MCTS Based on Simple Regret

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Hard to solve search problems

Search problems are often hard too solve in practice when:

- search space is extremely large;
- *and* good heuristics are unknown.

Easier to solve:

- Chess — search space size is manageable (10^{50}).
- Timetabling — good heuristics.

Hard to solve:

- Compute Go (10^{180}), Poker (10^{70}).
- Canadian Traveller Problem.

MCTS

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- Starts with the root only.

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    4. **Backpropagation:** values of each stored node are updated.

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Multi-armed Bandit Problem and UCB

Multi-armed Bandit Problem:

- We are given a set of K arms.
- Each arm can be pulled multiple times.
- The reward is drawn from an **unknown** (but normally *stationary* and *bounded*) distribution.
- The **total reward** must be maximized.

UCB is near-optimal for MAB — solves *exploration/exploitation* tradeoff.

- pulls an arm that maximizes **U**pper **C**onfidence **B**ound: $b_i = \overline{X}_i + \sqrt{\frac{c \log(n)}{n_i}}$
- the cumulative regret is $O(\log n)$.

UCT

UCT (**U**pper **C**onfidence Bounds applied to **T**rees) is based on UCB.

- Adaptive MCTS.
- Applies the UCB selection scheme at each step of the rollout.
- Demonstrated good performance in Computer Go (MoGo, CrazyStone, Fuego, Pachi, ...) as well as in other domains.

However, the first step of a rollout is different:

- The purpose of MCTS is to choose an action with the greatest utility.
- Therefore, the **simple regret** must be minimized.

SRCR

Simple **R**egret followed by **C**umulative **R**egret.

- Maximizes **simple regret** at the **first step**.
- Continues with UCT from the **second step on**.

```
procedure Rollout(node, depth=1)
    if IsLeaf(node, depth) then
        return 0
    else
        if depth=1 then action ← FirstAction(node)
        else action ← NextAction(node)
        next-node ← NextState(node, action)
        reward ← Reward(node, action, next-node)
                     + Rollout(next-node, depth+1)
        UpdateStats(node, action, reward)
```

Sampling for Simple Regret

Sampling schemes for miniminizing the simple regret:

1. ε-greedy sampling.
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- 1, 2 — heuristic selection criterion, theoretical upper bounds can be obtained.
- 3 — based on principles of *Rational Metareasoning*, but harder to analyze.

Heuristic sampling schemes

ε-greedy:

- Pulls the empirically best arm with probability ε.
- Any other arm with probability $\frac{1-epsilon}{K-1}$.
- Exhibits exponentially decreasing simple regret.
- Uniform sampling when $\varepsilon = \frac{1}{K}$, much better when $\varepsilon = \frac{1}{2}$.

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$UCB_{\sqrt{\cdot}}$ ($\sqrt{\cdot}$ instead of log):

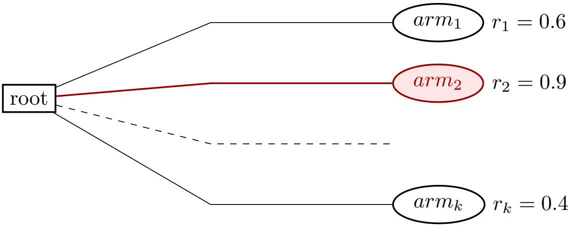
- Pulls arm i that maximizes $b_i = \overline{X}_i + \sqrt{\frac{c\sqrt{n}}{n_i}}$.
- Exhibits superpolynomially decreasing simple regret.

VOI-aware sampling

- Chooses an action with the maximum VOI estimate.
- Estimates the VOI based on bounds on:
  - the probability that one or more rollouts will make another action appear better than the current best;
  - the gain that may be incurred if such a change occurs.

$$
\begin{aligned}
VOI_\alpha &\approx \frac{\overline{X}_\beta}{n_\alpha + 1} \exp\left(-2(\overline{X}_\alpha - \overline{X}_\beta)^2 n_\alpha\right) \\
VOI_i &\approx \frac{1 - \overline{X}_\alpha}{n_i + 1} \exp\left(-2(\overline{X}_\alpha - \overline{X}_i)^2 n_i\right), \quad i \ne \alpha \\
\text{where} \quad & \alpha = \arg\max_i \overline{X}_i, \quad \beta = \arg\max_{i,\, i \ne \alpha} \overline{X}_i
\end{aligned}
$$

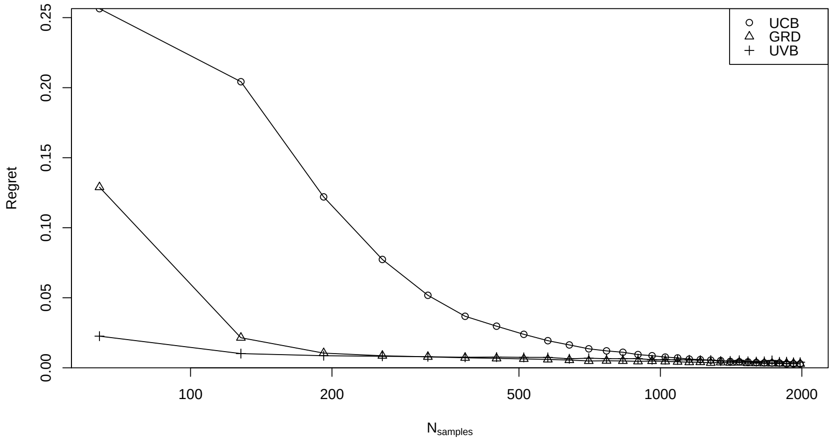
Doing better than UCT on sets



When an arm is selected based on the **sample mean**:

- Regret of UCB decreases *polynomially* with n.
- Regret of ϵ-greedy decreases *exponentially* with n.
- Regret of UVB: $\max V_i$, $V_{i_{best}} = \frac{1-1/k}{n_{i_{best}}}$, $\quad V_{i_{other}} = \frac{1/k}{n_{i_{other}}}$ decreases exponentially with n, faster than ϵ-greedy.

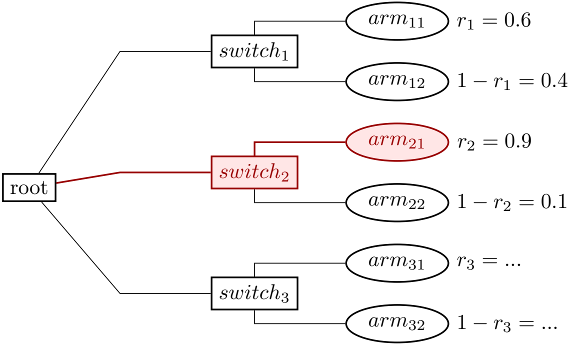
UCB vs. ϵ-greedy vs UVB



64 Bernoulli arms, randomly generated

Doing Better Than UCT on Trees

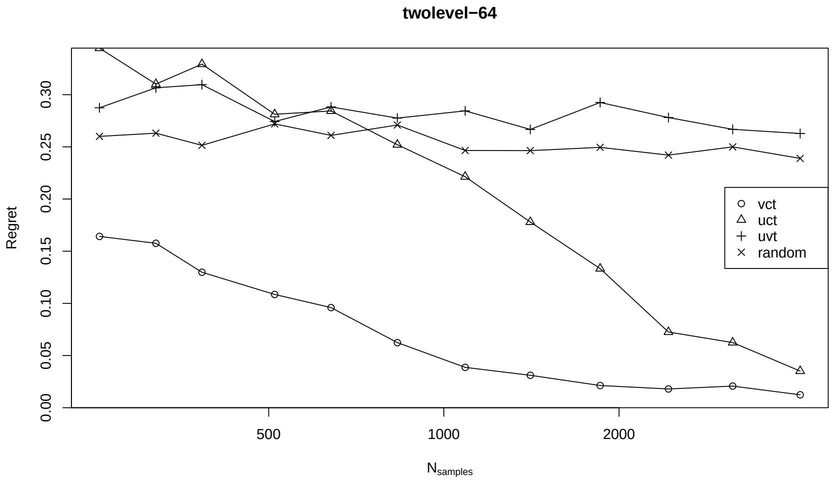
Uniform sampling is useless in this tree:



Rational sampling:

- first, choose an action that maximizes VOI (UVB);
- then, choose actions that maximize average reward (UCB).

UVT vs. VCT (UVB+UCT) vs. UCT



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