}, g_{ba}\right\} - g_{ba}\tau_b\left(g_{ab}, g_{ba}\right)\\ &= \left(g_{aa} - g_{ba}\right)^+ + g_{ba}\tau_a\left(g_{ab}, g_{ba}\right), \qquad (179)\end{aligned}$$

$$\begin{aligned}r_a^p(\widehat{G}_a) &\leq \max\left\{g_{aa} - g_{ab}, g_{ba}\right\} - r_b^c(\widehat{G}_b)\\ &\leq \max\left\{g_{aa} - g_{ab}, g_{ba}\right\} - g_{ba}\tau_b\left(g_{ab}, g_{ba}\right)\\ &= \left(g_{aa} - g_{ab} - g_{ba}\right)^+ + g_{ba}\tau_a\left(g_{ab}, g_{ba}\right), \qquad (180)\end{aligned}$$

$$\begin{aligned}r_a^c(\widehat{G}_a) + r_a^p(\widehat{G}_a) &\leq \max\left\{g_{aa}, g_{ba}\right\} - r_b^c(\widehat{G}_b)\\ &\leq \max\left\{g_{aa}, g_{ba}\right\} - g_{ba}\tau_b\left(g_{ab}, g_{ba}\right)\\ &= \left(g_{aa} - g_{ba}\right)^+ + g_{ba}\tau_a\left(g_{ab}, g_{ba}\right). \qquad (181)\end{aligned}$$

Which, when simplified under the assumption $r_a(\widehat{G}_a) = r_a^c(\widehat{G}_a) + r_a^p(\widehat{G}_a)$, corresponds with the outer bounds of (168)–(171). Similar analysis of Sender b's scheme yields the analogous result.

D) Proof for Views 3 and 5: As in View 1, by definition of the problem and noting the knowledge common to both transmitters, the following must hold for some $\tau_a(g_{aa}, g_{bb})$, $\tau_b(g_{aa}, g_{bb})$ summing to one:

$$r_a(\widehat{G}_a) \geq g_{aa}\tau_a(g_{aa}, g_{bb}),, \tag{182}$$

$$r_b(\widehat{G}_b) \geq g_{bb}\tau_b(g_{aa}, g_{bb}). \tag{183}$$

Our proof relies upon analysis of virtual Z-channels that provide structure to the uncertainty of each transmitter.

Let us first study the POV of Transmitter a. Recalling the assumption that $g_{aa} \geq g_{bb}$, we consider all possible weak interference gain values for Transmitter a's out-going interference: $g_{ab} \in \{0, 1, \ldots, g_{aa}\}$. At Receiver b, the achievability of desired rates $r_b(\widehat{G}_b)$ is dependent on the following conditions:

$$r_b(\widehat{G}_b)|_{g_{ab}=0} \leq \frac{1}{n}\sum_{j=1}^{g_{bb}} L_{b,j}(\widehat{G}_b)|_{g_{ab}=0}, \tag{184}$$

$$r_b(\widehat{G}_b)|_{g_{ab}=1} \leq \frac{1}{n}\Big(\sum_{j=1}^{g_{bb}-1} L_{b,j}(\widehat{G}_b)|_{g_{ba}=1} + n - L_{a,1}(\widehat{G}_a)\Big), \tag{185}$$

$$\vdots$$

$$r_b(\widehat{G}_b)|_{g_{ab}=g_{aa}-1} \leq \frac{1}{n}\left(ng_{bb} - \sum_{i=g_{aa}-g_{bb}}^{g_{aa}-1} L_{a,i}(\widehat{G}_a)\right), \tag{186}$$

$$r_b(\widehat{G}_b)|_{g_{ab}=g_{aa}} \leq \frac{1}{n}\left(ng_{bb} - \sum_{i=g_{aa}-g_{bb}+1}^{g_{aa}-1} L_{a,i}(\widehat{G}_a)\right). \tag{187}$$

Combining (184)–(187) with expression (183) implies more generally

$$\begin{aligned}\sum_{i=\kappa+1}^{\kappa+g_{bb}} L_{a,i}(\widehat{G}_a) &\leq n\left(g_{bb} - g_{bb}\tau_b(g_{aa}, g_{bb})\right)\\ &= n\left(g_{bb}\tau_a(g_{aa}, g_{bb})\right),\end{aligned} \tag{188}$$

for $\kappa \in \{0, \ldots, g_{aa} - g_{bb}\}$; i.e., any g_{bb} successive signal levels of Transmitter a's input are constrained to a TDM-like rate. Notice that if g_{aa} is a multiple of g_{bb}, we can select disjoint sets of successive signal level that span Transmitter a's input, thus completing the proof.

On the other hand, if g_{aa} is not evenly divisible by g_{bb}, we construct a virtual Z-channel with the actual Transmitter a as the initial interferer (the top link in Fig. 10(b)). Notice that in doing so, we neglect the incoming interference link gain g_{ab}; however, this can be rationalized as a genie providing the interference signal to Receiver a. Moreover, we will demonstrate that it is in fact the objective of not inhibiting the transmission of the other link that provides the active constraint on Transmitter a's input.

Let $\theta_0 = g_{aa} \bmod g_{bb}$, and notice that $\theta_0 < g_{bb} \leq g_{aa}$. For example, in Fig. 10(b) $\theta_0 = 1$. By properly selecting inequalities of the form (188) and also including the bound from (184)–(187) for $r_b(\widehat{G}_b)|_{g_{ab}=\theta_0}$, we have

$$\begin{aligned}nr_a(\widehat{G}_a) &\leq \sum_{i=1}^{g_{aa}} L_{a,i}(\widehat{G}_a)\\ &\leq n\left(g_{aa} - \theta_0\right)\tau_a\left(g_{aa}, g_{bb}\right) + \sum_{i=1}^{\theta_0} L_{a,i}(\widehat{G}_a)\end{aligned}$$

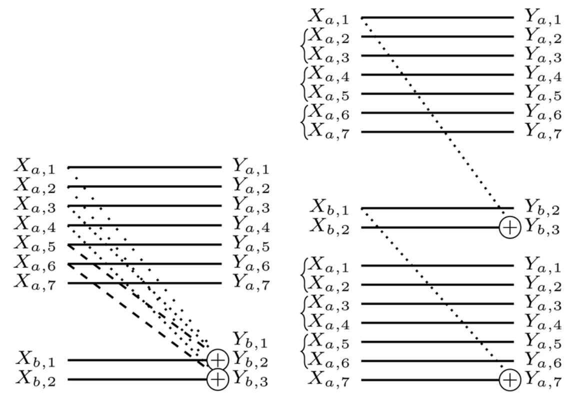


Fig. 10. Virtual Z-channels used to derive outer bound for View 3: (a) Some of the two-user virtual Z channels used to derive (184)–(187), and (b) larger virtual Z to bound relatively prime direct link channels. Bracketed inputs are bounded by constraints from (184)–(187) and other inputs are constrained by potential interference interactions.

$$\begin{aligned}&\leq n\left(g_{aa} - \theta_0\right)\tau_a\left(g_{aa}, g_{bb}\right)\\ &\quad + \sum_{j=1}^{g_{bb}-\theta_0} L_{b,j}(\widehat{G}_b)|_{g_{ab}=\theta_0}\\ &\quad + n\theta_0 - ng_{bb}\tau_b\left(g_{aa}, g_{bb}\right).\end{aligned} \tag{189}$$

In our virtual Z-channel, we now have a virtual b link where $g_{ba} = \theta_0$. We must now address the constraints on the un-interfered signal levels of the virtual link, $\sum_{j=1}^{g_{bb}-\theta_0} L_{b,j}(\widehat{G}_b)|_{g_{ab}=\theta_0}$. We bound the summation over j using a bound adapted from (37) coupled again with bounds of the form (188) and arrive at

$$\begin{aligned}&\sum_{j=1}^{g_{bb}-\theta_0} L_{b,j}(\widehat{G}_b)|_{g_{ab}=\theta_0}\\ &\leq \sum_{i=1}^{g_{aa}-(g_{bb}-\theta_0)} L_{a,i}(\widehat{G}_a)|_{g_{ba}=g_{bb}-\theta_0} + n\left(g_{bb} - \theta_0\right)\\ &\quad - nr_a(\widehat{G}_a)|_{g_{ba}=g_{bb}-\theta_0}\\ &\leq \sum_{i=1}^{g_{aa}-(g_{bb}-\theta_0)} L_{a,i}(\widehat{G}_a)|_{g_{ba}=g_{bb}-\theta_0} + n\left(g_{bb} - \theta_0\right)\\ &\quad - ng_{aa}\tau_a\left(g_{aa}, g_{bb}\right)\\ &\leq \sum_{i=1}^{\theta_1} L_{a,i}(\widehat{G}_a)|_{g_{ba}=g_{bb}-\theta_0}\\ &\quad + n\left(g_{aa} - (g_{bb} - \theta_0) - \theta_1\right)\tau_a\left(g_{aa}, g_{bb}\right)\\ &\quad + n\left(g_{bb} - \theta_0\right) - ng_{aa}\tau_a\left(g_{aa}, g_{bb}\right)\\ &= \sum_{i=1}^{\theta_1} L_{a,i}(\widehat{G}_a)|_{g_{ba}=g_{bb}-\theta_0} - n\theta_1\tau_a\left(g_{aa}, g_{bb}\right)\\ &\quad + n\left(g_{bb} - \theta_0\right)\tau_b\left(g_{aa}, g_{bb}\right),\end{aligned} \tag{190}$$

where

$$\begin{aligned}\theta_1 &= \left(g_{aa} - (g_{bb} - \theta_0)\right) \bmod g_{bb}\\ &= \left(g_{aa} + \theta_0\right) \bmod g_{bb}.\end{aligned}$$

If $\theta_1 = 0$, then we arrive a scenario like that in Fig. 10(b), where the second Link a (first virtual Link a) has a number of noninterfered signal levels that is evenly divisible by g_{bb}. If this is not the case, we may continue the growth of the virtual Z-channel and arrive at a bound on the remaining levels of the virtual Link a:

$$\begin{aligned}\sum_{i=1}^{\theta_1} & L_{a,i}(\widehat{G}_a)|_{g_{ba}=g_{bb}-\theta_0} \\ &\le \sum_{j=1}^{g_{bb}-\theta_1} L_{b,j}(\widehat{G}_b)|_{g_{ab}=\theta_1} + n\theta_1 - ng_{bb}\tau_b\left(g_{aa},g_{bb}\right) \\ &\le \sum_{j=1}^{g_{bb}-\theta_1} L_{b,j}(\widehat{G}_b)|_{g_{ab}=\theta_1} - n\left(g_{bb}-\theta_1\right)\tau_b\left(g_{aa},g_{bb}\right) \\ &\quad + n\theta_1\tau_a\left(g_{aa},g_{bb}\right), \end{aligned} \tag{191}$$

and

$$\begin{aligned}\sum_{j=1}^{g_{bb}-\theta_2} & L_{b,j}(\widehat{G}_b)|_{g_{ab}=\theta_1} \\ &\le \sum_{i=1}^{\theta_2} L_{a,i}(\widehat{G}_a)|_{g_{ba}=g_{bb}-\theta_1} - n\theta_2\tau_a\left(g_{aa},g_{bb}\right) \\ &\quad + n\left(g_{bb}-\theta_2\right)\tau_b\left(g_{aa},g_{bb}\right), \end{aligned} \tag{192}$$

where

$$\theta_2 = \left(g_{aa}+\theta_1\right) \bmod g_{bb}.$$

Though this process may seem cyclic, we note that

$$\theta_\ell = \left(g_{aa}+\ell\theta_0\right) \bmod g_{bb},$$

and that there exists some value for ℓ such that $\theta_\ell = 0$. When this is the case

$$\begin{aligned}\sum_{j=1}^{g_{bb}-\theta_{\ell-1}} & L_{b,j}(\widehat{G}_b)|_{g_{ab}=\theta_{\ell-1}} \\ &\le n\theta_\ell\tau_a\left(g_{aa},g_{bb}\right) - n\theta_\ell\tau_a\left(g_{aa},g_{bb}\right) \\ &\quad + n\left(g_{bb}-\theta_\ell\right)\tau_b\left(g_{aa},g_{bb}\right) \\ &= n\left(g_{bb}-\theta_{\ell-1}\right)\tau_b\left(g_{aa},g_{bb}\right), \end{aligned}$$

and

$$\sum_{i=1}^{\theta_1} L_{a,i}(\widehat{G}_a)|_{g_{ba}=g_{bb}-\theta_0} \le n\theta_{\ell-1}\tau_a\left(g_{aa},g_{bb}\right).$$

Carrying this process down to $\ell = 0$ and substituting into (189) yields

$$nr_a(\widehat{G}_a) \le ng_{aa}\tau_a\left(g_{aa},g_{bb}\right) \tag{193}$$

as desired.

To show that the input at Transmitter b is also constrained to its TDM allotted rate requires a single extension to the previously constructed Z-channel. If we now assume that the top link is a b-link and consider the possibility of $g_{ba} = g_{bb}$, then achievability of the TDM-like rate is reliant on

$$\begin{aligned} nr_b(\widehat{G}_b) &\le \sum_{j=1}^{g_{bb}} L_{b,j}(\widehat{G}_b) \\ &\le \sum_{i=1}^{g_{aa}-g_{bb}} L_{a,i}(\widehat{G}_a)|_{g_{ba}=g_{bb}} + ng_{bb} - nr_a(\widehat{G}_a)|_{g_{ba}=g_{bb}} \\ &\le n\left[\left(g_{aa}-g_{bb}\right)\tau_a\left(g_{aa},g_{bb}\right) + g_{bb} - r_a(\widehat{G}_a)|_{g_{ba}=g_{bb}}\right] \\ &\le n\left[\left(g_{aa}-g_{bb}\right)\tau_a\left(g_{aa},g_{bb}\right) + g_{bb} - g_{aa}\tau_a\left(g_{aa},g_{bb}\right)\right] \\ &= n\left[g_{bb} - g_{bb}\tau_a\left(g_{aa},g_{bb}\right)\right] \\ &= ng_{bb}\tau_b\left(g_{aa},g_{bb}\right). \end{aligned} \tag{194}$$

The statement for View 5 is a direct result of the result from View 3. In designing a policy let it be assumed that a genie will provide Transmitter a with knowledge of g_{ba}, and Transmitter b with knowledge of g_{ab}. The resulting genie-aided view is exactly the same as View 1, and thus the result that the capacity region is confined to that of TDM also holds.

E) Proofs for Views 4, 6, and 7: We prove the results for Views 4, 6, and 7 by applying the results and intuitions gained from View 2. Whereas View 2 had two bottleneck cases, to prove the statement for Views 4, 6, and 7, we only require one worst case potential channel state for each: the case where the unknown links form a fully contested Z-channel. In the case of View 4 at Sender a, we apply the possibility that

$$g_{ab} = g_{bb} = g_{aa},$$

which requires

$$r_a(\widehat{G}_a) + r_b(\widehat{G}_b)|_{g_{ab}=g_{bb}=g_{aa}} \le g_{aa}.$$

Notice that in order to satisfy the minimum performance criterion, this inequality must be an equality which also implies

$$\frac{r_a(\widehat{G}_a)}{g_{aa}} \le 1 - \frac{r_b(\widehat{G}_b)|_{g_{ab}=g_{bb}=g_{aa}}}{g_{aa}}.$$

We define

$$\begin{aligned}\tau_a^{g_{aa}} &\triangleq \frac{r_a(\widehat{G}_a)}{g_{aa}}, \\ \tau_b^{g_{aa}} &\triangleq \frac{r_b(\widehat{G}'_b)|_{g'_{ab}=g'_{bb}=g_{aa}}}{g_{aa}}, \end{aligned}$$

where $\tau_a^{g_{aa}} + \tau_b^{g_{aa}} = 1$, and now consider the response of Sender b to its view of the channel. By analyzing an analogous Z-channel (where the direct link is fully interfered), we have

$$\begin{aligned}\tau_a^{g_{bb}} &\triangleq \frac{r_a(\widehat{G}'_a)|_{g'_{ba}=g'_{aa}=g_{bb}}}{g_{bb}}, \\ \tau_b^{g_{bb}} &\triangleq \frac{r_b(\widehat{G}_b)}{g_{bb}}, \end{aligned}$$

where $\tau_a^{g_{bb}} + \tau_b^{g_{bb}} = 1$. The final step to completing the proof is to note that the minimum performance criterion $\tau_a^{g_{aa}} + \tau_b^{g_{bb}} \geq 1$, for all G, requires that the inequality be an equality. Therefore, the View 4 region is exactly that of TDM.

To demonstrate the theorem for Views 6 and 7, we need only analyze the proper worst case Z-channels and apply the same logic.

F) Gap Between LDIC and GIC Capacity Regions: We use heavily the result (79) from Section VI-B. Additionally, we make the following observation:

$$\sum_{\ell_1}^{\ell_2} \Lambda_a(\widehat{H}_a) - (\ell_2 - \ell_1)^+ \leq \log(3). \tag{195}$$

7) View 1: If $h_{aa} = h_{bb}$, then we refer the reader to the gap analysis for View 4, for which the worst case channel state assumes $h_{bb} = h_{aa}$. Otherwise, bounding the gap Δ_1 proceeds as follows.

By Lemma 3, a Gaussian policy based on HK schemes that uses rates prescribed by the associated LDIC policy less 4 bits per user is achievable.

In order to bound the gap between the LDIC and GIC outer bounds, we manipulate expression (41) while noting (79) and arrive at a bound on the interference component of Transmitter a's signal

$$\begin{aligned}
&\sum_{\ell=1}^{g_{ab}} \Lambda_{a,\ell}(\widehat{H}_a) \\
&\leq \left(h(Y_b^n | W_{bb,u_b^+}, W_{ab,u_b^-}) - n\log(2\pi e) - I(X_b^n; Y_b^n)\right) \\
&\quad + \left(\sum_{\ell=1}^{u_b^+} \Lambda_{b,\ell}(\widehat{H}_b') + \sum_{\ell=1}^{u_b^-} \Lambda_{a,\ell}\right) \\
&\leq n\min\{g_{bb}, g_{ab}\} + n\log(6) - nr_b(\widehat{H}_b') \\
&\quad + \sum_{\ell=1}^{u_b^+} \Lambda_{b,\ell}(\widehat{H}_b') + \sum_{\ell=1}^{u_b^-} \Lambda_{a,\ell}(\widehat{H}_a).
\end{aligned} \tag{196}$$

Similarly, a constraint on the amount of interference in Transmitter b's signal is given by

$$\begin{aligned}
\sum_{\ell=1}^{g_{ba}} \Lambda_{b,\ell}(\widehat{H}_b) &\leq n\min\{g_{aa}, g_{ba}\} + n\log(6) - nr_a(\widehat{H}_a') \\
&\quad + \sum_{\ell=1}^{u_a^+} \Lambda_{a,\ell}(\widehat{H}_a') + \sum_{\ell=1}^{u_a^-} \Lambda_{b,\ell}(\widehat{H}_b).
\end{aligned} \tag{197}$$

In Appendix B, expressions analogous to (196) and (196) [namely (94) and (96)] were used to add virtual users representing different policy responses to a virtual Z-channel in constructing an outer bound for View 1 of the LDIC. As alluded to in Section VI, each additional interference event considered (each additional virtual user added to the virtual Z-channel) increases the gap between the LDIC and GIC outer bounds by $\log(6)$.

TABLE II
GAP BETWEEN COMPONENT BOUNDS (123)–(136) AND GAUSSIAN COUNTERPARTS

Eq. #	Component	Gap	Max ℓ
(123)	r_a	$\log(3) + \log(6)$	—
(124)	r_b	$\log(3) + \log(6)$	—
(125)	$\overline{r}_a^c$	$\log(3) + (2\ell+1)\log(6)$	$\lceil \frac{g_{bb}}{\delta} \rceil$
(126)	$\overline{r}_a^c$	$\log(3)$	—
(127)	$\overline{r}_b^c$	$\log(3) + \log(6)$	—
(128)	$\overline{r}_b^c$	$\log(3) + (2\ell+1)\log(6)$	$\lceil \frac{g_{ba}}{\delta} \rceil$
(129)	$\overline{r}_a^c + r_b$	$\log(3) + (2\ell+1)\log(6)$	$\lceil \frac{g_{bb}}{\delta} \rceil$
(130)	$\overline{r}_b^c + r_a$	$\log(3) + \log(6)$	—
(131)	$\overline{r}_b^c + r_b$	$\log(3) + (2\ell)\log(6)$	$\lceil \frac{g_{ba}}{\delta} \rceil - 1$
(132)	$\overline{r}_a^c + r_b - \overline{r}_b^c$	$(2\ell+1)\log(6)$	$\lceil \frac{(g_{bb}-g_{ba})^+}{\delta} \rceil$
(133)	$\overline{r}_b^c + r_a - \overline{r}_a^c$	$(2\ell+1)\log(6)$	$\lceil \frac{g_{ba}}{\delta} \rceil$
(134)	$r_a - \overline{r}_a^c$	$\log(3) + (2\ell+1)\log(6)$	$\lceil \frac{g_{bb}}{\delta} \rceil$
(135)	$r_a - \overline{r}_a^c$	$\log(3) + \log(6)$	—
(136)	$r_b - \overline{r}_b^c$	$\log(3) + (2\ell-1)\log(6)$	$\lceil \frac{(g_{bb}-g_{ba})^+}{\delta} \rceil$

Adding additional virtual users accounts for any remaining sum of uninterfered signal levels (e.g., $\sum_{\ell=1}^{u_a^+} \Lambda_{a,\ell}(\widehat{H}_a')$ in (197)). When a virtual user is not added (when the Z channel terminates), an additional gap between the Gaussian outer bound and its linear deterministic equivalent results from the quantization of the channel gain. For example

$$\sum_{\ell=1}^{u_a^+} \Lambda_{a,\ell}(\widehat{H}_a') \leq \max_{p(X_a^n)} \sum_{\ell=1}^{u_a^+} \Lambda_{a,\ell}(\widehat{H}_a') \leq nu_a^+ + \log(3). \tag{198}$$

In summary, the outer bounds constructed for View 1 in the GIC and LDIC have a gap that increases by $\log(6)$ per interferer considered, and by $\log(3)$ at the end of the Z chain (component bounds that are nonterminating lack the $\log(3)$ gap). We find gaps between the component bounds (42)–(55) and their Gaussian IC equivalents using this result, and detail them in Table II.

From Table II, and referring to (123)–(136), we compute per-user gaps by combining the respective component bound gaps of Table II, note the potential gap in achievable policy rates, and arrive at the bound in Table I.

8) View 2: By Lemma 3, a Gaussian policy based on HK schemes that uses rates prescribed by the associated LDIC policy, less 4 bits per user, is achievable. As in the LDIC version, two interference cases suffice to define a set of outer bounds for the GIC. WLOG, we analyze Transmitter a's response and mimic the derivation of (56)–(59). Following the derivation of (56), we have

$$\begin{aligned}
nr_a(\widehat{H}_a) &\leq I(X_a^n; Y_a^n) \\
&\leq I(X_a^n; Y_a^n | X_b^n) \\
&\leq \sum_{i=1}^{g_{aa}} \Lambda_a(\widehat{G}_a) \\
&\leq n\log(1 + 2^{g_{aa}+1}) \\
&\leq ng_{aa} + n\log(3).
\end{aligned} \tag{199}$$

From the derivation of (57), we have

$$\begin{aligned}
nr_a(\widehat{H}_a) &\le I(X_a^n;Y_a^n)\\
&\le h(Y_a^n|W_{aa,u_a^+},W_{ba,u_a^-}) - n\log(2\pi e) - \sum_{\ell=1}^{g_{ba}}\Lambda_{b,\ell}(\widehat{H}_b)\\
&\quad + \sum_{\ell=1}^{u_a^+}\Lambda_{a,\ell}(\widehat{H}_a) + \sum_{\ell=1}^{u_a^-}\Lambda_{b,\ell}(\widehat{H}_b)\\
&\le h(Y_a^n|W_{aa,u_a^+},W_{ba,u_a^-}) - n\log(2\pi e) - nr_b(\widehat{H}_b)|_{h_{bb}=h_{ba}}\\
&\quad + \sum_{\ell=1}^{u_a^+}\Lambda_{a,\ell}(\widehat{H}_a) + \sum_{\ell=1}^{u_a^-}\Lambda_{b,\ell}(\widehat{H}_b)\\
&\le h(Y_a^n|W_{aa,u_a^+},W_{ba,u_a^-}) - n\log(2\pi e) - ng_{ba}\tau_b\left(h_{ab},h_{ba}\right)\\
&\quad + \sum_{\ell=1}^{u_a^+}\Lambda_{a,\ell}(\widehat{H}_a) + \sum_{\ell=1}^{u_a^-}\Lambda_{b,\ell}(\widehat{H}_b)\\
&\le n\log(6) + n\min\{g_{aa},g_{ba}\} - ng_{ba}\tau_b\left(h_{ab},h_{ba}\right)\\
&\quad + nu_a^+ + nu_a^- + n\log(3)\\
&\le n\left[g_{ba}\tau_a\left(h_{ab},h_{ba}\right) + (g_{aa}-g_{ba})^+ + \log(6) + \log(3)\right].
\end{aligned} \tag{200}$$

If we analyze Transmitter a's potential impact on Link b, we have

$$\begin{aligned}
nr_b(\widehat{H}_b)|_{h_{bb}=h_{ab}} &\le I(X_b^n;Y_b^n)\\
&\le h(Y_b^n|W_{bb,u_b^+},W_{ab,u_b^-}) - n\log(2\pi e) - \sum_{\ell=1}^{g_{ab}}\Lambda_{a,\ell}(\widehat{H}_a)\\
&\quad + \sum_{\ell=1}^{u_b^+}\Lambda_{b,\ell}(\widehat{H}_b) + \sum_{\ell=1}^{u_b^-}\Lambda_{a,\ell}(\widehat{H}_a)\\
&\le ng_{ab} - \sum_{\ell=1}^{g_{ab}}\Lambda_{a,\ell}(\widehat{H}_a) + n\log(6),
\end{aligned}$$

or

$$\begin{aligned}
\sum_{\ell=1}^{g_{ab}}\Lambda_{a,\ell}(\widehat{H}_a) &\le ng_{ab} - nr_b(\widehat{H}_b)|_{h_{bb}=h_{ab}} + n\log(6)\\
&\le n\left[g_{ab}\tau_a\left(h_{ab},h_{ba}\right) + \log(6)\right],
\end{aligned}$$

which gives us

$$\begin{aligned}
nr_a(\widehat{H}_a) &\le \max I(X_a^n;Y_a^n)\\
&\le \max I(X_a^n;Y_a^n|X_b^n)\\
&\le \max\sum_{i=1}^{g_{aa}}\Lambda_a(\widehat{H}_a)\\
&\le \max\sum_{i=1}^{g_{ab}}\Lambda_a(\widehat{H}_a) + \sum_{i=g_{ab}+1}^{g_{aa}}\Lambda_a(\widehat{H}_a)\\
&\le n\left[g_{ab}\tau_a\left(h_{ab},h_{ba}\right) + \log(6) + (g_{aa}-g_{ab})^+ + \log(3)\right],
\end{aligned} \tag{201}$$

and

$$\begin{aligned}
nr_a(\widehat{H}_a) &\le \max I(X_a^n;Y_a^n)\\
&\le \max I(X_a^n;Y_a^n|X_b^n)\\
&\le \max\sum_{i=1}^{g_{aa}}\Lambda_a(\widehat{H}_a)\\
&\le \max\sum_{i=1}^{g_{ab}}\Lambda_a(\widehat{H}_a) + \sum_{i=g_{ab}+1}^{g_{aa}}\Lambda_a(\widehat{H}_a)\\
&\le \max ng_{ab}\tau_a\left(h_{ab},h_{ba}\right) + n\log(6) + \sum_{i=g_{ab}+1}^{g_{aa}-g_{ba}}\Lambda_a(\widehat{H}_a)\\
&\quad + \sum_{i=g_{aa}-g_{ba}+1}^{g_{aa}}\Lambda_a(\widehat{H}_a),\\
&\le n\big[g_{ab}\tau_a\left(h_{ab},h_{ba}\right) + \log(6) + (g_{aa}-g_{ab}-g_{ba})^+\\
&\quad + \log(3) + g_{ba}\tau_a\left(h_{ab},h_{ba}\right) + \log(6)\big].
\end{aligned} \tag{202}$$

Consequently, we bound the Link a capacity gap (and Link b by parallel analysis) comparing (199)–(202) to (56)–(59), and arrive at the stated bound.

9) Views 3 and 5: Proof of Views 3 and 5 relies on bounding disjoint sets of g_{bb} consecutive levels of Transmitter a's input with the expression (188). In the Gaussian IC version, by applying (79), we have

$$\sum_{i=\kappa+1}^{\kappa+g_{bb}}\Lambda_{a,i}(\widehat{H}_a) \le n\left(g_{bb}\tau_a(h_{aa},h_{bb}) + \log(6)\right).$$

To bound the total per-user gap to the Gaussian IC capacity region, we track how the gap accumulates in the construction of the virtual Z-channel used in the linear deterministic proof. Let $\tilde{\Delta}_3[\ell]$ be the total gap after ℓ virtual Link a s have been addressed. When $\ell = 1$, we find

$$\tilde{\Delta}_3[1] = \left(\frac{g_{aa}-\theta_0}{g_{bb}}\right)\log(6). \tag{203}$$

Recall that if $\theta_0 = 0$, then g_{aa} was evenly divisible by g_{bb} and the proof ends. If $\theta_0 \neq 0$, we can find through induction

$$\begin{aligned}
\tilde{\Delta}_3[\ell] &= \left(\tilde{\Delta}_3[\ell-1]+2+\frac{g_{aa}-\theta_\ell-(g_{bb}-\theta_\ell-1)}{g_{bb}}\right)\log(6)\\
&= \left(\frac{\ell g_{aa}-\theta_{\ell-1}}{g_{bb}}+\ell-1\right)\log(6).
\end{aligned} \tag{204}$$

When $\theta_{\ell-1} = 0$, the chain of substitutions in the proof of Theorem 7 ends, implying ℓg_{aa} is evenly divisible by g_{bb}. Consequently, if we let $\ell^\star$ be the minimum value of ℓ where this occurs, $\ell^\star g_{aa}$ is the least common multiple of g_{aa} and g_{bb}, and it becomes clear that

$$\begin{aligned}
\Delta_3 &= \left(\frac{\ell^\star g_{aa}}{g_{bb}}+\ell^\star-1\right)\log(6)\\
&= \left(\frac{\ell^\star g_{aa}}{g_{bb}}+\frac{\ell^\star g_{aa}}{g_{aa}}-1\right)\log(6)\\
&= \left(\frac{\mathrm{LCM}(g_{aa},g_{bb})}{g_{bb}}+\frac{\mathrm{LCM}(g_{aa},g_{bb})}{g_{aa}}-1\right)\log(6).
\end{aligned} \tag{205}$$

10) Views 4, 6, and 7: For View 4, when Transmitter a considers the case $h_{ba} = h_{bb} = h_{aa}$

$$\begin{aligned} nr_a(\widehat{H}_a) &\le I(X_a^n; Y_a^n) \\ &\le h(Y_a^n | W_{aa,u_a^+}, W_{ba,u_a^-}) - n\log(2\pi e) - \sum_{\ell=1}^{g_{ba}} \Lambda_{b,\ell}(\widehat{H}_b) \\ &\quad + \sum_{\ell=1}^{u_a^+} \Lambda_{a,\ell}(\widehat{H}_a) + \sum_{\ell=1}^{u_a^-} \Lambda_{b,\ell}(\widehat{H}_b) \\ &\le n\log(6) + n\min\{g_{aa}, g_{ba}\} - nr_b(\widehat{H}_b)|_{h_{bb}=h_{aa}} \\ &\le (n\log(6) + ng_{aa} - ng_{aa}\tau_b) \\ &\le n\left[g_{aa}\tau_a + \log(6)\right]. \end{aligned} \tag{206}$$

For View 6, from the case $g_{ab} = g_{bb} = g_{aa}$

$$\begin{aligned} nr_a(\widehat{H}_a) &\le I(X_a^n; Y_a^n) \\ &\le \sum_{\ell=1}^{g_{aa}} \Lambda_{a,\ell}(\widehat{H}_a) \\ &\le ng_{aa} - nr_b(\widehat{H}_b)|_{h_{bb}=h_{aa}} + n\log(6) \\ &\le ng_{aa} - ng_{aa}\tau_b + n\log(6) \\ &\le n\left[g_{aa}\tau_a + \log(6)\right]. \end{aligned} \tag{207}$$

The analysis for View 7 may follow that of either View 4 or 6, and comparison of the resulting expression with the linear deterministic analogue confirms the stated claim.

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