

Climate Modeling

November 7, 2024

PRACTICAL REPORT

ADVECTION

By:

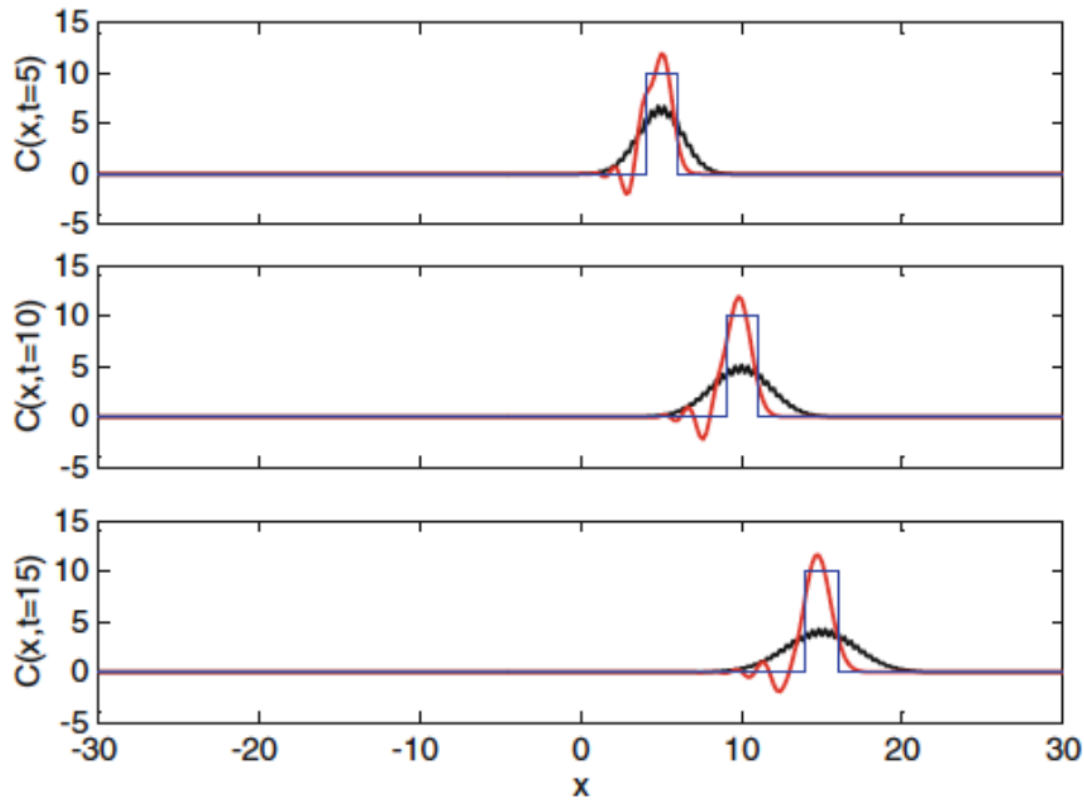
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1 Problem.



Advection of a rectangular profile (thin blue curve): $\Delta x=0.2$, $u=1$.

Initial condition: $C=10$ for $-1 \leq x \leq 1$ & $C=0$ else.

2 Reproduce the above figure using Lax scheme and Lax-Wendroff scheme.

```
[1]: import numpy as np
import matplotlib.pyplot as plt
```

The initial condition and analytical function is the same as previous practice.

```
[2]: dx = 0.2
dt = 0.1
u = 1
t = np.arange(0, 16, dt)
x = np.arange(-20, 20, dx)

C_L = np.zeros((len(x), len(t)))
C_LW = np.zeros((len(x), len(t)))
C_real = np.zeros((len(x), len(t)))
```

```
[3]: for j in range(len(t)):
      for i in range(len(x)):
          if x[i] >= -1 + u * t[j] and x[i] <= 1 + u * t[j]:
              C_real[i, j] = 10
          else:
              C_real[i, j] = 0
```

```
[4]: for i in range(len(x)):
      if x[i] <= 1 and x[i] >= -1:
          C_L[i, 0] = 10
          C_LW[i, 0] = 10
      else:
          C_L[i, 0] = 0
          C_LW[i, 0] = 0
```

Lax scheme:

$$C_{m,n+1} = C_{m,n} - \frac{u \Delta t}{2 \Delta x} (C_{m+1,n} - C_{m-1,n}) + \frac{1}{2} (C_{m+1,n} - 2C_{m,n} + C_{m-1,n})$$

Lax-Wendroff scheme:

$$C_{m,n+1} = C_{m,n} - \frac{u \Delta t}{2 \Delta x} (C_{m+1,n} - C_{m-1,n}) + \frac{1}{2} \left(\frac{u \Delta t}{\Delta x} \right)^2 (C_{m+1,n} - 2C_{m,n} + C_{m-1,n})$$

```
[5]: for n in range(0, len(t) - 1):
      for m in range(0, len(x) - 1):
          C_L[m, n + 1] = C_L[m, n] - u * dt / (2 * dx) * (C_L[m + 1, n] - C_L[m - 1, n]) + 1 / 2 * (C_L[m + 1, n] - 2 * C_L[m, n] + C_L[m - 1, n])
          C_LW[m, n + 1] = C_LW[m, n] - u * dt / (2 * dx) * (C_LW[m + 1, n] - C_LW[m - 1, n]) + 1 / 2 * (u * dt / dx) ** 2 * (C_LW[m + 1, n] - 2 * C_LW[m, n] + C_LW[m - 1, n])
```

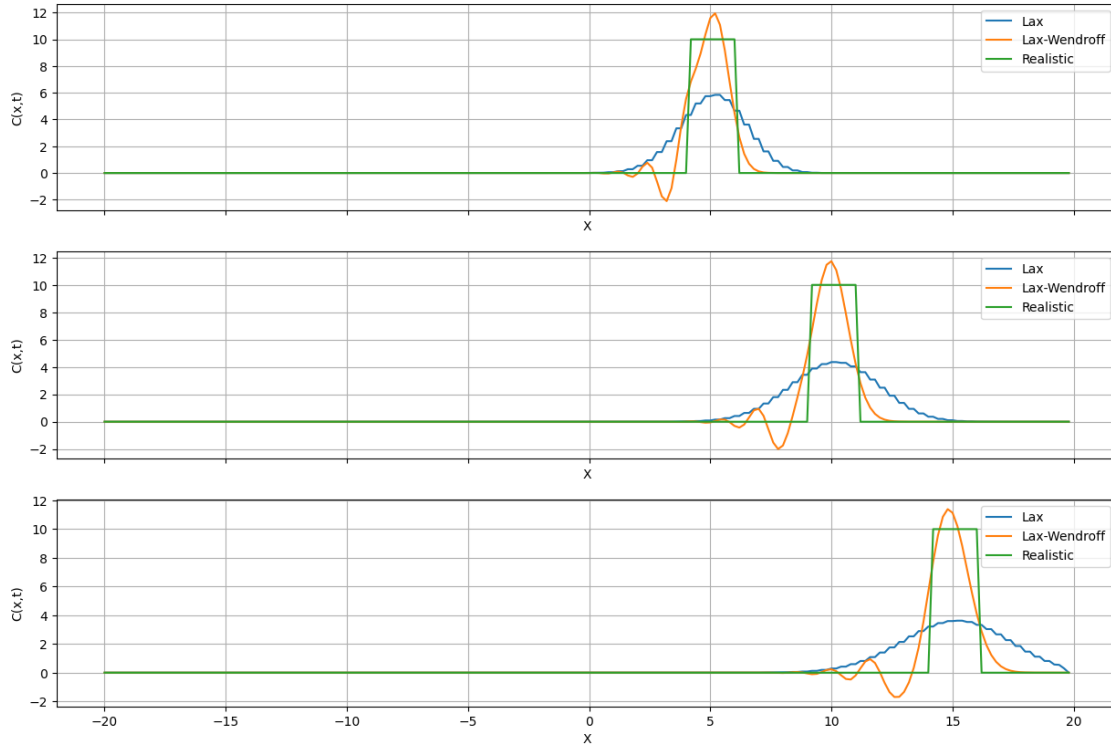
```
[6]: for i in range(len(t)):
      if t[i] == 5:
          t5 = i
      if t[i] == 10:
          t10 = i
      if t[i] == 15:
          t15 = i

tpoint = [t5, t10, t15]

fig, ax = plt.subplots(3, 1, figsize=(15, 10), sharex=True)

for i in range(0, 3):
    ax[i].plot(x, C_L[:, tpoint[i]], label='Lax')
    ax[i].plot(x, C_LW[:, tpoint[i]], label='Lax-Wendroff')
    ax[i].plot(x, C_real[:, tpoint[i]], label='Realistic')
```

```
ax[i].grid(True)
ax[i].legend()
ax[i].set_ylabel('C(x,t)')
ax[i].set_xlabel('X')
```



The two curves can show the moving peak but Lax curve cannot hold it for long and soon drop.

3 Testing the responses of the different schemes if the CFL criterion was not ensured.

CFL criterion:

$$|u| \frac{\Delta t}{\Delta x} \leq 1$$

→ change Δt to 0.5 to break the CFL criterion.

```
[7]: dt = 0.5
```

The following code to calculate CTCS, FCTS and FTUS are all from previous practice.

```
[8]: C_CTCS = np.zeros((len(x), len(t)))
      C_FTCS = np.zeros((len(x), len(t)))
      C_FTUS = np.zeros((len(x), len(t)))
```

```
[9]: for i in range(len(x)):
      if x[i] <= 1 and x[i] >= -1:
          C_CTCS[i, 0] = 10
          C_FTCS[i, 0] = 10
          C_FTUS[i, 0] = 10
      else:
          C_CTCS[i, 0] = 0
          C_FTCS[i, 0] = 0
          C_FTUS[i, 0] = 0
```

```
[10]: # CTCS
      for m in range(1, len(x) - 1):
          C_CTCS[m, 1] = C_CTCS[m, 0] - u * dt / (2 * dx) * (C_CTCS[m + 1, 0] -
          ↪ C_CTCS[m - 1, 0])

      for n in range(1, len(t) - 1):
          for m in range(1, len(x) - 1):
              C_CTCS[m, n + 1] = C_CTCS[m, n - 1] - (u * dt / dx) * (C_CTCS[m + 1, n]
              ↪ - C_CTCS[m - 1, n])
```

```
[11]: #FTCS
      for n in range(0, len(t) - 1):
          for m in range(1, len(x) - 1):
              C_FTCS[m,n+1]=C_FTCS[m,n]-u*dt/(2*dx)*(C_CTCS[m+1,n]-C_CTCS[m-1,n])
```

```
[12]: # FTUS
      for n in range(0, len(t) - 1):
          for m in range(0, len(x) - 1):
              C_FTUS[m, n + 1] = C_FTUS[m, n] - (u * dt / dx) * (C_FTUS[m, n] -
              ↪ C_FTUS[m - 1, n])
```

```
[13]: for i in range(len(t)):
      if t[i] == 5:
          t5 = i
      if t[i] == 10:
          t10 = i
      if t[i] == 15:
          t15 = i

      tpoint = [t5, t10, t15]

      fig, ax = plt.subplots(3, 1, figsize=(15, 10), sharex=True)

      for i in range(0, 3):
          ax[i].set_title(r't=' + str(t[tpoint[i]]))
          ax[i].plot(x, C_CTCS[:, tpoint[i]], label='CTCS')
          ax[i].plot(x, C_FTCS[:, tpoint[i]], label='FTCS')
```

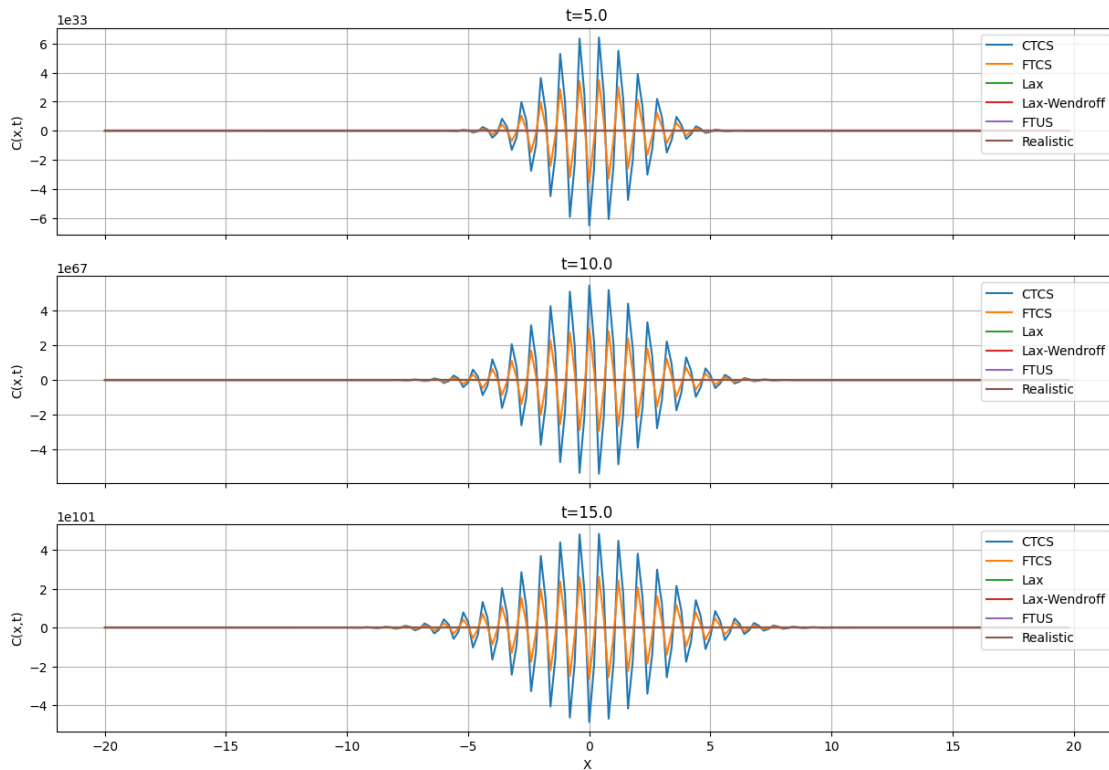
```

ax[i].plot(x, C_L[:, tpoint[i]], label='Lax')
ax[i].plot(x, C_LW[:, tpoint[i]], label='Lax-Wendroff')
ax[i].plot(x, C_FTUS[:, tpoint[i]], label='FTUS')
ax[i].plot(x, C_real[:, tpoint[i]], label='Realistic')
ax[i].grid(True)
ax[i].legend()
ax[i].set_ylabel('C(x,t)')

ax[2].set_xlabel('X')

```

[13]: Text(0.5, 0, 'X')



The CTCS curve are the most unstable with biggest oscillations. The FTCS curve also fluctuates so much that other curves cannot be seen. So I remove these two and plot others.

```

[14]: fig, ax = plt.subplots(3, 1, figsize=(15, 10), sharex=True)

for i in range(0, 3):
    ax[i].set_title(r't=' + str(t[tpoint[i]]))
    ax[i].plot(x, C_L[:, tpoint[i]], label='Lax')
    ax[i].plot(x, C_LW[:, tpoint[i]], label='Lax-Wendroff')
    ax[i].plot(x, C_FTUS[:, tpoint[i]], label='FTUS')
    ax[i].plot(x, C_real[:, tpoint[i]], label='Realistic')

```

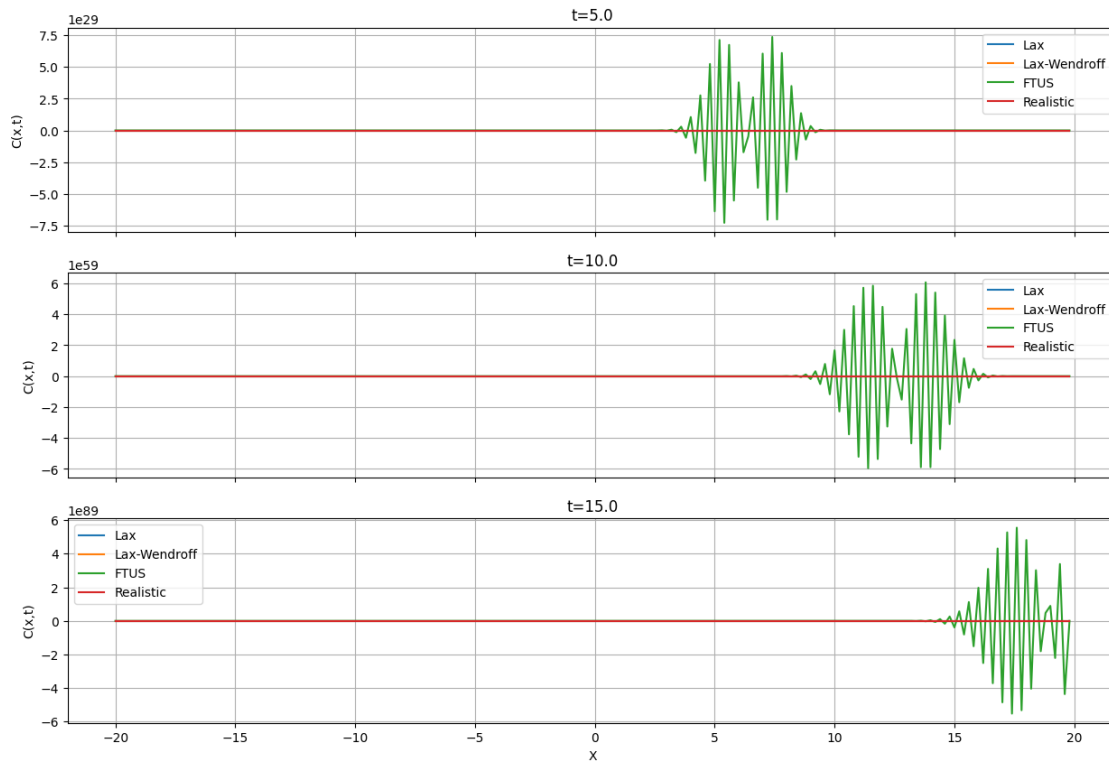
```

ax[i].grid(True)
ax[i].legend()
ax[i].set_ylabel('C(x,t)')

ax[2].set_xlabel('X')

```

```
[14]: Text(0.5, 0, 'X')
```



The FTUS curve can remove most of the fluctuations, and the range of oscillation values is also narrower. I continue to remove this.

```

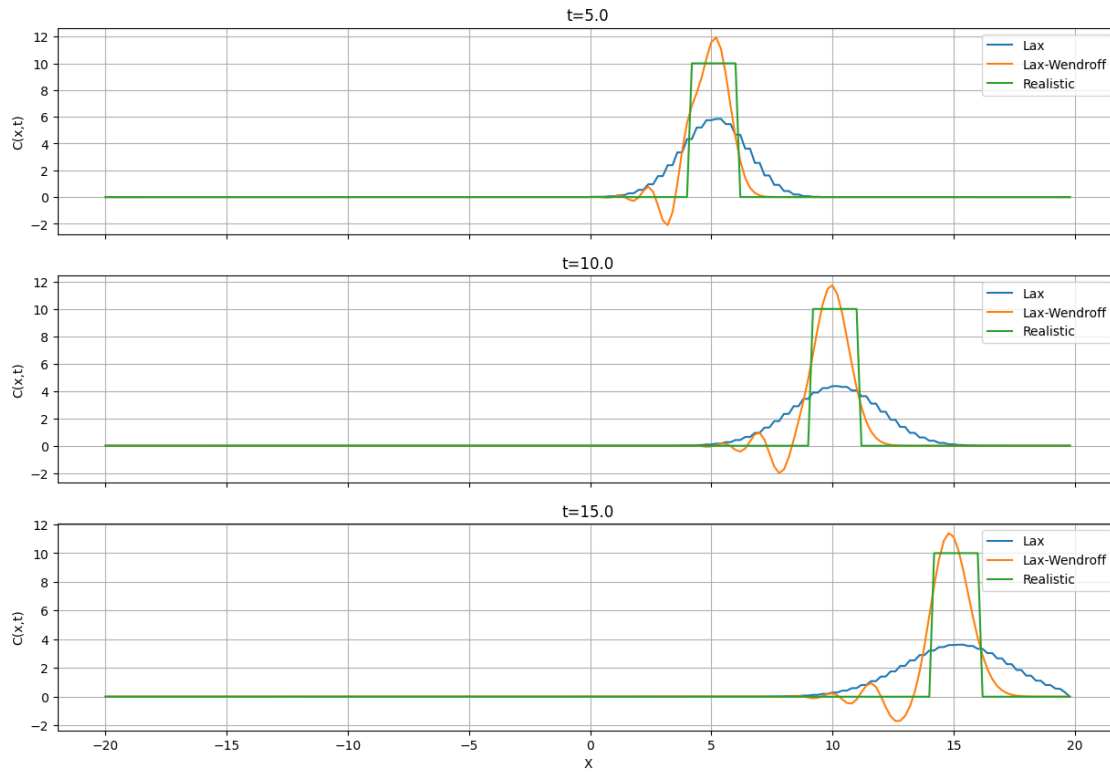
[15]: fig, ax = plt.subplots(3, 1, figsize=(15, 10), sharex=True)

for i in range(0, 3):
    ax[i].set_title(r't=' + str(t[tpoint[i]]))
    ax[i].plot(x, C_L[:, tpoint[i]], label='Lax')
    ax[i].plot(x, C_LW[:, tpoint[i]], label='Lax-Wendroff')
    ax[i].plot(x, C_real[:, tpoint[i]], label='Realistic')
    ax[i].grid(True)
    ax[i].legend()
    ax[i].set_ylabel('C(x,t)')

ax[2].set_xlabel('X')

```


[15]: Text(0.5, 0, 'X')



Compared to others above, these are the two most stable schemes.

4 Conclusion

After breaking the CFL criterion, among all schemes mentioned:

- CTCS and FTCS are the two most unstable.
- FTUS can remove most of the fluctuations.
- Lax and Lax-Wendroff are the two most stable.

[]: