

Calculus Quiz 5: Prof. Kaplan

March 13, 2025

Student name: _____.

Do what you can in 15 minutes.

Question 5.1: What is $\partial_y (a_0 + a_1x + a_2x^2)$?

- (i) $a_1 + a_2x$ (ii) $a_1 + 2a_2x$ (iii) $a_1 + 2a_2y$ (iv) 0

Question 5.2: What is $\partial_t A x e^{kt}$?

- (i) 0 (ii) $A k e^{kt}$ (iii) $A k x e^{kt}$ (iv) $A e^{kt}$

Question 5.3: What is

$$\partial_{yx} [a_0 + a_1x + b_1y + cxy + a_2x^2 + b_2y^2] ?$$

- (i) c (ii) $2a_2$ (iii) $2b_2$ (iv) 0

Question 5.4: What is $\partial_{yx} h(x, y) g(y)$?

- i. $\partial_{yx} h(x, y)$
ii. $g(y) \partial_{yx} h(x, y) + h(x, y) \partial_y g(y)$
iii. $(\partial_y g(y)) (\partial_x h(x, y)) + g(y) (\partial_{yx} h(x, y))$
iv. $(\partial_x g(y)) (\partial_x h(x, y)) + g(y) (\partial_{xx} h(x, y))$

Question 5.5: What is $\partial_t (7 + 8t^2 + 3t^4)$?

- i. $16t + 12t^3$
ii. $8t + 4t^3$
iii. $16t^2 + 9t^3$
iv. $4t + 12t^2$

Question 5.6: What is $\partial_y h(x, y) g(y)$?

- i. $\partial_y g(y)$
ii. $g(y) \partial_y h(x, y)$
iii. 0
iv. $g(y) \partial_y h(x, y) + h(x, y) \partial_y g(y)$

Question 5.7: Which of the derivative rules should you use to find

$$\partial_t e^{t^2} ?$$

- i. The constant multiplier rule
ii. The linear combination rule
iii. The product rule
iv. The chain rule
v. No rule needed, it is so basic.

Question 5.8: Which of the derivative rules should you use to find

$$\partial_t e^t \sin(x) ?$$

- i. The constant multiplier rule
ii. The linear combination rule
iii. The product rule
iv. The chain rule
v. No rule needed, it is so basic.

Question 5.9: For the function

$$g(P) \equiv \sin\left(\frac{2\pi}{P}(t - t_0)\right)$$

is the interior function linear?

- (i) Yes (ii) No

Question 5.10: Here are several functions that are related by differentiation and integration:

- a. $\frac{1}{a} e^{ax+b}$
b. $a^2 e^{ax+b}$
c. $\frac{1}{a^2} e^{ax+b}$
d. e^{ax+b}
e. $a e^{ax+b}$

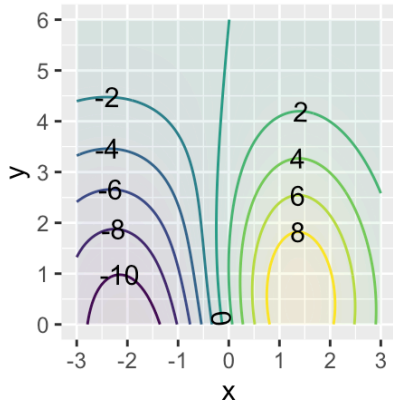
Put these functions in order that the derivative of each function precedes the anti-derivative.

- i. The order is b-e-d-a-c
ii. The order is b-d-e-c-a
iii. The order is b-d-c-e-a
iv. The order is e-b-d-c-a
v. The order is e-b-d-a-c

Question 5.11: There are two pattern-book functions whose **second** derivative is proportional to the function itself. Which are they?

- i. Exponential and sinusoid
ii. Exponential and sigmoid
iii. Exponential and logarithm
iv. Sinusoid and gaussian

Question 5.12: Consider the function shown in the following contour plot:



At which of these inputs is the function practically flat?

- i. $(x = 0, y = 6)$
- ii. $(x = 1, y = 2)$
- iii. $(x = -2, y = 3)$
- iv. $(x = 0, y = 1)$

Question 5.13: Imagine a second-order polynomial in three inputs: x , y , and z , like this:

$$b_0 + b_x x + b_y y + b_z z + b_{xy} xy + b_{xz} xz + b_{xx} x^2 + b_{yy} y^2 + b_{zz} z^2.$$

All of the possible second-order (or less) terms are shown, except for one. Which term is missing?

- i. the interaction between y and z
- ii. the quadratic term in z
- iii. the linear term in y
- iv. the constant term

Question 5.14: Suppose you know only this one fact about $f(x)$, that

$$[\partial_{xx} f(x)]_{x=7.3} = 1.6.$$

Which of these statements **must** be true?

- i. $f(x)$ is increasing at $x = 7.3$.
- ii. $f(x)$ is concave up and decreasing at $x = 7.3$
- iii. $f(x)$ is concave up at $x = 7.3$
- iv. $f(x)$ is concave up at $x = 7.3$, but eventually it will become concave down.

Question 5.15: Which of the following is the correct construction for $\partial_t g(t)$?

- i. $\lim_{h \rightarrow 0} \frac{g(t+h) - g(t)}{h}$
- ii. $\lim_{h \rightarrow 0} \frac{g(t+h) - g(t)}{t}$
- iii. $\lim_{h \rightarrow 0} \frac{g(t) - g(t+h)}{h}$
- iv. $\lim_{x \rightarrow 0} \frac{g(t+h) - g(t)}{h}$

Question 5.16: The derivative

$$\partial_x \text{dnorm}(x) = -x \text{dnorm}(x).$$

What is

$$\partial_x \text{dnorm}\left(\frac{x^2}{4}\right)?$$

- i. $-\frac{x^3}{8} \text{dnorm}\left(\frac{x^2}{4}\right)$
- ii. $-\frac{x}{2} \text{dnorm}\left(\frac{x^2}{4}\right)$
- iii. $-\frac{x}{8} \text{dnorm}\left(\frac{x^2}{4}\right)$
- iv. $-\frac{x^2}{2} \text{dnorm}\left(\frac{x^2}{4}\right)$