Calculus Quiz 5: Prof. Kaplan

March 20, 2025

Student name: _____________________. Do what you can in 30 minutes.

Question 6.1:

Which of these is a positive-going zero crossing of $g(t)$ where

$$g(t) \equiv \sin\left(\frac{2\pi}{5}t - 3\right) ?$$

(i) $t = 15/2\pi$ (ii) $t = 2\pi/15$ (iii) $t = 3$ (iv) None of the above

Question 6.2:

What is the **change** in the value of $f()$ when the input goes from 2 to 4?
Assume $f(x) \equiv 2x + 1$

(i) -4 (ii) -2 (iii) 2 (iv) 4 (v) 9

Question 6.3:

What is the **rate of change** in the value of $f()$ when the input goes from 2 to 4?
Assume $f(x) \equiv 2x + 1$

(i) -2 (ii) -4 (iii) 2 (iv) 4 (v) 9

Question 6.4:

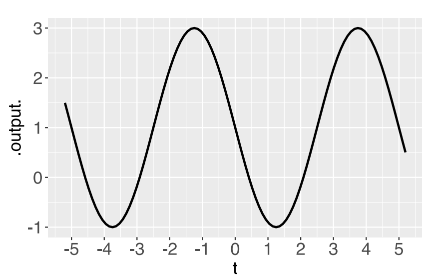

Figure 1: A periodic function

What is the period of the function graphed in Figure 1?

(i) 1 (ii) 2 (iii) 3 (iv) 4 (v) 5

Question 6.5:

If t is measured in seconds and A is measured in feet, what will be the dimension of $A\sin(2\pi t/P)$ when P is two hours?

(i) L (ii) T (iii) L/T

Question 6.6:

Which one of these is **not** the derivative of a pattern-book function?

(i) Reciprocal (ii) Zero (iii) One (iv) Sigmoid

Question 6.7:

Imagine a second-order polynomial in three inputs: x, y, and z, like this:

$$b_0 + b_x x + b_{xy} xy + b_{xz} xz + b_{xx} x^2 + b_z z +$$
$$+ b_y y + b_{yy} y^2 + b_{zz} z^2 .$$

All of the possible second-order (or less) terms are shown, except for one. Which term is missing?

i. the interaction between y and z
ii. the quadratic term in z
iii. the linear term in y
iv. the constant term

Question 6.8:

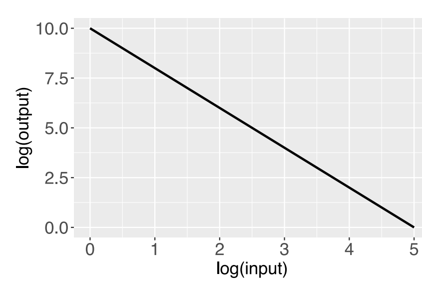

Figure 2: Pay attention to the axis scale.

What is the correct form for the relationship shown in Figure 2?

i. $g(x) \equiv 10x^{-2}$
ii. $g(x) \equiv e^{10}e^{-1.5x}$
iii. $g(x) \equiv e^{10}x^{-2}$
iv. $g(x) \equiv e^{10}x^{-1.5}$

Question 6.9:

Suppose $a = 25\text{ft}$ and $d = 1\text{meter}$

Is this combination dimensionally valid?

$$\sqrt[3]{a^2 d}$$

Why or why not?

i. Invalid. You cannot raise a dimensionful quantity to a non-integer power.
ii. Valid. $a^2 d$ is a volume: L^3. The cube root of L^3 is L.
iii. Invalid. 25 feet squared is 625 square feet. It makes no sense to multiply square feet by meters.

Question 6.10:

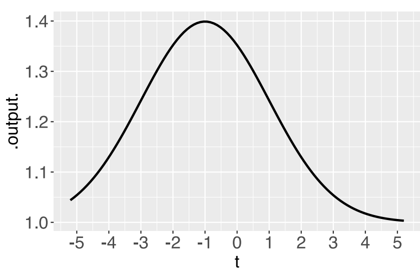

Figure 3: A bump function

One of the following choices is the standard deviation of the function graphed in Figure 3. Which one?

(i) 0 (ii) 1 (iii) 2 (iv) 3 (v) 4

Question 6.11:

Engineers often prefer to describe sinusoids in terms of their *frequency* ω, writing the function as $\sin(2\pi\omega t)$, where t is time.

What is the dimension of ω?

(i) T^{-1} (ii) T (iii) T^2

Question 6.12:

Which pattern-book function is the **anti-derivative** of the gaussian dnorm()?

Question 6.13:

What is $\partial_x \ln(x)/x^2$? (Hint: A simple trick will turn the division into multiplication!)

i. $-2x^{-1}\ln(x)$
ii. $-2x^{-3}\ln(x)$
iii. $x^{-3}(1 - 2\ln(x))$
iv. $-2x^{-3}(1/x - 1)$

Question 6.14:

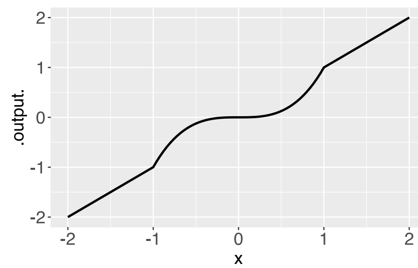

Figure 4: A wiggly function

Which of the following tilde-expressions could be used to generate the graph in Figure 4?

i. `ifelse(x > 1, 1, x^2) ~ x`
ii. `ifelse(x > 0, sin(x), x) ~ x`
iii. `ifelse(abs(x) > 1, x, x^3) ~ x`
iv. `ifelse(abs(x) > 1, x^3, x) ~ x`
v. `ifelse(abs(x) > 1, x, exp(x^2)) ~ x`

Question 6.15:

Which of the following shapes of functions is **not** allowed? If the shape is allowed, make a tiny drawing of the shape next to the item.

1. Increasing and concave up.
2. Decreasing and concave up.
3. Increasing and concave down.
4. Decreasing and concave down.
5. None of them are allowed.
6. All of them are allowed.

Question 6.16:

1. Which of the following is the reason that

$$\lim_{x \to 0} \sin(1/x)$$

does not exist?

i. Because no matter how close x gets to 0, there are x's near 0 for which $\sin(1/x) = 1$, and some for which $\sin(1/x) = -1$.
ii. Because the function values oscillate around 0
iii. Because $1/0$ is undefined.
iv. all of the above

Question 6.17:

You're not expected to get this one, but write down an idea about how you might go about finding the answer.

Find the derivative of $g(x)^{f(x)}$

Question 6.18:

Using $f'()$ and $g'()$ to stand for the derivatives $\partial_x f(x)$ and $\partial_x g(x)$, write down the derivative w.r.t. x of each of these functions:

1. $Af(x) + B$
2. $f(x)g(x)$
3. $f(x) + g(x)$
4. $\log(g(x))$
5. $\exp(f(x))$
6. $f(g(x))$
7. $1/g(x)$
8. $\mid g(x)) \mid$

Question 6.19:

$H()$ is the Heaviside ("step") function. Write down the derivative of $H(x)g(x)$.

Question 6.20:

For the function

$$h(u) \equiv \ln\left(a^2 u - \sqrt{b}\right)$$

is the interior function linear?

(i) Yes (ii) No

Question 6.21:

Which of these is $\partial_t(ln(6) + t^4 - e^t)$?

i. $\frac{1}{6} + 4t^3 - e^t$
ii. $\frac{1}{6} + 4t^3 - e^{-t}$
iii. $4t^3 - e^{-t}$
iv. $4t^3 - e^t$

Question 6.22:

Which of these is a reasonable definition of a derivative?

i. A derivative is a function whose value tells, for any input, the instantaneous rate of change of the function from which it was derived.
ii. A derivative is the slope of a function.
iii. A derivative is a function whose value tells, for any input, the instantaneous change of the function from which it was derived.

Question 6.23:

What is $\partial_y \partial_x [\, a_0 + a_1 x + b_1 y + cxy + a_2 x^2 + b_2 y^2 \,]$?

(i) c (ii) $2a_2$ (iii) $2b_2$ (iv) 0

Question 6.24:

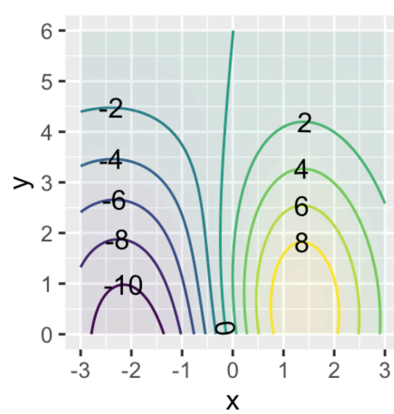

Figure 5: An imagined terrain.

You are standing in the terrain shown in Figure 5, the position $(x = -1, y = 4)$. In terms of the compass points (where north would be up and east to the right), which direction points most steeply uphill from where you are standing.

(i) NE (ii) SE (iii) SW (iv) NW

Question 6.25:

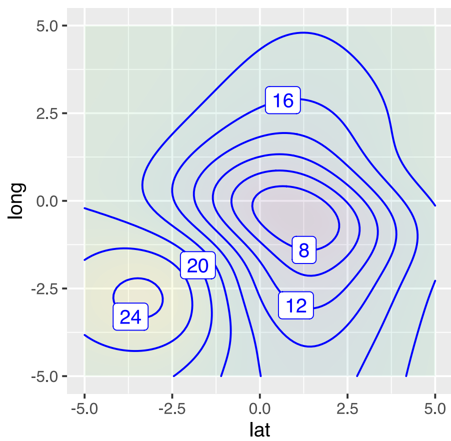

Figure 6: A different terrain.

On Figure 6, mark all local maximum with the letter "T" and all local minima with the letter "B."

Question 6.26:

Again with respect to Figure 6, draw 10 gradient vectors at scattered points of your choice. Make the length of the vector (roughly) proportional to the steepness of the terrain.

Question 6.27:

Using Figure 7 mark every local maximum with "T" and every local minimum with "B."

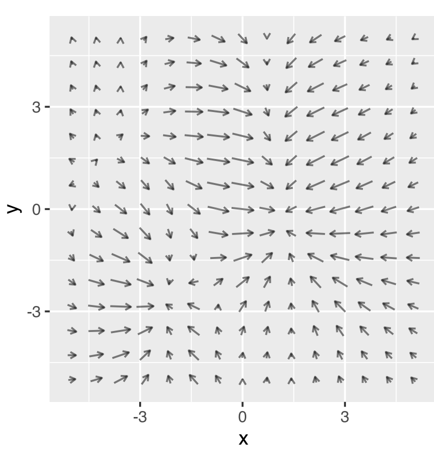

Figure 7: A gradient field.

Question 6.28:

For the function $f(x)$, write down the first five terms of the Taylor polynomial expansion around $x_0 = 0$. Use prime notation, for instance $f''(x)$

Question 6.29:

Here is a Taylor polynomial for a familiar, even, pattern-book function, centered on $x_0 = 2$.

$$1 - \frac{(x-2)^2}{2!} + \frac{(x-2)^4}{4!} + \ldots$$

Paying attention only to the zeroth-order and second-order terms (that is, ignoring $(x-2)^4/4!$ and everything that might follow), expand out the second-order term algebraically and re-write the result in standard polynomial order (that is, $a_0 + a_1 x + a_2 x^2$).

Is the resulting function even around $x = 0$? Briefly explain the reasoning behind your conclusion?

Question 6.30:

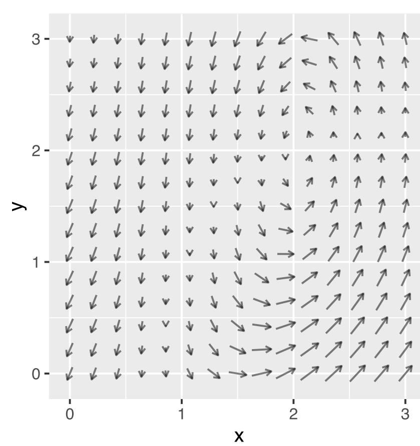

Figure 8: A vector field.

Does the vector field in Figure 8 show the gradient of a function. Explain your reasoning.