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# Impact of Network Topology Knowledge on Fairness: A Geometric Approach

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***Abstract*—In this paper, we examine how the precision of network topology knowledge impacts the achievable degree of max-min fairness. We focus on time-division multiple access (TDMA) networks, and employ a model based on physical-layer events that sufficiently describes the topology effects with respect to TDMA. Using Jain's fairness index, our key contribution is a characterization of the fairness loss resulting from allocation of resources (in our case time-divisions) based on imprecise knowledge of topology. We find loss is more pronounced when a single link has low signal-to-noise ratio (SNR); i.e. links which have poor throughput also make the allocation more unfair. Conversely, our analysis suggests that if the relative error in estimating link qualities is identical for all links in the network, no one link dominates the fairness loss.**

## I. Introduction

The concept of fairness in wireless networks has been studied from numerous perspectives. Many definitions of fairness and corresponding metrics have been defined [1]–[6], along with protocols designed to meet the various objectives in mind [7]–[12]. Generally, protocols are designed under the assumption that the underlying network topology either is known perfectly or remains static over a timescale sufficiently long such that iterative methods converge. In these cases where the network topology is known (or will eventually be known) perfectly, a resource allocator can optimally divide resources for any fairness objective. Such analyses have been crucial in building our understanding of fairness in wireless networks. However, wireless network topologies naturally change due to mobility, and the cost of overhead limits how much knowledge can be pooled at a central location. Hence, topology estimates suffer from limited precision, clearly inhibiting complete satisfaction of fairness criteria. Only little attention [13], [14] has been paid to this important generalization, which applies directly to practical systems.

In this paper, we use a geometric approach to characterize the loss in fairness (measured by Jain's fairness index [1]) resulting from errors in knowledge of the network topology. Our work focuses on a multiuser TDMA uplink, where many nodes communicate with a central receiver; see Figure 1. Notice the topology of this network is specified by not only the transmitter-receiver pairs, but also the average received signal-to-noise ratio, $\overline{\mathsf{SNR}}$, of those links. The $\overline{\mathsf{SNR}}$ in turn is directly related to the packet delivery success rate, $q_i$, which is the metric we use to quantify link quality. Thus, the relevant topology characteristics are perfectly known if the central resource allocator knows without error the delivery success probability, $q_i$, for each link. Any error in the knowledge of any $q_i$ constitutes an error in the knowledge of network topology.

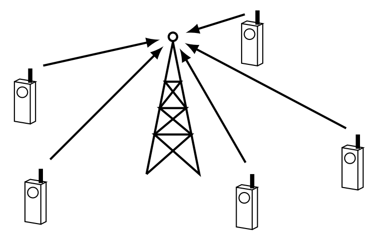

Fig. 1. A multiuser uplink in a cellular network.

For our problem, we show that determining the loss in fairness can be interpreted geometrically. An error ball in the topology knowledge space is mapped to an ellipsoid in the space of throughput allocations, whose form has varying implications on the resulting fairness loss. This geometric formulation allows us to exactly characterize the worst case fairness loss for an $N$-user system, which can be computed in a closed form for the two-user case.

A key observation is that for the same degree of precision in network topology, the loss in fairness is larger at low-$\overline{\mathsf{SNR}}$. That is, when errors are considered, the network which has poor network throughput also has a lower Jain's fairness index. Conversely, higher $\overline{\mathsf{SNR}}$ networks have a smaller fairness loss. It has been established that a poor link reduces the overall network throughput significantly if the allocation is fair [15]; for example, the tradeoff between system utilization and fairness becomes sharper if the weakest link in a network becomes weaker. However, links with poor quality have a doubly negative effect on network operation. From a practical standpoint, our findings imply that links with lowest $\overline{\mathsf{SNR}}$ (for example, when nodes are at the edge of the cells) largely determine both the achievable network efficiency *and* fairness.

Our analysis also points to a solution to avoid weak links from dominating network fairness properties, by proposing that all link qualities be measured with the same *relative* error. That is, if a single user's error normalized by its link quality is the same for all links in the network, no single link affects the resulting loss in fairness more than any other link. [1]

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[1]Due to lack of space, we do not discuss measurement schemes which produce the identical relative error.

The rest of the paper is organized as follows. In Section II, we formulate the problem. The main results for the two-user network are derived in Section III and its extensions are studied in Section IV. We numerically demonstrate the presented concepts in Section V and conclude in Section VI.

## II. Problem Formulation

In this section, we first state the system and link model then define the model of topology precision error. Finally, we state our fairness loss metric and how it pertains to TDMA network resource allocation.

### A. System Model

We consider $N$-user time-division multiple access (TDMA) networks, as in Figure 1. For the purpose of this analysis, it will be sufficient to assume the network is centrally managed.

In the network, the throughput experienced by each user, indexed by $i \in \{1, \ldots, N\}$, is defined by three parameters. First, we assume each user transmits packets encoded at a fixed data rate, $r_i > 0$. Second, the probability of User $i$ correctly receiving a transmitted packet, $q_i \in (0, 1)$, represents the quality of the transmission channel (and hence topology), and is described in more detail in Sections II-B and II-C. Finally, within this TDMA network, each user transmits for an allocated fraction of time, $t_i$, where it is the only user transmitting. These time fractions $t_i$ are normalized such that

$$\sum_{i=1}^{N} t_i \leq 1.$$

Consequently, the average throughput of a user is

$$x_i = t_i r_i q_i, \tag{1}$$

and the achievable throughput region, $\mathcal{X}$, is defined by a simplex bounded region:

$$\mathcal{X} = \left\{ \mathbf{x} \in \mathbb{R}^N : \sum_{i=1}^{N} \frac{x_i}{r_i q_i} \leq 1,\ x_i \geq 0\ \forall\ i \right\}.$$

Points in the throughput region are achieved by selecting different time-division vectors, $\mathbf{t}$, and the choice of a time-division vector can be viewed as the choice of a linear map from the network topology space, $\mathcal{Q} = (0, 1)^N$, to the space of throughput vectors, $\mathcal{X}$, as shown in Figure 2 for a two-user case.

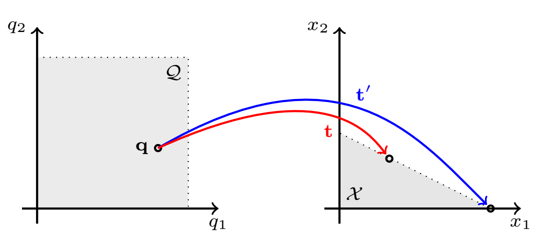


Fig. 2. For a 2-user network, two different time-division vectors map the same topology to different throughput allocations. Time-division $\mathbf{t}$ allocated half the temporal resources to each user, whereas $\mathbf{t}'$ allocates all to User 1.

### B. Physical Link Model

In a fading wireless network, the probability of Link $i$ correctly receiving a packet, $q_i$, is largely determined by its average SNR, denoted by $\overline{\mathsf{SNR}}_i$. In mobile networks, $\overline{\mathsf{SNR}}_i$ can also vary with time. For our purposes, we assume that the change in $\overline{\mathsf{SNR}}_i$ is much slower than a packet duration and hence we can assume $\overline{\mathsf{SNR}}_i$ stays constant. Furthermore, we assume that the probability $q_i$ is monotonically increasing with $\overline{\mathsf{SNR}}_i$, but is less than 1 for finite $\overline{\mathsf{SNR}}_i$.

As a practical example, consider the scenario where the instantaneous (per-packet) gain experiences Rayleigh fading. In this case, User $i$'s packet is received correctly when its instantaneous SNR is high enough to support the code rate used. The alternative event, losing a packet, is commonly known as an outage [16]. In this case, the packet delivery success rate, $q_i$, is

$$q_i = e^{-\frac{2^{R_i}-1}{\overline{\mathsf{SNR}}_i}}. \tag{2}$$

Several subtleties regarding this sort of relationship will be discussed in the context of the main result in Section III.

### C. Precision in Knowledge of Topology

With the above definition of per link channel quality per link, the network topology in Figure 1 is completely specified by $N$ channel quality indicators $\{q_i\}_{i=1}^N$. We organize $\{q_i\}_{i=1}^N$ as a vector, $\mathbf{q}$, in the $N$-dimensional space, $\mathcal{Q} = (0, 1)^N$. Thus, the vector $\mathbf{q}$ is the full description of network topology relevant to TDMA data transmission.

With perfect knowledge of the topology, the behavior of a TDMA network is well understood. Unfortunately, due to noisy and time-limited measurement, topology information is subject to error. In order to gain a more general understanding, we refrain from proposing any specific measurement methods, and assume this error is a precision error in the sense that its magnitude is known to be bounded by some value, and is characteristic of the measurement method.

Let $\hat{\mathbf{q}}$ denote the topology estimate vector that is used to (deterministically) select a time-division, and $\Delta\mathbf{q}$ denote the vector of estimation error, i.e.

$$\Delta\mathbf{q} = \mathbf{q} - \hat{\mathbf{q}}. \tag{3}$$

Furthermore, let $\delta$ represent the precision or Euclidean magnitude bound on the error, $\Delta\mathbf{q}$,

$$\|\Delta\mathbf{q}\| \leq \delta. \tag{4}$$

We assume $\delta$ is sufficiently small such that for any $\Delta\mathbf{q}$ satisfying (4), $\Delta\mathbf{q} + \hat{\mathbf{q}} \in \mathcal{Q}$.

Finally, we note that our model of error is deterministic and not stochastic. In other words, we are not specifying a distribution on the error, $\Delta q_i$. Our work in this paper can be used as a building block for computing statistical results for the problem. Due to lack of space, those extensions are not presented in this paper.

### D. Fair Allocation in TDMA Systems

We assume the network enacts an allocation policy with the objective of throughput max-min fairness [17] within a *finite* coherence time. This temporal constraint is often necessitated by the mobility of a network, which over time may not be a stationary process, thus varying the average SNRs in an unmanageable way.

With the throughput region previously described, this objective amounts to computation of the time-division that results in a throughput vector on the simplex boundary where all users experience the same throughput. With perfect knowledge of the topology, $\mathbf{q}$, the time division that achieves the max-min fair throughput vector, $\mathbf{t}^*(\mathbf{q})$, can be found by solving a straightforward linear system of equations. However, when only a topology estimate, $\hat{\mathbf{q}}$, is available, computation of time-division results in the allocation

$$t_i^*\left(\hat{\mathbf{q}}\right) = \frac{1}{\hat{q}_i r_i \sum_{j=1}^{N} \frac{1}{\hat{q}_j r_j}}. \tag{5}$$

Since any error in topology information produces a time-division that may not be max-min fair, in order to quantify any such deviation, we employ the commonly used Jain's index [1] as the measure of fairness. By applying the expression for throughput (1), we denote the value of the index as a function of the topology estimate, $\hat{\mathbf{q}}$, and error, $\Delta\mathbf{q}$,

$$J\left(\Delta\mathbf{q}, \hat{\mathbf{q}}\right) = \frac{\left(\sum_{i=1}^{N} t_i^*\left(\hat{\mathbf{q}}\right) r_i \left(\hat{q}_i + \Delta q_i\right)\right)^2}{N \sum_{i=1}^{N} \left(t_i^*\left(\hat{\mathbf{q}}\right) r_i \left(\hat{q}_i + \Delta q_i\right)\right)^2}. \tag{6}$$

Using this index, and given a bound on topology error, we seek to establish the loss in fairness when the worst-case error occurs. We define the function $\epsilon(\cdot)$ as the fairness loss (or discrimination [1]) resulting from topology estimate, $\hat{\mathbf{q}}$, and precision, $\delta$,

$$\epsilon(\delta, \hat{\mathbf{q}}) = 1 - \min_{\Delta\mathbf{q} \text{ s.t. } \|\Delta\mathbf{q}\| \leq \delta} J(\Delta\mathbf{q}, \hat{\mathbf{q}}). \tag{7}$$

This concept best quantifies the ability of a central authority to enforce and guarantee fair policies.

## III. The Impact of Topology Knowledge on Fairness in a Two-User Network

We begin by establishing how error in topology knowledge impacts fairness in a two-user TDMA network. Specifically, we will derive the fairness loss as a function of the precision.

First, let the topology estimate, $\hat{\mathbf{q}} = (\hat{q}_1, \hat{q}_2)$, and error bound, $\delta$, be given. This provides us with the 2-D ball shown in Figure 3(a). Using the estimates, a time-division is chosen such that the point $(\hat{q}_1, \hat{q}_2)$ in the topology space, $\mathcal{Q}$, is mapped to some point $(\hat{x}, \hat{x})$ in the throughput space, $\mathcal{X}$. From the computation of time-divisions in (5),

$$\hat{x} = \left(\sum_{i=1}^{N} \frac{1}{r_i \hat{q}_i}\right)^{-1}. \tag{8}$$

Because the TDMA mapping is linear, the circular error ball in $\mathcal{Q}$ is transformed into an ellipse in $\mathcal{X}$ with major and minor axes parallel to the user-throughput axes, as seen in Figure 3(b).

In order to determine the worst case error, first an observation on the nature of Jain's index is required. Note that the following derivation is for a general $N$-user case, not just the two-user case.[2] By examining (6), we derive an alternate geometric interpretation in the throughput-space by using the equivalent expression

$$J = \left(\sum_{i=1}^{N} x_i\right)^2 \Big/ \left(N \sum_{i=1}^{N} x_i^2\right) \tag{9}$$

$$= \left(\sum_{i=1}^{N} \frac{x_i}{\|\mathbf{x}\|} \frac{1}{\sqrt{N}}\right)^2 \tag{10}$$

$$= \left\langle \frac{\mathbf{x}}{\|\mathbf{x}\|}, \frac{\mathbf{1}}{\|\mathbf{1}\|} \right\rangle^2 \tag{11}$$

$$= \cos(\Theta)^2, \tag{12}$$

where $\mathbf{1}$ is the length $N$ vector of ones, and $\Theta$ is the angle between the experienced and estimated throughput vectors. Then for any number of users, the loss in fairness is monotone with this angle; maximization of the angle results in minimization of fairness.

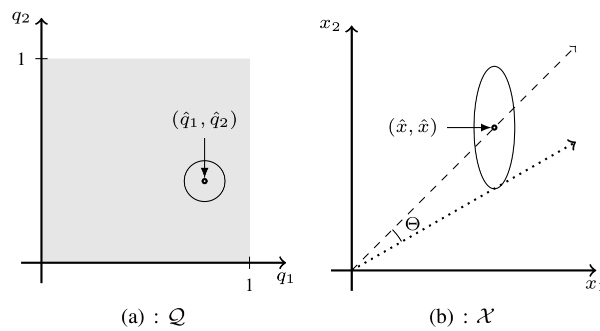


Fig. 3. Transformation of uncertainty region from $\mathcal{Q}$ to $\mathcal{X}$. The fairness minimizing tangent ray is also shown in (b).

If for each feasible throughput point, a ray from the origin through the feasible point is considered, the throughput vector maximizing $\Theta$ is one whose corresponding ray is one of the two rays tangent to the ellipse (Figure 3(b)). Since the mapping (and inverse mapping) between $\mathcal{Q}$ and $\mathcal{X}$ is linear, and linear transformations preserve tangency, the tangent rays in $\mathcal{X}$ must be linear transformations of similarly tangent rays in $\mathcal{Q}$. This intuition allows us to arrive at the following result.

[2] We use simplified notation and express a user's thoughput as $x_i$ to emphasize the geometry of Jain's index in $\mathcal{X}$.

**Theorem 1.** *Let the topology error be bounded by $\delta$, and let a topology estimate $\hat{\mathbf{q}} = (\hat{q}_1, \hat{q}_2)$, with $\hat{q}_1 > \hat{q}_2$ be given. The worst case error, $\Delta\mathbf{q}^{(WC)}$, is*

$$\Delta\mathbf{q}^{(WC)} = -\left(\frac{\delta}{\|\hat{\mathbf{q}}\|}\right)^2 \begin{bmatrix}\hat{q}_1\\ \hat{q}_2\end{bmatrix} + \frac{\delta}{\|\hat{\mathbf{q}}\|}\sqrt{1-\left(\frac{\delta}{\|\hat{\mathbf{q}}\|}\right)^2}\begin{bmatrix}\hat{q}_2\\ -\hat{q}_1\end{bmatrix}, \tag{13}$$

*and the fairness loss is*

$$\epsilon(\delta, \hat{\mathbf{q}}) = \frac{\left(\frac{\hat{q}_1}{\hat{q}_2} + \frac{\hat{q}_2}{\hat{q}_1}\right)^2}{\left(2\gamma - \left[\frac{\hat{q}_1}{\hat{q}_2} - \frac{\hat{q}_2}{\hat{q}_1}\right]\right)^2 + \left(\frac{\hat{q}_1}{\hat{q}_2} + \frac{\hat{q}_2}{\hat{q}_1}\right)^2}, \tag{14}$$

*where*

$$\gamma = \frac{\|\hat{\mathbf{q}}\|}{\delta}\sqrt{1-\left(\frac{\delta}{\|\hat{\mathbf{q}}\|}\right)^2}. \tag{15}$$

*Proof:* We have established from our geometric intuition tangency as a necessary condition for a worst-case error. Furthermore, since tangency is preserved in the mapping from $\mathcal{Q}$ to $\mathcal{X}$, we can search for errors that satisfy the tangency condition while working in $\mathcal{Q}$. To find such errors, we must satisfy two quadratic equations:

$$\langle \Delta\mathbf{q}, \hat{\mathbf{q}} + \Delta\mathbf{q}\rangle = 0 \tag{16}$$

$$\langle \Delta\mathbf{q}, \Delta\mathbf{q}\rangle = \delta^2. \tag{17}$$

As the surface normal of the error ball in $\mathcal{Q}$ is proportional to $\Delta\mathbf{q}$, (16) means the position vector of $\hat{\mathbf{q}}+\Delta\mathbf{q}$ is orthogonal to the surface normal. The second equation, (17) constrains us to the surface of the error ball (boundary of the circle).

Equations (16) and (17) form a quadratic system of equations, which for two dimensions have two unknowns ($\Delta q_1$ and $\Delta q_2$). When solved, we arrive at two possible error vectors that satisfy tangency,

$$\Delta\mathbf{q} = -\left(\frac{\delta}{\|\hat{\mathbf{q}}\|}\right)^2 \begin{bmatrix}\hat{q}_1\\ \hat{q}_2\end{bmatrix} \pm \frac{\delta}{\|\hat{\mathbf{q}}\|}\sqrt{1-\left(\frac{\delta}{\|\hat{\mathbf{q}}\|}\right)^2}\begin{bmatrix}\hat{q}_2\\ -\hat{q}_1\end{bmatrix}. \tag{18}$$

Using (6), and noting our assumption that $\hat{q}_1 > \hat{q}_2$, we see in

$$J(\Delta\mathbf{q}, \hat{\mathbf{q}}) = \frac{\left(2\gamma \mp \left[\frac{\hat{q}_1}{\hat{q}_2} - \frac{\hat{q}_2}{\hat{q}_1}\right]\right)^2}{\left(2\gamma \mp \left[\frac{\hat{q}_1}{\hat{q}_2} - \frac{\hat{q}_2}{\hat{q}_1}\right]\right)^2 + \left(\frac{\hat{q}_1}{\hat{q}_2} + \frac{\hat{q}_2}{\hat{q}_1}\right)^2}, \tag{19}$$

where $\gamma$ is as defined in (15), that the choice of sign that decreases fairness gives us (13). Similarly we can calculate (14) directly from (19). ■

**Remark 1:** Due to the symmetry of the problem, no generality is lost by assuming $\hat{q}_1 > \hat{q}_2$.

**Remark 2:** Fixed coding rates have no impact on fairness loss. This is a result of the time-division, $\mathbf{t}$, being chosen to balance out the perfectly known effects of system defined coding rates.

**Remark 3:** The worst-case error places more emphasis (the magnitude of the component in the error vector is larger) on the user estimated to have poorer channel conditions.

**Remark 4:** The expression (14) is a function of the relative packet success rates, $\frac{\hat{q}_1}{\hat{q}_2}$, and the ratio $\frac{\delta}{\|\hat{\mathbf{q}}\|}$. Notice that if the channel is more symmetric, the loss in fairness is also reduced compared to a more asymmetric case. Additionally, the normalization of topology precision by the estimate magnitude suggests that in order to achieve the same level of fairness, weaker channels require more precise measurement.

That the parameter relevant to fairness is relative error, not absolute error, also suggests that if the error ball in $\mathcal{Q}$ was not a circle but an ellipse such that the weaker of the two channels was estimated with higher precision, then the time-division chosen may produce a circle in $\mathcal{X}$, signifying that the two channel quality estimates and their differing degrees of precision contribute equally to loss in fairness.

**Remark 5:** Recall in Section II-B our discussion relating $\overline{\mathsf{SNR}}_i$ to link quality $q_i$. Notice that if we keep increasing $\overline{\mathsf{SNR}}_i$ for all $i$, all link quality values approach 1. As a result, at high SNR the asymmetry between link qualities is reduced, and the system is inherently more fair. Furthermore, at higher SNR, the magnitude of the topology estimate, $\|\hat{\mathbf{q}}\|$, is larger and for fixed $\delta$, fairness is again increased. Thus, higher SNR networks lead to more fair solutions.

## IV. Extensions to an $N$-User Network

The two-user case provides all the tools and intuition for the $N$-user network, the solution of we present here.

**Theorem 2.** *The loss in fairness in an $N$-user TDMA network is given by the optimization problem*

$$\epsilon(\delta, \hat{\mathbf{q}}) = \max_{\mathbf{u}} \frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left(\frac{u_i}{\hat{q}_i} - \frac{u_j}{\hat{q}_j}\right)^2}{\left(N\gamma + \sum_{i=1}^{N}\frac{u_i}{\hat{q}_i}\right)^2 + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left(\frac{u_i}{\hat{q}_i} - \frac{u_j}{\hat{q}_j}\right)^2}. \tag{20}$$

*where $\gamma$ is as defined in (15) and $\mathbf{u}$ is such that $\sum_{i=1}^{N}\frac{u_i}{\hat{q}_i} \le 0$, $\|\mathbf{u}\| = \|\hat{\mathbf{q}}\|$ and $\langle \mathbf{u}, \hat{\mathbf{q}}\rangle = 0$.*

*Proof:* The proof and further discussion are given in [18]. ■

**Remark 6:** The characterization of fairness in Theorem 2 provides a method of reducing the search space for the worst case error. Selection of a particular vector $\mathbf{u} \neq \pm\hat{\mathbf{q}}$, confines the search within some two-dimensional subspace of the larger $N$-dimensional space. The first constraint on $\mathbf{u}$ (the validity of which can be confirmed by comparing the objective function in (20) evaluated for any $\mathbf{u}$ and $-\mathbf{u}$) further reduces the search to a two-dimensional half-space, and the final two constraints allow us to focus only on points satisfying the previously mentioned tangency criterion.

**Remark 7:** Notice that the form of the expression for the $N$-user network, (20), is exactly that of (14). However, for the two-user network there exists only one vector ($\mathbf{u} = (\hat{q}_2, -\hat{q}_1)$) satisfying the three constraints on $\mathbf{u}$.

**Remark 8:** Just as in the two-user network, error in estimating the weakest link carries a greater potential for fairness loss than error in estimating any other link, due to fair solutions compensating most for this user. Similarly, asymmetry in link qualities again plays a role in the potential for fairness loss, though quantifying the asymmetry for an $N$-user network is significantly more complex than for the two-user network.

## V. Numerical Results

In this section we demonstrate the concepts presented in this paper, by considering the fairness loss function for a number of topology estimates, listed in Table I. Included in the estimates presented are two symmetric (labeled SYMM) and two asymmetric (labeled ASYM) topologies. For each case, both a high SNR (large $\|\hat{\mathbf{q}}\|$, labeled HI) and low SNR (labeled LO) case is considered.

| Label | $\hat{q}_1$ | $\hat{q}_2$ | $\|\hat{\mathbf{q}}\|$ |
|---|---|---|---|
| ASYM-LO | 0.1000 | 0.4123 | 0.4243 |
| ASYM-HI | 0.2000 | 0.8246 | 0.8485 |
| SYMM-LO | 0.3000 | 0.3000 | 0.4243 |
| SYMM-HI | 0.6000 | 0.6000 | 0.8485 |

TABLE I
Topology estimates for two-user simulation

Notice that in Table I, the estimates ASYM-LO can be expressed as a scaled version of ASYM-HI; this is true also of the two symmetric topologies, SYMM-LO and SYMM-HI. This is a means for fair comparison. These topology estimates are "similar" in the sense that they will produce the same time-division solution, and the expected throughput vector of one is a scaled version of the other.

The expression (20) suggests that if one were to consider normalized error, $\frac{\delta}{\|\hat{\mathbf{q}}\|}$, similar channels behave identically. Figure 4 demonstrates this intuition by presenting the fairness loss versus normalized error. As the plot shows, two networks where the topology estimate vector differ only by a multiplicative constant behave identically with respect to the normalized error. That similar topologies behave identically with respect to normalized error reiterates the importance of a "percent error", rather that absolute error, in estimating network topology effects. The simulations also confirm that the asymmetric network is much more unfair than the symmetric one. Similar behavior is exhibited in larger networks.

## VI. Conclusions

In this paper we found the fairness loss in TDMA networks when errors in topology knowledge are considered. In deriving the fairness loss function, we saw that the *relative error* and *asymmetry* of the network are determining factors. In future work, our results can be used to derive more intelligent protocols to learn the network topology and our geometric methodology can be applied to any number of more complex resource allocation schemes. Finally, the intuition gained can facilitate examination of other fairness metrics and stochastic error models.

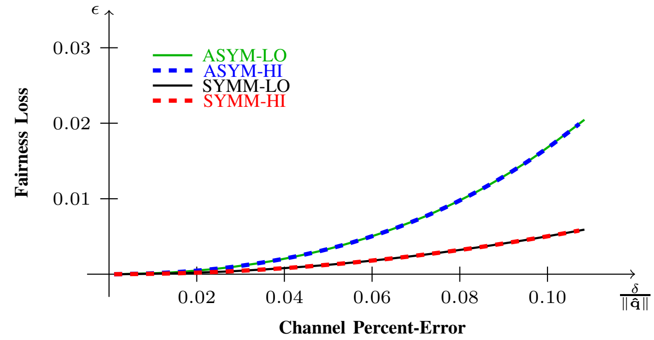


Fig. 4. Fairness loss versus normalized error in a two-user network with topology estimates as listed in Table I.

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