

CSE 3380 – Homework #1

Assigned: Thursday, January 25, 2018

Due: Thursday, February 1, 2018 at the end of class

Note the following about the homework:

1. You must show your work to receive credit.
2. If your submission has more than one page, staple the pages. **If I have to staple it, the cost is 10 points.**

Assignment:

Process

1. Use Gauss-Jordan elimination to find a solution to the following system of equations if it is consistent. If a solution cannot be found because the system is inconsistent, state so.

$$\begin{aligned}x_1 + 2x_2 + 3x_3 &= -10 \\2x_1 \quad \quad + 1x_3 &= -7 \\4x_1 - x_2 + 1x_3 &= -11\end{aligned}$$

2. Use Gauss-Jordan elimination to find a solution to the following system of equations if it is consistent. If a solution cannot be found because the system is inconsistent, state so.

$$\begin{aligned}x_1 \quad \quad + 2x_3 &= 2 \\x_2 + 3x_3 &= 3 \\2x_1 - x_2 + 2x_3 &= 2\end{aligned}$$

3. Use Gauss-Jordan elimination to find a solution to the following system of equations if it is consistent. If a solution cannot be found because the system is inconsistent, state so.

$$\begin{aligned}2x_1 \quad \quad + 2x_3 &= 4 \\4x_1 + x_2 + 5x_3 &= 11 \\x_1 - x_2 \quad \quad &= 1\end{aligned}$$

Theory

4. If A and B are both $n \times n$ matrices of real numbers, determine if each of the following is necessarily true. That is, will the relationships always be true regardless of the specific values in A and B . Prove this using the rules of matrix arithmetic as applied to A and B . Do not use counter examples or specific matrices. Show enough steps that it is clear why you have reached your conclusion.

(a) $(A + B)^2 \stackrel{?}{=} A^2 + 2AB + B^2$

(b) $ABA \stackrel{?}{=} A^2B$

(c) $ABAB \stackrel{?}{=} (AB)^2$

Applications

5. Alex bought 3 gallons of red paint, 4 gallons of white paint, and 5 gallons of blue paint for a total of \$185. Bob bought 2 gallons of red paint and 4 gallons of blue paint for a total of \$80. Charles bought one gallon of red, one gallon of white, and one gallon of blue paint, for a total of \$45. Create a system of equations and solve it to determine the cost per gallon of each color of paint.
6. **(CS application: algorithms)** One of the properties of matrix multiplication is that it is associative. That is, if we have matrices A , B , and C with dimensions that allow them to be multiplied as ABC , then $A(BC) = (AB)C$. However, the amount of multiplications and additions may not be identical for both orderings.
 - (a) If A has dimensions 3×2 , B has dimensions 2×4 , and C has dimensions 4×5 , then what is the total number of multiplications and the total number of additions for each of the two orderings?
 - (b) Also, if A , B , and C have dimensions $m \times n$, $n \times p$, and $p \times q$, respectively, what are the formulae for the number of multiplications and additions for each ordering? You should show that you derived these formulae.

You should produce separate values and formulae for the multiplications and additions; don't combine them. For example, you might find that $A(BC)$ with the dimensions given above requires 100 multiplications and 80 additions with the generic formula for multiplications being $mn + pq$ and the generic formula for additions being $m + n + p$.

Review Questions

These are not for credit, but instead are intended to test your understanding of the concepts. You should be able to answer these without simply regurgitating equations.

1. If $AB = BA$, what shape must A and B have?
2. Is it ever possible for a linear system of equations to have exactly two solutions? Why or why not?