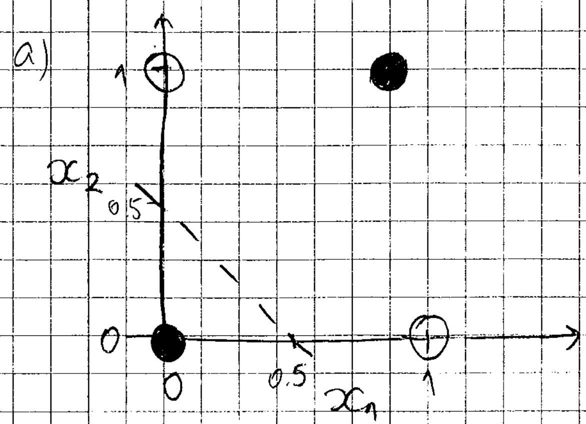
7. Linear separability of Boolean functions.

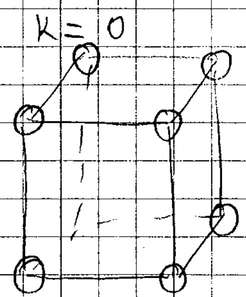
a)



In order for the problem to be solved by a simple perceptron with 2 input terminals and one output unit (no hidden layers) it must be linearly separable, that is, there must exist a plane such that all inputs mapping to target outputs equal to +1 have to be on one side of the plane, and all inputs mapping to target outputs −1 have to be on the other side of the plane. This is not possible for the Boolean XOR problem. See an example decision boundary in the figure; it obviously does not solve the problem, because one pattern $\binom{1}{1}$ is wrongly classified.

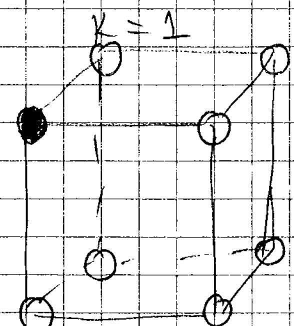
b) 3-d Boolean functions. Consider a function with $\underline{k}$ input patterns that map to 1 (target output), and 8−k input patterns that map to −1.

k = 0



Linearly separable.
Symmetric to k=8 case } 2 functions

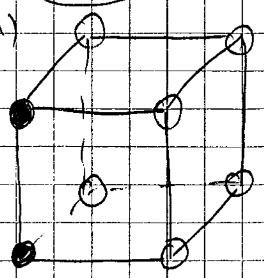
k = 1



→ Linearly separable
8 functions × 2 (case k=7)
= 16 functions

K=2 (total $\binom{8}{2} = \frac{8\cdot 7}{2} = 28$)

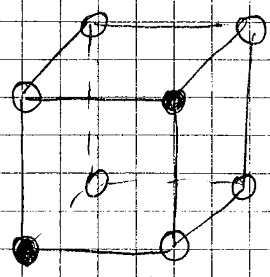
a)



Linearly separable

12 cases $\times \underset{\uparrow \atop (K=6)}{2}$ = 24 functions

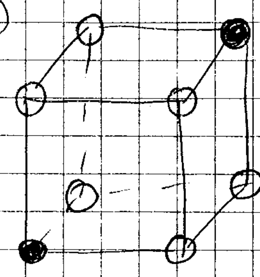
b)



Not lin. sep.

12 cases for K=2
(12 cases for K=6)

c)



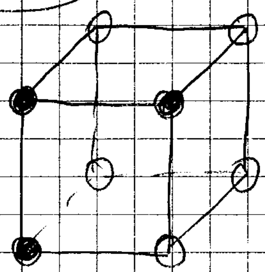
Not lin. separable

4 cases (for K=2)
4 cases (for K=6)

// Check : for K=2, there are 28 functions in total, 12 LS + 16 NLS //

K=3 total : $\binom{8}{3} = \frac{8\cdot 7\cdot 6}{3\cdot 2} = 56$

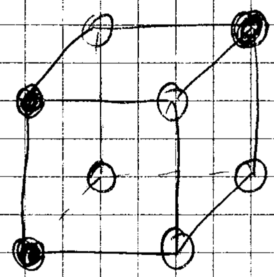
a)



Linearly separable

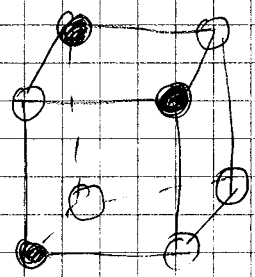
$\binom{4}{3}\cdot 6$ cases $= \frac{4\cdot 3\cdot 2}{3\cdot 2}\cdot 6 = 24$ for K=3
+24 for K=5 } = 48

b)



NLS: $2\cdot 12 = 24$
(and 24 for K=5)

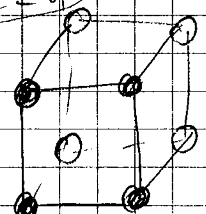
c)



NLS: $2\cdot 4 = 8$
(and 8 for K=5)

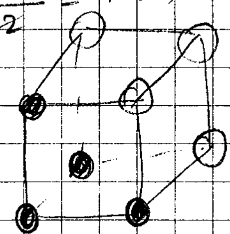
K=4 (Total $\binom{8}{4} = \frac{8\cdot 7\cdot 6\cdot 5}{4\cdot 3\cdot 2} = 70$)

a)



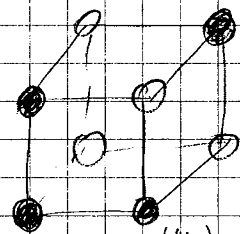
LS: 6 functions

b)



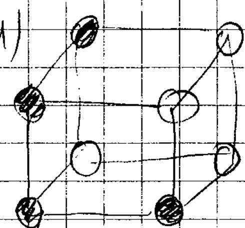
LS: 8 functions

c)



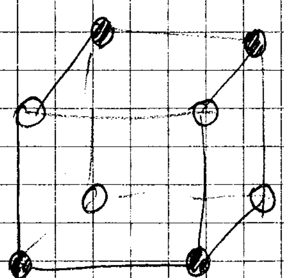
NLS: $6\cdot\binom{4}{3} = 6\cdot 4 = 24$

d)

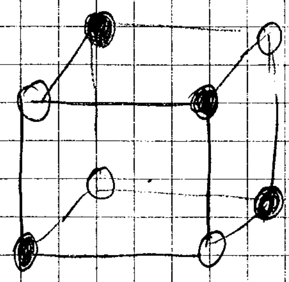


NLS: 24

e)



NLS: 4



NLS: 4

Total:

LS = 6 + 8 = 14
NLS = 24 + 24 + 8 = 56 } = 70

In total, there are 2 + 16 + 24 + 48 + 14 = 104 linearly separable 3-dimensional Boolean functions.

// Note: it suffices to show and count only linearly separable cases to obtain full points for the task. //