

In[4370]:=

```
minxplot = -5;
maxxplot = 5;
minyplot = -5;
maxyplot = 5;

μ = 1;

sol[u_, v_] :=
  NDSolve[{x'[t] == μ * x[t] - 5 y[t] - x[t]^3, y'[t] == 5 x[t] + μ * y[t] + 3 y[t]^3,
    x[0] == u, y[0] == v}, {x, y}, {t, 10}, Method → "StiffnessSwitching"]

miny = -0.01;
maxy = 0.01;
minx = -0.01;
maxx = 0.01;
step = 0.005;

Table11 = Table[{minx, y}, {y, miny, maxy, step}];
Table21 = Table[{x, maxy}, {x, minx, maxx, step}];
Table31 = Table[{maxx, y}, {y, miny, maxy, step}];
Table41 = Table[{x, miny}, {x, minx, maxx, step}];

miny = -4;
maxy = 4;
minx = -4;
maxx = 4;
step = 0.2;

Table12 = Table[{minx, y}, {y, miny, maxy, step}];
Table22 = Table[{x, maxy}, {x, minx, maxx, step}];
Table32 = Table[{maxx, y}, {y, miny, maxy, step}];
Table42 = Table[{x, miny}, {x, minx, maxx, step}];

TableFinal = Join[Table11, Table21, Table31, Table41, Table12, Table32];

Show[Table[ParametricPlot[
  Evaluate[{x[t], y[t]} /. sol[TableFinal[[i, 1]], TableFinal[[i, 2]]],
  {t, 0, 5}, PlotRange → {{minxplot, maxxplot}, {minyplot, maxyplot}},
  PlotLabel → Style["System 1. μ = 1. Arrows denote starting
    points and direction. Unstable origin.", FontSize → 15]] /.
  Line[x_] → {Arrowheads[{0.01, 0.}], Arrow[x]}, {i, Length[TableFinal]}]]
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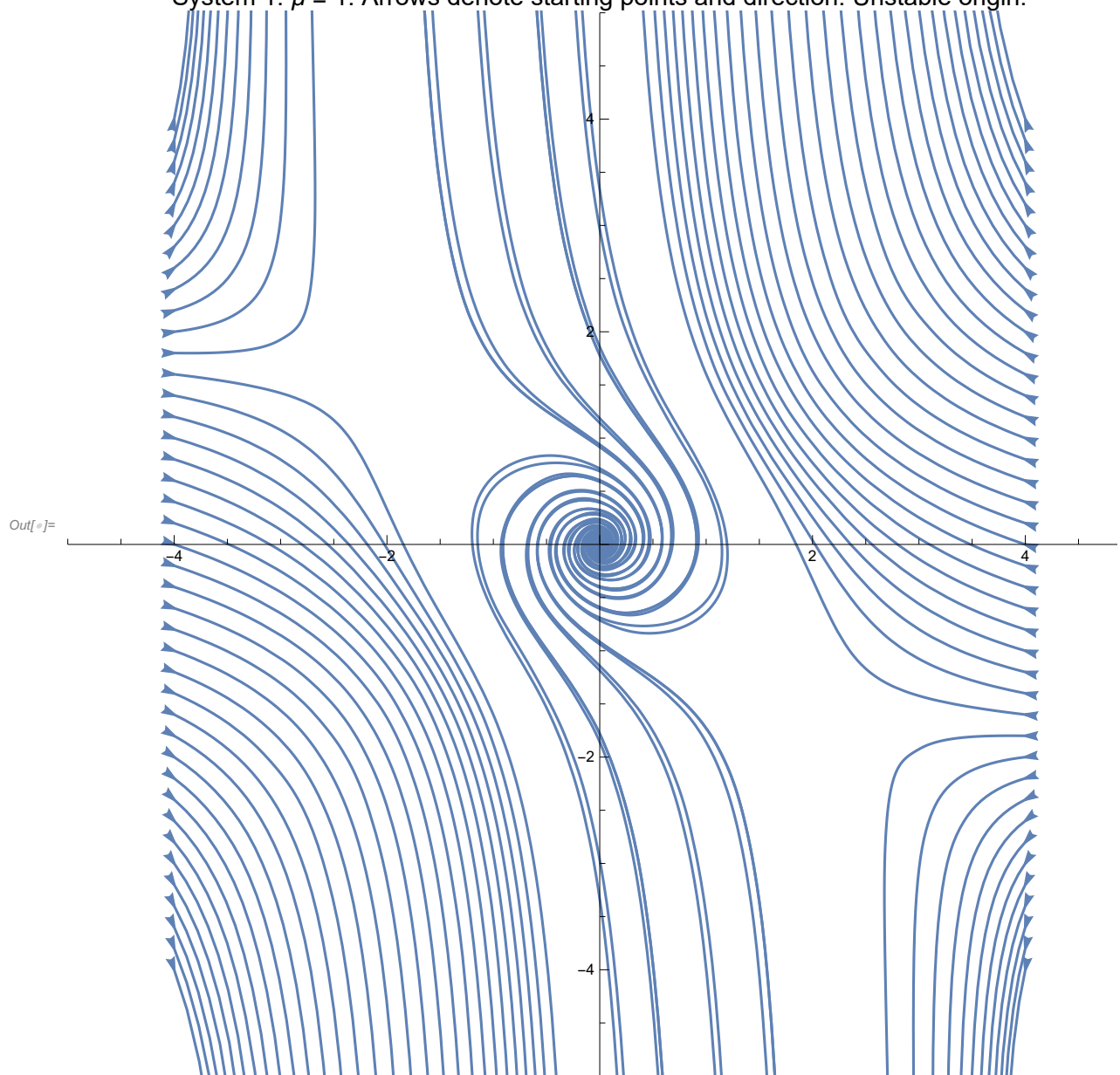
... **NDSolve**: At t == 4.13400248900072, step size is effectively zero; singularity or stiff system suspected.

... **NDSolve**: At t == 4.622847994309177, step size is effectively zero; singularity or stiff system suspected.

... **NDSolve**: At t == 4.717209143655243, step size is effectively zero; singularity or stiff system suspected.

... **General**: Further output of NDSolve::ndsiz will be suppressed during this calculation.

System 1.  $\mu = 1$ . Arrows denote starting points and direction. Unstable origin.



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In[707]:= minxplot = -5;
          maxxplot = 5;
          minyplot = -5;
          maxyplot = 5;

           $\mu = -1$ ;

          sol[u_, v_] :=
            NDSolve[{x'[t] ==  $\mu$  * x[t] - 5 y[t] - x[t]^3, y'[t] == 5 x[t] +  $\mu$  * y[t] + 3 y[t]^3,
                    x[0] == u, y[0] == v}, {x, y}, {t, 10}, Method -> "StiffnessSwitching"]

          miny = -2;
          maxy = 2;
          minx = -2;
          maxx = 2;
          step = 0.2;

```

```

Table11 = Table[{minx, y}, {y, miny, maxy, step}];
Table21 = Table[{x, maxy}, {x, minx, maxx, step}];
Table31 = Table[{maxx, y}, {y, miny, maxy, step}];
Table41 = Table[{x, miny}, {x, minx, maxx, step}];

```

```

miny = -4;
maxy = 4;
minx = -4;
maxx = 4;
step = 0.2;

```

```

Table12 = Table[{minx, y}, {y, miny, maxy, step}];
Table22 = Table[{x, maxy}, {x, minx, maxx, step}];
Table32 = Table[{maxx, y}, {y, miny, maxy, step}];
Table42 = Table[{x, miny}, {x, minx, maxx, step}];

```

```

miny = -1;
maxy = 1;
minx = -1;
maxx = 1;
step = 0.2;

```

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Table13 = Table[{minx, y}, {y, miny, maxy, step}];
Table23 = Table[{x, maxy}, {x, minx, maxx, step}];
Table33 = Table[{maxx, y}, {y, miny, maxy, step}];
Table43 = Table[{x, miny}, {x, minx, maxx, step}];

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TableFinal = Join[Table11, Table21, Table31, Table41, Table12,
  Table22, Table32, Table42, Table13, Table23, Table33, Table43];

```

```

Show[Table[ParametricPlot[
  Evaluate[{x[t], y[t]} /. sol[TableFinal[[i, 1]], TableFinal[[i, 2]]],
  {t, 0, 5}, PlotRange -> {{minxplot, maxxplot}, {minyplot, maxyplot}},
  PlotLabel -> Style["System 1.  $\mu = -1$ . Arrows denote starting
    points and direction. Stable origin.", FontSize -> 15]] /.
  Line[x_] -> {Arrowheads[{0.01, 0.}], Arrow[x]}, {i, Length[TableFinal]}]]

```

... NDSolve: At t == 0.037882367181830655, step size is effectively zero; singularity or stiff system suspected.

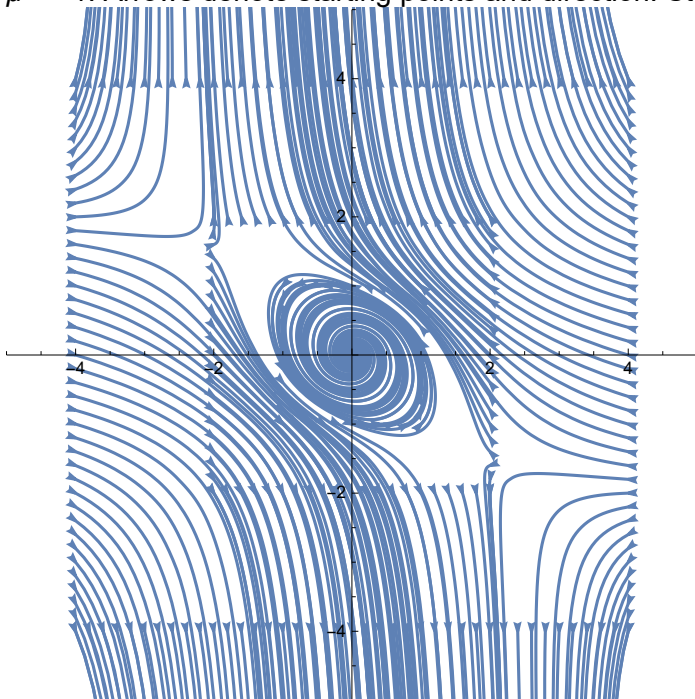
... NDSolve: At t == 0.045258189821241454, step size is effectively zero; singularity or stiff system suspected.

... NDSolve: At t == 0.05464811135174331, step size is effectively zero; singularity or stiff system suspected.

... General: Further output of NDSolve::ndsiz will be suppressed during this calculation.

$\mu = -1$ . Arrows denote starting points and direction. Sta

Out[741]=



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In[462]:= minxplot = -1.1;
maxxplot = 1.1;
minyplot = -1.1;
maxyplot = 1.1;

μ = 0.2;

sol[u_, v_] :=
  NDSolve[{x'[t] == μ * x[t] + y[t] - x[t]^2, y'[t] == -x[t] + μ * y[t] + 2 * x[t]^2,
    x[0] == u, y[0] == v}, {x, y}, {t, 100}, Method → "StiffnessSwitching"]

miny = -0.01;
maxy = 0.01;
minx = -0.01;
maxx = 0.01;
step = 0.005;

Table11 = Table[{minx, y}, {y, miny, maxy, step}];
Table21 = Table[{x, maxy}, {x, minx, maxx, step}];
Table31 = Table[{maxx, y}, {y, miny, maxy, step}];
Table41 = Table[{x, miny}, {x, minx, maxx, step}];

miny = -1;
maxy = 1;
minx = -1;
maxx = 1;
step = 0.1;

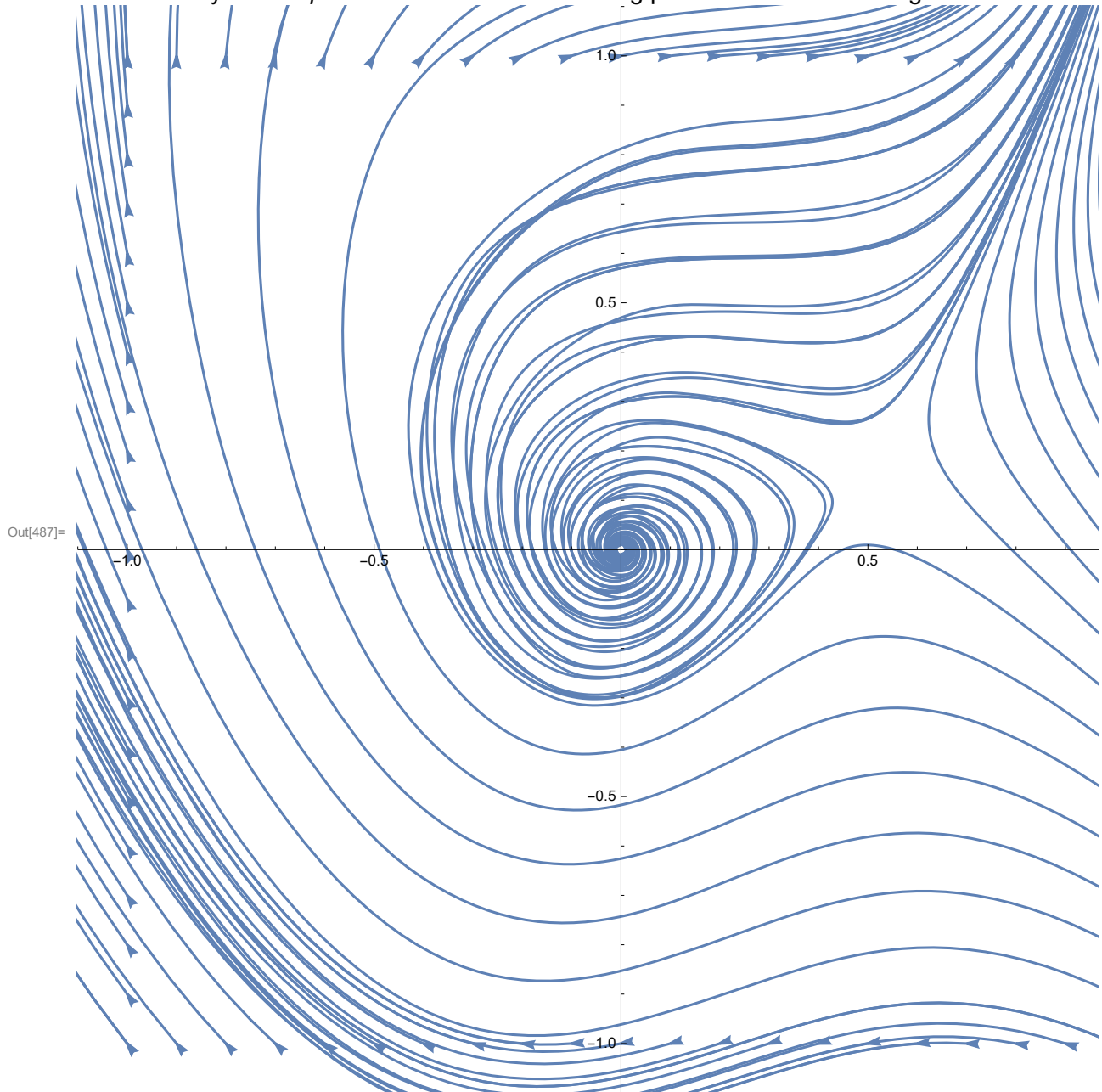
Table13 = Table[{minx, y}, {y, miny, maxy, step}];
Table23 = Table[{x, maxy}, {x, minx, maxx, step}];
Table33 = Table[{maxx, y}, {y, miny, maxy, step}];
Table43 = Table[{x, miny}, {x, minx, maxx, step}];

TableFinal =
  Join[Table11, Table21, Table31, Table41, Table13, Table23, Table33, Table43];

Show[Table[ParametricPlot[
  Evaluate[{x[t], y[t]} /. sol[TableFinal[[i, 1]], TableFinal[[i, 2]]],
  {t, 0, 100}, PlotRange → {{minxplot, maxxplot}, {minyplot, maxyplot}},
  PlotLabel → Style["System 2. μ = 0.2. Arrows denote starting
    points and direction. Origin unstable.", FontSize → 15]] /.
  Line[x_] → {Arrowheads[{0.01, 0.}], Arrow[x]}, {i, Length[TableFinal]}]]

```

System 2.  $\mu = 0.2$ . Arrows denote starting points and direction. Origin unstable.



```

In[488]:= minxplot = -1.1;
maxxplot = 1.1;
minyplot = -1.1;
maxyplot = 1.1;

 $\mu = -0.2;$ 

sol[u_, v_] :=
NDSolve[{x'[t] ==  $\mu$  * x[t] + y[t] - x[t]^2, y'[t] == -x[t] +  $\mu$  * y[t] + 2 * x[t]^2,
  x[0] == u, y[0] == v}, {x, y}, {t, 100}, Method -> "StiffnessSwitching"]

miny = -1;
maxy = 1;
minx = -1;
maxx = 1;
step = 0.1;

Table13 = Table[{minx, y}, {y, miny, maxy, step}];
Table23 = Table[{x, maxy}, {x, minx, maxx, step}];
Table33 = Table[{maxx, y}, {y, miny, maxy, step}];
Table43 = Table[{x, miny}, {x, minx, maxx, step}];

TableFinal = Join[Table13, Table23, Table33, Table43];

Show[Table[ParametricPlot[
  Evaluate[{x[t], y[t]} /. sol[TableFinal[[i, 1]], TableFinal[[i, 2]]],
  {t, 0, 100}, PlotRange -> {{minxplot, maxxplot}, {minyplot, maxyplot}},
  PlotLabel -> Style["System 2.  $\mu = -0.2$ . Arrows denote starting
    points and direction. Origin stable.", FontSize -> 15]] /.
  Line[x_] -> {Arrowheads[{0.01, 0.}], Arrow[x]}, {i, Length[TableFinal]}]]

```

System 2.  $\mu = -0.2$ . Arrows denote starting points and direction. Origin stable.

