BAT - Bolt Analysis Tool

User Manual

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Issue:	0.1
Date:	xx.xx.2021

Contents

Symbols and Abbreviations

Symbols

$\mathbf{0}$	zero vector
∇^2	Laplacian or Laplace operator
α	damping parameter for Levenberg-Marquardt algorithm
$\Gamma(\cdot)$	Gamma function
δ_{nm}	Kronecker delta
δ_{per}	small perturbation for $\boldsymbol{J}$ evaluation
$\boldsymbol{\Delta}_{dLM}$	right-hand side of deterministic LM algorithm
$\boldsymbol{\Delta}$	right-hand side of VB LM algorithm
ε	numerical error for LM algorithm termination
ζ	i.i.d. Gaussian random variable
η	correlation length of stochastic process
$\boldsymbol{\Theta}, \boldsymbol{\theta}$	(model) parameters
$\boldsymbol{\Theta}^{(i)}$	drawn samples for MCMC
$\varkappa$	offset parameter
κ	regularization parameter (Tikhonov)
λ_n	eigenvalues of spectral decomposed stochastic process
λ_1, λ_2	model parameters for biexponential example
$\boldsymbol{\Lambda}$	precision matrix of parameters $\boldsymbol{\theta}$
$\boldsymbol{\Lambda}_0$	prior precision matrix of parameters $\boldsymbol{\theta}$
$\boldsymbol{\Lambda}_{dLM}$	gradient for deterministic LM algorithm
$\xi_n(\omega)$	orthogonal, random variables
σ	standard deviation
σ_Y^2	variance of stochastic process
$\underline{\boldsymbol{\sigma}}$	stress tensor
$\boldsymbol{\Sigma}$	covariance matrix
ϕ	precision of added Gaussian noise
$\psi(\cdot)$	di-gamma function
$\boldsymbol{\psi}$	linearization of nonlinear model $\boldsymbol{\psi} = \boldsymbol{y} - \boldsymbol{g}(\boldsymbol{m})$
ω	stochastic outcome
Ω	domain
$\partial\Omega$	boundary of domain

A_i, B_i	probabilistic events
$\boldsymbol{A}$	matrix for FDM in Poisson problem
$b, \boldsymbol{b}$	right-hand side of Poisson equation
$\underline{\hat{\boldsymbol{b}}}$	body loads
c	shape parameter of Gamma distribution for model noise
c_0	prior shape parameter of Gamma distribution for model noise
$C(\boldsymbol{x}, \boldsymbol{x}')$, C_Y	covariance function
D	dimension
$\mathcal{D}$	space domain
$\boldsymbol{e}$	additive Gaussian noise vector
$e_{\boldsymbol{\psi}}$	error norm for result comparison
$\mathbb{E}[\cdot]$	expectation or mean
$\mathcal{F}$	set of events
$F(\cdot)$	cumulative distribution function
$f(\cdot)$	arbitrary function
f_n	eigenfunctions of spectral decomposed stochastic process
$\boldsymbol{g}(\cdot)$	nonlinear forward model
$\mathcal{GP}(m(\boldsymbol{x}), C(\boldsymbol{x}, \boldsymbol{x}'))$	Gaussian process
h	mesh size
$\boldsymbol{h}$	search direction for optimization
$\mathbb{H}[\cdot] = \mathbb{H}[p, p]$	differential or relative entropy
$\mathbb{H}[p, q]$	cross entropy
i_{max}	maximum number of iterations for algorithms
$\boldsymbol{I}$	identity matrix
$\mathcal{J}$	dimension of $\boldsymbol{y}$
$\boldsymbol{J}(\cdot)$	Jacobi matrix, Jacobian
$k(\boldsymbol{x})$	material parameter
$\mathrm{KL}(\cdot\|\|\cdot)$	Kullback-Leibler divergence
l_i	length for geometrical pipe transformation
L	length/dimension of domain
$\boldsymbol{\mathcal{L}}(\cdot)$	lower bound
$m(\boldsymbol{x})$	mean function
$m, \boldsymbol{m}$	mean (vector), optimization vector in algorithms
$\boldsymbol{m}_0$	prior mean vector of parameters $\boldsymbol{\theta}$
$\boldsymbol{m}_{new}$	new mean/optimization vector for parameters $\boldsymbol{\theta}$ in iteration
$\boldsymbol{m}_{old}$	old mean/optimization vector for parameters $\boldsymbol{\theta}$ in iteration
M_1, M_2	model parameters for biexponential example
N	number of used terms in KL-expansion
$\mathcal{O}$	higher-order-terms in Taylor approximation

$p(\boldsymbol{\Theta})$	prior distribution
$p(\boldsymbol{\Theta}\|\boldsymbol{y})$	posterior distribution
$p(\boldsymbol{y})$	normalization constant, model evidence, marginal likelihood, Bayes factor
$p(\boldsymbol{y}\|\boldsymbol{\Theta})$	likelihood function
$p(\cdot)$	probability measure, probability distribution/density, marginal probability
$p(\cdot\|\cdot)$	conditional probability
$p(\cdot,\cdot) = p(\cdot \cap \cdot)$	joint probability
$p_n(x)$	Taylor polynomial of n-th order
$\tilde{p}(\cdot,\cdot)$	defined distribution in VB derivation
$q(\cdot)$	arbitrary or factorized probability distribution
$q_j^{opt}(\boldsymbol{\Theta}_j)$	optimal solution for variational inference in VB derivation
$q(\boldsymbol{\Theta}\|\boldsymbol{y})$	approximate posterior distribution in VB derivation
$q_{\boldsymbol{\theta}}(\boldsymbol{\theta}\|\boldsymbol{y})$	factorized prior for model parameters in VB derivation
$q_{\phi}(\phi\|\boldsymbol{y})$	factorized prior for model noise in VB derivation
r_P	radius of pipe model
s	scale parameter of Gamma distribution for model noise
s_0	prior scale parameter of Gamma distribution for model noise
$\mathcal{S}$	sample space
$(\mathcal{S}, \mathcal{F}, p)$	probability space
t	time
$\mathcal{T}$	time domain
$u, \boldsymbol{u}$	solution (vector) of Poisson problem
$\mathrm{var}[\cdot]$	variance
w_n	positive roots of characteristic equation in KL-expansion
$X, X(\cdot)$	random variable
$\boldsymbol{X}$	random vector
$\boldsymbol{y}$	measurement
Y	stochastic process

Abbreviations

AWGN	additive white Gaussian noise
BACI	C++ research code
CDF	cumulative distribution function
(d)LM	(deterministic) Levenberg-Marquardt
dTikLM	deterministic, elementwise optimization with Tikhonov regularization
FE(A)	finite element (analysis)
Gam	Gamma distribution
GN	Gauss-Newton
GP	Gaussian process
i.i.d.	independent, identically distributed
KL-divergence	Kullback-Leibler-divergence
KL-expansion	Karhunen-Loève-expansion
NLLS	nonlinear least squares
MCMC	Markov Chain Monte Carlo
MVN	multivariate normal distribution
SNR	signal-to-noise ratio
VB	Variational Bayes

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1 Introduction

This document will include the BAT (Bolt Analysis Tool) User Manual [1] [2] [3].

$$p(\boldsymbol{\Theta}|\boldsymbol{y}) = \frac{p(\boldsymbol{y}|\boldsymbol{\Theta})\ p(\boldsymbol{\Theta})}{p(\boldsymbol{y})}\ , \tag{1.1}$$

2 References

[1] Guidelines for threaded fasteners. ESA Guideline ESA PSS-03-208 Issue 1, Structures and Mechanism Division ESTEC, December 1989.

[2] Space engineering - threaded fasteners handbook. ECSS Handbook ECSS-E-HB-32-23A, ECSS European Cooperation for Space Standardization, 16 April 2010.

[3] Systematic calculation of highly stressed bolted joints - joints with one cylindrical bolt. VDI Guideline VDI2230 Part 1, VDI - Verein Deutscher Ingenieure, November 2015.