2a. GENERAL GEOMETRY AND THE METHOD OF UNIVERSALITY

[1674]

Since theorems serve only to abbreviate or guide the solution of problems and since all theory should assist practise, all we need to do in order to judge the variety of kinds of geometry is to consider its problems. The problems of geometry are about straight lines or curves, and presuppose only the magnitude of some lines or figures, straight or curved. To the latter sort belong the problems of finding centers of gravity, and consequently, a good many problems of mechanics. Thus we may say that there are two kinds of geometry, an Apollonian and an Archimedean; the first was revived by Viète and Descartes, the second by Galileo and Cavalieri.

Problems of straight lines reduce to the solution of some equation whose roots must be extracted analytically by means of calculation, or geometrically by means of the intersection of loci, either exactly or approximately. Problems of curved figures in general are not yet subjected to a known method of analysis,* and if we wanted to reduce them to an equation, we should find it to be of infinite degree.

. . . I do not repeat here what I have recently said in a separate paper on the *Method of Universals,* which shortens calculation by including several cases under a single one, and which leads us to discover harmonies in the figures and gives us a means of ordering them in classes through general ideas.

* This was written a year before Leibniz discovered the infinitesimal calculus independently of Newton's prior but yet unpublished method of fluxions.

3. DIALOGUE ON THE CONNECTION BETWEEN THINGS AND WORDS

[1677]

A. If you were asked to bend a string so that it would enclose the largest possible area for its length, how would you bend it?

B. Into a circle; for, as geometry shows, of all figures with equal perimeters, the circle encloses the maximum area. Given two islands which can be circumnavigated in the same time, one having the shape of a circle and the other that of a square, the circular one would contain more land.

A. Do you believe this remains true even when you are not thinking about it?

B. Why, yes—even when geometers have not yet proven it, or people have not yet taken notice of it.

A. So in your opinion *truth* and *falsity* both reside in things, and not in thoughts?

B. Yes, of course.

A. But can we ever call any *thing* false?

B. No, I suppose only thoughts or statements about a thing are called false.

A. Then falsity is in thoughts, not things.

B. I must grant you that.

A. And, accordingly, truth as well?

B. So it seems; but I am still not thoroughly convinced about that, and still in some doubt about your conclusion.

A. When you are confronted with a problem and your opinion is not yet exactly settled as certain, are you not said to be in doubt whether you have the truth or falsity of the matter?

B. Certainly.

A. Then you recognize that it means the same to come to a decision about a question as to say we have a true or false answer to it?

B. I see now, and I concede that truth and falsity both pertain to thoughts and not to things.

A. But this contradicts your previous opinion that a proposition remains true even when you are not thinking about it.

B. Now you have me completely befuddled!

A. Well, then, we must look for a common ground in the two propositions. Would it be this: thoughts which might possibly be true are realized somewhere in fact, or to express myself more clearly, do you believe that every possible thought is actually thought?

B. No.

A. Then you see that truth really belongs to the class of thoughts which are possible, and that there is certainty only when somebody takes one or the other of the contradictory forms of his thought to be true or false.

B. It seems you have now found the correct answer to our difficulty.

A. However, since it is necessary to assume that a thought is to be called true or false, where, I ask, are we to find this true thought?

B. Well, I suppose by thinking about the nature of things.

A. How would it be, if it should come out of your own nature?

B. But surely not from it alone. For outside of my own nature there must be also the nature of the things I am thinking about, so that by pursuing the right method I may discover whether the proposition I finally arrive at is valid and true.

A. Quite right; yet there remain many difficulties.

B. Which do you have in mind?

A. Many scholars are of the opinion that truth originates in the human will and belongs to names or characters.

B. A very paradoxical position.

A. Nevertheless they prove it in the following way: The basis of every proof is definition.

B. Of course, we can prove many theorems simply by connecting definitions.

A. Does the truth of such theorems then depend on the definitions?

B. Certainly.

A. But definitions depend on our will?

B. How is that?

A. Is it not true that it is only the arbitrary will of the mathematician which makes him use the word "ellipse" to designate a defined figure? And again, is it not the arbitrary will of the Latin scholar which leads him to choose certain words to define "circulus"?

B. Well, what about it? Can thoughts not exist without words?

A. But not without some sign or other Ask yourself whether you can perform any arithmetical calculation without making use of any number-signs.

B. You have me quite confused, for I do hold characters or signs as indispensable in reckoning.

A. The truths of arithmetic are then expressed by some kind of signs or characters?

B. I cannot deny that.

A. Then they depend on man's will?

B. You are resolved to trick me with a curious sort of word juggling!

A. Not I but some very acute thinkers started this.

B. How can anyone be so irrational as to hold truth to be arbitrary and make it depend on names, when surely the same geometrical truth is expressible in Greek, Latin, and German?

A. That is correct, and for that reason we must meet the difficulty.

B. This one only makes me realize that in my thinking I never recognize, discover, or prove any truth without calling up to mind words or some other kind of signs.

A. Quite so; yes, if there were no signs, we should never think or conclude anything intelligibly.

B. However, when we look at geometrical diagrams, we frequently go ahead and bring to light many truths simply by studying the diagrams.

A. Quite so, but we must not forget that these diagrams are also to be regarded as characters. For the circle drawn on paper is not the true circle: but it is not necessary that it should be, for it suffices to substitute the drawn figure for the circle.

B. Yet it has a definite similarity to the circle, and that is not arbitrary.

A. By all means, and just for that reason diagrams are the most universal characters. But what similarity is there between the number of ten things and the mark "10"?

B. There exists among the marks, apart from whether they are properly chosen, a relation or order which corresponds to the order in things.

A. That may be, but what similarity do the first elements have to the objects they designate, for example, "O" to nothing, or the letter "a" to a line? You must admit that at least these elements need have no similarity to things. This holds, for example, for the word-stems "lux" and "fero," because their composite "lucifer" has certainly a definite relation to them, and, in fact, the relation is the one holding between the *objects* designated by *lucifer, lux, fero.*

B. In Greek, however, φώσφορος has the same relation to φῶς and φέρω.

A. Yes, yet the Greeks were also able to use another word here.

B. Quite right; I only mean that characters must show, when they are used in demonstrations, some kind of connection, grouping and order which are also found in the objects, and that this is required, if not in the single words—though it were better so—then at least in their union and connection. This order and correspondence at least must be present in all languages, though in different ways. And that leaves me with hope for a solution of the difficulty. For even though characters are as such arbitrary, there is still in their application and connection something valid which is not arbitrary; namely, a relationship which exists between them and things, and consequently, definite relations among all the different characters used to express the same things. And this relationship, this connection is the foundation of truth. For this explains why no matter which characters we use, the result remains the same, or at least, the results which we find are equivalent and correspond to one another in definite ways. Some kind of characters is surely always required in thinking.

A. Excellent! You have gotten yourself out of the difficulty marvelously. Your view is corroborated by analytical and arithmetical calculation; for, in the case of numbers, we arrive at the same results whether we use the decimal system or duodecimal system. If we apply the results of different kinds of systems of counting to grain or to any other countable materials, the result always remains the same. This is also true of algebraic analysis, although it is easier here to use a variety of characters than to modify things themselves in their relations to one another. Here also, the firm foundation of