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# MIDTERM SIGNAL & IMAGE PROCESSING

**Q1** $x[n] = \{\underset{\uparrow}{1}\ 2\ 3\ 4\}$

$h[n] = \{-1\ \underset{\uparrow}{2}\ 1\ 3\}$

a) $x[n] * h[n]$ in 3 ways?

① $\boxed{y[n] = \sum_{k=-\infty}^{\infty} x[k].\ h[n-k]}$ $\quad h[-n] = \{3\ 1\ 2\ -1\}$

(extra 0 just for shifting)

$y[-1] = $ 0 0 0 1 2 3 4 0 0 0

3 1 2 -1

$= 1 \times -1 = 0$

$y[0] = $ 0 0 0 1 2 3 4 0 0 0

3 1 2 -1

$= 1 \times -2 + 2 \times -1 = 0$

$y[1] = $ 0 0 0 1 2 3 4 0 0 0

3 1 2 -1

$= 1 \times 1 + 2 \times 2 + 3 \times -1 = 2$

$y[2] = $ 0 0 0 1 2 3 4 0 0 0

3 1 2 -1

$= 1 \times 3 + 2 \times 1 + 3 \times 2 + 4 \times -1 = 7$

$y[3] = $ 0 0 0 1 2 3 4 0 0 0

3 1 2 -1

$= 2 \times 3 + 3 \times 1 + 4 \times 2 = 17$

$y[4] = $ 0 0 0 1 2 3 4 0 0 0

3 1 2 -1

$= 3 \times 3 + 4 \times 1 = 13$

$y[5] = $ 0 0 0 1 2 3 4 0 0 0

3 1 2 -1

$= 4 \times 3 = 12$

$\rightarrow y[n] = \{-1 \quad \underset{\uparrow}{0} \quad 2 \quad 7 \quad 17 \quad 13 \quad 12\}$

(2) Mutiply polyminals

$x[n] \rightarrow x^3 + 2x^2 + 3x + 4$

$h[n] \rightarrow -x^3 + 2x^2 + x + 3$

$x[n] * h[n] \rightarrow (x^3 + 2x^2 + 3x + 4) \times (-x^3 + 2x^2 + x + 3)$

$= -x^6 + 2x^5 + x^4 + 3x^3$

$- 2x^5 + 4x^4 + 2x^3 + 6x^2$

$- 3x^4 + 6x^3 + 3x^2 + 9x$

$- 4x^3 + 8x^2 + 4x + 12$

$= -x^6 + 0x^5 + 2x^4 + 7x^3 + 17x^2 + 13x + 12$

$\rightarrow y[n] = \{-1 \quad 0 \quad 2 \quad 7 \quad 17 \quad 13 \quad 12\}$

(3) $\boxed{X(z) = \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n}}$

$X(z) = x[0]z^0 + x[1]z^{-1} + x[2]z^{-2} + x[3]z^{-3}$

$= 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$

$H(z) = h[0]z^1 + h[1]z^0 + h[2]z^{-1} + h[3]z^{-2}$

$= -1z^1 + 2z^0 + 1z^{-1} + 3z^{-2}$

$Y(z) = X(z) \cdot H(z) = -z^0 + 0z^{-1} + 2z^{-2} + 7z^{-3} + 17z^{-4} + 13z^{-5} + 12z^{-6}$

$\rightarrow y[n] = \{-1 \quad 0 \quad 2 \quad 7 \quad 17 \quad 13 \quad 12\}$

(4) I use MATLAB command

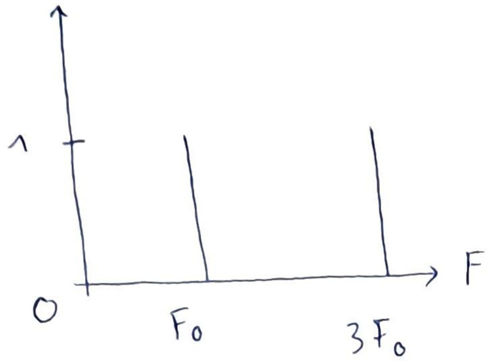
input: conv([1 2 3 4], [-1 2 1 3])

output: ans = -1 0 2 7 17 13 12

$\rightarrow y[n] = \{-1 \quad 0 \quad 2 \quad 7 \quad 17 \quad 13 \quad 12\}$

b) z-transform of $h[n] \rightarrow H(z) = -z^1 + 2z^0 + z^{-1} + 3z^{-2}$

Q2




- two frequency components, at $F_0$ and $3F_0$, each with magnitude of 1
- signal in the time domain will consist of 2 sinusoidal waves:

at $F_0$ : $S_1(t) = A_1 \cos(2\pi F_0 t + \phi_1)$

at $3F_0$ : $S_2(t) = A_2 \cos(2\pi 3F_0 t + \phi_2)$

$A_1 = A_2 = 1$, called amplitudes

- There is no repeating pattern in the frequency domain.
- The signal is aperiodic in frequency domain (no periodic repetition)
  → the signal is continuous in time domain.
- The lowest frequency in the signal is $F_0$ → the overall periodicity in the time domain is related to $\frac{1}{F_0}$.

- Inverse Fourier transform: frequency domain → time domain

$$\boxed{x(t) = \sum_n C_n e^{j.n\Omega_0 t}}$$

Let $\Omega_0 = 2\pi F_0$ is the fundamental angular frequency of the signal

$$x(t) = 1 . e^{j\Omega_0 t} + 1 e^{j3\Omega_0 t}$$

$$= e^{j2\Omega_0 t} . e^{-j\Omega_0 t} + e^{j2\Omega_0 t} . e^{j\Omega_0 t}$$

$$= e^{j2\Omega_0 t} (e^{-j\Omega_0 t} + e^{j\Omega_0 t})$$

$$= e^{j2\Omega_0 t} . 2\cos(\Omega_0 t)$$

$$= (\cos(2\Omega_0 t) + j\sin(2\Omega_0 t)) . 2\cos(\Omega_0 t)$$

$$= \underbrace{2\cos(2\Omega_0 t)\cos(\Omega_0 t)}_{\text{real part}} + \underbrace{2j\sin(2\Omega_0 t)\cos(\Omega_0 t)}_{\text{imaginary part}}$$

Q3 $H(z) = \dfrac{5}{1 - 2\cos(5)z^{-1} + z^{-2}}$ . Find its spectra?

$$H(e^{jw}) = \frac{5}{1 - 2\cos(5)e^{-jw} + e^{-2jw}}$$

(transfer function on the unit circle in the z-plane by substituting $z = e^{jw}$)

$$= \frac{5 \cdot e^{jw}}{e^{jw} - 2\cos(5) + e^{jw}}$$

$$= \frac{5(\cos w + j\sin w)}{2\cos w - 2\cos(5)}$$

$$= \underbrace{\frac{5\cos w}{2\cos w - 2\cos(5)}}_{Re[H(e^{jw})]} + j\underbrace{\frac{5\sin w}{2\cos w - 2\cos(5)}}_{Im[H(e^{jw})]}$$

Magnitude spectrum

$$|H(e^{jw})| = \sqrt{Re[H(e^{jw})]^2 + Im[H(e^{jw})]^2}$$

$$= \sqrt{\left(\frac{5\cos w}{2\cos w - 2\cos(5)}\right)^2 + \left(\frac{5\sin w}{2\cos w - 2\cos(5)}\right)^2}$$

$$= \sqrt{\frac{25\cos^2 w + 25\sin^2 w}{[2\cos w - 2\cos(5)]^2}} = \frac{5}{|2\cos w - 2\cos(5)|}$$

Phase spectrum

$$\angle H(e^{jw}) = \operatorname{atan}\left(\frac{Im[H(e^{jw})]}{Re[H(e^{jw})]}\right)$$

$$= \operatorname{atan}\left(\frac{\dfrac{5\sin w}{2\cos w - 2\cos(5)}}{\dfrac{5\cos w}{2\cos w - 2\cos(5)}}\right)$$

$$= \operatorname{atan}\left(\frac{\sin w}{\cos w}\right) = \operatorname{atan}.\tan w = w$$