MOSAIC Calculus Quiz 6: Prof. Kaplan

April 22, 2025

Student name: ____________________. Do what you can in 20 minutes.

Question 7.1:

Refer to Figure 1. Write down the coefficients that solve for $\vec{b}$ in terms of $\vec{a}$ and $\vec{c}$.

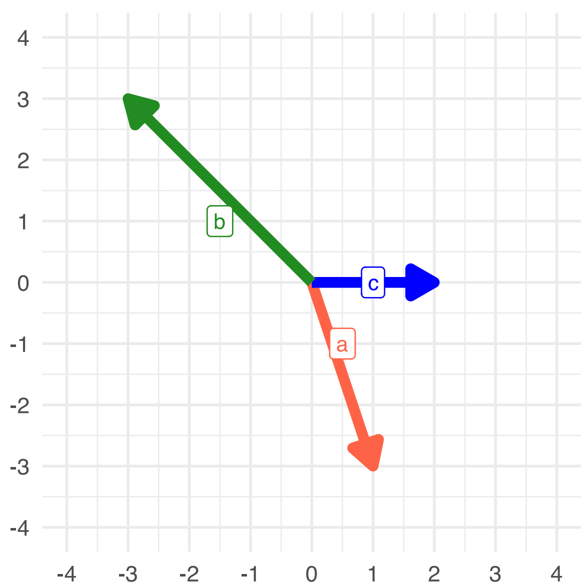

Figure 1

Question 7.2:

Using dot products (by hand), compute numerically the projection of $\vec{b}$ onto $\vec{a}$ and the residual from that projection. Write down numerically the two vectors that you found.

$$\vec{a} \equiv \begin{pmatrix} -4 \\ 1 \\ 3 \\ 7 \\ 5 \end{pmatrix} \quad \text{and} \quad \vec{b} \equiv \begin{pmatrix} 2 \\ 1 \\ 0 \\ -3 \\ 2 \end{pmatrix}$$

Question 7.3:

Referring to Figure 1, and rounding off the vector positions to integer values,

i. draw the projection of $\vec{b}$ onto $\vec{c}$.
ii. Using dot products, find the cosine of the angle between the two vectors.
iii. Estimate the R^2 of the projection.

Question 7.4:

Consider this matrix M and vector b and the task of solving $M\,\vec{x} = \vec{b}$

$$M \equiv \begin{pmatrix} 1 & 3 & 4 & -6 \\ 4 & -4 & 0 & 8 \\ 8 & 0 & 8 & 0 \end{pmatrix} \quad \text{and} \quad \vec{b} \equiv \begin{pmatrix} -5 \\ -2 \\ 1 \end{pmatrix}$$

i. After defining `M` and `b` in R, I tried `qr.solve(M, b)`. The result was an error message,

Error in qr.solve(M, b) : singular matrix 'a' in solve.

Explain what went wrong.

ii. You could fix the problem by crossing out two of the vectors in M. Figure out two that will do the job and X-them out.

iii. The result of the deletion in (ii) means that there will be a non-zero residual. Pencil in a new (third) vector for M that would permit *zero residual.* (Hint: don't overthink it!)

[**Note**: Flip the sheet for another question.]

Question 7.5:

Construct a matrix Q with mutually orthogonal vectors that spans the same space as the given M. (The vectors do not need to be unit length. Let them be whatever length is easier for you.)

$$M \equiv \begin{array}{c} \begin{matrix} \vec{x} & \vec{y} & \vec{z} \end{matrix} \\ \begin{pmatrix} 8 & 1 & 1 \\ 4 & 0 & -2 \\ 0 & 4 & 2 \\ 1 & -8 & 1 \end{pmatrix} \end{array}$$

For the sake of convenience, you can refer to the columns of M by the names $\vec{x}, \vec{y}$ and $\vec{z}$ respectively. (Hint: It might be easier than you are thinking.)