# MOSAIC Calculus Quiz 9: Prof. Kaplan

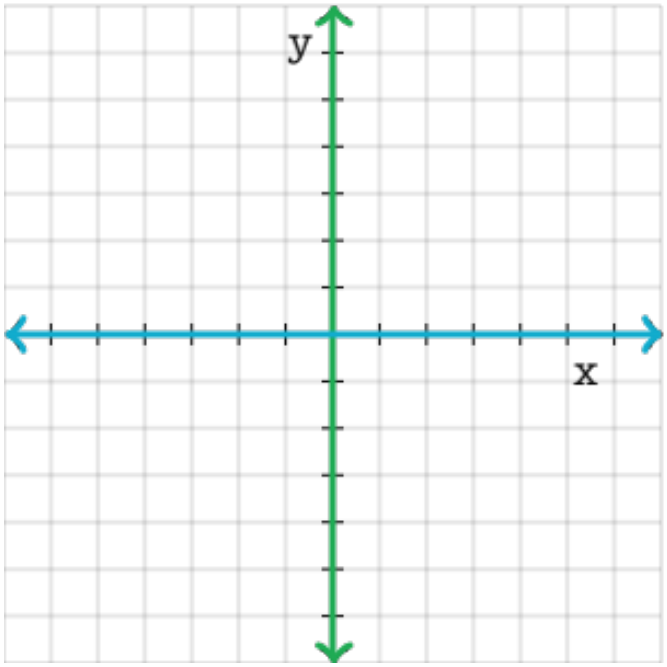
## June 6, 2025

Student name: _____. Do what you can in 30 minutes.

This quiz is about the simple dynamical system

$$\partial_t x \equiv y, \quad \partial_t y \equiv -x.$$

**Question 7.1** Draw the nullclines of the system on the graph plane below. Make sure to label clearly which is the x-nullcline and which is the y-nullcline.




Double check to make sure you haven't reversed the the nullclines. Hint: the coordinate point $(x = 1, y = 0)$ is on one of the nullclines.

**Question 7.2** Draw in flow arrows on each of the nullclines to each side of the origin. (There will be 4 flow arrows altogether.)

**Question 7.3** Draw in another four flow arrows, one in each of the four quadrants of the plane.

**Question 7.4** Bye eye, draw a plausible trajectory for the flow, starting at whatever initial condition you choose (but **not** $(x = 0, y = 0)$).

**Question 7.5** Consider the time series $x(t) = \sin(t)$ as a **candidate** solution starting at the initial condition $(x = 0, y = 1)$. Use anti-differentiation to find the corresponding $y(t)$.

**Question 7.6** Modify the differential equations so that $x(t) = A\sin(\omega t)$ and its corresponding $y(t)$ are solutions, where $\omega$ describes the frequency of the oscillation. Write the modified equations here.

Turn the sheet over for an extra-credit problem.

**Extra credit**: Figure 1 shows another dynamical system, a pendulum bob at the end of a rigid rod hung from a pivot. The state is (angle, velocity). Angle 0 means the bob is directly below the pivot. Angles $$180 put the bob straight above the pivot. Positive velocity corresponds to a counter-clockwise swing.

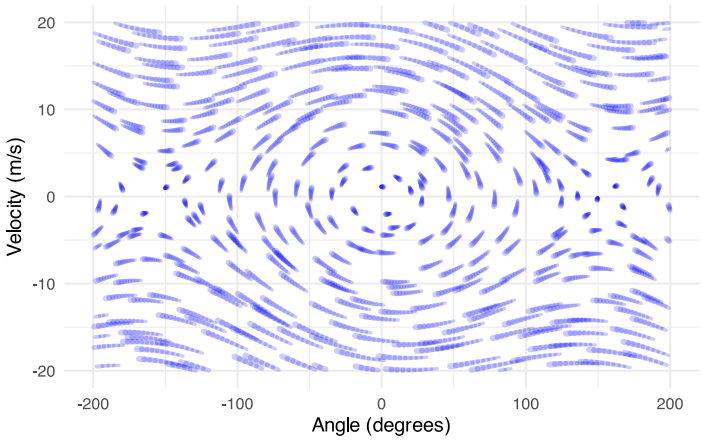



Figure 1: Dynamics of a pendulum

**Question 7.7** Draw a trajectory from the initial condition where the angle is $-120°$ and the velocity is zero. (Note: the initial angle is *negative*.)

**Question 7.8** Draw in the angle- and velocity-nullclines. They may have multiple segments.

**Question 7.9** Mark each of the fixed points in the graphic domain and indicate whether they are stable or unstable.