Exploring the limits of mixed precision FEM based computations on the Tegra-K1 micro-architecture

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Date: 2016/06/01

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A brief history overview

Trends:

- Memory clock speeds are increasing
  - GTX 980 Ti Memory clock speed: 2x 1753 MHz
  - GTX 1080 Memory clock speeds: 4x 2500 MHz (+185%)
- Alternative computing architectures
  - APU's
  - SoC's such as the NVIDIA Tegra K1 micro-architecture
- NVIDIA pushing the use of half precision.
  - half and half2 were announced as important new features in the CUDA Toolkit version 7.5 [1]

→ **Sharing memory between CPU and GPU is becoming easier and more common**

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Definition

Definition: Mixed precision algorithm

An algorithm that uses different precisions in its computation

Goal:

Obtain the **same accuracy** by using high precision

but **better performance** by utilizing low precision computations

Performance gains for bandwidth bound algorithms

- 64 bit = 1 double = 2 floats = 4 halfs
- More variables per **bandwidth** and variables per **storage**
- Applies to all memory levels: network, disc, cache, main, device, local, register

Performance gains for compuation bound algorithms

- 1 double addition $\approx$ 2 float additions $\approx$ 4 half additions (linear)
- 1 double multip. $\approx$ 4 float multip. $\approx$ 16 half multip. (quadratic)
- $\rightarrow$ up to 16 times better computational efficiency

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Challenges and past achievements

Challenges when it comes to Mixed precision

- Data has to be converted
- When using GPU accelleration data has to be transferred from the host to the device (usually) over the relatively slow PCIe bus
- We are being bottlenecked be memory interfaces
  → a lot of time is wasted while waiting for data

Past achievements:

In the year 2013. M. Madesen, S. L. Glimberg and A. P. Engsig-Karup were able to achieve a 38% performance increase using GPU accelerated mixed precision algorithms for full nonlinear water wave computation.[2]

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Roundoff and Cancellation

Definition Machine Precision

The smallest positive number ϵ for wich a floating point calculation evaluates the expression $1 + \epsilon > 1$ to be true.

Examples:

- $\epsilon_{double} \approx 2.220446049250313 \cdot 10^{-16}$
- $\epsilon_{float} \approx 1.1920929 \cdot 10^{-7}$
- $\epsilon_{half} \approx 9.765625 \cdot 10^{-4}$
- So more precision is usually better

Examples for cancellation effects in float

additive roundoff	$a = 1 + 0.00000004 = 1.00000004$		$=_{fl} 1$
multiplicative roundoff	$b = 1.0002 \cdot 0.9998 = 0.99999996$		$=_{fl} 1$
cancellation	$c \in \{a, b\}$	$\pm 4 = (c - 1) \cdot 10^8$	$=_{fl} 0$
order of operations	$1 + 0.00000004 - 1 =_{fl} 0$		
	$1 - 1 + 0.00000004 =_{fl} 0.0000004$		

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Computational precision vs accuracy of result

Instructive Example [S.M. Rump, 1988]

$f(x, y) = (333.75 - x^2)y^6 + x^2(11x^2y^2 - 121y^4 - 2) + 5.5y^8 + 0.5x/y$
$x_0 = 77617, y_0 = 33096$

float s23e8	1.1726
double s52e11	1.17260394005318

The correct result is:

-0.82739605994682136814116509547981629

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Floting point operations. A deeper analysis

Number representation→ almost all numbers have to be truncated

half s10e5 a	1 bit sign s_a	10 bit mantissa m_a	5 bit exp. e_a
float s23e8 b	1 bit sign s_b	23 bit mantissa m_b	8 bit exp. e_b
double s52e11 c	1 bit sign s_c	52 bit mantissa m_c	11 bit exp. e_c

multiplication		
precision	exact format	mantissa truncation:
half(a) $\cdot$ half(b)	s20e6	from 20 to 10 bit
float(a) $\cdot$ float(b)	s46e9	from 46 to 23 bit
double(a) $\cdot$ double(b)	s104e12	from 104 to 52 bit
addition		
precision	exact format	mantissa truncation:
half(a) + half(b)	s41e5	from 41 to 10 bit
float(a) + float(b)	s278e8	from 278 to 23 bit
double(a) + double(b)	s2099e11	from 2099 to 52 bit

Content

Algorithm

mixed precision iterative refinement

Algorithm 1 mixed precision iterative refinement

1: **while** $\|r_{m-1}\| \cdot \|r_0\|^{-1} > TOL$ **do**
2: $\quad r_m^h = b - Ax_m$
3: $\quad r_m^l = r_m^h$ *//convert from high to low precision*
4: $\quad Ad_m^l = r_m^l$ *//solve using a fixed number of iterations*
5: $\quad d_m^h = d_m^l$ *//convert from low to high precision*
6: $\quad x_{m+1} = x_m + d_m$
7: **end while**

- Mixed precision iterative refinement deals with the truncation errors of lower precision data formats
- The optimal number of inner iterations was empirically determined

Problem definition and Hardware configuration

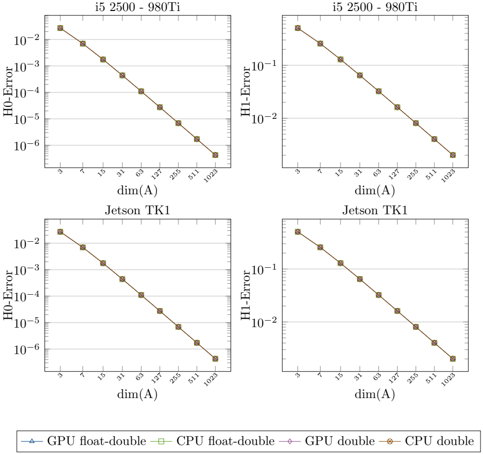
Problem

Solve the Poission problem $-\Delta u = 1,\ x \in \Omega$ with dirichlet boundary conditions $u \equiv 0,\ x \in \partial\Omega$ using conforming quadrilateral elements for the finite-element discretization of the unit square $\Omega = (-1, 1)^2$

- Method 1: Using a V-cycle MG Solver
- Method 2 : Using a V-cycle MG Solver inside of an iterative refinement loop

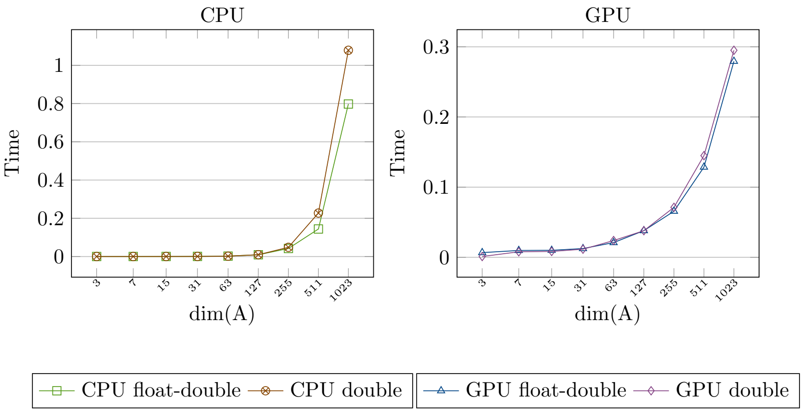
- Hardware configuration 1: i5 2500 + 980Ti
  →representing a high preformance convetional setup
- Hardware configuration 2: Tegra K1
  → representing alternative computing hardware

Testresults - Accuracy



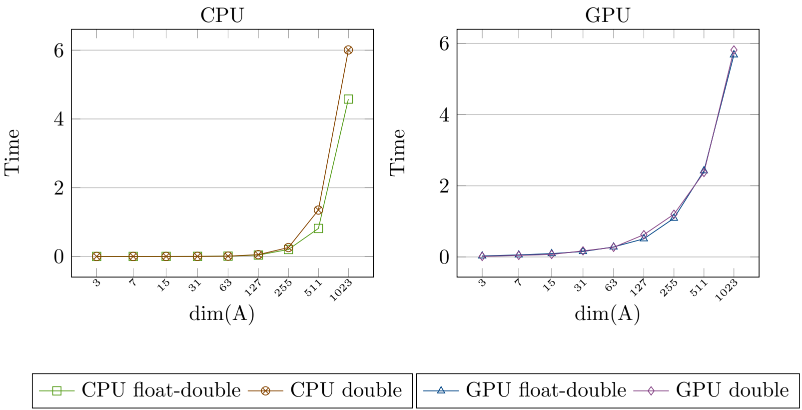
- (virtually) same accuracy as double calculation
- H0-Error is $O(m^2)$, H1-Error is $O(m^1)$. $[V_h = Q_1]$

Testresults - Performance on comodity Hardware



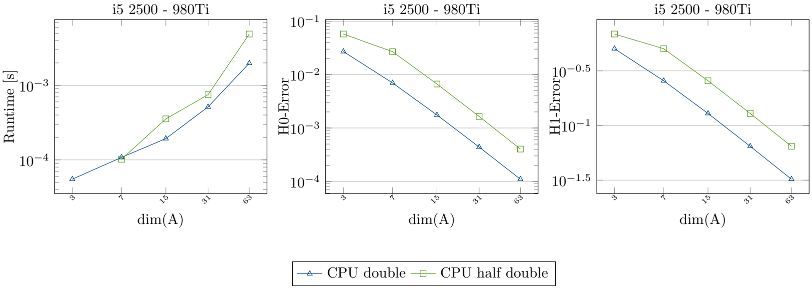
- **up to 57%** increase in performance on x86

Testresults - Performance on the Tegra K1



- **up to 65%** increase in performance on a Jetson TK1
- → **Speedup of about 1.6**
- → performance model show that a speedup of 2 is optimal
- → for longer calculations a speedup of 1.6 can save multiple days

Testresults - Half



- Accuracy loss. Inner solver diverges at more than 63^2 DOF's.

Discussion and Future work

Future work

- Try combining half, float and double precision in order to achieve an even greater performance increase

Thank you for your attention!

Bibliography

New features in cuda 7.5.
https://devblogs.nvidia.com/parallelforall/new-features-cuda-7-5/.
Accessed: 2016-05-27.

Madsen M. Glimberg S. L., Engsig-Karup A. P.
A fast gpu-accelecrated mixed-precision strategy for full nonlinear water wave computation.
Technical report, ResearchGate, January 2013.