Bolt Analysis Tool

User Manual

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Contents

Symbols and Abbreviations

Symbols

α_A	tightening factor (assembly uncertainty factor)
α_b	coefficient of linear thermal expansion of the bolt
α_{ci}	coefficient of linear thermal expansion of the clamped part i
δ_b	elastic compliance of the bolt
δ_c	elastic compliance of the clamped part (plate)
ε_{th}	linear thermal elongation
θ	half angle of thread groves
$\varkappa$	parameter for elastic or plastic shear stress behaviour
λ	under-head bearing angle of bolt
μ_T	coefficient of friction in the clamped part interfaces
μ_{th}	coefficient of friction in bolt thread
μ_{uh}	coefficient of friction under bolt head
ν	bolt utilization factor
φ	helix angle / slope of bolt thread
φ_c	compression cone half angle
Φ	load factor of concentric joint (also: force ratio or relative compliance factor)
Φ_n	load factor for concentric clamping and concentric force load introduction via the clamped parts
ρ	friction angle in bolt thread
σ_M	normal stress in the bolt at F_M
$\sigma_{v,M}$	von-Mises equivalent stress in the bolt at F_M
σ_u	material ultimate strength
σ_y	material yield strength
τ_M	shear stress in the bolt at F_M
A_0	relevant stress cross section (BAT: $A_0 = A_s$)
A_1	nominal cross section of threaded bolt
A_3	minimal thread cross section
A_i	cross sectional area of component / segment i
A_p	pitch cross section of threaded bolt
A_s	stress cross section of threaded bolt

A_{sub}	substitutional compliance area (BAT specific)
c	stiffness
c_b	bolt stiffness
c_c	clamp part stiffness
d	nominal threaded bolt diameter
d_0	relevant stress diameter (BAT: $d_0 = d_s$)
d_2	pitch diameter of threaded bolt
d_3	minimal diameter of threaded bolt
d_h	minimal contact diameter under bolt head
d_s	stress diameter of threaded bolt
D_{avail}	available diamter for compression zone
D_{hole}	through-hole diameter (drilled bolt hole)
D_{Km}	effective diameter of under head/nut friction torque
D_{lim}	limiting diameter of compression zone
E_b	Young's Modulus of bolt
E_c	Young's Modulus of clamped part
F_A	external, axial bolt load
F_K	clamp load
F_{KR}	residual clamp load at the interface during loading in service
F_{Kreq}	required clamping force for friction grip per bolt
F_M	preload after tightening / assembly preload
f_P	clamped part compression
f_{PA}	additional deformation of clamped part due to loading
F_{PA}	additional axial plate load
F_Q	external, shear bolt load
f_S	bolt elongation
F_S	bolt load
f_{SA}	additional elongation of bolt due to loading
F_{SA}	additional axial bolt load
ΔF_{Vth}	thermal preload change
F_V	service preload incl. embedding and thermal influence
f_Z	plastic deformation due to embeddding
F_Z	preload loss due to embedding
K	joint coefficient
k_τ	shear stress reduction coefficient
L_i	length of component/segment i
ΔL	total change in length after temperature loading (bolt and clamped parts)
Δl_b	change in length of bolt after temperature loading
l_{0b}	initial length of bolt ($= l_K$) without temperature loading
Δl_c	change in clamped part thickness after temperature loading

l_{ci}	thickness of clamp part i
l_{0ci}	initial thickness of clamp part i without temperature loading
$L_{eng,sub}$	substitutional length of engaged thread
$L_{h,sub}$	substitutional length of bolt head
l_K	joint clamped length
$L_{n,sub}$	substitutional length of nut (locking device)
M_{th}	friction torque at thread interface
M_p	prevailing torque of bolt locking device
M_{uh}	under-head torque due to friction
n	loading plane factor / load introduction factor
p	pitch of bolt thread
q_F	number of shear force transmitting interfaces
ΔT	temperature gradient
T_A	total installation torque of bolt
$T_{scatter}$	torque scatter of tightening device (torque wrench)
W_p	polar section modulus / polar moment of resistance
w	parameter for δ_c calculation (TBJ: $w = 1$, TTJ: $w = 2$)
x	first parameter for φ_c calculation
x_c	parameter for δ_c calculation for multiple clamped parts (BAT specific)
y	second parameter for φ_c calculation

Abbreviations

BAT	Bolt Analysis Tool
CTE	Coefficient of Thermal Expansion
MOS	Margin of Safety
TBJ	Through-Bolt Joint
TTJ	Tapped Thread Joint

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1 Introduction

This document includes the Bolt Analysis Tool **BAT** user manual and a description of the used analyis methodologies and algorithms.

The *Bolt Analyis Tool* **BAT** is an input file based Python3 command line tool for multi-bolt analyses. A graphical user interface *GUI* designed in pyQt5 is available for easy useability. **BAT** is an open source project and is released under GNU General Public License v3.0.

BAT Info:
The current design status of the *Bolt Analysis Tool* **BAT** only includes *Concentric Axially Loaded Bolted Joints*.

2 Bolt and Thread Geometry

D_{Km} is the *effective diameter of under head/nut friction torque* and is defined by

$$D_{Km} = \frac{D_{hole} + d_h}{2} \tag{2.1}$$

where D_{hole} is the *through-hole diameter* in the clamped parts and d_h is the *minimum bearing surface outer diamter* of the bolt head or nut. An other input value to calculate the under-head friction torque (4.2) is the *under head bearing angle* λ seen in Figure 2.1.

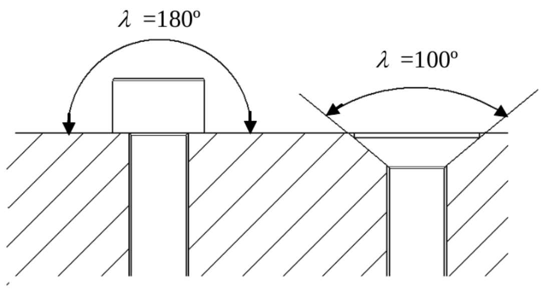
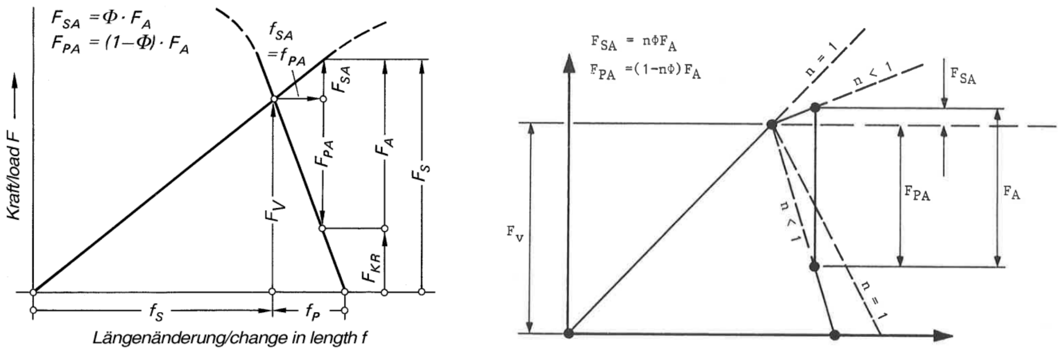


Figure 2.1: Definition of under head bearing angle [2]

3 Joint Diagram

The *joint diagram* seen in Figure 3.1(a) [3] visualizes the forces and displacements and helps to understand the loading conditions of a concentrically loaded bolted joint.



(a) without loading plane effect $n = 1$ [3] (b) with loading plane effect $n < 1$ [1]

Figure 3.1: Joint diagram for the working state of a concentrically loaded bolted joint

The preload force F_V (see §4.1) compresses the clamped parts f_P and elongates the bolt f_S during tightening. If an external load F_A is acting on the bolted joint the bolt is streched much further and the clamped parts are relaxed ($f_{SA} = f_{PA}$) due to the loading. The *additional axial bolt load* F_{SA} and the *additional axial plate load*[1] F_{PA} are given to

$$F_{SA} = n\Phi F_A, \qquad F_{PA} = (1 - n\Phi)F_A \tag{3.1}$$

where Φ is the load factor. n is the *loading plane factor* and is crucial for determining the size of the additional bolt loads (for detailed information see §3.1). As seen in Figure 3.1(a) F_{KR} is the residual clamping load at the interface during relief or loading by F_{PA} and after embedding in service and F_S is the maximum bolt load.

[1]If the bolt is loaded in tension F_{PA} is better described as an *axial plate relaxation force.*

The load factor Φ is the quotient of the addition bolt load F_{SA} and the axial working load component F_A.

$$\Phi = \frac{F_{SA}}{F_A} \tag{3.2}$$

The elastic compliance of the bolt δ_b (see §4.4.1), the elastic compliance of the clamped parts δ_c (see §4.4.2) and an estimation of the load introduction factor n (see §3.1) are required in order to deter- mine the load factor Φ [3]. For a *concentric loading and clamping* the load factor Φ_n with included loading plane factor n is calculed as follows.

$$\Phi_n = n \cdot \frac{\delta_c}{\delta_b + \delta_c} \tag{3.3}$$

3.1 Loading Plane Factor

A simple explanation of the *loading plane effect* is given in [1] and this text is used directly for the description in this document.

Figure 3.2 shows the external load applied at planes under the bolt head and under the nut. The extreme case where the external bolt force F_A is acting directly under head / nut is rare ($n = 1$), usually the effective loading planes are considered to be within the joint as seen in Figure 3.2, nl_K apart ($n < 1$).

Between the loading planes the joint is reliefed by F_A but outside of the loading planes the clamped material is subjected to F_A in addition to F_K the clamping load. The effect of this to the loading diagram is shown in Figure 3.1(b). The effective bolt stiffness is reduced and the effective joint stiffness increased as n reduces. Hence the closer the load application is to the joint interface the smaller is the external force seen by the bolt F_{SA}.

For a more detailed explanation of the *loading plane factor* and for the correct derivation and analysis of n see [3, 2].

Approximate Method [2]:
The loading plane factor depends on the deformation of the joint caused by preload. For uncritical verification purposes with simple joint geometry the loading plane factor may be set to $n = 0.5$, which assumes that the loading planes are at the center of each flange.

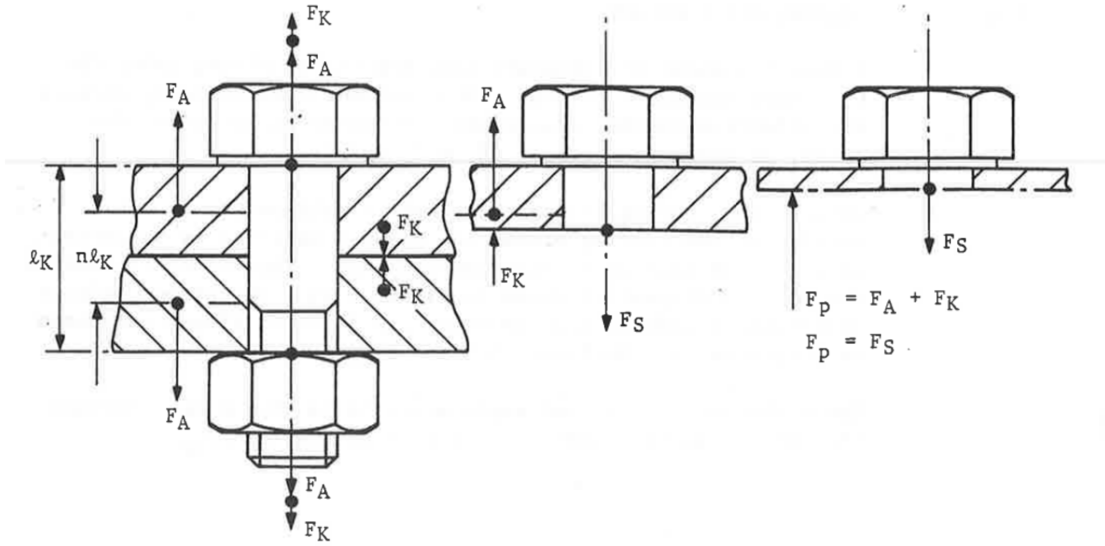


Figure 3.2: Tension joint loading planes and the forces acting within the joint [1]

4 Method B: ECSS-E-HB-32-23A

This chapter provides a quick overview and summary of the equations used in **BAT**. A detailed description can be found in the complete ECSS-E-HB-32-23A ESA handbook [2]. Some used variables in the following equations have been changed compared to [2] by the author to increase clarity and consistency.

4.1 Preload and Torques

The torque present at the thread interface M_{th} is dependent of the *axial bolt preload* F_V and is given by

$$M_{th} = F_V \tan(\varphi + \rho) \frac{d_2}{2} \tag{4.1}$$

and the *under-head torque* M_{uh} due to friction between bolt head or nut and the adjacent clamped part (or shim) is defined by

$$M_{uh} = F_V \frac{\mu_{uh} D_{Km}}{2} \frac{1}{\sin {}^{\lambda}\!/_{2}} \tag{4.2}$$

where λ is the *under head bearing angle* seen in Figure 2.1. It is assumed that the friction force for M_{uh} is acting at mean bearing radius of the bolt head D_{Km} (2.1). φ is the helix angle of the thread and ρ is given by the relation

$$\tan \rho = \frac{\mu_{th}}{\cos {}^{\theta}\!/_{2}} \tag{4.3}$$

where θ is the half angle of the thread groves (for Unified or Metric threads $\theta = 60°$).

The *total installation torque* T_A (without torque device scatter) applied to bolt head or nut during tightening to produce the axial bolt preload F_V is

$$T_A = M_{th} + M_{uh} + M_p \tag{4.4}$$

where M_p is the *prevailing torque* of the locking device. With the approximation $\tan\varphi \tan\rho \ll 1$ the expression $\tan(\varphi+\rho)$ can be written as $\tan(\varphi+\rho) \approx \tan\varphi + \tan\rho$. Now equation (4.4) can be rewritten to

$$T_A = F_V \underbrace{\left[\frac{d_2}{2}\left(\tan\varphi + \frac{\mu_{th}}{\cos\theta/2}\right) + \frac{\mu_{uh} D_{Km}}{2\sin\lambda/2}\right]}_{K} + M_p \tag{4.5}$$

where K is the *joint coefficient*.

BAT Info:
For calculation of the minimum and maximum axial bolt preload, BAT implements the *experimental coefficient method* [2] with an explicit torque scatter torque of the tightening device $T_{scatter}$.

The minimum and maximum total installation torques with included torque scatter are defined

$$T_A^{min} = T_A - T_{scatter}, \qquad T_A^{max} = T_A + T_{scatter}. \tag{4.6}$$

To calculate the minimum and maximum *axial bolt preload after tightening* $F_M^{min/max}$, (4.5) and (4.6) are combined

$$F_M^{min} = \frac{T_A^{min} - M_p^{max}}{K^{max}}, \qquad F_M^{max} = \frac{T_A^{max} - M_p^{min}}{K^{min}}. \tag{4.7}$$

If also the thermal influence and embedding is considered, this leads to the minimum and maximum *axial bolt preload at service* $F_V^{min/max}$

$$F_V^{min} = \frac{T_A^{min} - M_p^{max}}{K^{max}} + \Delta F_{Vth} - F_Z \tag{4.8a}$$

$$F_V^{min} = F_M^{min} + \Delta F_{Vth} - F_Z \tag{4.8b}$$

$$= \frac{T_A^{min} - M_p^{max}}{\frac{d_2}{2}\left(\tan\varphi + \frac{\mu_{th}^{max}}{\cos\theta/2}\right) + \frac{\mu_{uh}^{max} D_{Km}}{2\sin\lambda/2}} + \Delta F_{Vth} - F_Z \tag{4.8c}$$

$$F_V^{max} = \frac{T_A^{max} - M_p^{min}}{K^{min}} + \Delta F_{Vth} \tag{4.9a}$$

$$F_V^{max} = F_M^{max} + \Delta F_{Vth} \tag{4.9b}$$

$$= \frac{T_A^{max} - M_p^{min}}{\frac{d_2}{2}\left(\tan\varphi + \frac{\mu_{th}^{min}}{\cos \theta/2}\right) + \frac{\mu_{uh}^{min} D_{Km}}{2 \sin \lambda/2}} + \Delta F_{Vth} \tag{4.9c}$$

where ΔF_{Vth} is thermal preload change (see §4.2) and F_Z is the preload loss due to embedding (see §4.3).

The tightening factor α_A (assembly uncertainty factor) which takes into account the scatter of the achievable assembly preload between F_M^{max} and F_M^{min} is introduced in the following form

$$\alpha_A = \frac{F_M^{max}}{F_M^{min}} \tag{4.10}$$

4.2 Thermal Influcence

If a thermal load is applied to a bolted joint, the bolt sees a change ΔF_{Vth} in the preload force due to the CTE mismatch of bolt and clamped parts seen in (4.8c) and (4.9c).

4.2.1 Linear Thermal Influence

For the following derivation it is assumed that the Young's Modulus E of bolt and clamped parts does not change with temperature (temperature independent material properties). $c_b = 1/\delta_b$ and $c_c = 1/\delta_c$ are defined as bolt stiffness and clamp-part stiffness respectively. Linear thermal elongation is defined

$$\varepsilon_{th} = \alpha \Delta T = \frac{\Delta l}{l_0}$$

The thermal elongation for bolt (index b) and clamped-parts (index c) are given to

$$\Delta l_b = \alpha_b \cdot \Delta T \cdot l_{0b}$$
$$\Delta l_c = \sum_i \alpha_{ci} \cdot \Delta T \cdot l_{0ci}$$

where $l_K = l_{0b} = \sum_i l_{0ci}$ is the clamping length of the joint.

BAT Info:
The sign definition in BAT is $\Delta L = \Delta l_c - \Delta l_b$ and this leads to

$$\alpha_c > \alpha_b \Rightarrow +\Delta F_{Vth}$$
$$\alpha_c < \alpha_b \Rightarrow -\Delta F_{Vth}$$

where $+\Delta F_{Vth}$ defines an increase and $-\Delta F_{Vth}$ a loss in bolt preload.

If the standard stiffness equation $F = c \cdot \Delta x$ is used for the bolt / clamp-part joint, this leads to

$$\Delta F_{Vth} = c \cdot \Delta L \tag{4.13a}$$

$$= \frac{\Delta L}{\frac{1}{c_b} + \frac{1}{c_c}} \tag{4.13b}$$

$$= \Delta L \frac{c_b c_c}{c_b + c_c} = \Delta L \frac{1}{\delta_b + \delta_c} \tag{4.13c}$$

4.2.2 VDI Method

to be filled

4.3 Embedding

When bolts are first tightened the male and female thread, the under-head and under-nut surfaces and the clamped parts interface contact each other only on microscopically small high spots (surface roughness). The material at these hight spots will be overloaded, well past their yield point, during initial tightening and will subsequently creep until a large enough area of the available contact surface has been engaged to stabilize the process. In addition, plastic flow will often occur at the highest stressed points such as thread roots or at the first engaged thread in the nut. These relatively short-term relaxation effects are known as *embedding*. After tightening the rate of relaxation is a maximum, reducing exponentially, usually over the first few minutes, to a constant very low rate of creep. Typically embedding accounts for only a few percent loss of initial preload, however 5% to 10% are not uncommon [2]. The value significantly depends on the amount and hardness of the clamping parts.

The *embedding preload loss* F_Z depends on the plastic deformation f_Z of the joint.

$$\frac{F_Z}{F_V} = \frac{f_Z}{(\delta_b + \delta_c)\, F_V}$$

This it follows that,

$$F_Z = \frac{f_Z}{\delta_b + \delta_c} \tag{4.14}$$

where f_Z depends on surface roughness, the number of clamped parts interfaces in the joint and the material type (and hardness). For uncritical cases a value of 5% of the preload can be used for calculation purposes [2].

BAT Info:
In BAT, the 5% embedding preload loss is defined for the maximum bolt preload after tightening.

$$F_Z = 0.05 F_M^{max}$$

It is always recommended that the correct embedding preload loss is determined by experiment. If no experimental data is available Table 4.1 can be used to find approximate values. This table may only be used if the service temperatures are below 50% of the recrystallization temperatures of the used materials. The table used in ECSS-E-HB-32-23A [2] is used out of the VDI 2230 guideline [3].

Average roughness height	**Loading**	**Guide values for amounts of embedding** in µm		
R_z according to DIN 4768		in the thread	per head or nut bearing area	per inner interface
< 10 µm	tension/compression	3	2,5	1,5
	shear	3	3	2
10 µm up to < 40 µm	tension/compression	3	3	2
	shear	3	4,5	2,5
40 µm up to < 160 µm	tension/compression	3	4	3
	shear	3	6,5	3,5

Table 4.1: Guide values for amounts of embedding of bolts, nuts and compact clamped parts made of steel, without coatings [3]

BAT Info:
Only the maximum embedding values listed in Table 4.1 for tension/compression and shear are used in BAT (no distinction between external loading types).

4.4 Compliance of Bolt and Clamped Parts

4.4.1 Compliance of Bolt

The bolt consists of different sections with dedicated compliances seen in Figure 4.1.

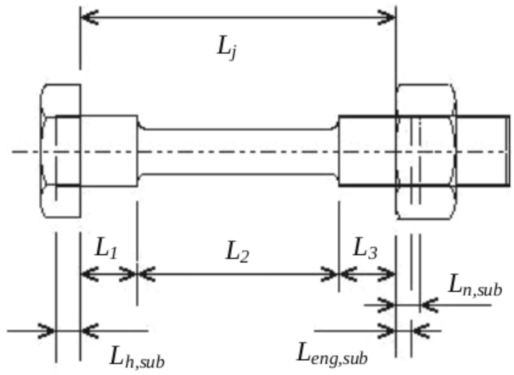


Figure 4.1: Dimensioning of the bolt for compliance calculation [2]

Applying the Hook's law to each segment of the bolt and combining the equations, the compliance of the bolt δ_b can be written

$$\delta_b = \frac{1}{c_b} = \frac{1}{E_b} \sum \frac{L_i}{A_i} \tag{4.15}$$

where $c_b = 1/\delta_b$ is the bolt stiffness, A_i and L_i are the segment cross section and segement lengths respectively. E_b is the bolt Young's Modulus and A_i are the cross sections for each bolt segement. Expanding (4.15) and introducing substitution lengths for deformation in the bolt head and engaged region in thread and nut the equation leads to

$$\delta_b = \frac{1}{E_b}\left[\frac{L_{h,sub}}{A_1} + \frac{L_{eng,sub}}{A_3} + \left(\frac{L_1}{A_{L_1}} + \frac{L_2}{A_{L_2}} + \cdots + \frac{L_i}{A_{L_i}}\right)\right] + \frac{L_{n,sub}}{E_n A_1} \tag{4.16}$$

where $L_{h,sub}$ and $L_{n,sub}$ are substitutional lengths of bolt head and nut (locking device) respectively. $L_{eng,sub}$ is the substitution length for the engaged thread and the value depends on the type of joint. For through bolt joint (TBJ) $L^{TBJ}_{eng,sub} = 0.4d$ and for tapped thread joints (TTJ) $L^{TTJ}_{eng,sub} = 0.33d$.

BAT Info:
The current BAT implementation does not include *shank bolts* with different diameter sections; complete shaft length is threaded ($\sum {}^{L_i}/{}_{A_i} = {}^{l_K}/{}_{A_3}$). Also for the bolt head compliance only cylindrical head is considered $L_{h,sub} = 0.4d$ for simplicity. The same substitution length is used for the locking device (nut) $L_{n,sub} = 0.4d$ according to [2]. It is assumed that the nut has the same Young's Modulus than the bolt $E_n = E_b$.

$$\delta_b = \frac{1}{E_b}\left(\frac{L_{h,sub}}{A_1} + \frac{L_{eng,sub}}{A_3} + \frac{L_i}{A_{L_i}} + \frac{L_{n,sub}}{A_1}\right) \tag{4.17a}$$

$$= \frac{1}{E_b}\left(\frac{0.4d}{A_1} + \frac{L_{eng,sub}}{A_3} + \frac{l_K}{A_3} + \frac{0.4d}{A_1}\right) \tag{4.17b}$$

4.4.2 Compliance of Clamped Parts

The calculation of the compliance of the clamped parts δ_c is more complicated than that of the bolts presented in §4.4.1 because of the 3-dimensional stress state in the joint that is induced by the preload. The presented method [2] neglects the compliance of the interfaces and therefore it is most accurate for joints with a small number of clamped parts. If more clamped parts are required the correct compliance can be determined by test or finite element analysis.

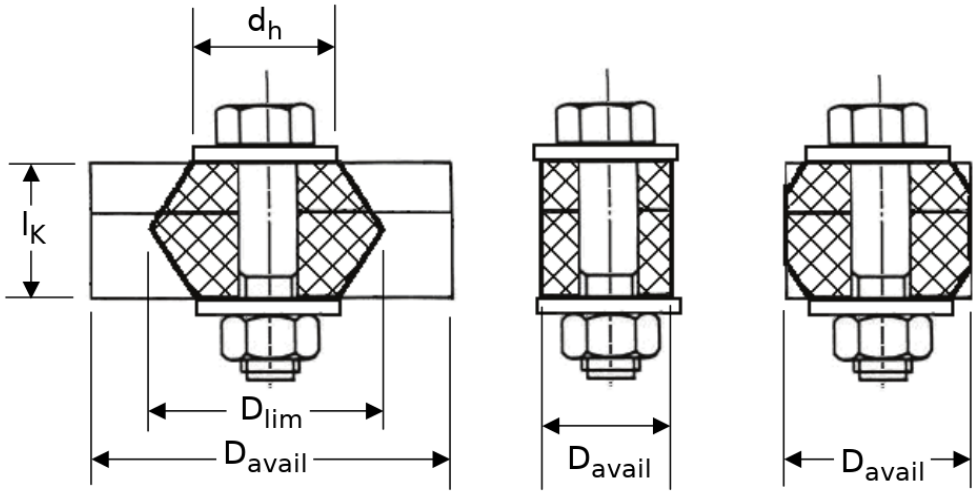


Figure 4.2: Compression zones in cylindrical clamped parts [2]

The configuration of the *compression zone* depends on the geometry of the clamped parts (joint geometry). Figure 4.2 shows the three possible compression zones for cylindrical clamped parts[3]. The configuration on the left of Figure 4.2 has flanges that are sufficiently wide to allow full spreading of the compression zone to the limiting diameter D_{lim}. In reality, the shape of the 3-dimensional zone of compression in an isotropic material is a paraboloid. The used approximate method simplifies the shape to a pair of compression cones. The two cones are symmetric about the mid-point of the clamped parts length l_K, which does not necessarily correspond to the interface between the flanges. If the flanges are too small for the compression cone to fully develop $D_{avail} < D_{lim}$, a full or partial *compression sleeve* develops. For *tapped thread joints* (TTJ) the analysis model assumes only one compression cone in the non-threaded clamped parts. For the general case with clamped parts that are not axially symmetric about the bolt axis, multiple edge distances are present; here the configuration of the deformation zone D_{avail}^{min} should be determined by the minimum edge distance (inscribed diameter).

Analysis Method

If the edge distance of the flanges and hence the available diameter for the compression zone D_{avail} is known, it can be compared with the *limit diameter of the compression cone* D_{lim}, given by

$$D_{lim} = d_h + w l_K \tan \varphi_c \tag{4.18}$$

where φ_c is the *compression cone half angle* and the parameter w is defined according to the joint type. For through bolt joints $TBJ : w = 1$ and for tapped thread joints $TTJ : w = 2$. The compression cone half angle φ_c can be derived with an empirical equation [2]

$$TBJ: \qquad \tan \varphi_c = 0.362 + 0.032 \ln (x/2) + 0.153 \ln y \tag{4.19a}$$

$$TTJ: \qquad \tan \varphi_c = 1.295 - 0.246 \ln x + 0.94 \ln y \tag{4.19b}$$

where the following non-dimensional parameters are used $x = l_K/d_h$, $y = D_{avail}/d_h$. The existence of a compression sleeve is determined as follows:

CASE 1: $\mathbf{D_{avail} > D_{lim}}$ the compression zone is fully developed into a cone or a pair of cones for TBJ joints. The compliance of the clamped parts δ_c for a compression cone is given by

$$\delta_c = \frac{2 \ln \left[\frac{(d_h + d)(D_{lim} - d)}{(d_h - d)(D_{lim} + d)} \right]}{w E_c \pi d \tan \varphi_c} \tag{4.20}$$

[3] The figure assumes that the flanges are compressed between infinitely stiff washers with the diameter equal to the *minimal diameter under bolt head* d_h for the compression zone. In BAT the washer is considered as clamp part.

CASE 2: $\mathbf{D_{avail}} < \mathbf{d_h}$ only a compression sleeve and no compression cone develops due to small clamped part dimesions. The compliance of the clamped parts δ_c for a compression sleeve is given by

$$\delta_c = \frac{4 l_K}{E_c \pi \left(D_{avail}^2 - d^2\right)} \tag{4.21}$$

CASE 3: $\mathbf{d_h} < \mathbf{D_{avail}} < \mathbf{D_{lim}}$ partial compression sleeve and cone(s) develops. The compliance of the clamped parts δ_c for a combined cone / sleeve compression zone is given by

$$\delta_c = \frac{\frac{2}{wd\tan\varphi_c} \ln\left[\frac{(d_h+d)(D_{avail}-d)}{(d_h-d)(D_{avail}+d)}\right] + \frac{4}{D_{avail}^2 - d^2}\left[l_k - \frac{D_{avail}-d_h}{w\tan\varphi_c}\right]}{E_c \pi} \tag{4.22}$$

This equation includes the effect of both the compression cone(s) and sleeve and is therefore appropriate when the clamped parts have the same Young's Modulus. If the compliances of the cone and sleeve should be calculated separately (multiple materials / different Young's Moduli present in the clamped parts) the clamped part compliances should be calculated by a different method [2] → **OPEN ISSUE IN BAT - TO BE CHECKED**.

BAT Info:
Multiple clamped parts with different materials can be defined in BAT. To calculate the overall clamped part compliance δ_c the equations described above for *CASE 1, 2 and 3* are restructured. For each case a parameter x_c is evaluated, which is simply the δ_c-equations (4.20), (4.21) and (4.22) without the Young's Modulus E_c

$$\delta_c = \frac{l_K}{E_c A_{sub}} = \frac{x_c}{E_c} \qquad \rightarrow \qquad A_{sub} = \frac{l_K}{x_c} \tag{4.23}$$

where A_{sub} is the derived *substitutional compliance area* of the clamped parts. It is assumed that A_{sub} can be used to calculate the overall clamped part compliance for multiple clamped parts with different Young's Moduli E_{ci} and thicknesses l_{ci}

$$\delta_c = \frac{l_K}{\tilde{E}_c A_{sub}} = \sum_i \frac{l_{ci}}{E_{ci} A_{sub}} \tag{4.24}$$

5 Stesses in Bolt and Margins

5.1 Stress in bolt after tightening

Based on §4.1 the stresses in the bolt after tightening can be derived. The minimum and maximum shear stress in the bolt $\tau_M^{min/max}$ after tightening can be calculated as follows

$$\tau_M^{min} = \frac{T_A^{min} - M_{uh}^{max}}{W_p}, \qquad \tau_M^{max} = \frac{T_A^{max} - M_{uh}^{min}}{W_p} \tag{5.1}$$

where T_A is defined in (4.6), M_{uh} in (4.2) with $F_V = F_M$ and $W_p = {}^{\pi d_s^3}/_{16}$ is the polar section modulus. The minimum and maximum normal stress $\sigma_M^{min/max}$ in the bolt caused by the preload is defined

$$\sigma_M^{min/max} = \frac{F_M^{min/max}}{A_s} \tag{5.2}$$

The minimum and maximum von-Mises equivalent stress $\sigma_{v,M}^{min/max}$ in the bolt after tightening (with 100% shear stress contribution / 0% shear stress relaxation) is defined

$$\sigma_{v,M}^{min/max} = \sqrt{\left(\sigma_M^{min/max}\right)^2 + 3\left(\tau_M^{min/max}\right)^2} \tag{5.3}$$

Based on the von-Mises equivalent stress in the bolt the minimum and maximum *bolt utilization* $\nu^{min/max}$ is defined

$$\nu^{min/max} = \frac{\sigma_{v,M}^{min/max}}{\sigma_y} \tag{5.4}$$

where σ_y is the bolt material yield strength and this defines the tightening status of the joint.

5.2 Margins of safety in bolt at service

The bolted joint can be loaded with an axial force F_A and a shear force F_Q. For the axial loading the additional bolt load is defined in (3.1) and for the shar loading the required clamping force for friction grip per bolt is defined

$$F_{Kreq} = \frac{F_Q}{q_F \mu_T} \tag{5.5}$$

where q_F is the number of shear force transmitting interfaces and μ_T the coefficient of friction in the clamped part interfaces.

Local Slippage Margin MOS_{slip}^{local}

The *local slippage margin* is defined for a minimum preload F_V^{min}

$$MOS_{slip}^{local} = \frac{F_V^{min} - F_{PA}}{F_{Kreq} FOS_{slip}} - 1 \tag{5.6}$$

where FOS_{slip} is the factor of safety against slippage. It is also possible that the local slippage margin is defined with the mean preload $F_V^{mean} = 0.5(F_V^{min} + F_V^{max})$, based on engineering judgedment.

Local Gapping Margin MOS_{gap}

The *local gapping margin* is defined for a minimum preload F_V^{min}

$$MOS_{gap} = \frac{F_V^{min}}{F_{PA} FOS_{gap}} - 1 \tag{5.7}$$

where FOS_{gap} is the factor of safety against gapping.

Yield and Ultimate Bolt Margin $MOS_{y/u}$

The *yield and ultimate margins* are defined for a maximum preload F_V^{max}

$$MOS_y = \frac{\sigma_y}{\sqrt{\left(\frac{F_V^{max} + F_{SA} FOS_y}{A_s}\right)^2 + 3\left(k_\tau \tau^{max}\right)^2}} - 1 \tag{5.8a}$$

$$MOS_u = \frac{\sigma_u}{\sqrt{\left(\frac{F_V^{max} + F_{SA} FOS_u}{A_s}\right)^2 + 3\left(k_\tau \tau^{max}\right)^2}} - 1 \tag{5.8b}$$

where $FOS_{y/u}$ is the factor of safety against yield and ultimate. For the $MOS_{y/u}$ calculation a 50% shear load reduction $k_\tau = 0.5$ is considered[2] .

[2] After tightening the shear stress in the bolt relaxes significantly after a few minutes and a conservative 50% relaxation is assumed for margin evaluation [2, 3].

6 Standard Tightening Torque (Torque Table)

The maximum possible preload (load at yield point) of the bolt is influenced by the simultaneously acting tension and torsional stresses σ_M and τ_M. These two stresses are combined to en equivalent uniaxial stress state with the deformation energy theory (von-Mises)

$$\sigma_{v,M} = \sqrt{\sigma_M^2 + 3\tau_M^2} \tag{6.1}$$

where $\sigma_{v,M}$ (5.3), σ_M (5.2) and τ_M (5.1) are defined in §5.1 in detail. For the further derivation the following equations apply

$$\sigma_M = \frac{F_M}{A_0}, \qquad \tau_M = \frac{M_{th}}{W_P} \tag{6.2}$$

where d_0 is the diameter of the relevant stress cross section $A_0 = {d_0^2\pi}/{4}$ of the bolt and with the assocaiated polar moment of resistance W_P. The torque present at the thread interface M_{th} is defined in (4.1). Equation (6.1) can be restructured to

$$\frac{\sigma_{v,M}}{\sigma_M} = \sqrt{1 + 3\left(\frac{\tau_m}{\sigma_M}\right)^2} \tag{6.3}$$

and if the equivalent uniaxial stress is set to the minimum yield point $\sigma_{v,M} = \sigma_y^{min}$ of the bolt material the *permissible assembly normal stress* σ_M^{allow} can be defined

$$\sigma_M^{allow} = \frac{\sigma_y^{min}}{\sqrt{1 + 3\left(\frac{\tau_m}{\sigma_M}\right)^2}} \tag{6.4}$$

If the two equations (6.2) are combined, the factor ${\tau_m}/{\sigma_m}$ can be defined

$$\frac{\tau_M}{\sigma_M} = \frac{M_{th}A_0}{W_P F_M} \tag{6.5}$$

where for *metric threads* the torque M_{th} can be simplified with the approximation $\tan(\varphi+\rho) \approx \tan\varphi + \tan\rho$ to

$$M_{th} = F_M \frac{d_2}{2} \tan(\varphi+\rho) \tag{6.6a}$$

$$= F_M \frac{d_2}{2} \left(\tan\varphi + \tan\rho\right) \tag{6.6b}$$

$$= F_M \frac{d_2}{2} \left(\frac{p}{\pi d_2} + 1.155\mu_{th}\right) \tag{6.6c}$$

Generally, for the elastic reagion applies $W_P = W_{P,el} = {d_0^3\pi}/{16}$ and for the *fully plastic state*[4] a correction to the polar moment of resistance is applied $W_P = W_{P,pl} = {d_0^3\pi}/{12}$. All these equations can be combined to

$$\sigma_M^{allow} = \frac{\sigma_y^{min}}{\sqrt{1 + 3\left[\varkappa \frac{d_2}{d_0}\left(\frac{p}{\pi d_2} + 1.155\mu_{th}^{min}\right)\right]^2}} \tag{6.7}$$

where

$$\varkappa = 2 \qquad \text{for elastic region}^5 \qquad W_P = \frac{d_0^3\pi}{16}$$

$$\varkappa = \frac{2}{3} \qquad \text{for plastic region}^5 \qquad W_P = \frac{d_0^3\pi}{12}$$

The *permissible assembly preload* F_M^{allow} is defined

$$F_M^{allow} = \sigma_M^{allow} A_0 \nu \tag{6.9}$$

with the *bolt utilization factor* ν (5.4)

$$\nu = \frac{\sigma_{v,M}^{allow}}{\sigma_y^{min}}$$

Frequently, especially in the case of torque-controlled tightening only one proportional utilization (normally 90%) is permitted in order to exclude the possibility of the yield point being exceeded in service [3].

[4] Full plasticity of cross section: constant torsional stress over the cross section

[5] The elastic region is used in ECSS-E-HB-32-23A [2] and is more conservative than the plastic region, which is used in VDI 2230 [3].

BAT Info:
In BAT the equation (6.7) is used with both options for elastic $\varkappa = 2$ and full plastic region $\varkappa = {}^2\!/_3$. Based on the fact that only fully threaded bolts are used in BAT (no shank bolts) the critical cross section is set to stress area $A_0 = A_s$ with the corresponding stress diameter d_s. . For the analysis of the correct tightening torque T_A based on the permissible assembly preload the simplified version of (4.5) is used

$$T_A = F_M^{allow} \left(0.16p + 0.58 d_2 \mu_{th} + \frac{\mu_{uh} D_{Km}}{2 \sin {}^\lambda\!/_2} \right) \tag{6.10}$$

with $D_{Km} = {}^{(d_h+d)}\!/_2$ (D_{hole} is neglected and the nominal bolt diameter d is used instead for simplicity) and $\mu_{th} = \mu_{uh} = \mu^{min}$ is used, which leads to the limiting torque.
In calculating the tightening torque, it is **always the minimum coefficient of friction which should be used**, assuming a necessary maximum permissible assembly preload [3].

7 References

[1] Guidelines for threaded fasteners. ESA Guideline ESA PSS-03-208 Issue 1, Structures and Mechanism Division ESTEC, December 1989.

[2] Space engineering - threaded fasteners handbook. ECSS Handbook ECSS-E-HB-32-23A, ECSS European Cooperation for Space Standardization, 16 April 2010.

[3] Systematic calculation of highly stressed bolted joints - joints with one cylindrical bolt. VDI Guideline VDI2230 Part 1, VDI - Verein Deutscher Ingenieure, November 2015.