CSE 3380 – Homework #7

Assigned: Thursday, March 29, 2018
Due: Thursday, April 5, 2018 at the end of class

Note the following about the homework:

1. You must show your work to receive credit.
2. If your submission has more than one page, staple the pages. If I have to staple it, the cost is 10 points.

Assignment:
Process

1. Given basis $\mathfrak{B}$ and $[\vec{x}]_{\mathfrak{B}}$, find $\vec{x}$.
$$\mathfrak{B} = \left\{ \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}, \qquad [\vec{x}]_{\mathfrak{B}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$
2. Given basis $\mathfrak{B}$ and $\vec{x}$, find $[\vec{x}]_{\mathfrak{B}}$.
$$\mathfrak{B} = \left\{ \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}, \qquad \vec{x} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$
3. Find the coordinates of the polynomial $-2t^2 + 20t - 10$ with respect to the basis
$$\mathfrak{B} = \{(2-t)^2, (3+t)^2, t^2 + 2t + 3\}$$
that is a basis for $\mathbb{P}_2$.
4. Given bases $\mathfrak{B}$ and $\mathfrak{C}$, find $P_{\mathfrak{B}\leftarrow\mathfrak{C}}$ and $P_{\mathfrak{C}\leftarrow\mathfrak{B}}$. Be clear as to which is which.
$$\mathfrak{B} = \left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}, \qquad \mathfrak{C} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$$
5. Given bases $\mathfrak{B}$ and $\mathfrak{C}$, find $P_{\mathfrak{B}\leftarrow\mathfrak{C}}$ and $P_{\mathfrak{C}\leftarrow\mathfrak{B}}$. Be clear as to which is which.
$$\mathfrak{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} \right\}, \qquad \mathfrak{C} = \left\{ \begin{bmatrix} 3 \\ 3 \\ 8 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 10 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 10 \end{bmatrix} \right\}$$
6. If
$$\mathfrak{B} = \left\{ \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}, \qquad \mathfrak{C} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\} \qquad \text{and} \qquad [\vec{x}]_{\mathfrak{B}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$
what is $[\vec{x}]_{\mathfrak{C}}$?
7. If
$$\mathfrak{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \\ -10 \end{bmatrix} \right\}, \quad \mathfrak{C} = \left\{ \begin{bmatrix} 1 \\ -1 \\ -5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \quad \text{and} \quad [\vec{x}]_{\mathfrak{B}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix},$$
what is $[\vec{x}]_{\mathfrak{C}}$?

Applications

8. In the last few assignments, you will begin learning to use Python. Note that Python doesn't have built-in functions for performing many of the math and plotting functions we will need for this course, so you will also need the NumPy and Matplotlib libraries. Both of these are included with Anaconda (get the 3.6 or later version).

   On the course website are two Python programs: `hw07main.py` and `hw07student.py`. `hw07main.py` is a driver program that calls the function `hw07student()` within `hw07student.py`. Replace the sample code within `hw07student()` with your own code to perform the tasks given below. `hw07main()` will perform the printing of the solutions and should not be modified.

   (a) Find the solution $\vec{x}$ of $A\vec{x} = \vec{b}$. This will be performed via a single function call.

   (b) Produce $C = A + I_3$, where I_3 is a 3×3 identity matrix. When producing I_3, use the function that produces an identity matrix without hard-coding the dimension of 3 rows and 3 columns. The passed matrix A has a property that you can use to get its dimensions.

   (c) Produce $\vec{d} = \vec{b} + [1, 1, 1]^T$. When doing this, use the function that produces a vector of all ones without hard-coding the dimension of 3 rows.

   (d) Find the solution $\vec{y}$ of $C\vec{y} = \vec{d}$.

   (e) Find the norm of the difference in the two solutions, $||\vec{x} - \vec{y}||$. This will be performed via a single function call.

General requirements about the Python problems:

a) As a comment in your source code, include your name.

b) The Python program should do the work. Don't perform the calculations and then hard-code the values in the code or look at the data and hard-code to this data unless instructed to do so.

c) Your function should use the data passed to it and should work if I were to change the data.

d) The program should not prompt the user for values or read from files unless instructed to do so.

e) I don't want your function to cause anything to print; leave it to the `XXXmain.py` file to do the printing of what is returned.

To submit the Python portion, do the following:

a) Create a directory using your net ID in lowercase characters. This should be something of the form `abc1234`.

b) Place your .py files in this directory.

c) Zip the directory, not just the files within the directory. You must use the zip format and the name of the file, assuming your net ID is `abc1234`, will be `abc1234.zip`.

d) Upload the zip'd file to Blackboard.

Review Questions

These are not for credit, but instead are intended to test your understanding of the concepts. You should be able to answer these without simply regurgitating equations.

1. What is the purpose of a change-of-basis matrix?