MOSAIC Calculus Quiz 9: Prof. Kaplan

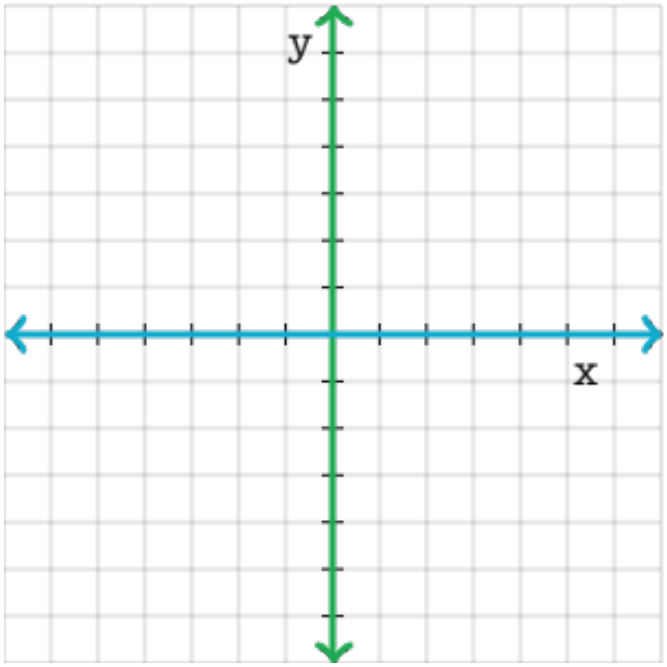
June 6, 2025

Student name: _____. Do what you can in 30 minutes.

This quiz is about the simple dynamical system

$$\partial_t x \equiv y, \quad \partial_t y \equiv -x.$$

Question 7.1 Draw the nullclines of the system on the graph plane below. Make sure to label clearly which is the x-nullcline and which is the y-nullcline.



Double check to make sure you haven't reversed the the nullclines. Hint: the coordinate point $(x = 1, y = 0)$ is on one of the nullclines.

Question 7.2 Draw in flow arrows on each of the nullclines to each side of the origin. (There will be 4 flow arrows altogether.)

Question 7.3 Draw in another four flow arrows, one in each of the four quadrants of the plane.

Question 7.4 Bye eye, draw a plausible trajectory for the flow, starting at whatever initial condition you choose (but **not** $(x = 0, y = 0)$).

Question 7.5 Consider the time series $x(t) = \sin(t)$ as a **candidate** solution starting at the initial condition $(x = 0, y = 1)$. Use anti-differentiation to find the corresponding $y(t)$.

Question 7.6 Modify the differential equations so that $x(t) = A\sin(\omega t)$ and its corresponding $y(t)$ are solutions, where ω describes the frequency of the oscillation. Write the modified equations here.

Turn the sheet over for an extra-credit problem.

Extra credit: Figure 1 shows another dynamical system, a pendulum bob at the end of a rigid rod hung from a pivot. The state is (angle, velocity). Angle 0 means the bob is directly below the pivot. Angles $$180 put the bob straight above the pivot. Positive velocity corresponds to a counterclockwise swing.

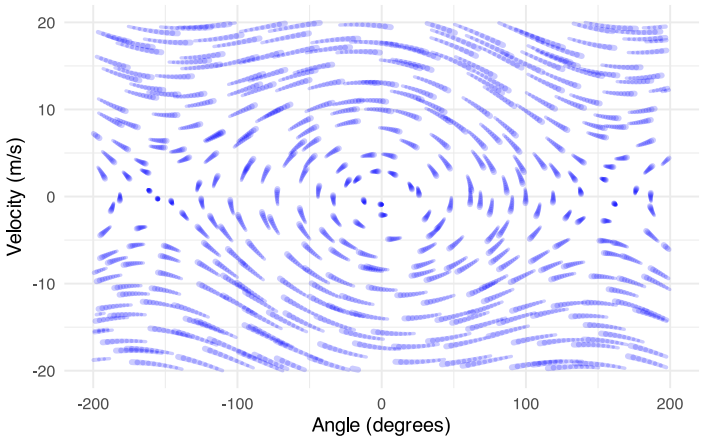


Figure 1: Dynamics of a pendulum

Question 7.7 Draw a trajectory from the initial condition where the angle is −120° and the velocity is zero. (Note: the initial angle is *negative*.)

Question 7.8 Draw in the angle- and velocity-nullclines. They may have multiple segments.

Question 7.9 Mark each of the fixed points in the graphic domain and indicate whether they are stable or unstable.