Climate Modeling

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PRACTICAL REPORT

INTRODUCTION & GENERAL CONCEPTS

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Contents

Hydrostatic pressure in the ocean:

$$\frac{dp}{dz} = -g \cdot \rho(z)$$

Where $g = 9.81\,\mathrm{m/s}^2$, ρ is water density, given by

$$\rho(T, S) = \rho_0 \left(1 - \alpha \cdot (T - T_0) + \beta \cdot (S - S_0)\right)$$

- $\rho_0 = 1028\,\mathrm{kg/m}^3$, $T_0 = 0°\mathrm{C}$, $S_0 = 35$ is water density.
- $\alpha = 5.4 \times 10^{-5}\,\mathrm{K}^{-1}$ is the thermal expansion coefficient.
- $\beta = 7.6 \times 10^{-4}$ is the haline expansion coefficient.
- At high latitudes, $T = -1°\mathrm{C}$ at the depth $z = 1000\,\mathrm{m}$, and increases linearly to $T = 10°\mathrm{C}$ at the surface $z = 0$.
- At the surface, $p = 1.013 \times 10^5\,\mathrm{Pa}$.
- Assume $S = 37$.
- $\Delta z = -100\,\mathrm{m}$.

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

[2]:
```
g = 9.81
rho_0 = 1028
T_0 = 0
S_0 = 35
alpha = 5.4e-5
beta = 7.6e-4
T_surface = 10
T_depth = -1
z_depth = 1000
S = 37
```

1 Plot rho(z) against z.

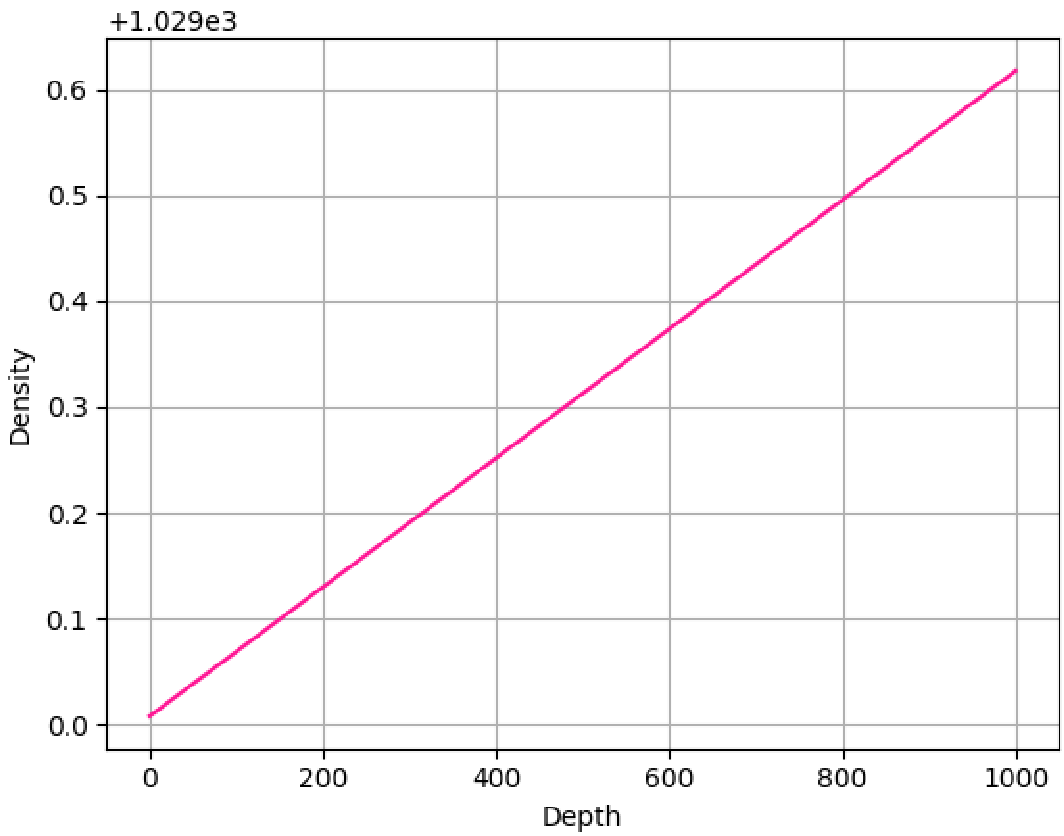
Let T be the linear gradient between the surface temperature T=10 C and depth temperature T=−1 C at 1000 meters.

$$T(z) = T_s + \left(\frac{T_d - T_s}{z_d}\right) \cdot z$$

For each depth z, I calculate T(z) and ρ(z) for it.

[3]:
```
def T(z):
    return T_surface + (T_depth - T_surface) * (z / z_depth)

def density(T, S):
    return rho_0 * (1 - alpha * (T - T_0) + beta * (S - S_0))
```

```
z = np.arange(0, z_depth + 1, 100)

rho = density(T(z), S)

plt.figure()
plt.plot(z, rho, color = 'deeppink')
plt.xlabel('Depth')
plt.ylabel('Density')
plt.grid(True)
plt.show()
```



As it goes deeper, the density also increases linearly. As it goes deeper into the ocean, both pressure and the cooling effect cause the water to become denser.

2 Using the Euler forward scheme to estimate the pressure profile and plot it against z.

Given that:

$$\frac{dp}{dz} = -g \cdot \rho(z)$$

Euler forward scheme:

$$\frac{p_{i+1} - p_i}{dz} \approx \frac{dp}{dz}$$

$\Rightarrow$

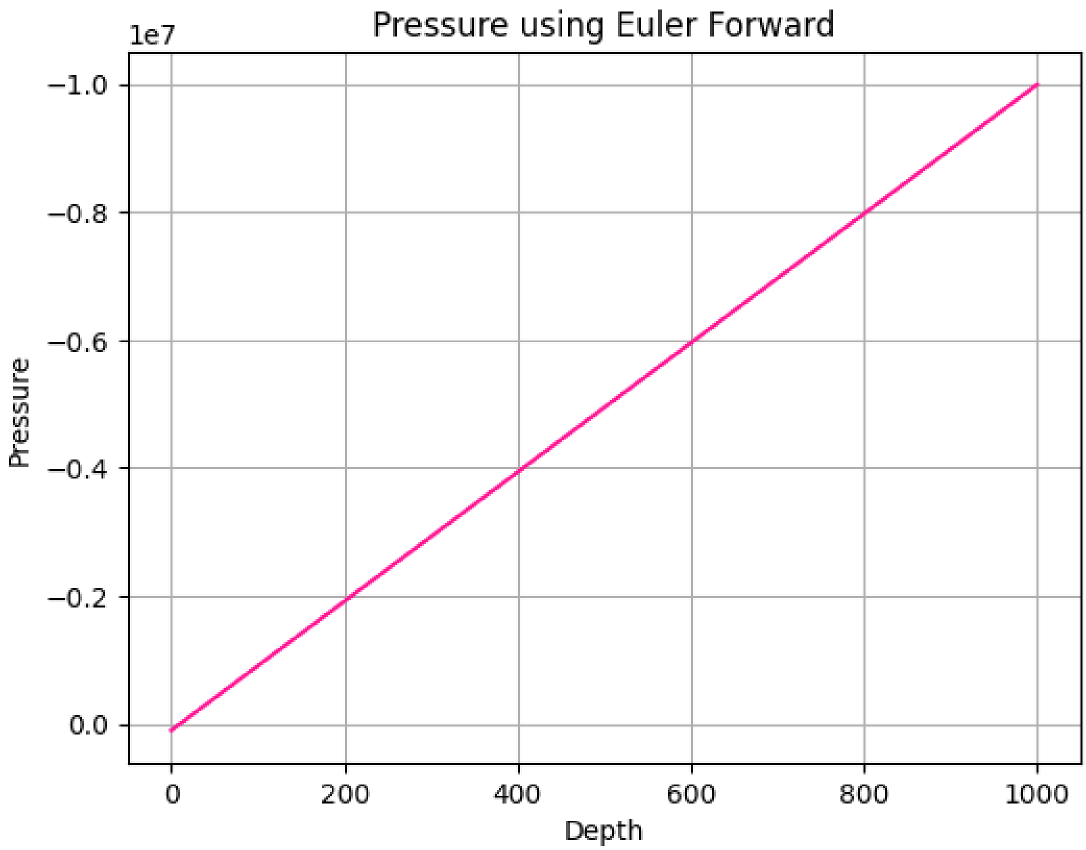
$$\frac{p_{i+1} - p_i}{dz} = -g \cdot \rho(z_i)$$

$\Rightarrow$

$$p_{i+1} = p_i - g \cdot \rho(z_i) \cdot dz$$

In this practice, dz is positive while still represents that it is going deep down into the ocean. Normally, in the Euler formula, when going down, the pressure gradient is represented with a negative sign. So when visualizing, the pressure axis is inverted: a value of -1 in the equation corresponds to an actual value of 1 in real life.

[4]:
```python
p_surface = 1.013e5
dz = 100

def euler(z, rho, dz, p_surface):
    pressures = [p_surface]
    for i in range(1, len(z)):
        dp = -g * rho[i-1] * dz
        pressures.append(pressures[-1] + dp)
    return np.array(pressures)

p_euler = euler(z, rho, dz, p_surface)

plt.figure()
plt.plot(z, p_euler, color='deeppink')
plt.gca().invert_yaxis()
plt.xlabel('Depth')
plt.ylabel('Pressure')
plt.title('Pressure using Euler Forward')
plt.grid(True)
plt.show()
```



As depth increases, pressure also increases. The pressure at the surface is 0 and at the depth of 1000m is 1e7 Pa.

3 Calculate pressure by integrating Eq. 1. Show the difference between a) and b).

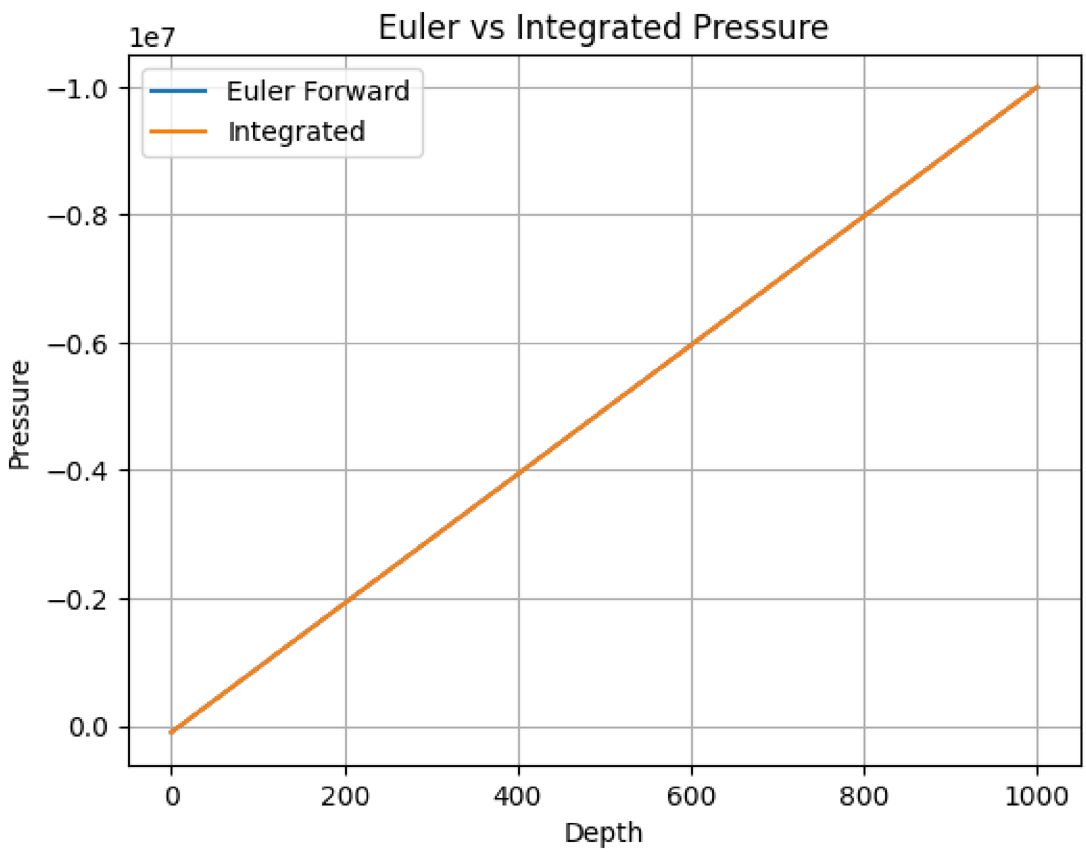
Integrating the function, we got:

$$p(z) = p_{\text{surface}} + \int_{z_{\text{surface}}}^{z} -g \cdot \rho(z')\, dz'$$

I use the cumulative_trapezoid function to calculate the the integral and set the initial parameter to 0. Then I overlay it with the Euler forward result for comparison and calculate the mean difference.

[5]:
```
from scipy.integrate import cumulative_trapezoid

def integrated_p(z, rho, p_surface):
    dp_integrated = cumulative_trapezoid(-g * rho, z, initial=0)
    pressures = p_surface + dp_integrated
    return pressures
```

```
p_integrated = integrated_p(z, rho, p_surface)

plt.figure()
plt.plot(z, p_euler, label="Euler Forward")
plt.plot(z, p_integrated, label="Integrated")
plt.gca().invert_yaxis()
plt.xlabel('Depth')
plt.ylabel('Pressure')
plt.title('Euler vs Integrated Pressure')
plt.legend()
plt.grid(True)
plt.show()
```



As being seen, two lines overlap and no signìcant differences observed in the plot.

[6]:
```
difference = np.abs(p_euler - p_integrated)
print(f"Average difference between Euler and Integrated: {np.mean(difference):.
↪2f} Pa")
```

```
Average difference between Euler and Integrated: 149.76 Pa
```

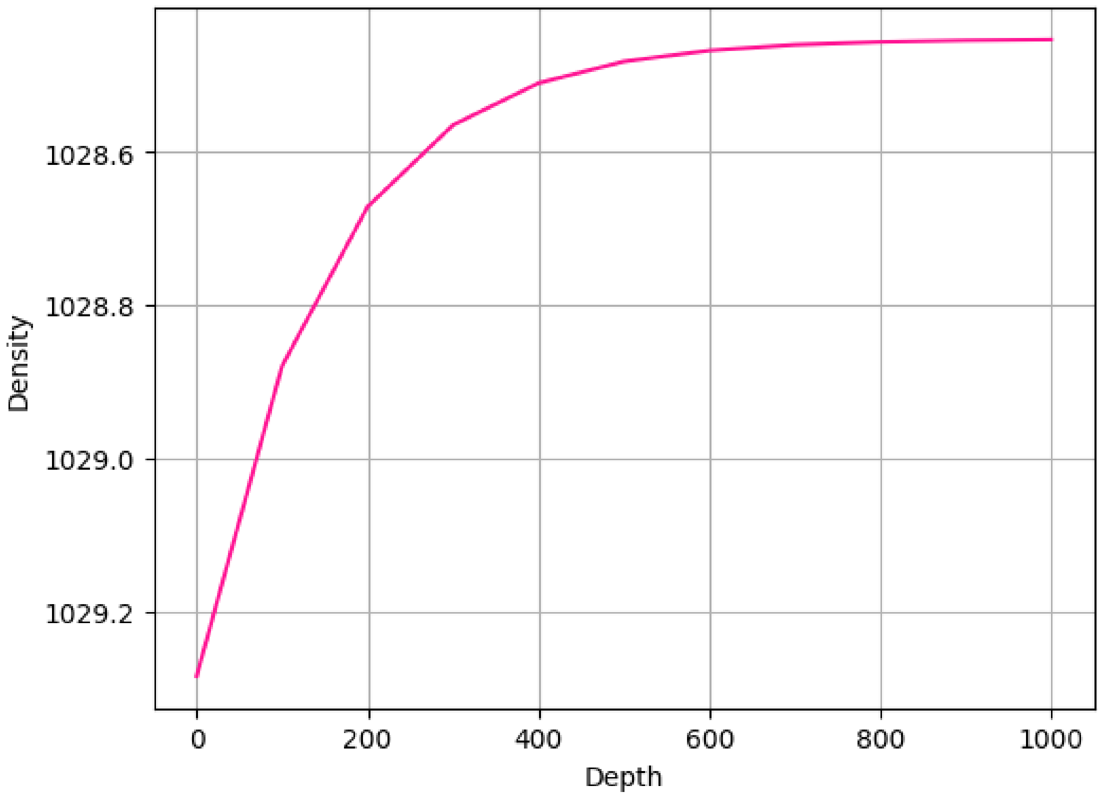
Compared to the large value of pressure, the average difference is quite small. It can be concluded that the Euler forward occurs very tiny error.

4 In lower latitudes, ocean temperatures vary approximately exponentially from the surface at T = 20°C to T = 5°C at 1000 m depth with a scale depth of d = 150 m. Repeat a), b), and c) but with the exponential temperature profile and plot rho(z) and p(z).

Use the same algorithm as above exercises.

[7]:
```
T_surface = 20
T_depth = 5
scale_depth = 150
```

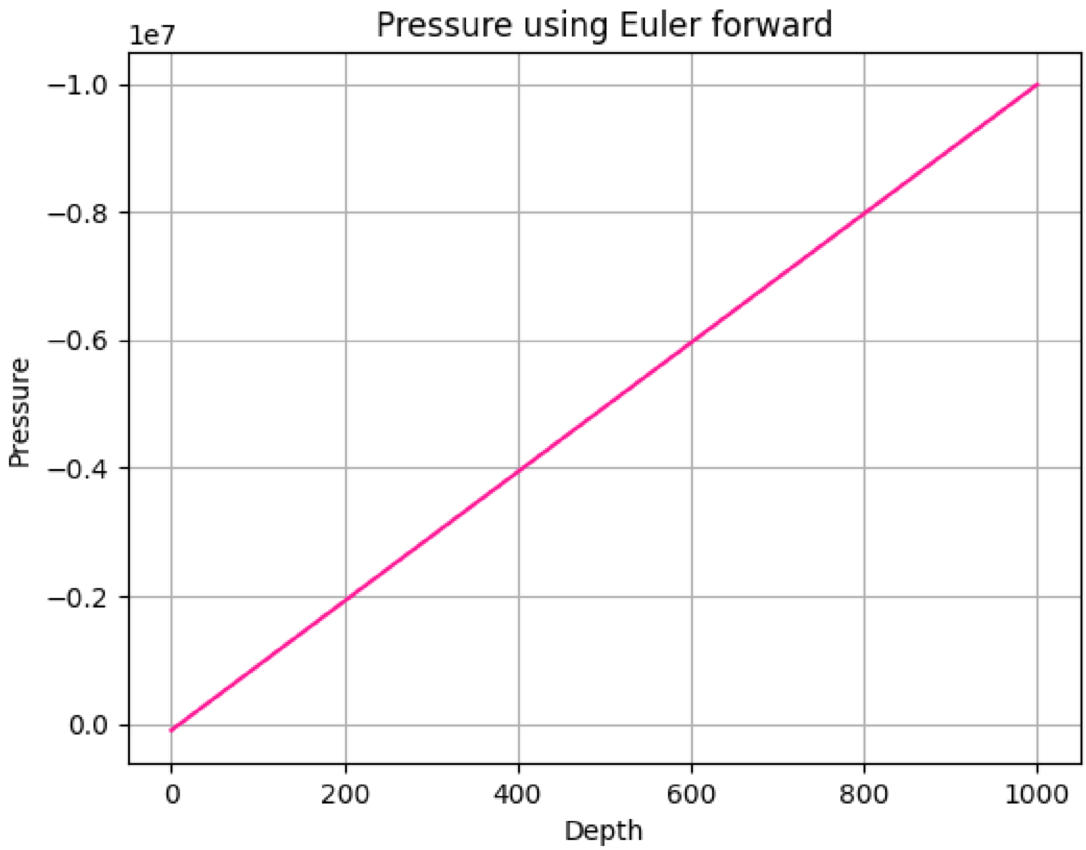
[8]:
```
def T(z):
    return T_surface + (T_depth - T_surface) * np.exp(-z / scale_depth)

rho = density(T(z), S)

plt.figure()
plt.plot(z, rho, color = 'deeppink')
plt.gca().invert_yaxis()
plt.xlabel('Depth')
plt.ylabel('Density')
plt.grid(True)
plt.show()
```



The density rises quickly at first but then levels off, indicating that most of the significant changes in density is in the upper layers, while the deeper layers have smaller changes.

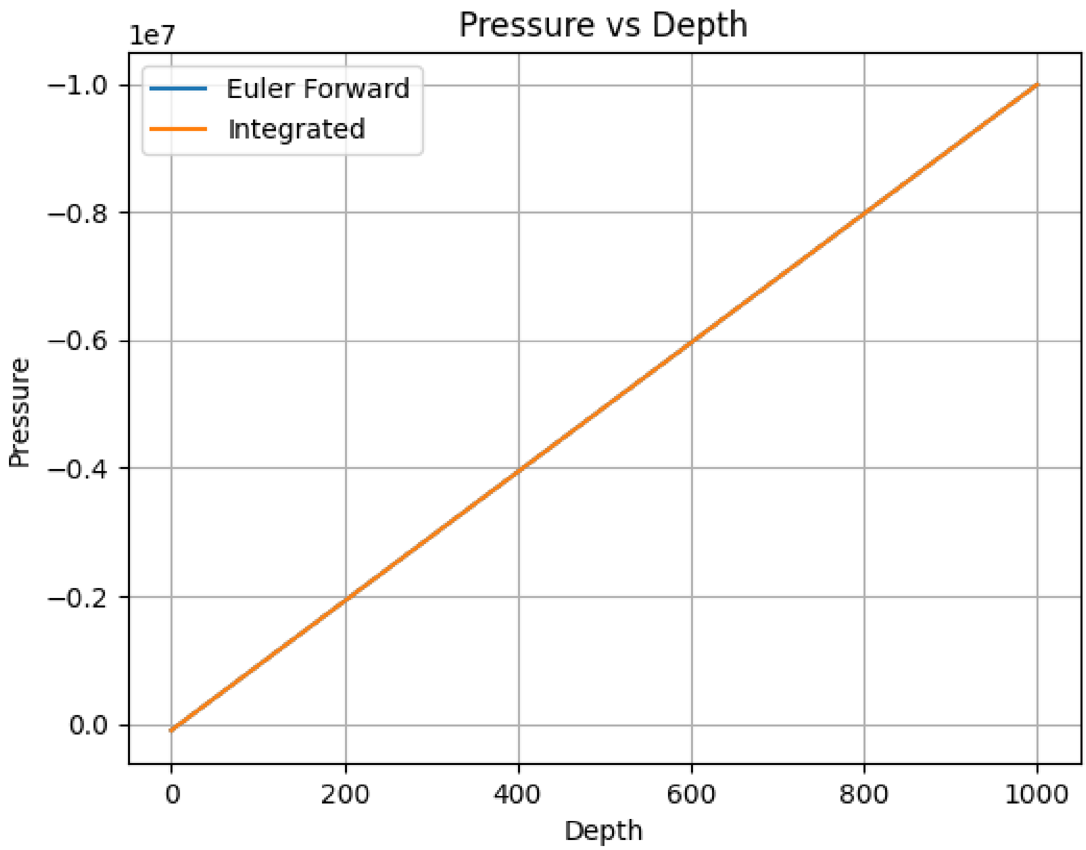
[9]:
```
p_euler = euler(z, rho, dz, p_surface)
p_integrated = integrated_p(z, rho, p_surface)
```

[10]:
```
plt.figure()
plt.plot(z, p_euler, color = 'deeppink')
plt.xlabel('Depth')
plt.gca().invert_yaxis()
plt.ylabel('Pressure')
plt.title('Pressure using Euler forward')
plt.grid(True)
plt.show()
```



As depth increases, pressure also increases.

[11]:
```
plt.figure()
plt.plot(z, p_euler, label="Euler Forward")
plt.plot(z, p_integrated, label="Integrated")
plt.gca().invert_yaxis()
plt.xlabel('Depth')
plt.ylabel('Pressure')
plt.title('Pressure vs Depth')
plt.legend()
plt.grid(True)
plt.show()
```



[12]:
```
difference = np.abs(p_euler - p_integrated)
print(f"Average difference between Euler and Integrated: {np.mean(difference):.
 ↪2f} Pa")
```

```
Average difference between Euler and Integrated: 332.17 Pa
```

Both lines overlap completely and the difference is very small.

5 Conclusion

- The Euler Forward method and the Integration method give almost identical results for pressure calculation.
- Both methods are accurate and have very small difference in the results.