Homework

3.4 $dp.dA = -\rho.dA.ds.a$

$\rightarrow \frac{dp}{ds} = -\rho.a = -1000(kg/m^3) \times (30 m/s^2)$

$= -3 \times 10^4 (Pa/m) = -30 kPa/m$

3.30 Bernoulli : $\frac{V^2}{2} + gz + \frac{P}{\rho} = const$

energy state 1 = energy state 2

$$\frac{V_1^2}{2} + gz_1 + \frac{P_1}{\rho} = \frac{V_2^2}{2} + gz_2 + \frac{P_2}{\rho}$$

we have: $v_1 = 0$

$z_2 - z_1 = 0$

$P_1 - P_2 = 0$

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$\rightarrow gz_1 = \frac{V_2^2}{2} \rightarrow (9{,}81 m/s^2) \times z_1 = \frac{(8 m/s)^2}{2} \rightarrow z_1 = 3{,}362 m$

3.41 $Q = A_1 V_1 = A_2 V_2$

$$V_1 = \frac{\frac{\pi D_2^2}{4}}{\frac{\pi D_1^2}{4}} V_2 = \left(\frac{D_2}{D_1}\right)^2 V_2$$

$$V_1 = \left(\frac{3in}{4in}\right)^2 \times (150 ft/s) = 84{,}4 ft/s$$

$\rightarrow z_1 = z_2 = 0$; $P_2 = 0$; $v_2 = 150 ft/s$

$\rightarrow P_1 + \frac{\rho V_1^2}{2} = \frac{\rho V_2^2}{2}$

$\rightarrow P_1 = \frac{\rho}{2}(V_2 - V_1)^2 = \frac{0{,}00238 \frac{slugs}{ft^3}}{2} \times \left[(150 ft/s) - (84{,}4 ft/s)\right]^2$

$= 18{,}3054 \ lb/ft^2$

4.8 Given: $u = x + y$

$v = xy^3 + 16$

$w = 0$

at a stagnation point:

$u = v = w = 0$

$x + y = 0 \rightarrow x = -y$
$xy^3 + 16 = 0$
$\rightarrow (-y)y^3 + 16 = 0$
$\rightarrow y = \pm 2$
$\rightarrow x = \pm 2$

$\Rightarrow$ 2 stagnation points are $(-2; 2)$ & $(2; -2)$

4.27 $$\vec{V} = \frac{-2xyz}{(x^2+y^2)^2}\hat{i} + \frac{(x^2-y^2)z}{(x^2+y^2)^2}\hat{j} + \frac{y}{x^2+y^2}\hat{k}$$

we have $$\nabla \vec{V} = \begin{bmatrix} \partial V_x/\partial x & \partial V_x/\partial y & \partial V_z/\partial z \\ \partial V_y/\partial x & \partial V_y/\partial y & \partial V_y/\partial z \\ \partial V_z/\partial x & \partial V_z/\partial y & \partial V_z/\partial z \end{bmatrix}$$

$$\nabla \vec{V} = \begin{bmatrix} \frac{8x^2yz}{(x^2+y^2)^3} - \frac{2yz}{(x^2+y^2)^2} & \frac{8xy^2z}{(x^2+y^2)^3} - \frac{2xz}{(x^2+y^2)^2} & -\frac{2xy}{(x^2+y^2)^2} \\ -\frac{4x(x^2-y^2)z}{(x^2+y^2)^3} + \frac{2xz}{(x^2+y^2)^2} & -\frac{4y(x^2-y^2)z}{(x^2+y^2)^3} - \frac{2yz}{(x^2+y^2)^2} & \frac{x^2-y^2}{(x^2+y^2)^2} \\ -\frac{2xy}{(x^2+y^2)^2} & -\frac{2y^2}{(x^2+y^2)^2} + \frac{1}{x^2+y^2} & 0 \end{bmatrix}$$

$$\vec{a} = \nabla \vec{V} . V$$

$$\begin{bmatrix} \frac{8x^2yz}{(x^2+y^2)^3} - \frac{2yz}{(x^2+y^2)^2} & \frac{8x^2yz}{(x^2+y^2)^3} - \frac{2xz}{(x^2+y^2)^2} & -\frac{2xy}{(x^2+y^2)^2} \\ -\frac{4x(x^2-y^2)z}{(x^2+y^2)^3} + \frac{2xz}{(x^2+y^2)^2} & -\frac{4y(x^2-y^2)z}{(x^2+y^2)^3} - \frac{2yz}{(x^2+y^2)^2} & \frac{x^2-y^2}{(x^2+y^2)^2} \\ -\frac{2xy}{(x^2+y^2)^2} & -\frac{2y^2}{(x^2+y^2)^2} + \frac{1}{x^2+y^2} & 0 \end{bmatrix} \times \begin{bmatrix} -\frac{2xy}{(x^2+y^2)^2} \\ \frac{(x^2-y^2)z}{(x^2+y^2)^2} \\ \frac{y}{x^2+y^2} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{2xy^2}{(x^2+y^2)^3} + \frac{(x^2-y^2)z\left(\frac{8xy^2z}{(x^2+y^2)^3} - \frac{2xz}{(x^2+y^2)^2}\right)}{(x^2+y^2)^2} - \frac{2xyz\left(\frac{8xy^2z}{(x^2+y^2)^3} - \frac{2yz}{(x^2+y^2)^2}\right)}{(x^2+y^2)^2} \\ \frac{y(x^2-y^2)}{(x^2+y^2)^3} - \frac{2xyz\left(-\frac{4x(x^2-y^2)z}{(x^2+y^2)^3} + \frac{2xz}{(x^2+y^2)^2}\right)}{(x^2+y^2)^2} + \frac{(x^2+y^2)z\left(-\frac{4y(x^2-y^2)z}{(x^2+y^2)^3} - \frac{2yz}{(x^2+y^2)^2}\right)}{(x^2+y^2)^2} \\ \frac{4x^2y^2z}{(x^2+y^2)^4} + \frac{(x^2-y^2)\left(-\frac{2y^2}{(x^2+y^2)^2} + \frac{1}{x^2+y^2}\right)z}{(x^2+y^2)^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-2x(y^2+z^2)}{(x^2+y^2)^3} \\ \frac{(x^2-y^2)z}{(x^2+y^2)^2} \\ \frac{y}{x^2+y^2} \end{bmatrix} = \frac{-2x(y^2+z^2)}{(x^2+y^2)^3}\hat{i} - \frac{(x^2-y^2)z}{(x^2+y^2)^2}\hat{j} + \frac{y}{x^2+y^2}\hat{k}$$

5.3 a) $\int_{sys} \rho \forall$: unchanging mass of system

from conservation of mass principle $\rightarrow \frac{D}{Dt}\int_{sys} \rho d\forall = 0$

b) $\dot{m}_2 = \rho A_2 V_2 \rightarrow V_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{(10 \text{slugs/s})}{(1,94 \text{slugs/ft}^3)(0,3 \text{ft}^2)}$

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$\rightarrow V_2 = 17{,}182 \text{ ft/s}$

c) $\int_3 \rho \vec{V} \cdot \hat{n} \cdot dA$

from conservation of mass principle

$\rightarrow \dot{m}_1 = \dot{m}_2 + \dot{m}_3$

$\rightarrow \dot{m}_3 = \dot{m}_1 - \dot{m}_2 = \rho A_1 V_1 - \dot{m}_2$

$= \left(1{,}94 \frac{\text{slugs}}{\text{ft}^3}\right) \times (0{,}7 \text{ ft}^2) \times (15 \text{ ft/s}) - (10 \text{ slugs/s})$

$= 10{,}37 \text{ slugs/s}$

5.19 $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = \dot{m}_{0,8V} + \dot{m}_V$

$\rightarrow \rho A_1 V_1 + \rho A_2 V_2 = \rho A_{0,8V} \cdot 0{,}8 V + \rho \cdot A_V \cdot V$

$\rightarrow V = \frac{A_1 V_1 + A_2 V_2}{A_{0,8V}(0{,}8) + A_V} = \frac{(50 \times 3 \times 3) + (80 \times 5 \times 4)}{(30 \times 6 \times 0{,}8) + (70 \times 6)} = 3{,}63 \text{ ft/s}$

7.1 $G = \sqrt{\frac{\dot{W}}{\mu \cdot \forall}} = \sqrt{\frac{\frac{\text{ft} \cdot \text{lb}}{\text{s}}}{\frac{\text{lb} \cdot \text{s}}{\text{ft}^2} \times \text{ft}^3}} = \sqrt{\frac{1}{\text{s}^2}} = \frac{1}{\text{s}} = \text{s}^{-1}$

7.7 $\dot{W} = \frac{F \cdot L}{T} = F L T^{-1}$ $\qquad \forall = L^3$

$\mu = F T L^{-2}$ $\qquad G = T^{-1}$

number of variables = 4

number of repeatable variables = 3

number of π terms = 4 - 3 = 1

$\pi_1 = \dot{W} \times \mu^a \times \forall^b \times G^e$

$= F L T^{-1} \times (F L T^{-2})^a \times (L^3)^b \times (T^{-1})^e$

$= F^{a+1} \times L^{-2a+3b+1} \times T^{a-c-1}$

$$\begin{cases} a + 1 = 0 \\ -2a + 3b + 1 = 0 \\ a - c - 1 = 0 \end{cases} \rightarrow \begin{cases} a = -1 \\ b = 1 \\ c = -2 \end{cases}$$

$\rightarrow \pi_1 : \mu^{-1} \cdot \forall^{-1} \cdot G^{-2} \cdot \dot{W} = \frac{\dot{W}}{\mu \forall G^2}$