CSE 3380 – Homework #4

Assigned:	Tuesday, February 20, 2018
Due:	Thursday, March 1, 2018 at the end of class

Note the following about the homework:

1. You must show your work to receive credit.
2. If your submission has more than one page, staple the pages. If I have to staple it, the cost is 10 points.

Assignment:
Process

1. For each of the following, is $\vec{b}$ in $span(A)$? If so, what linear combination of the columns of A produces $\vec{b}$? Refer to the columns of A as $\vec{a_1}$, $\vec{a_2}$ and $\vec{a_3}$ and give your answer in a form like $\vec{b} = 3\vec{a_1} - 5\vec{a_2} + 7\vec{a_3}$.

$$a)\ A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 10 \\ 10 \\ 20 \end{bmatrix}, \qquad b)\ A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 6 \\ 9 \\ 10 \end{bmatrix}$$

$$c)\ A = \begin{bmatrix} 3 & 0 & 6 \\ 2 & 4 & 0 \\ 1 & 6 & -4 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} -9 \\ 6 \\ 15 \end{bmatrix}, \qquad d)\ A = \begin{bmatrix} 3 & 0 & 6 \\ 2 & 4 & 0 \\ 1 & 6 & -4 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} -9 \\ 6 \\ 10 \end{bmatrix}$$

2. For each of the following, find $Nul(A)$:

$$a)\ A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}, \qquad b)\ A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix}, \qquad c)\ A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 5 & -1 \\ 6 & 0 & 6 \end{bmatrix}$$

$$d)\ A = \begin{bmatrix} 2 & 4 & 6 \\ 6 & 12 & 15 \end{bmatrix}, \qquad e)\ A = \begin{bmatrix} 3 & 4 & 7 & 9 \\ 1 & 2 & 3 & 3 \end{bmatrix}$$

Applications

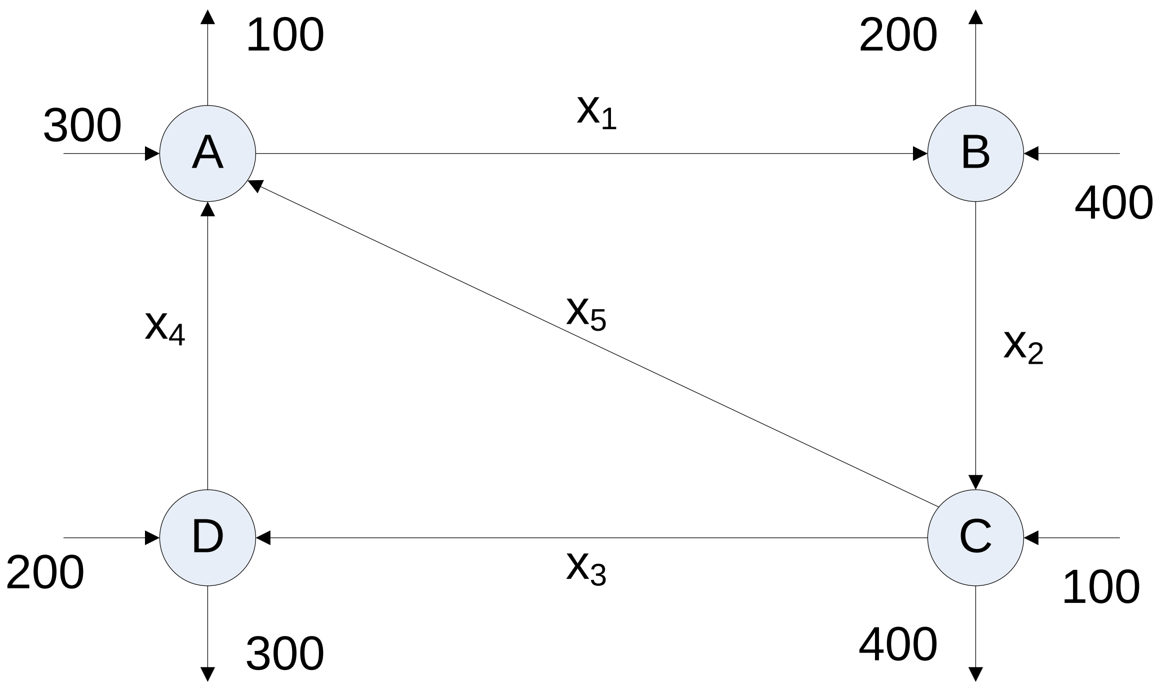

Figure 1: Network for problem 3

3. **(CS application: networks)** This problem is based on an example in *Herman and Pepe*. Figure 1 shows a flow network in which each node has input values and output values. Examples could be pipes with fluids flowing in and out, roads with cars moving in and out, and so forth–anything in which things flow through points. This means that the sum of all input to a node matches the sum of output from the node.

   (a) Set up the system of equations representing the network flow of the nodes and solve the system. To do so, for each node the sum of the input values should equal the sum of the output values. For example, the equation for node A is

   $$300 + x_4 + x_5 = 100 + x_1$$

   which simplifies to

   $$x_1 - x_4 - x_5 = 300 - 100 = 200$$

   The system of equations will consist of five variables but only four equations, so there will be free variables in the answer. The answer should have a form similar to

   $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 12 \\ 0 \\ 14 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

   The specific values and free variables on the right-hand side of the equal sign may differ.

(b) Assign a value of 100 to each free variable in the solution and use these to find the values of the pivot variables. You are finding a specific solution to the network. Using these values for $x_1, x_2, \ldots, x_5$, confirm that the total input to each node is equal to the total output from each node.

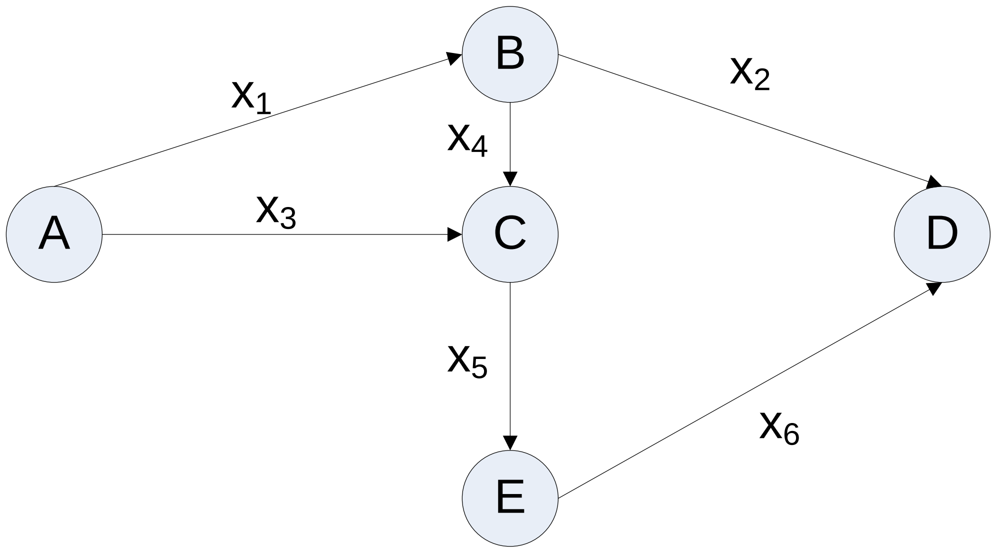

Figure 2: Network for problem 4

4. **(CS application: graph theory)** This problem is based on an example in *Herman and Pepe*. Figure 2 shows a network in which the nodes A, B, C, and E are input nodes and node D an output node. For example, some type of liquid may enter the network at nodes A, B, C, or E but ultimately leaves it at node D. There is a lot of redundancy in the network and we would like to eliminate the redundant arcs while still ensuring that anything entering one of the input nodes can ultimately reach the output node. In the language of graph theory, we are producing a spanning tree. To do this:

(a) Set up the system of equations representing the network flow of the nodes and represent it as a matrix; this will be the A in $A\vec{x} = \vec{0}$. To do so, for each node the sum of the input values should equal the sum of the output values. For example, the equation for node A is

$$x_1 + x_3 = 0$$

Note that the system of equations will consist of six variables but only five equations.

(b) Find the reduced row echelon form (RREF) of the matrix.

(c) Eliminate from the network the arcs corresponding to the free variables; that is, keep the arcs corresponding to the pivot variables. Draw the final network with the redundant arcs eliminated.

Note that the directions of the arcs matters when constructing the system of equations, but don't matter in the final network since we assume that whatever is flowing through the network can flow in either direction of an arc.

References

Visual Linear Algebra, Eugene A. Herman and Michael D. Pepe, John Wiley & Sons, 2005.

Review Questions

These are not for credit, but instead are intended to test your understanding of the concepts. You should be able to answer these without simply regurgitating equations.

1. Given a set A of vectors, what does $span(A)$ mean?
2. Given a set A of vectors, is there a minimum number of vectors in A to guarantee that a particular vector $\vec{b}$ is in $span(A)$?
3. What is the column space of a matrix?
4. What is the nullspace of a matrix?