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POD Lecture 07

Quenching sphere w/ Biot #

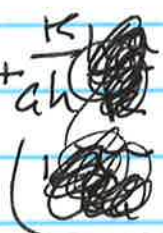
$$T|_{t=0} = T_1, \quad q_r|_{r=a} = h(T|_{r=a} - T_0) = -k \frac{\partial T}{\partial r}|_{r=a}$$

$$\therefore \rho \hat{C}_p \frac{\partial T}{\partial t} = k \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right)$$

$$T^* = \frac{T - T_0}{T_1 - T_0}, \quad t^* = \frac{t k}{\rho \hat{C}_p a^2} = \frac{\alpha t}{a^2}, \quad r^* = \frac{r}{a}$$

$$\therefore h T^*|_{r^*=1} = -\frac{k}{a} \frac{\partial T^*}{\partial r^*}|_{r^*=1}$$

$$\text{or } T^*|_{r^*=1} + \frac{k}{ah} \frac{\partial T^*}{\partial r^*}|_{r^*=1} = 0$$


 $\rightarrow \frac{1}{Bi}$

$$\frac{\partial T^*}{\partial r^*}|_{r^*=0} = 0 \quad (\text{symmetry, finite value})$$

$$\text{So: } \frac{\partial T^*}{\partial t^*} = \frac{1}{r^{*2}} \frac{\partial}{\partial r^*} \left(r^{*2} \frac{\partial T^*}{\partial r^*} \right)$$

$$T^* = F(r^*) G(t^*)$$

$$\therefore \frac{G'}{G} = \frac{(r^{*2} F')'}{r^{*2} F} = -\lambda \quad ; \quad G = e^{-\lambda t^*} \quad (2)$$

$$(r^{*2} F')' - \lambda r^{*2} F = 0$$

$$F'(0) = 0 \quad F(1) + \frac{1}{B_i} F'(1) = 0$$

$$\text{So } T^* = \sum_{n=1}^{\infty} A_n e^{-\lambda_n t^*} F_n(r^*)$$

$$\text{we seek } T^* \Big|_{r^*=0} \text{ as } f^n(t^*)$$

This has an analytic solution, but it is a bit messy! (many such sol. are found in Carslaw & Jaeger - nice compendium)
Do it numerically instead!