Calculus Quiz 5: Prof. Kaplan

March 13, 2025

Student name: _____. Do what you can in 15 minutes.

Question 5.1: What is $\partial_y(a_0 + a_1 x + a_2 x^2)$?

(i) $a_1 + a_2 x$ (ii) $a_1 + 2a_2 x$ (iii) $a_1 + 2a_2 y$ (iv) 0

Question 5.2: What is $\partial_t A x e^{kt}$?

(i) 0 (ii) Ake^{kt} (iii) $Akxe^{kt}$ (iv) Ae^{kt}

Question 5.3: What is

$$\partial_{yx}[a_0 + a_1 x + b_1 y + cxy + a_2 x^2 + b_2 y^2] \text{ ?}$$

(i) c (ii) $2a_2$ (iii) $2b_2$ (iv) 0

Question 5.4: What is $\partial_{yx} h(x,y) g(y)$?

i. $\partial_{yx} h(x,y)$

ii. $g(y)\partial_{yx} h(x,y) + h(x,y)\ \partial_y g(y)$

iii. $\left(\partial_y g(y)\right)\left(\partial_x h(x,y)\right) + g(y)\left(\partial_{yx} h(x,y)\right)$

iv. $\left(\partial_x g(y)\right)\left(\partial_x h(x,y)\right) + g(y)(\partial_{xx} h(x,y))$

Question 5.5: What is $\partial_t(7 + 8t^2 + 3t^4)$?

i. $16t + 12t^3$
ii. $8t + 4t^3$
iii. $16t^2 + 9t^3$
iv. $4t + 12t^2$

Question 5.6: What is $\partial_y h(x,y) g(y)$?

i. $\partial_y g(y)$
ii. $g(y)\partial_y h(x,y)$
iii. 0
iv. $ g(y) _y h(x,y) + h(x,y) _y g(y)$

Question 5.7: Which of the derivative rules should you use to find

$$\partial_t e^{t^2} \text{ ?}$$

i. The constant multiplier rule
ii. The linear combination rule
iii. The product rule
iv. The chain rule
v. No rule needed, it is so basic.

Question 5.8: Which of the derivative rules should you use to find

$$\partial_t e^t \sin(x) \text{ ?}$$

i. The constant multiplier rule
ii. The linear combination rule
iii. The product rule
iv. The chain rule
v. No rule needed, it is so basic.

Question 5.9: For the function

$$g(P) \equiv \sin\left(\frac{2\pi}{P}(t - t_0)\right)$$

is the interior function linear?

(i) Yes (ii) No

Question 5.10: Here are several functions that are related by differentiation and integration:

a. $\frac{1}{a} e^{ax+b}$
b. $a^2 e^{ax+b}$
c. $\frac{1}{a^2} e^{ax+b}$
d. e^{ax+b}
e. ae^{ax+b}

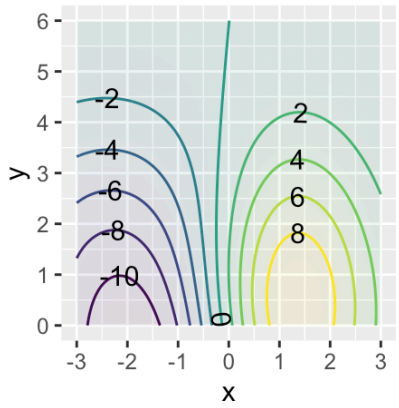
Put these functions in order that the derivative of each function preceeds the anti-derivative.

i. The order is b-e-d-a-c
ii. The order is b-d-e-c-a
iii. The order is b-d-c-e-a
iv. The order is e-b-d-c-a
v. The order is e-b-d-a-c

Question 5.11: There are two pattern-book functions whose **second** derivative is proportional to the function itself. Which are they?

i. Exponential and sinusoid
ii. Exponential and sigmoid
iii. Exponential and logarithm
iv. Sinusoid and gaussian

Question 5.12: Consider the function shown in the following contour plot:



At which of these inputs is the function practically flat?

i. $(x = 0, y = 6)$
ii. $(x = 1, y = 2)$
iii. $(x = -2, y = 3)$
iv. $(x = 0, y = 1)$

Question 5.13: Imagine a second-order polynomial in three inputs: x, y, and z, like this:

$$b_0 + b_x x + b_y y + b_z z + b_{xy} xy + b_{xz} xz + b_{xx} x^2 + b_{yy} y^2 + b_z z z^2 \, .$$

All of the possible second-order (or less) terms are shown, except for one. Which term is missing?

i. the interaction between y and z
ii. the quadratic term in z
iii. the linear term in y
iv. the constant term

Question 5.14: Suppose you know only this one fact about $f(x)$, that

$$\left[\partial_{xx} f(x)\right]_{x=7.3} = 1.6 \, .$$

Which of these statements **must** be true?

i. $f(x)$ is increasing at $x = 7.3$.
ii. $f(x)$ is concave up and decreasing at $x = 7.3$
iii. $f(x)$ is concave up at $x = 7.3$
iv. $f(x)$ is concave up at $x = 7.3$, but eventually it will become concave down.

Question 5.15: Which of the following is the correct construction for $\partial_t g(t)$?

i. $\lim_{h \to 0} \dfrac{g(t+h) - g(t)}{h}$
ii. $\lim_{h \to 0} \dfrac{g(t+h) - g(t)}{t}$
iii. $\lim_{h \to 0} \dfrac{g(t) - g(t+h)}{h}$
iv. $\lim_{x \to 0} \dfrac{g(t+h) - g(t)}{h}$

Question 5.16: The derivative

$$\partial_x \text{ dnorm } (x) = -x \, \text{dnorm } (x) \, .$$

What is

$$\partial_x \text{ dnorm } \left(\frac{x^2}{4} \right) ?$$

i. $-\frac{x^3}{8} \text{ dnorm } \left(\frac{x^2}{4}\right)$
ii. $-\frac{x}{2} \text{ dnorm } \left(\frac{x^2}{4}\right)$
iii. $-\frac{x}{8} \text{ dnorm } \left(\frac{x^2}{4}\right)$
iv. $-\frac{x^2}{2} \text{ dnorm } \left(\frac{x^2}{4}\right)$