CSE 3380 – Homework #11

Assigned: Thursday, April 26, 2018
Due: Thursday, May 3, 2018; drop off at my office by 2pm (note the time and location), submitting early is fine.

Note the following about the homework:

1. You must show your work to receive credit.
2. If your submission has more than one page, staple the pages. If I have to staple it, the cost is 10 points.

Assignment:
Process

1. (a) For the matrix
$$A = \begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$$
find
   i. the matrix 1-norm
   ii. the matrix ∞-norm
   iii. the Frobenius norm
   iv. the condition number. Use the matrix 1-norm for this.

   (b) For the matrix
$$B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$
find
   i. the matrix 1-norm
   ii. the matrix ∞-norm
   iii. the Frobenius norm
   iv. the condition number. Use the matrix 1-norm for this.

   (c) For the matrix
$$C = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$
find
   i. the matrix 1-norm
   ii. the matrix ∞-norm
   iii. the Frobenius norm
   iv. the condition number. Use the matrix 1-norm for this.

2. We have the following system of equations, which can be represented as $A\vec{x} = \vec{b}$.

$$\begin{aligned} x_1 + 2x_2 &= 2 \\ 3x_1 + 4x_2 &= 2 \end{aligned}$$

(a) Use Gauss-Jordan elimination to solve for $\vec{x}$.

(b) Produce $\vec{\hat{b}}$ by perturbing $\vec{b}$ by $(-0.5, 0)^T$ and solve for $\vec{\hat{x}}$ (i.e., x hat).

(c) Then calculate

$$r_b = \frac{||\Delta\vec{b}||_\infty}{||\vec{b}||_\infty} \quad , \quad r_x = \frac{||\Delta\vec{x}||_\infty}{||\vec{x}||_\infty} \quad , \quad \text{and} \quad \frac{r_x}{r_b}$$

(d) Find the condition number of A using the matrix ∞-norm.

Pay attention to which norms you are being asked to use here.

3. We have the following system of equations, which can be represented as $A\vec{x} = \vec{b}$.

$$\begin{aligned} x_1 + x_2 &= 5.0000 \\ x_1 + 0.9999x_2 &= 4.9997 \end{aligned}$$

(a) Use Gauss-Jordan elimination to solve for $\vec{x}$.

(b) Produce $\vec{\hat{b}}$ by perturbing $\vec{b}$ by $(0, 0.0003)^T$ and solve for $\vec{\hat{x}}$ (i.e., x hat).

(c) Then calculate

$$r_b = \frac{||\Delta\vec{b}||_1}{||\vec{b}||_1} \quad , \quad r_x = \frac{||\Delta\vec{x}||_1}{||\vec{x}||_1} \quad , \quad \text{and} \quad \frac{r_x}{r_b}$$

(d) Find the condition number of A using the matrix 1-norm.

Pay attention to which norms you are being asked to use here.

4. (hand solution) Use the graphical approach of linear programming to solve this problem; draw a graph and identify the feasible region.

Maximize $f(x, y) = 10x - 8y$

subject to

$$\begin{aligned} -x + y &\leq 3 \\ 3x + y &\geq 10 \\ x &\leq 5 \\ x, y &\geq 0 \end{aligned}$$

5. (hand solution) Use the graphical approach of linear programming to solve this problem; draw a graph and identify the feasible region.

   A business provides two services: A and B. Service A requires 60 minutes to complete, costs \$5 in material (ignore labor costs), and produces \$3 in profit. Service B requires 60 minutes to complete, costs \$12 in material (ignore labor costs), and produces \$12 in profit. The total value of materials that can be kept on hand is \$120 and the total labor minutes available is 720.

   How much of each service should be performed in order to maximize profits?

6. (hand solution) Use the simplex method of linear programming to solve this problem:

$$\begin{array}{rrcl}
\text{maximize} & 2x_1 + 3x_2 + 4x_3 & = & z \\
\text{subject to} & & & \\
& x_1 + \;\; x_2 \qquad\quad & \leq & 10 \\
& 2x_1 \qquad\quad + 3x_3 & \leq & 12 \\
& x_1 + \;\; x_2 + 2x_3 & \leq & 14 \\
& x_1 & \geq & 0 \\
& x_2 & \geq & 0 \\
& x_3 & \geq & 0
\end{array}$$

   Be clear about the final values of the variables.

Applications

7. **(CS application: computer vision)** When we look at distant objects, the distance between them appears smaller than it really is. For example, when the distant object is the top of a tall building, the sides of the building appear to be moving toward each other. At times, we would like to transform a picture in which the relationship between points is skewed due to distance into a picture showing the correct relationships. For this, we can find a change-of-basis matrix.

   Our change-of-basis matrix will have the form

$$H = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$$

   Given four points $[(x_1, y_1),\ (x_2, y_2),\ (x_3, y_3),\ (x_4, y_4)]$ in the skewed picture and four points $[(x_1', y_1'),\ (x_2', y_2'),\ (x_3', y_3'),\ (x_4', y_4')]$ in the corrected picture, we can solve for the eight values

$a, b, \ldots, h$ using

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & 0 & 0 & -x_1' & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & 0 & 0 & -y_1' & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_1 & y_1 & -1 & 0 & 0 & 0 \\ x_2 & y_2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -x_2' & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 & 0 & 0 & 0 & -y_2' & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_2 & y_2 & 0 & -1 & 0 & 0 \\ x_3 & y_3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -x_3' & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 & 0 & 0 & 0 & 0 & -y_3' & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_3 & y_3 & 0 & 0 & -1 & 0 \\ x_4 & y_4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -x_4' \\ 0 & 0 & 0 & x_4 & y_4 & 1 & 0 & 0 & 0 & 0 & 0 & -y_4' \\ 0 & 0 & 0 & 0 & 0 & 0 & x_4 & y_4 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

After constructing H from the values for $a, b, \ldots, h$ in the solution (note that the w_i values are not used), given a point (x, y) in the skewed picture we can produce

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix}$$

Our corrected point (x', y') is found by dividing by w:

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} \Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

(a) On the course website is a program, `homographyMain.py`, that contains the coordinate points for learning the mapping:

$$\text{points\_skewed} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{bmatrix}, \qquad \text{points\_corrected} = \begin{bmatrix} x_1' & x_2' & x_3' & x_4' \\ y_1' & y_2' & y_3' & y_4' \end{bmatrix}$$

The skewed (x, y) values are the corners of the blue trapezoid in Figure 1. It also contains a matrix of the points to be mapped from the skewed coordinate system to the corrected coordinate system; these points represent the red shape in Figure 1. Each column represents an (x, y) coordinate in homogeneous coordinates, so it's of the form

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(b) You will write the Python function `homographyStudent()`, which has the signature

```
H, projected = homographyStudent(points_skewed, points_corrected, test_points)
```

This function is given the skewed corner points, the corrected corner points, and the points to transform, and returns the transformation matrix H as well as a $3 \times n$ matrix of the n transformed points in homogeneous coordinates. This function will be placed in the file `homographyStudent.py`. Do not hard-code your logic to the data in the driver file.

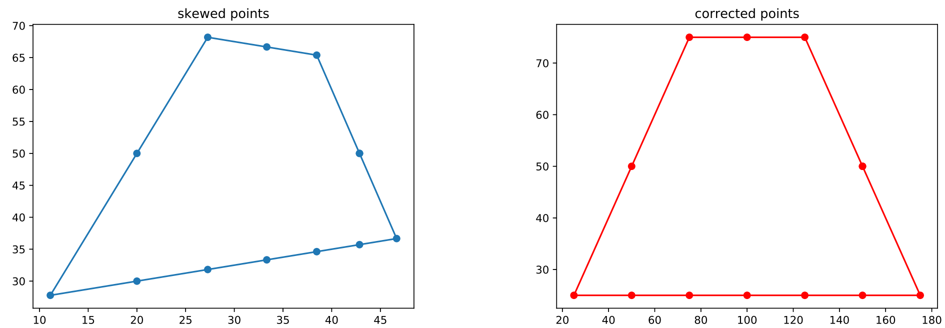

Figure 1: Plots of points before (left) and after (right) transformation.

General requirements about the Python problems:

a) As a comment in your source code, include your name.

b) The Python program should do the work. Don't perform the calculations and then hard-code the values in the code or look at the data and hard-code to this data unless instructed to do so.

c) Your function should use the data passed to it and should work if I were to change the data.

d) The program should not prompt the user for values or read from files unless instructed to do so.

e) I don't want your function to cause anything to print; leave it to the `XXXmain.py` file to do the printing of what is returned.

To submit the Python portion, do the following:

a) Create a directory using your net ID in lowercase characters. This should be something of the form `abc1234`.

b) Place your .py files in this directory.

c) Zip the directory, not just the files within the directory. You must use the zip format and the name of the file, assuming your net ID is `abc1234`, will be `abc1234.zip`.

d) Upload the zip'd file to Blackboard.

Review Questions

These are not for credit, but instead are intended to test your understanding of the concepts. You should be able to answer these without simply regurgitating equations.

1. What is the condition number of a matrix and what does it tell us about a system of equations?
2. What is the relationship between the condition number of a matrix and the relative changes in $\vec{b}$ and $\vec{x}$?