Climate Modeling

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PRACTICAL REPORT

ADVECTION

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Contents

1 Advection of a triangle profile.

1. Plot the CTCS solution for different t values with FTCS at Δt
2. Overlay the realistic curve
3. Overlay the FTCS solution for all time steps

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

u=1, Δx=1, Δt=0.1

[2]:
```
dx = 1
dt = 0.1
u = 1
ts = np.arange(0, 11, dt)
xs = np.arange(-20, 20, dx)

C = np.zeros((len(xs), len(ts)))
C_FTCS = np.zeros((len(xs), len(ts)))
C_real = np.zeros((len(xs), len(ts)))
```

C(0,0)=10; C(x,0)=0 elsewhere

[3]:
```
for i in range(len(xs)):
    if xs[i] == 0:
        C[i, 0] = 10
        C_FTCS[i, 0] = 10
    else:
        C[i, 0] = 0
        C_FTCS[i, 0] = 0
```

The realistic curve is a pulse of height 10 moving with speed u so its function is:

$$x = u \cdot t$$

[4]:
```
for j in range(len(ts)):
    for i in range(len(xs)):
        if xs[i] == u * ts[j]:
            C_real[i, j] = 10
        else:
            C_real[i, j] = 0
```

FCTS:

$$C_{m,n+1} = C_{m,n} - \frac{u\Delta t}{2\Delta x}(C_{m+1,n} - C_{m-1,n})$$

[5]:
```
for n in range(0, len(ts) - 1):
    for m in range(1, len(xs) - 1):
        C_FTCS[m, n + 1] = C_FTCS[m, n] - u * dt / (2 * dx) * (C_FTCS[m + 1, n]
 ↪- C_FTCS[m - 1, n])
```

CTCS can't be run directly because there is a missing value at the "edged" so there is not enough value so I run a for loop with the FCTS formula first and then calculate the loop with CTCS formula later:

$$C_{m,1} = C_{m,0} - \frac{u\Delta t}{2\Delta x}(C_{m+1,0} - C_{m-1,0})$$

$$C_{m,n+1} = C_{m,n-1} - \frac{u\Delta t}{\Delta x}(C_{m+1,n} - C_{m-1,n})$$

[6]:
```
for m in range(1, len(xs) - 1):
    C[m, 1] = C[m, 0] - u * dt / (2 * dx) * (C[m + 1, 0] - C[m - 1, 0])

for n in range(1, len(ts) - 1):
    for m in range(1, len(xs) - 1):
        C[m, n + 1] = C[m, n - 1] - u * dt / dx * (C[m + 1, n] - C[m - 1, n])
```

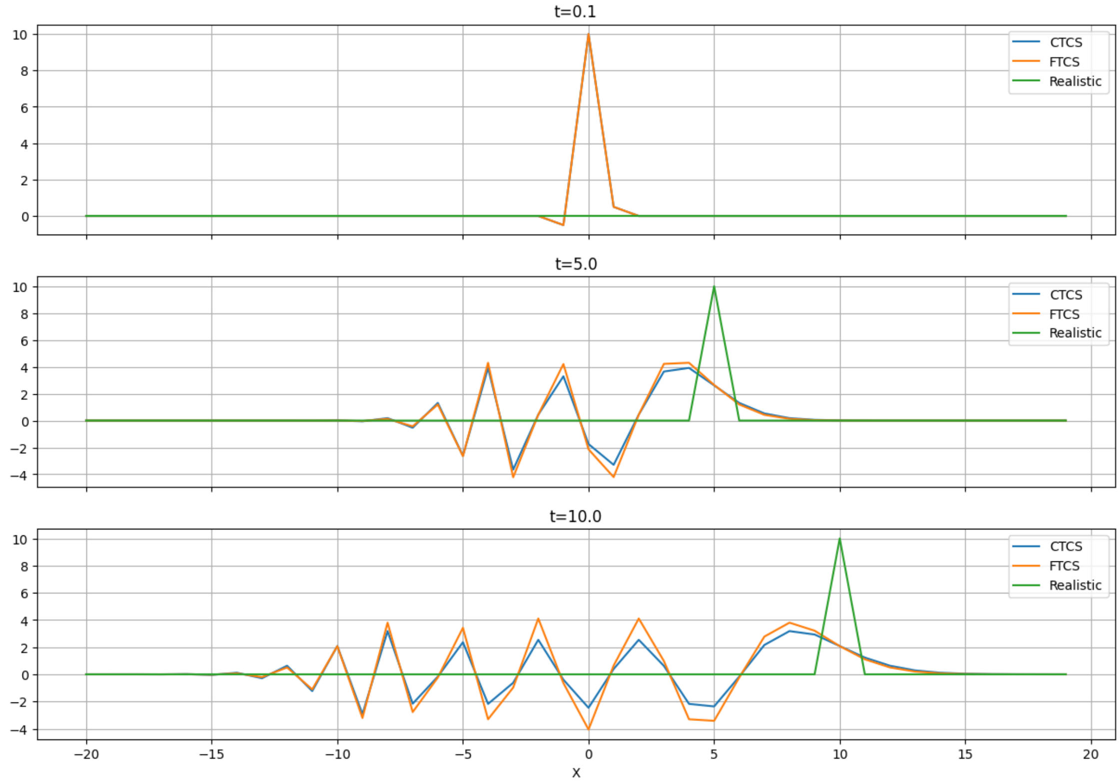
Plot the time steps asked in the task.

[7]:
```
for i in range(len(ts)):
    if ts[i] ==0.1:
        t1 = i
    if ts[i] == 5:
        t2 = i
    if ts[i] == 10:
        t3 = i
tpoint = [t1, t2, t3]

fig, ax = plt.subplots(3, 1, figsize=(15,10),sharex=True)

for i in range(3):
    ax[i].set_title(r't=' + str(ts[tpoint[i]]))
    ax[i].plot(xs, C[:, tpoint[i]], label='CTCS')
    ax[i].plot(xs, C_FTCS[:, tpoint[i]], label='FTCS')
    ax[i].plot(xs, C_real[:, tpoint[i]], label='Realistic')
    ax[i].grid(True)
    ax[i].legend()
ax[2].set_xlabel('X')
```

[7]:
```
Text(0.5, 0, 'X')
```



At t=0.1, the realistic curve is a clean straight line, the FTCS and CTCS curve have small fluctuation and overlay each other. At t=5 and t=10, the realistic curve maintain its shape and value while moving at a constant speed, while the FTCS and CTCS curve fluctuate a lot and perform quite inaccurate.

2 Advection of a rectangular profile.

1. Plot the realistic curves
2. Overlay the CTCS solutions
3. Overlay the FTCS results
4. Overlay the FTUS results
5. Plot the figure for Δt=0.5

Δx=0.2, Δt=0.1, u=1.

[8]:
```
dx = 0.2
dt = 0.1
u = 1
tspace = np.arange(0, 16, dt)
xs = np.arange(-20, 20, dx)
```

```
C = np.zeros((len(xs), len(tspace)))
C_real = np.zeros((len(xs), len(tspace)))
C_FTCS = np.zeros((len(xs), len(tspace)))
C_FTUS = np.zeros((len(xs), len(tspace)))
```

C=10 for -1 x 1 & C=0 else.

[9]:
```
for i in range(len(xs)):
    if xs[i] <= 1 and xs[i] >= -1:
        C[i, 0] = 10
        C_FTCS[i, 0] = 10
    else:
        C[i, 0] = 0
        C_FTCS[i, 0] = 0
```

$$C_{m,1} = C_{m,0} - \frac{u\Delta t}{2\Delta x}(C_{m+1,0} - C_{m-1,0})$$

$$C_{m,n+1} = C_{m,n-1} - \frac{u\Delta t}{\Delta x}(C_{m+1,n} - C_{m-1,n})$$

[10]:
```
for m in range(1, len(xs) - 1):
    C[m,1]=C[m,0]-u*dt/(2*dx)*(C[m+1,0]-C[m-1,0])

for n in range(1, len(tspace) - 1):
    for m in range(1, len(xs) - 1):
        C[m,n+1]=C[m,n-1]-u*dt/dx*(C[m+1,n]-C[m-1,n])
```

$$C_{m,n+1} = C_{m,n} - \frac{u\Delta t}{2\Delta x}(C_{m+1,n} - C_{m-1,n})$$

[11]:
```
for n in range(0, len(tspace) - 1):
    for m in range(1, len(xs) - 1):
        C_FTCS[m,n+1]=C_FTCS[m,n]-u*dt/(2*dx)*(C[m+1,n]-C[m-1,n])
```

$$C_{m,n+1} = C_{m,n} - \frac{u\Delta t}{\Delta x}(C_{m,n} - C_{m-1,n})$$

[12]:
```
for n in range(0, len(tspace) - 1):
    for m in range(1, len(xs) - 1):
        C_FTUS[m,n+1]=C_FTCS[m,n]-u*dt/(dx)*(C[m,n]-C[m-1,n])
```

The realistic curve is a pulse of height 10 that shifts right by $u \cdot t$ at each time, define its left edge as $-1 + u \cdot t$ and its right edge as $1 + u \cdot t$

[13]:
```
for j in range(len(tspace)):
    for i in range(len(xs)):
        if xs[i] >= -1 + u * tspace[j] and xs[i] <= 1 + u * tspace[j]:
            C_real[i, j] = 10
```

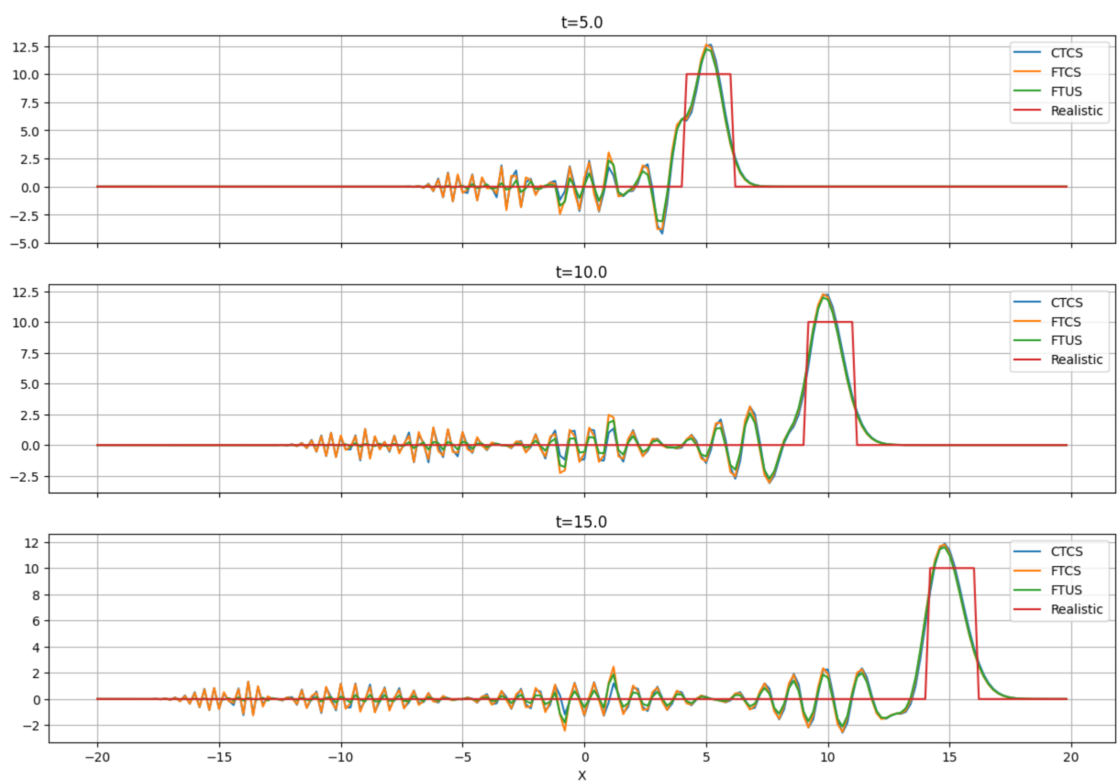
```
        else:
            C_real[i, j] = 0
```

[14]:
```
for i in range(len(tspace)):
    if tspace[i] == 5:
        t5 = i
    if tspace[i] == 10:
        t10 = i
    if tspace[i] == 15:
        t15 = i
tpoint = [t5, t10, t15]

fig, ax = plt.subplots(3, 1, figsize=(15,10), sharex=True)

for i in range(3):
    ax[i].set_title(r't=' + str(tspace[tpoint[i]]))
    ax[i].plot(xs, C[:, tpoint[i]], label='CTCS')
    ax[i].plot(xs, C_FTCS[:, tpoint[i]], label='FTCS')
    ax[i].plot(xs, C_FTUS[:, tpoint[i]], label='FTUS')
    ax[i].plot(xs, C_real[:, tpoint[i]], label='Realistic')
    ax[i].grid(True)
    ax[i].legend()
ax[2].set_xlabel('X')
```

[14]: Text(0.5, 0, 'X')



Compare to the previous part, the fluctuations seems to be smaller and more controlled. All schemes peaks are close to the square peak of realistic.

3 CFL criterion.

Same code as above but change dt to 0.5 to plot.

[15]:
```
dx = 0.2
dt = 0.5
u = 1
tspace = np.arange(0, 16, dt)
xs = np.arange(-20, 20, dx)

C = np.zeros((len(xs), len(tspace)))
C_real = np.zeros((len(xs), len(tspace)))
C_FTCS = np.zeros((len(xs), len(tspace)))
C_FTUS = np.zeros((len(xs), len(tspace)))

for i in range(len(xs)):
    if xs[i] <= 1 and xs[i] >= -1:
        C[i, 0] = 10
        C_FTCS[i, 0] = 10
    else:
        C[i, 0] = 0
        C_FTCS[i, 0] = 0

# CTCS
for m in range(1, len(xs) - 1):
    C[m,1]=C[m,0]-u*dt/(2*dx)*(C[m+1,0]-C[m-1,0])

for n in range(1, len(tspace) - 1):
    for m in range(1, len(xs) - 1):
        C[m,n+1]=C[m,n-1]-u*dt/dx*(C[m+1,n]-C[m-1,n])

# FTCS
for n in range(0, len(tspace) - 1):
    for m in range(1, len(xs) - 1):
        C_FTCS[m,n+1]=C_FTCS[m,n]-u*dt/(2*dx)*(C[m+1,n]-C[m-1,n])

# FTUS
for n in range(0, len(tspace) - 1):
    for m in range(1, len(xs) - 1):
        C_FTUS[m,n+1]=C_FTCS[m,n]-u*dt/(dx)*(C[m,n]-C[m-1,n])

# Realistic
```

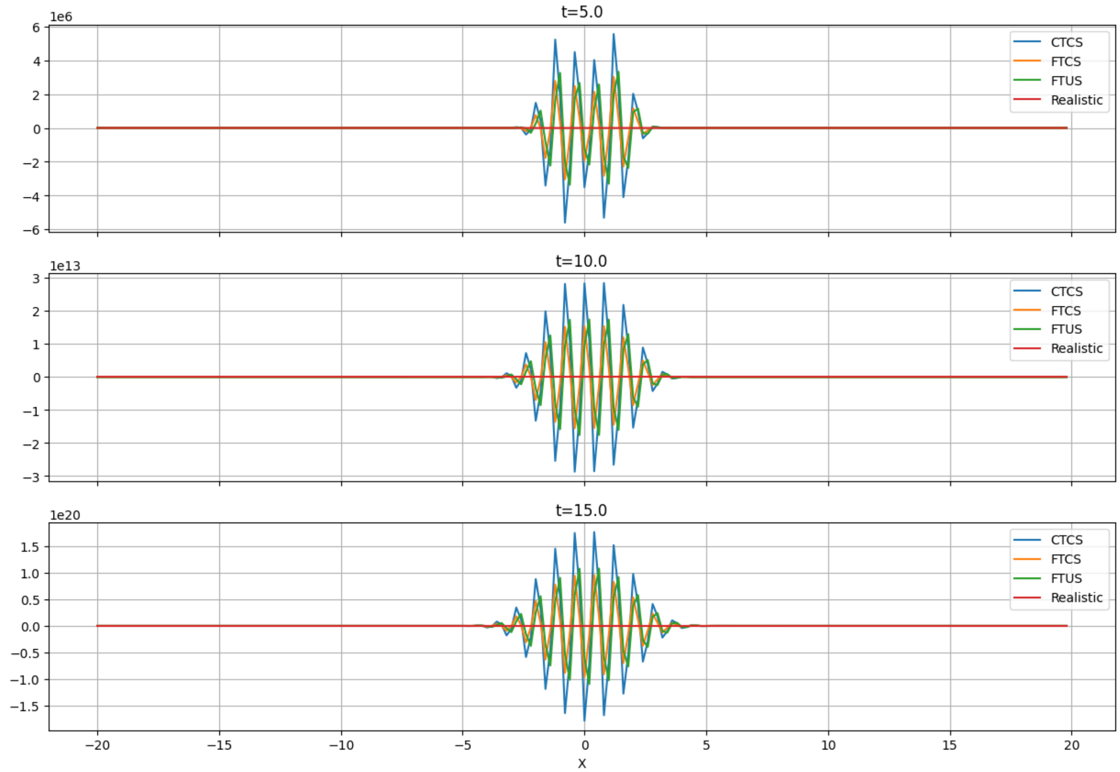
```
for j in range(len(tspace)):
    for i in range(len(xs)):
        if xs[i] >= -1 + u * tspace[j] and xs[i] <= 1 + u * tspace[j]:
            C_real[i, j] = 10
        else:
            C_real[i, j] = 0

for i in range(len(tspace)):
    if tspace[i] == 5:
        t5 = i
    if tspace[i] == 10:
        t10 = i
    if tspace[i] == 15:
        t15 = i
tpoint = [t5, t10, t15]

fig, ax = plt.subplots(3, 1, figsize=(15,10), sharex=True)

for i in range(3):
    ax[i].set_title(r't=' + str(tspace[tpoint[i]]))
    ax[i].plot(xs, C[:, tpoint[i]], label='CTCS')
    ax[i].plot(xs, C_FTCS[:, tpoint[i]], label='FTCS')
    ax[i].plot(xs, C_FTUS[:, tpoint[i]], label='FTUS')
    ax[i].plot(xs, C_real[:, tpoint[i]], label='Realistic')
    ax[i].grid(True)
    ax[i].legend()
ax[2].set_xlabel('X')
```

```
[15]: Text(0.5, 0, 'X')
```



When change dt to 0.5, the CFL condition was greater than 1, so the scheme is diverge and don't show the peak as previous. The higher t is, the more extreme the oscillations are. In all three plots, CTCS curve is the most unstable one, while CTCS and FTUS are more stable although not correct.

[ ]: