CSE 3380 – Homework #2

Assigned: Thursday, February 1, 2018
Due: Thursday, February 8, 2018 at the end of class

Note the following about the homework:

1. You must show your work to receive credit.
2. If your submission has more than one page, staple the pages. If I have to staple it, the cost is 10 points.

Assignment:

Process
Find the inverse of A if it exists.

1. $A = \begin{bmatrix} 1 & 4 \\ 4 & 2 \end{bmatrix}$

2. $A = \begin{bmatrix} 5/3 & -1/3 \\ -2/3 & 1/3 \end{bmatrix}$,

3. $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 6 & 4 \\ 3 & 9 & 6 \end{bmatrix}$

4. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

Theory

5. Matrix multiplication is not commutative. That is, generally $AB \neq BA$. If

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

produce AB and BA and compare them to see if $AB = BA$. Use the A and B given with the letters $a, b, \ldots, h$; don't replace them with specific numbers.

6. If

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

use row operations to change $[A|I]$ to $[I|A^{-1}]$. That is, produce A^{-1} for a 2×2 matrix via row operations. Use the matrix A given; don't replace the variables a, b, c, and d with numbers. Make sure that your answer is simplified; do not leave the arithmetic for me to do.

Applications

7. **(CS application: networks)** Figure 1 shows a directed graph, which represents a network. This can be represented in matrix form by producing an adjacency matrix, whose rows represent the start nodes and the columns represent the nodes that they are directly connected to. Being a directed graph, the arrows indicate the direction of the connection and may not go both ways. In this example, A is connected to C but C is not connected to A.

   Our standard adjacency matrix shows which nodes are reachable in one step from any particular node, but what if we wish to know which nodes are reachable in two steps, three steps,

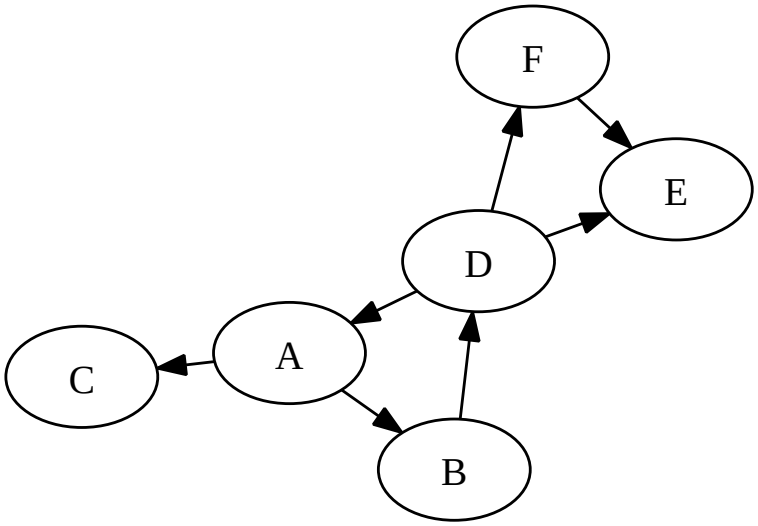

Figure 1: Directed graph

and so forth? For example, A can reach E in three steps via $A \to B \to D \to E$. To answer our question, we can use matrix multiplication. To determine which nodes are reachable in x number of steps, we can produce A^x and then read it like any other adjacency matrix.

Produce the adjacency matrix A of the given graph and use matrix multiplication to show which nodes are reachable in two steps or three steps. That is, produce A^2 and A^3.

8. **(CS application: computer graphics)** Some drawing packages allow the user to click on several points on a canvas and then will fit a curve to these points. Find the quadratic function that passes through the points $(5, 2)$, $(7, -2)$, and $(9, 0)$. Use this function to find $f(6)$ and $f(8)$ and then plot the graph through these five points.

Review Questions

These are not for credit, but instead are intended to test your understanding of the concepts. You should be able to answer these without simply regurgitating equations.

1. What is the inverse of a matrix by definition? If A^{-1} is the inverse of matrix A, what is the analogous relationship in terms of real numbers?

2. Does the relationship $(AB)^{-1} = B^{-1}A^{-1}$ hold for all matrices A and B? If not, what conditions must hold for the relationship to be true?