CHALMERS, GÖTEBORGS UNIVERSITET

Sample questions for exam in
ARTIFICIAL NEURAL NETWORKS

COURSE CODES: **FFR 135, FIM720GU, PhD**

The maximum score in the exam is 12 points. The sample questions below give 5 points.

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1. Initial stability in deterministic Hopfield model.
Consider a deterministic Hopfield model with deterministic update rule

$$S_i \leftarrow \mathrm{sgn}(\sum_{j=1}^{N} w_{ij} S_j) \ . \tag{1}$$

Here N is the number of neurons (= number of bits in a pattern), S_i stands for the state of neuron i, and w_{ij} are synaptic weights given by

$$w_{ij} = \frac{1}{N} \sum_{\mu=1}^{p} \zeta_i^{(\mu)} \zeta_j^{(\mu)} \quad \text{for } i \neq j \ , \text{ and } w_{ii} = 0 \ . \tag{2}$$

In Eq. (2) p is the total number of patterns stored in the network, and $\zeta_i^{(\mu)}$ denotes the state of bit i of stored pattern $\boldsymbol{\zeta}^{(\mu)}$.

a) Assuming that you feed in the network pattern $\boldsymbol{\zeta}^{(\nu)}$, derive the condition for a randomly chosen bit $\zeta_i^{(\nu)}$ of this pattern to be stable after a single step of deterministic asynchronous updating. Rewrite this stability condition using the "cross-talk term". (**0.5**p)

b) Assume that the patterns stored in the network described in a) are random ($\zeta_j^{(\mu)} = 1$ or -1 with equal probabilities), and that $p \gg 1$ and $N \gg 1$. Under these conditions, derive an approximate expression for the probability that a randomly chosen bit $\zeta_i^{(\nu)}$ is stable after a single step of asynchronous updating. (**0.5**p)

2. Hopfield model: recognition of one pattern.

Consider a deterministic Hopfield model with stored pattern shown in Fig. 1. The update rule is given by Eq. (1), and the weights w_{ij} satisfy Hebb's rule

$$w_{ij} = \frac{1}{N} \sum_{\mu=1}^{p} \zeta_i^{(\mu)} \zeta_j^{(\mu)} \quad \text{for } i, j = 1, \ldots, N \ . \tag{3}$$

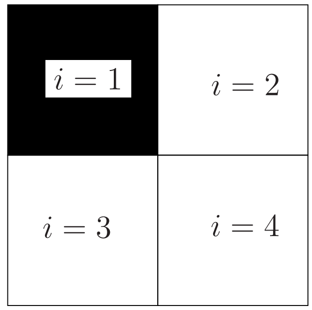

Figure 1: Question 2. Stored pattern $\boldsymbol{\zeta}^{(1)}$ with $N = 4$ bits, $\zeta_1^{(1)} = 1$, and $\zeta_i^{(1)} = -1$ for $i = 2, 3, 4$.

This network is expected to recognise the stored pattern provided that the number of bits that differ between the stored pattern and a fed pattern is $\leq N/2$. In this question you are asked to check this expectation as follows.

Separately feed into the network each of the 2^4 possible patterns with 4 bits. In each case, use deterministic synchronous updating and find the network output after one iteration. Discuss differences between the 2^4 cases analysed. (**1**p)

3. Backpropagation I

To train a multilayer perceptron using backpropagation one needs update formulae for the weights and thresholds in the network. Your task is to derive the corresponding update formulae for the network shown in Fig. 2. The weights for the first and second hidden layer, and for the output layer are denoted by $w_{ji}^{(1)}$, $w_{kj}^{(2)}$ and W_{1k}, respectively. The corresponding thresholds are denoted by $\theta_j^{(1)}$, $\theta_k^{(2)}$, and Θ_1. Denote the activation function by $g(\cdot)$. Denote the target value for input pattern $\boldsymbol{\xi}^{(\mu)}$ by $\zeta_1^{(\mu)}$. (**2**p)

4. Backpropagation II

Explain the idea behind backpropagation in a multilayer perceptron. Discuss an appropriate algorithm for this method. In the discussion, refer to and explain the following terms: “forward propagation”, “backward propagation”, “hidden layers”, “energy function”, “gradient descent”, “local energy minima”, “stochastic mode”, “batch mode”, “training set”, “validation set”, “classification error”, “over fitting”. Your answer must not be longer than one A4 page. (**1**p)

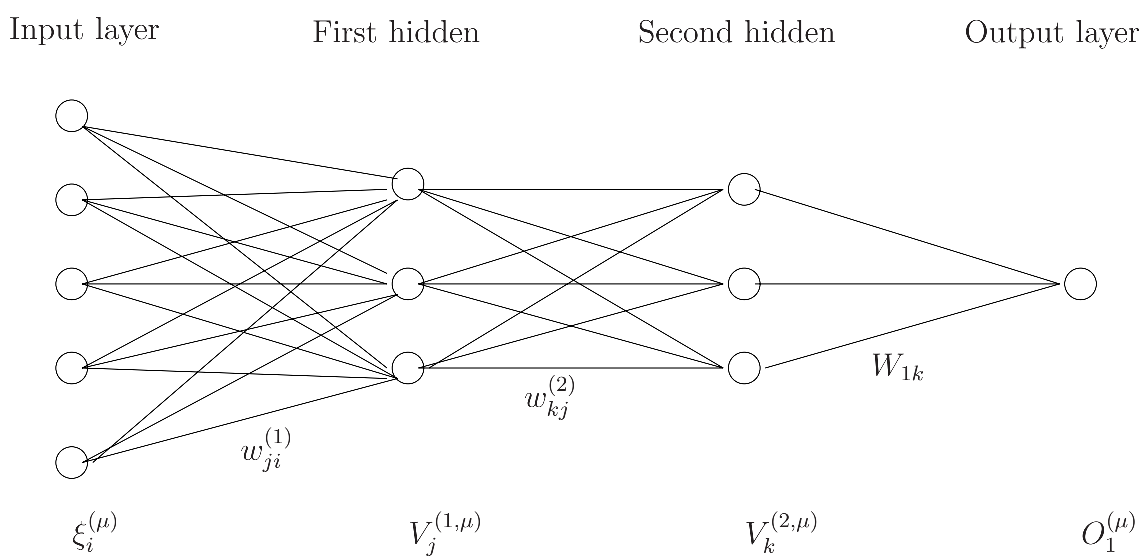

Figure 2: Question 3. Multilayer perceptron with two hidden layers.