MOSAIC Calculus Quiz 6: Prof. Kaplan

April 22, 2025

Student name: ____________________. Do what you can in 20 minutes.

Question 7.1:

Refer to Figure 1. Write down the coefficients that solve for $\vec{\mathbf{b}}$ in terms of $\vec{\mathbf{a}}$ and $\vec{\mathbf{c}}$.

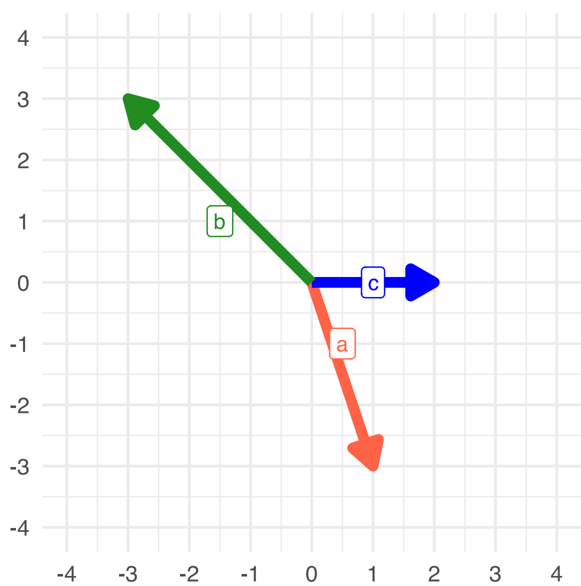

Figure 1

Question 7.2:

Using dot products (by hand), compute numerically the projection of $\vec{\mathbf{b}}$ onto $\vec{\mathbf{a}}$ and write down numerically the two vectors that result.

$$\vec{\mathbf{a}} \equiv \begin{pmatrix} -4 \\ 1 \\ 3 \\ 7 \\ 5 \end{pmatrix} \quad \text{and} \quad \vec{\mathbf{b}} \equiv \begin{pmatrix} 2 \\ 1 \\ 0 \\ -3 \\ 2 \end{pmatrix}$$

Question 7.3:

Referring to Figure 1, and rounding off the vector positions to integer values,

i. draw the projection of $\vec{\mathbf{b}}$ onto $\vec{\mathbf{c}}$.
ii. Using dot products, find the cosine of the angle between the two vectors.
iii. Estimate the R^2 of the projection.

Question 7.4:

Consider this matrix $\mathbf{M}$ and vector $\mathbf{b}$ and the task of solving $\mathbf{M}\,\vec{\mathbf{x}} = \vec{\mathbf{b}}$

$$\mathbf{M} \equiv \begin{pmatrix} 1 & 3 & 4 & -6 \\ 4 & -4 & 0 & 8 \\ 8 & 0 & 8 & 0 \end{pmatrix} \quad \text{and} \quad \vec{\mathbf{b}} \equiv \begin{pmatrix} -5 \\ -2 \\ 1 \end{pmatrix}$$

i. After defining `M` and `b` in R, I tried `qr.solve(M, b)`. The result was an error message, [Error in qr.solve(M, b) : singular matrix 'a' in solve`]{style = "color: blue;"}. Explain what went wrong.

ii. You could fix the problem by crossing out two of the vectors in $\mathbf{M}$. Figure out two that will do the job and X-them out.

iii. The result of the deletion in (ii) means that there will be a non-zero residual. Pencil in a new (third) vector for $\mathbf{M}$ that would permit *zero residual.* (Hint: don't overthink it!)

[**Note**: Flip the sheet for another question.]

<u>Question 7.5</u>:

Construct a matrix $\mathbf{Q}$ with mutually orthogonal vectors that spans the same space as the given $\mathbf{M}$. (The vectors do not need to be unit length. Let them be whatever length is easier for you.)

$$\mathbf{M} \equiv \begin{array}{c} \begin{matrix} \vec{x} & \vec{y} & \vec{z} \end{matrix} \\ \begin{pmatrix} 8 & 1 & 1 \\ 4 & 0 & -2 \\ 0 & 4 & 2 \\ 1 & -8 & 1 \end{pmatrix} \end{array}$$

For the sake of convenience, you can refer to the columns of $\mathbf{M}$ by the names $\vec{x}, \vec{y}$ and $\vec{z}$ respectively. (Hint: It might be easier than you are thinking.)