Thinking about skills/objectives

The idea is to divide along two directions:

1. The kind of action the student should be able to perform.
2. The topic to which (1) applies

Basic categories of action

- Graphical interpretation: **Gi**
- Graphics construction: **Gc**
- Symbolic computation: **Sc**
- Notation reading: **Nr**
- Numeric computation: **Co**mputer
- Hand computation: **Ha**nd
- Establish consistency: **Co**nfirm
- Conceptual relationship: **Re**
- Recognize and use vocabulary appropriately: **Vo**

Topics

Functions Fun

100. Notation for function creation: inputs, output, parameters, $\equiv$, `makeFun()`
101. Notation conventions: scalars vs functions
102. Evaluation of function
    a. from formula
    b. on computer
103. Shapes of functions
    a. slope, concavity, curvature, discontinuity, extrema, monotonic, inflection point
104. Pattern-book functions
105. Domain and range
106. Parameters (basic modeling functions)
    a. parameters with 'makeFun()
107. Input centering e.g $[x - x_0]$
108. Input scaling
109. Slope at a point
    a. Directional slope at a point
    b. Working with contours
110. Piecewise functions
111. Vector-valued functions (e.g. gradient)
112. Table as a function, graph as a function [be able to evaluate, find slope, accumulate]
113. Assembling functions
    a. linear combination (coefficients, scalar multipliers)
    b. composition
    c. product
114. Partial evaluation (e.g. in anti-derivatives or taking a slice)
115. Functions from data
    a. Splines and other interpolants
        i. constructing by software
        ii. when is smoothness necessary (cubic spline) or not wanted (Bezier)
    b. Fitting by eye for basic modeling functions
    c. Least squares fitting
    d. Evaluate function from a table (including multivariate)

116. Exponentials
    a. Interpretation as half-life, doubling-time
    b. Complex exponentials and interpretation as damped (or growing) sinusoid
        i. determine frequency from eigenvalues
117. Logarithms
    a. Graphical uses
    b. Graphics for functions
    c. slice plot
    d. contour plot
    e. surface plot
    f. vector field
    g. graphical domain
    h. paths and constraints
118. Parametric description of path as a set of functions of t.

Optimization Opt

650. Vocabulary: argmax vs max
651. Objective function
652. Incommensurate objectives
653. Constraints
    a. Active or not
    b. Equality and inequality
    c. Shadow price (Lagrange multiplier)
    d. constraint functions
    e. parallel gradients for constraint function and objective function at argmax
654. Numerical techniques
655. Derivatives and critical points
656. Gradient vector points uphill, not necessarily toward optimum
657. Gradient ascent/descent algorithm
658. Find argmax/min from gradient field
659. Find argmax/min from contour plot

Data Dat

200. Organization of a data frame
201. Point plot
202. Estimating function parameters (linear and nonlinear for basic modeling functions)
    a. straight-line
    b. exponential
    c. sinusoid
    d. gaussian and sigmoid
    e. distinguish between power-law and exponential
    f. log axes
203. Function as a table
204. Residual between response var and output of a model function

Dimension analysis Dim

300. Fundamental dimensions
301. Compound dimensions and dimensional arithmetic
    a. invalid operations on dimensional quantities
    b. Know dimension of basic physical quantities (acceleration, force, energy, power, pressure, area, volume, work=force x distance, energy = power x time, power = force x velocity)

302. Determine dimensional consistency
303. Unit conversion and flavors of 1
304. Dimension of derivatives and anti-derivatives
305. Power-law functions

Calculus operations Cal

400. Vocabulary
401. With-respect-to input
402. Notations: $\partial_t x$, $\dot{x}$, $\frac{dx}{dt}$, $\frac{\partial x}{\partial t}$
403. Accumulation **Acc**
    a. Symbolic anti-differentation
        i. pattern book functions
        ii. basic modeling functions (simple chain rule)
    b. Appropriate notation
    c. Constant of integration (also form from `antiD()`)
    d. Definite integral, net change
    e. Numerical integration and anti-differentiation
    f. "Area under a curve" and why it's signed area
    g. Fundamental theorem
        i. relationship between differentiation and anti-differentiation
        ii. confirm anti-derivative by differentiation
    h. Estimate from graph
    i. anti-differentiation: output is a function
    j. integration: output is a quantity
    k. `antiD()` in R and how to evaluate it for definite integral
    l. calculating work by integrating force over distance
    m. calculate energy by integrating power over time
    n. calculate center of mass, centroid, moment of inertia
    o. find probability that random variable is in a range
    p. discounting
404. Differentiation **Dif**
    a. Symbolic differentiation
        i. pattern book functions
        ii. basic modeling functions (simple chain rule)
        iii. chain rule
        iv. product rule
        v. sum rule
        vi. by computer
    b. Numerical differentiation
    c. Second and higher-order derivatives
    d. Partial derivatives
        i. first order
        ii. second-order, including mixed partials
    e. Gradient vector
        i. relationship to contours
    f. Order of smoothness
    g. Relationship to velocity and acceleration
    h. output is a function
    i. identifying monotonicity, concavity, curvature, critical points
    j. tangents: line, circle, polynomial
405. Argmax finding

Other operations

500. Zero finding (and inversion, solution)
501. Find zeros of linear functions by hand
502. Simultaneous zeros (e.g. crossings of nullclines)
503. Iteration
    a. improvement
    b. Newton method

Calculus theory Theory

600. Vocabulary
601. Slope function
602. Finite-difference approximation to derivative
    a. limit definition of a derivative
603. Euler accumulation
604. Value/Existence for a limit
605. Taylor polynomials
    a. Pick center x_0
    b. Memorize for sin(), cos(), exp()
    c. Compute error for a given input, characterize error as a function of $x - x_0$.
606. Local linear approximation
607. Differentials
608. Instantaneous vs average rate of change

Approximation Approx

700. Vocabulary
701. Polynomials
    a. linear, interaction, quadratic terms
702. Linearization
703. Low-order polynomial approximation

Linear algebra Lal

800. Vocabulary
801. Acceleration, position, momentum, velocity are vector quantities
802. Vectors are rootless, have just direction and magnitude
803. Vector operations (add, scale, dot product)
804. Matrices
805. Target problem
806. Eigenvectors and eigenvalues
807. Subspace
    a. Is a vector a member of a subspace? (numerically)
    b. Projection
    c. Residual

Dynamics Dyn

900. Dynamical system as set of first-order differential equations
901. Numerical integration from initial condition
902. Symbolic integration:
    a. first-order linear
    b. eigenvalues, characteristic values and stability
    c. exponential ansatz

    d. linear combinations of solutions are solutions in linear systems
903. Flows
904. Trajectories
905. Time series
906. Nullclines
907. Unlimited and limited growth models, logistic growth
908. Bifurcation (appearance and disappearance of fixed points in 1-D)
909. Classic models: predator-prey, SIR
910. Force-balance and equivalence to pair of first-order differential equations
911. Identify physical quantities (resistance, restoring force) in force-balance

Probability Prob

1000. meaning of random variable, event
1001. probability density vs relative density
1002. normalization of relative density
1003. expected value, variance, standard deviation,

Computer Comp

1100. Translate math formula into R formula and vice versa
1101. Construct a function with parameters
    a. set default values for parameters
1102. Graphics layers

Modeling concepts Mod

1200. Different models for different purposes
1201. Frameworks:
    a. low-order polynomials
        i. which terms are appropriate
        ii. constructing from first principles
        iii. constructing from data
        iv. limitations for modeling (dog chasing squirrel)
    b. differential equations
1202. Modeling cycle