Distributed Spectrum Sharing via Graph Coloring

David T. Kao and Ashutosh Sabharwal
Department of Electrical & Computer Engineering
Rice University
{davidkao,ashu}@rice.edu

Abstract

In a wireless environment with different devices it is often difficult to introduce any sort of centralization. Thus we examine the problem of completely distributed spectrum sharing among wireless single-hop links and we

- *Abstract the problem into a graph-coloring framework.*
- *Present an algorithm describing a distributed spectrum sharing protocol.*
- *Demonstrate through analysis and simulation convergence to Nash equilibria.*

1. The Scenario

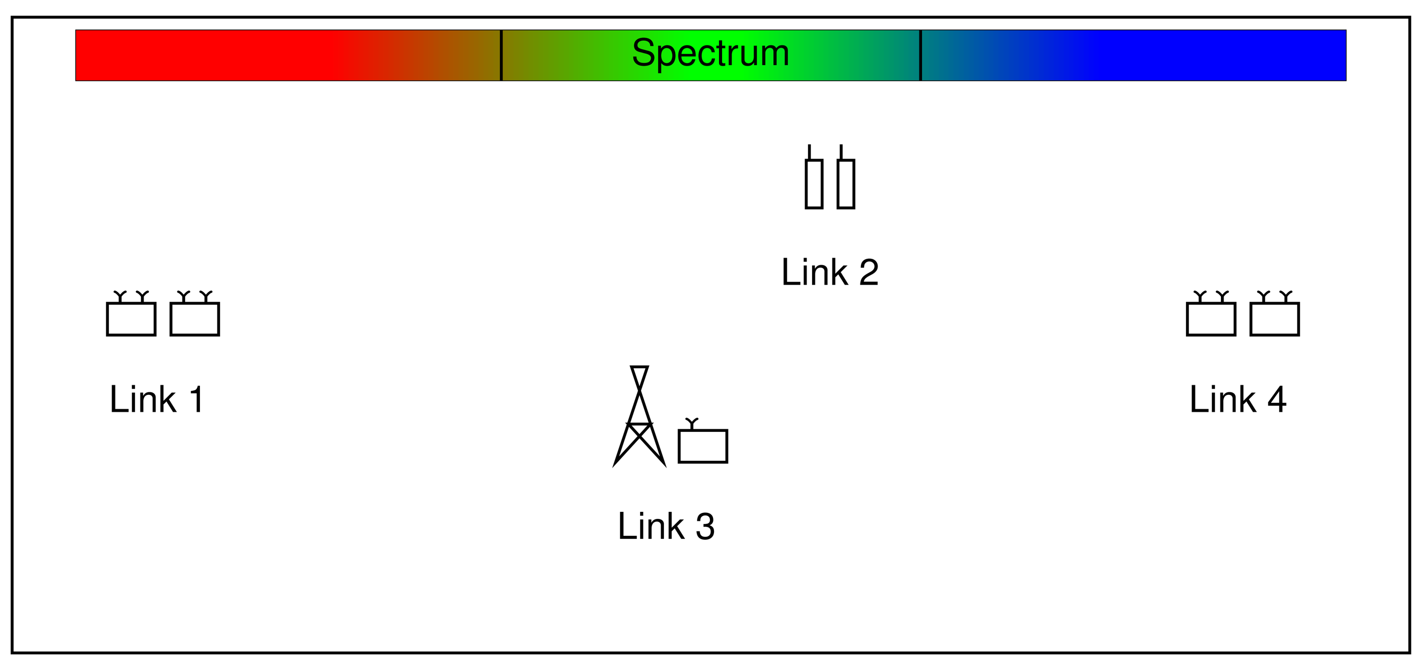

Figure 1: *A Wireless Environment (Geographic Topology and Available Spectrum)*

- Different transmitter-receiver pairs → different usable channels and no coordination
- Choose channel w/ minimum interference based on sensing

Mission: Allocate the spectral resources in a way that is Completely Distributed & In Some Sense Optimal

Question: What optimality can we expect from such a scenario?
Answer: Links optimize selfishly with respect to own channel options → **Nash Equilibria**

2. Abstract System Model

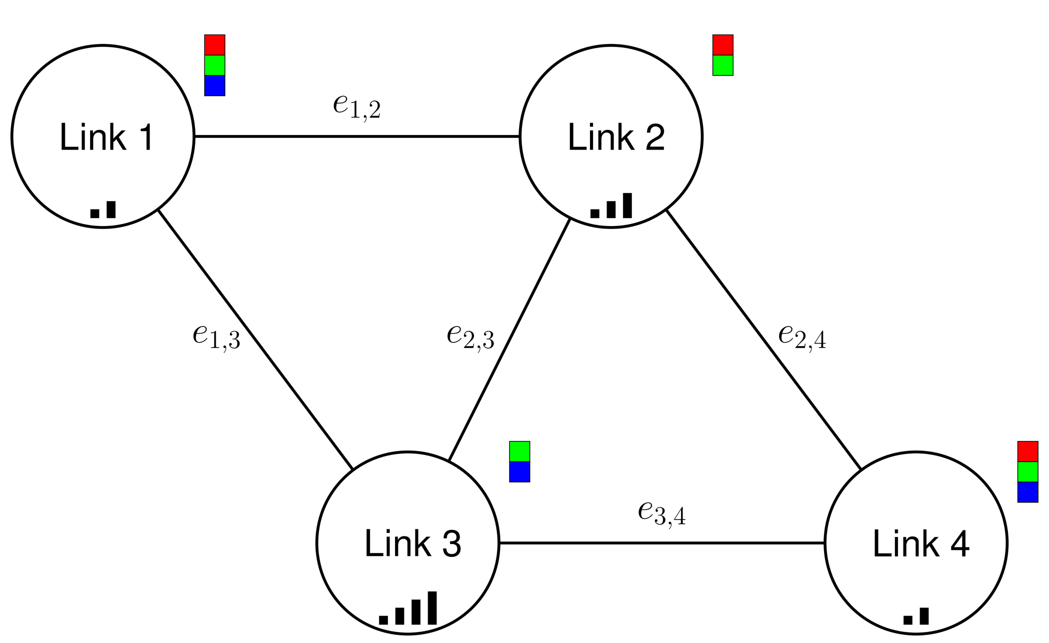

Figure 2: *Distributed Soft List Graph Coloring – The graph representation of the environment seen in Figure 1*

- Comm. Links → Vertices
- Attenuation → Edge Weights
- Usable Channels → Color Lists
- Trans. Power → Vertex Mag.

3. A Distributed Algorithm

Algorithm 1: *A distributed graph coloring algorithm with limited information. This algorithm is explained as a completely distributed protocol for spectrum access with sensing constraints.*

1. Begin with a seed distribution on the channels for each link. This distribution describes the access rule ($\underline{p}_i(k)$) that link i will use to select channels during block k.
2. For a finite block of time instances, use the access rule to randomly select the a channel to use during each time instance.
3. Record the interference level when a channel is used.
4. At the end of the block, make an estimate of the expected interference in each channel based on sensed data.
5. Update access rule with update equation ($F(\cdot)$) and repeat from step 2

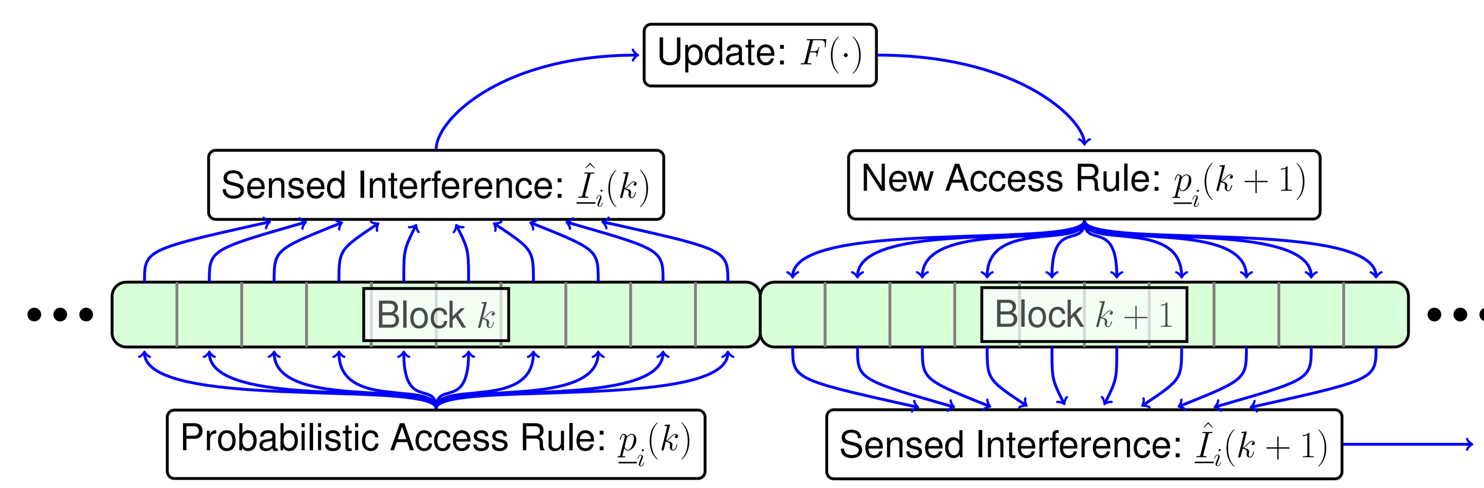

Figure 3: *Single-link POV of Distributed Protocol*

Update Function: Gradient Descent

$$\begin{aligned}\underline{p}_i(k+1) &= F\left(\underline{p}_i(k), \hat{\underline{I}}_i(k)\right)\\ &= \underline{p}_i(k) - \mu\left(\hat{\underline{I}}_i - \frac{1}{|C_i|}\sum_{\lambda \in C_i} \hat{I}_{i,\lambda}\right)\\ &= \underline{p}_i(k) - \mu \underline{G}\end{aligned}$$

4. Convergence Analysis

Theorem 1 *A network of cognitive links using this protocol converges in probability to an allocation $\underline{\lambda}^*$ that is a Nash Equilibrium.*

Proof Outline

- Stochastic approximation
- Differential equation based on expected value of gradient
- Analyze stable points → Nash Equilibria

5. Simulation Results

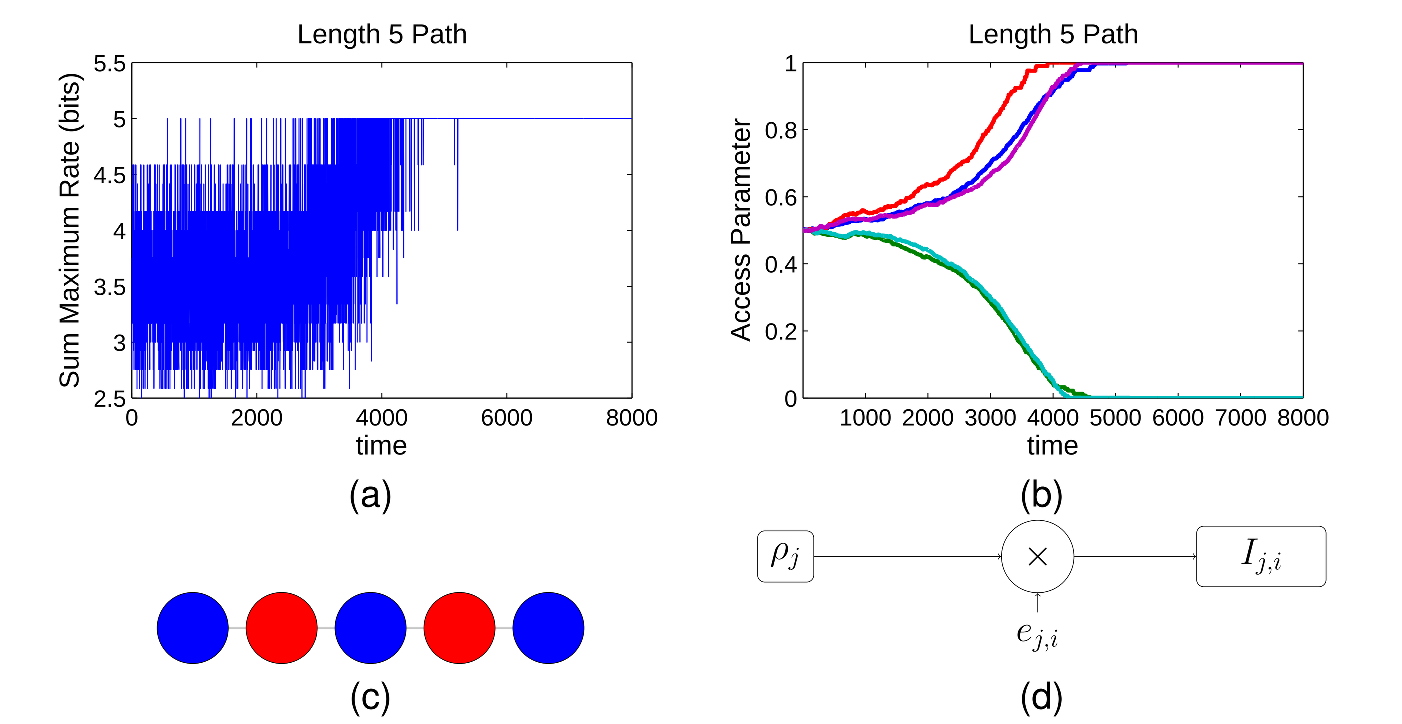

Figure 4: *Typical Performance: Deterministic Interference with Two Colors – An example of global utility converging (a), the evolution of each link's preference for the red instead of blue channel (b) and the final allotment (c) based on interference model shown in (d).*

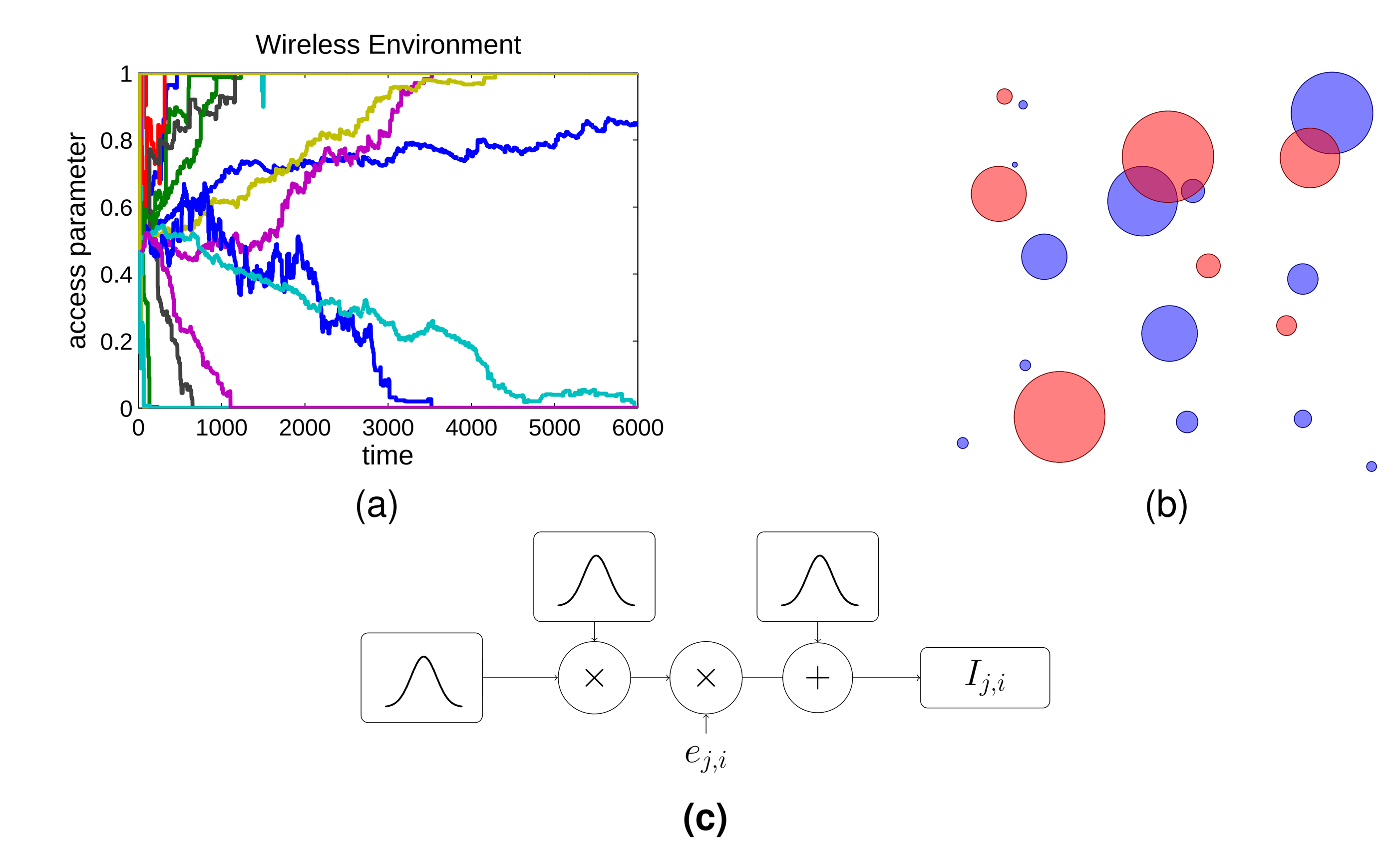

Figure 5: *Wireless Environment with Two Channels – The evolution of access parameters shown in (a) and the geographic topology, relative power of links and final allocation shown in (b) are the result of a more stochastic and realistic wireless model (c).*

6. Conclusion

- Presented a Limited Information Distributed Graph Coloring Algorithm
- Guaranteed Convergence to Nash Equilibria
- Application to & Implementation in **Realistic, Distributed** Spectrum Sharing

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