MCTS Based on Simple Regret

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Hard to solve search problems

Search problems are often hard too solve in practice when:

- search space is extremely large;
- *and* good heuristics are unknown.

Easier to solve:

- Chess — search space size is manageable (10^{50}).
- Timetabling — good heuristics.

Hard to solve:

- Compute Go (10^{180}), Poker (10^{70}).
- Canadian Traveller Problem.

MCTS

Monte **C**arlo **T**ree **S**earch helps in large search spaces.

- Starts with the root only.
- Repeats:
  1. **Selection:** select a branch to explore.
  2. **Expansion:** adds children of the leaf to the stored tree.
  3. **Simulation:** continues search (using a simple strategy) until a goal state is reached.
  4. **Backpropagation:** values of each stored node are updated.

Adaptive MCTS samples 'good' moves more frequently,
but sometimes **explores** new directions.

Multi-armed Bandit Problem and UCB

Multi-armed Bandit Problem:

- We are given a set of K arms.
- Each arm can be pulled multiple times.
- The reward is drawn from an **unknown** (but normally *stationary* and *bounded*) distribution.
- The **total reward** must be maximized.

UCB is near-optimal for MAB — solves *exploration/exploitation* tradeoff.

- pulls an arm that maximizes **U**pper **C**onfidence **B**ound: $b_i = \overline{X}_i + \sqrt{\frac{c \log(n)}{n_i}}$
- the cumulative regret is $O(\log n)$.

UCT

UCT (**U**pper **C**onfidence Bounds applied to **T**rees) is based on UCB.

- Adaptive MCTS.
- Applies the UCB selection scheme at each step of the rollout.
- Demonstrated good performance in Computer Go (MoGo, CrazyStone, Fuego, Pachi, ...) as well as in other domains.

However, the first step of a rollout is different:

- The purpose of MCTS is to choose an action with the greatest utility.
- Therefore, the **simple regret** must be minimized.

SRCR

Simple **R**egret followed by **C**umulative **R**egret.

- Maximizes **simple regret** at the **first step**.
- Continues with UCT from the **second step on**.

```
1:  procedure ROLLOUT(node, depth=1)
2:      if ISLEAF(node, depth) then
3:          return 0
4:      else
5:          if depth=1 then action ← FIRSTACTION(node)
6:          else action ← NEXTACTION(node)
7:          next-node ← NEXTSTATE(node, action)
8:          reward ← REWARD(node, action, next-node)
9:                    + ROLLOUT(next-node, depth+1)
10:         UPDATESTATS(node, action, reward)
```

Sampling for Simple Regret

Sampling schemes for miniminizing the simple regret:

1. ε-greedy sampling.
2. a modified version of UCB (worse for cumulative, better for simple regret).
3. VOI-based sampling.

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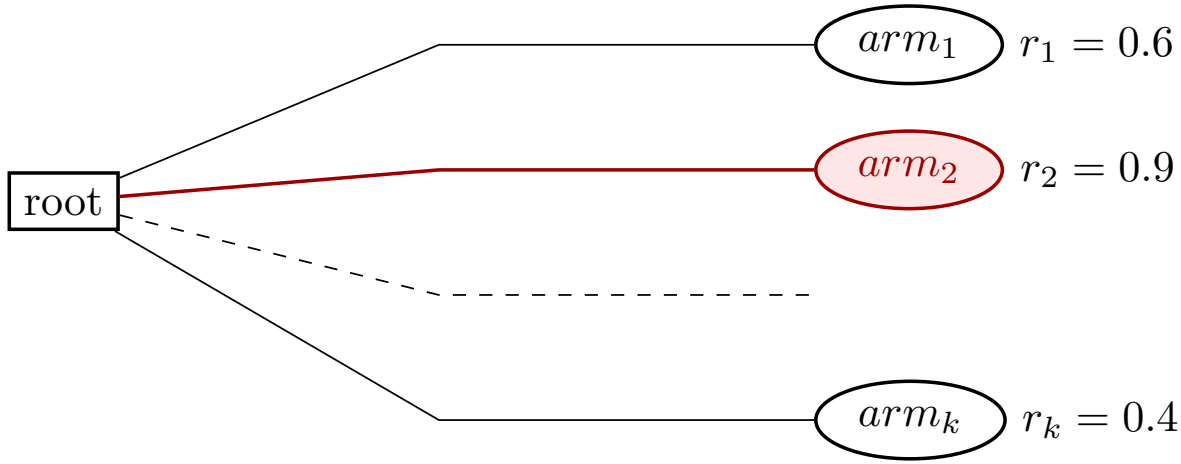
- 1, 2 — heuristic selection criterion, theoretical upper bounds can be obtained.
- 3 — based on principles of *Rational Metareasoning*, but harder to analyze.

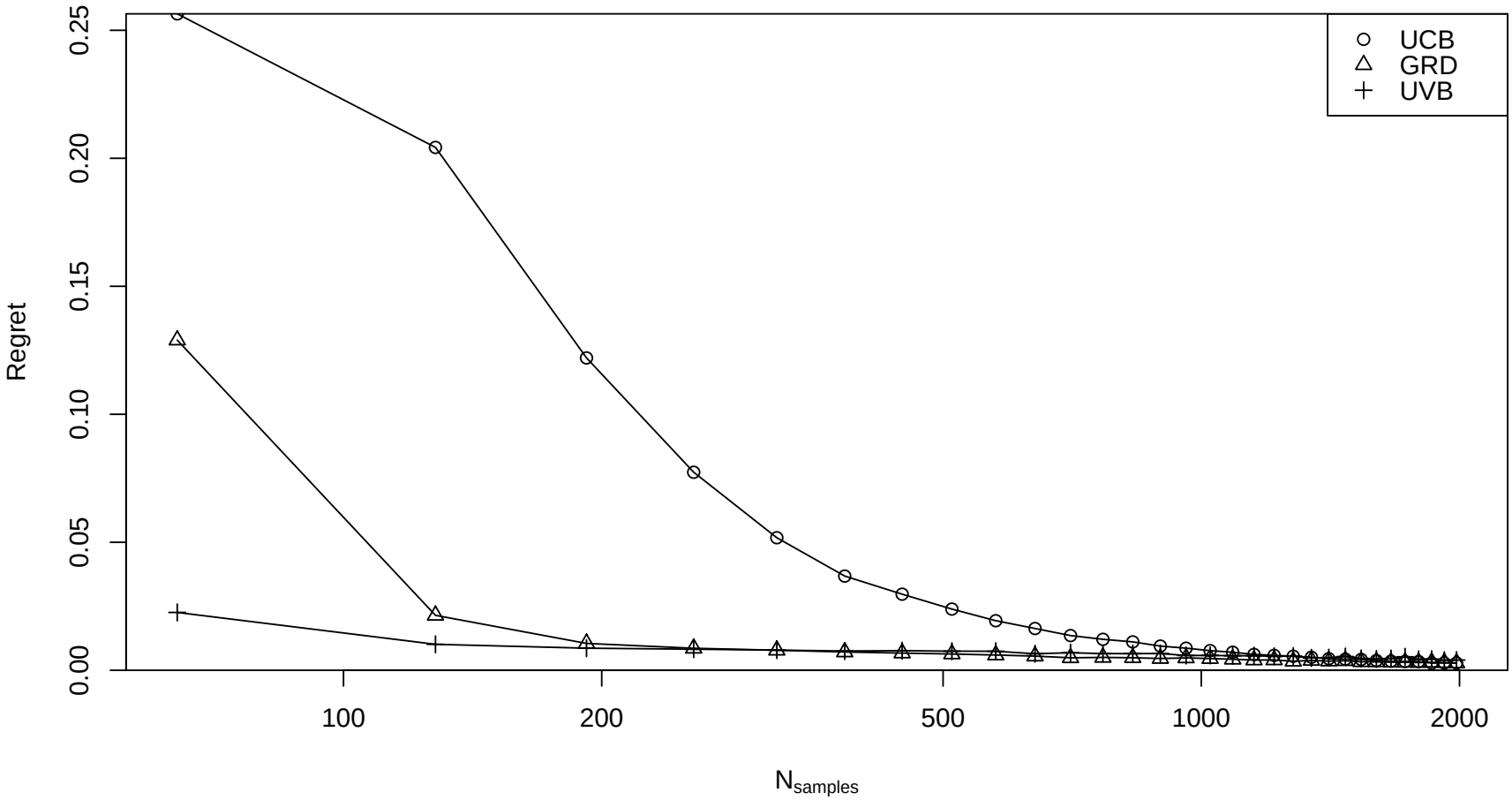
Doing better than UCT on sets



When an arm is selected based on the **sample mean**:

- Regret of UCB decreases *polynomially* with n.
- Regret of ϵ-greedy decreases *exponentially* with n.
- Regret of UVB: max V_i, $V_{i_{best}} = \frac{1-1/k}{n_{i_{best}}}$, $V_{i_{other}} = \frac{1/k}{n_{i_{other}}}$ decreases exponentially with n, faster than ϵ-greedy.

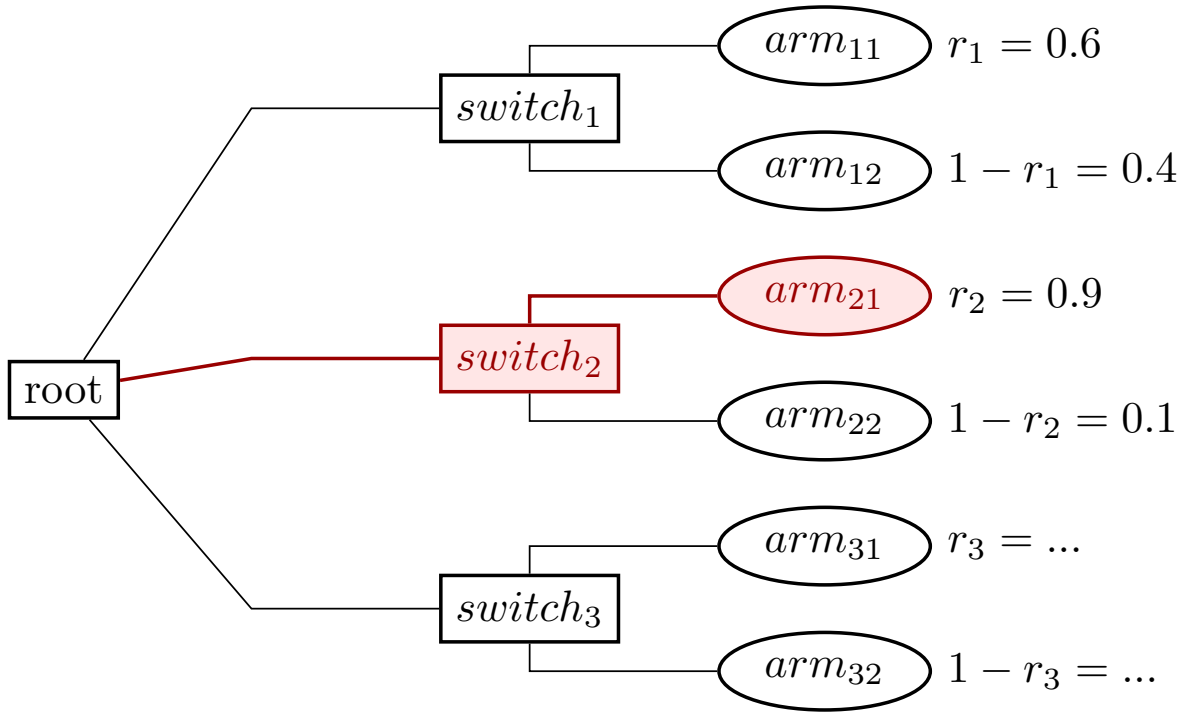
UCB vs. ϵ-greedy vs UVB



64 Bernoulli arms, randomly generated

Doing Better Than UCT on Trees

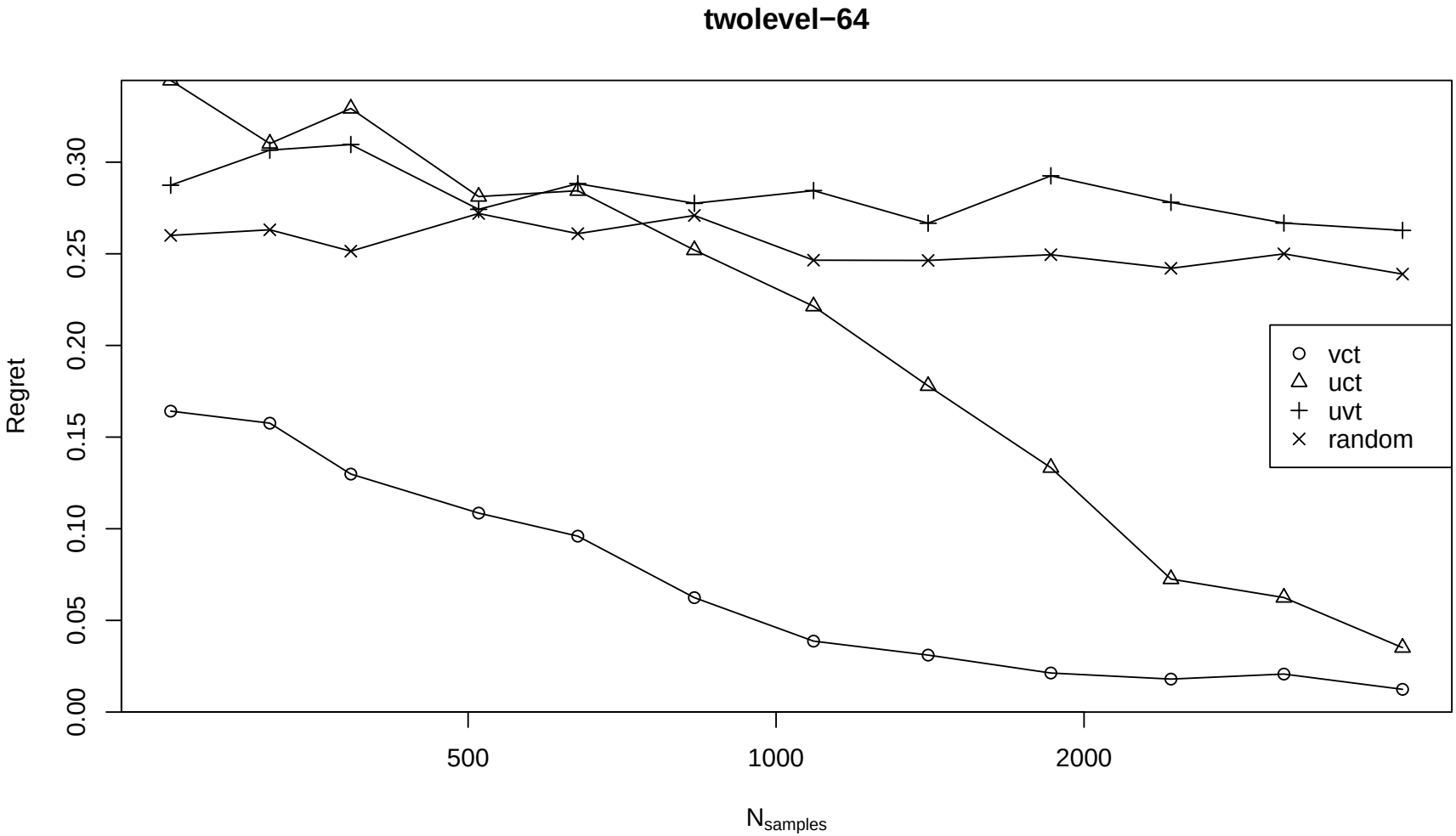
Uniform sampling is useless in this tree:



Rational sampling:

- first, choose an action that maximizes VOI (UVB);
- then, choose actions that maximize average reward (UCB).

UVT vs. VCT (UVB+UCT) vs. UCT



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