Climate Modeling

November 7, 2024

PRACTICAL REPORT

ADVECTION

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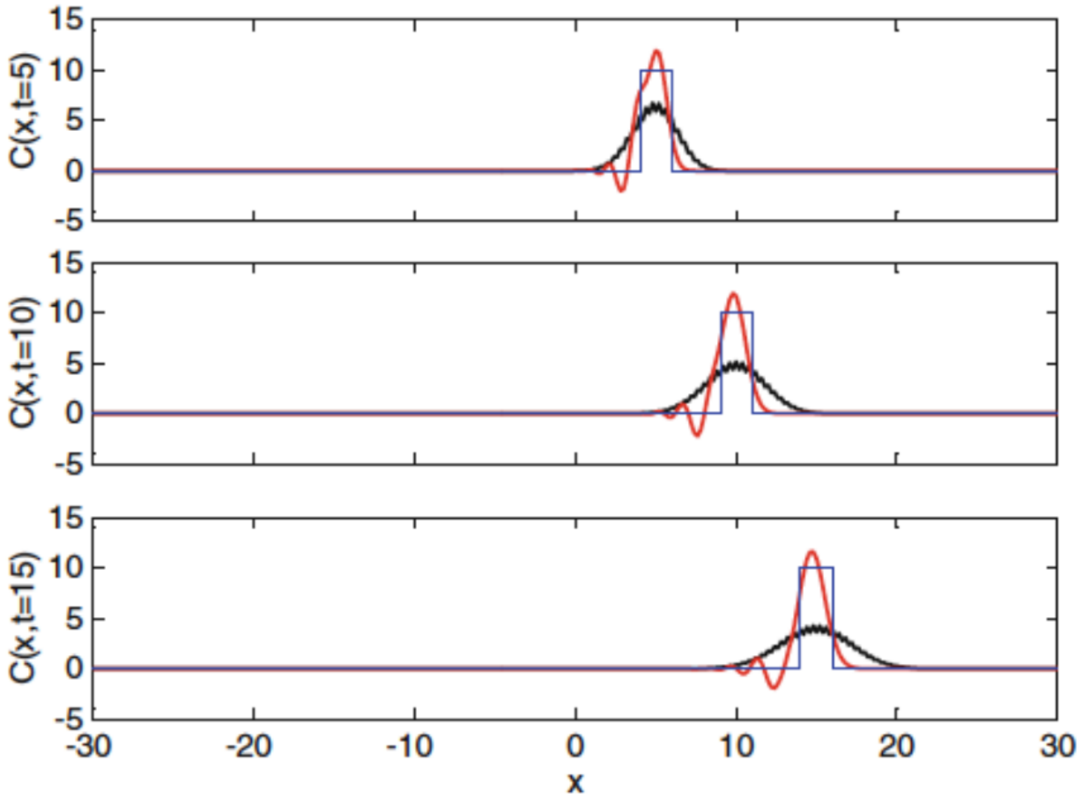
1 Problem.



Advection of a rectangular profile (thin blue curve): Δx=0.2, u=1.

Initial condition: C=10 for -1 x 1 & C=0 else.

2 Reproduce the above figure using Lax scheme and Lax-Wendroff scheme.

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

The initial condition and analytical function is the same as previous practice.

[2]:
```
dx = 0.2
dt = 0.1
u = 1
t = np.arange(0, 16, dt)
x = np.arange(-20, 20, dx)

C_L = np.zeros((len(x), len(t)))
C_LW = np.zeros((len(x), len(t)))
C_real = np.zeros((len(x), len(t)))
```

[3]:
```
for j in range(len(t)):
    for i in range(len(x)):
        if x[i] >= -1 + u * t[j] and x[i] <= 1 + u * t[j]:
            C_real[i, j] = 10
        else:
            C_real[i, j] = 0
```

[4]:
```
for i in range(len(x)):
    if x[i] <= 1 and x[i] >= -1:
        C_L[i, 0] = 10
        C_LW[i, 0] = 10
    else:
        C_L[i, 0] = 0
        C_LW[i, 0] = 0
```

Lax scheme:

$$C_{m,n+1} = C_{m,n} - \frac{u\,\Delta t}{2\,\Delta x}\left(C_{m+1,n} - C_{m-1,n}\right) + \frac{1}{2}\left(C_{m+1,n} - 2C_{m,n} + C_{m-1,n}\right)$$

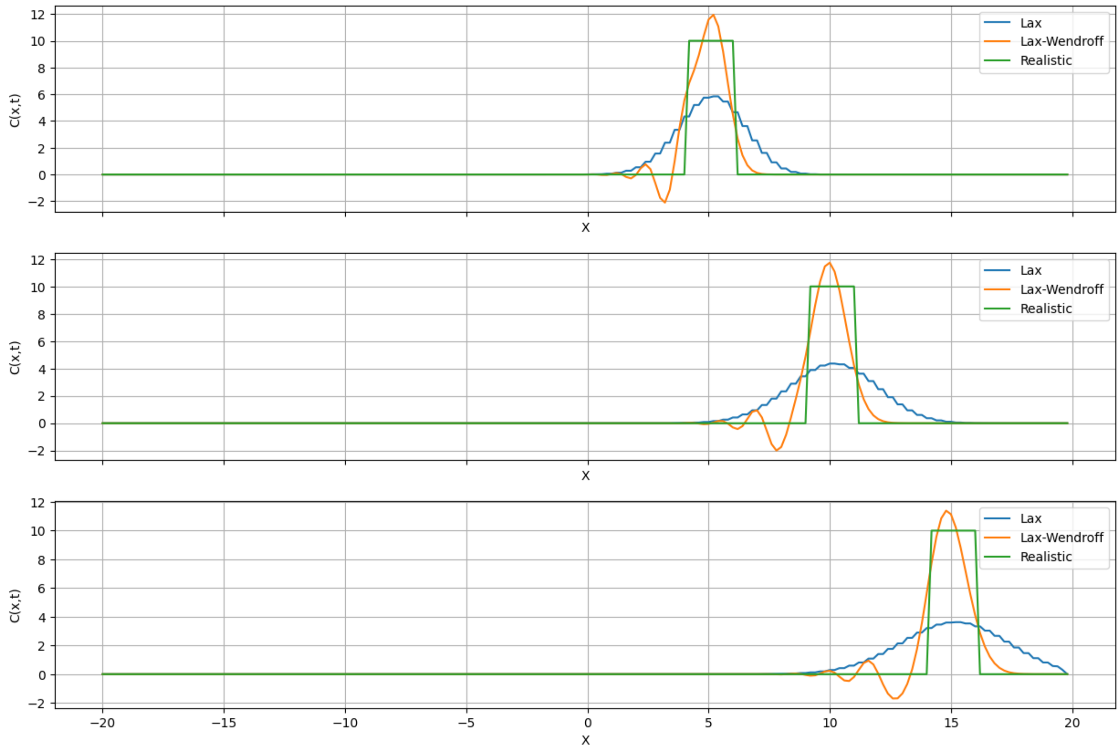
Lax-Wendroff scheme:

$$C_{m,n+1} = C_{m,n} - \frac{u\,\Delta t}{2\,\Delta x}\left(C_{m+1,n} - C_{m-1,n}\right) + \frac{1}{2}\left(\frac{u\,\Delta t}{\Delta x}\right)^2\left(C_{m+1,n} - 2C_{m,n} + C_{m-1,n}\right)$$

[5]:
```
for n in range(0, len(t) - 1):
    for m in range(0, len(x) - 1):
        C_L[m, n + 1] = C_L[m, n] - u * dt / (2 * dx) * (C_L[m + 1, n] - C_L[m␣
↪- 1, n]) + 1 / 2 * (C_L[m + 1, n] - 2 * C_L[m, n] + C_L[m - 1, n])
        C_LW[m, n + 1] = C_LW[m, n] - u * dt / (2 * dx) * (C_LW[m + 1, n] -␣
↪C_LW[m - 1, n]) + 1 / 2 * (u * dt / dx) ** 2 * (C_LW[m + 1, n] - 2 * C_LW[m,␣
↪n] + C_LW[m - 1, n])
```

[6]:
```
for i in range(len(t)):
    if t[i] == 5:
        t5 = i
    if t[i] == 10:
        t10 = i
    if t[i] == 15:
        t15 = i

tpoint = [t5, t10, t15]

fig, ax = plt.subplots(3, 1, figsize=(15, 10), sharex=True)

for i in range(0, 3):
    ax[i].plot(x, C_L[:, tpoint[i]], label='Lax')
    ax[i].plot(x, C_LW[:, tpoint[i]], label='Lax-Wendroff')
    ax[i].plot(x, C_real[:, tpoint[i]], label='Realistic')
```

```
    ax[i].grid(True)
    ax[i].legend()
    ax[i].set_ylabel('C(x,t)')
    ax[i].set_xlabel('X')
```



The two curves can show the moving peak but Lax curve cannot hold it for long and soon drop.

3 Testing the responses of the different schemes if the CFL criterion was not ensured.

CFL criterion:

$$| u\frac{\Delta t}{\Delta x} | \leq 1$$

$\rightarrow$ change Δt to 0.5 to break the CFL criterion.

[7]:
```
dt = 0.5
```

The following code to calculate CTCS, FCTS and FTUS are all from previous practice.

[8]:
```
C_CTCS = np.zeros((len(x), len(t)))
C_FTCS = np.zeros((len(x), len(t)))
C_FTUS = np.zeros((len(x), len(t)))
```

[9]:
```
for i in range(len(x)):
    if x[i] <= 1 and x[i] >= -1:
        C_CTCS[i, 0] = 10
        C_FTCS[i, 0] = 10
        C_FTUS[i, 0] = 10
    else:
        C_CTCS[i, 0] = 0
        C_FTCS[i, 0] = 0
        C_FTUS[i, 0] = 0
```

[10]:
```
# CTCS
for m in range(1, len(x) - 1):
    C_CTCS[m, 1] = C_CTCS[m, 0] - u * dt / (2 * dx) * (C_CTCS[m + 1, 0] -␣
 ↪C_CTCS[m - 1, 0])

for n in range(1, len(t) - 1):
    for m in range(1, len(x) - 1):
        C_CTCS[m, n + 1] = C_CTCS[m, n - 1] - (u * dt / dx) * (C_CTCS[m + 1, n]␣
 ↪- C_CTCS[m - 1, n])
```

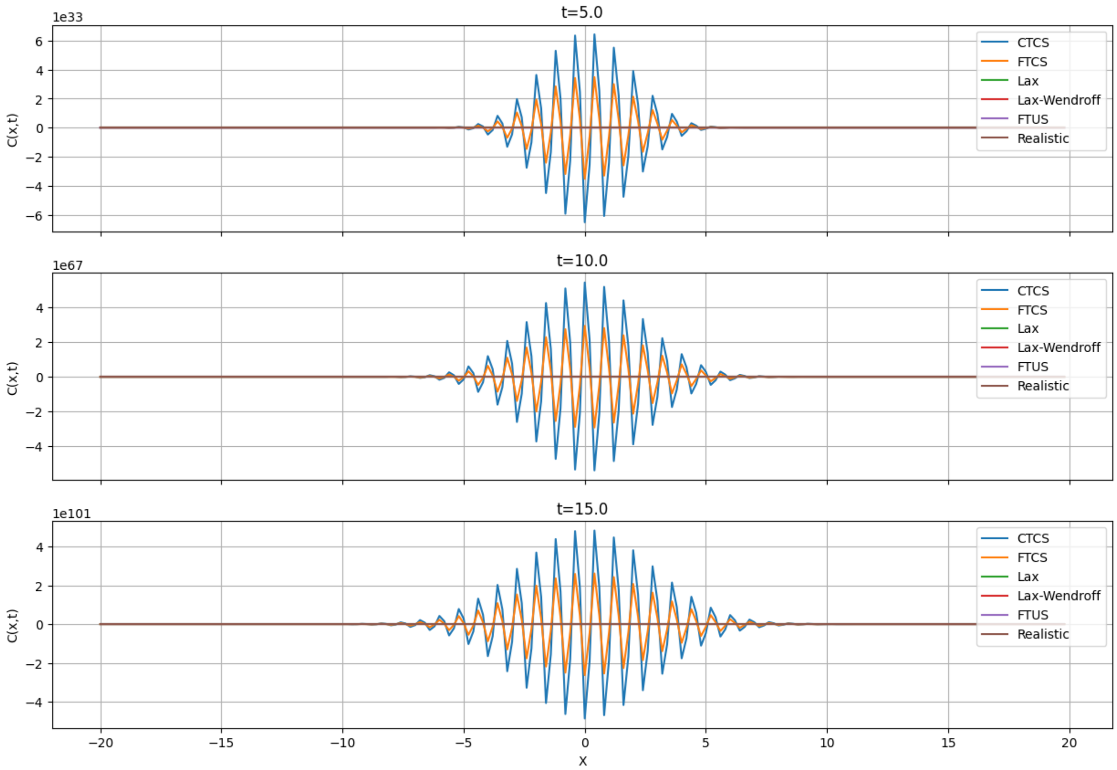
[11]:
```
#FTCS
for n in range(0, len(t) - 1):
    for m in range(1, len(x) - 1):
        C_FTCS[m,n+1]=C_FTCS[m,n]-u*dt/(2*dx)*(C_CTCS[m+1,n]-C_CTCS[m-1,n])
```

[12]:
```
# FTUS
for n in range(0, len(t) - 1):
    for m in range(0, len(x) - 1):
        C_FTUS[m, n + 1] = C_FTUS[m, n] - (u * dt / dx) * (C_FTUS[m, n] -␣
 ↪C_FTUS[m - 1, n])
```

[13]:
```
for i in range(len(t)):
    if t[i] == 5:
        t5 = i
    if t[i] == 10:
        t10 = i
    if t[i] == 15:
        t15 = i

tpoint = [t5, t10, t15]

fig, ax = plt.subplots(3, 1, figsize=(15, 10), sharex=True)

for i in range(0, 3):
    ax[i].set_title(r't=' + str(t[tpoint[i]]))
    ax[i].plot(x, C_CTCS[:, tpoint[i]], label='CTCS')
    ax[i].plot(x, C_FTCS[:, tpoint[i]], label='FTCS')
```

```
    ax[i].plot(x, C_L[:, tpoint[i]], label='Lax')
    ax[i].plot(x, C_LW[:, tpoint[i]], label='Lax-Wendroff')
    ax[i].plot(x, C_FTUS[:, tpoint[i]], label='FTUS')
    ax[i].plot(x, C_real[:, tpoint[i]], label='Realistic')
    ax[i].grid(True)
    ax[i].legend()
    ax[i].set_ylabel('C(x,t)')

ax[2].set_xlabel('X')
```

[13]: Text(0.5, 0, 'X')

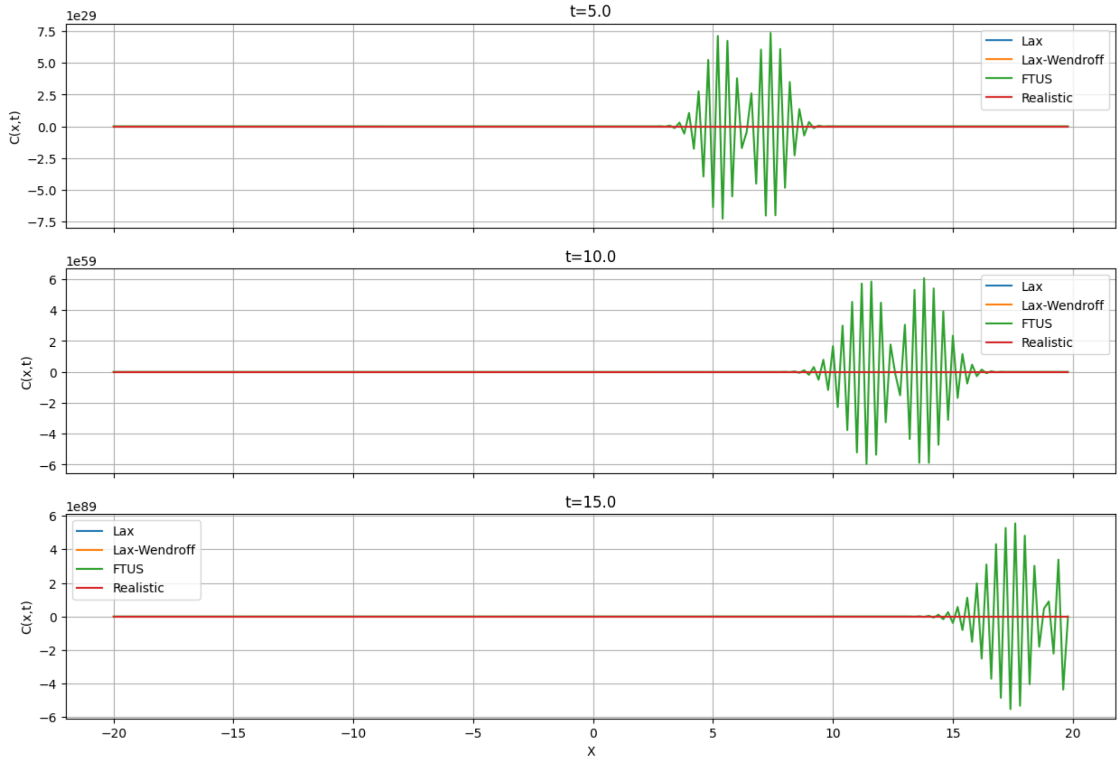


The CTCS curve are the most unstable with biggest oscillations. The FTCS curve also fluctuates so much that other curves cannot be seen. So I remove these two and plot others.

[14]:
```
fig, ax = plt.subplots(3, 1, figsize=(15, 10), sharex=True)

for i in range(0, 3):
    ax[i].set_title(r't=' + str(t[tpoint[i]]))
    ax[i].plot(x, C_L[:, tpoint[i]], label='Lax')
    ax[i].plot(x, C_LW[:, tpoint[i]], label='Lax-Wendroff')
    ax[i].plot(x, C_FTUS[:, tpoint[i]], label='FTUS')
    ax[i].plot(x, C_real[:, tpoint[i]], label='Realistic')
```

```
    ax[i].grid(True)
    ax[i].legend()
    ax[i].set_ylabel('C(x,t)')

ax[2].set_xlabel('X')
```

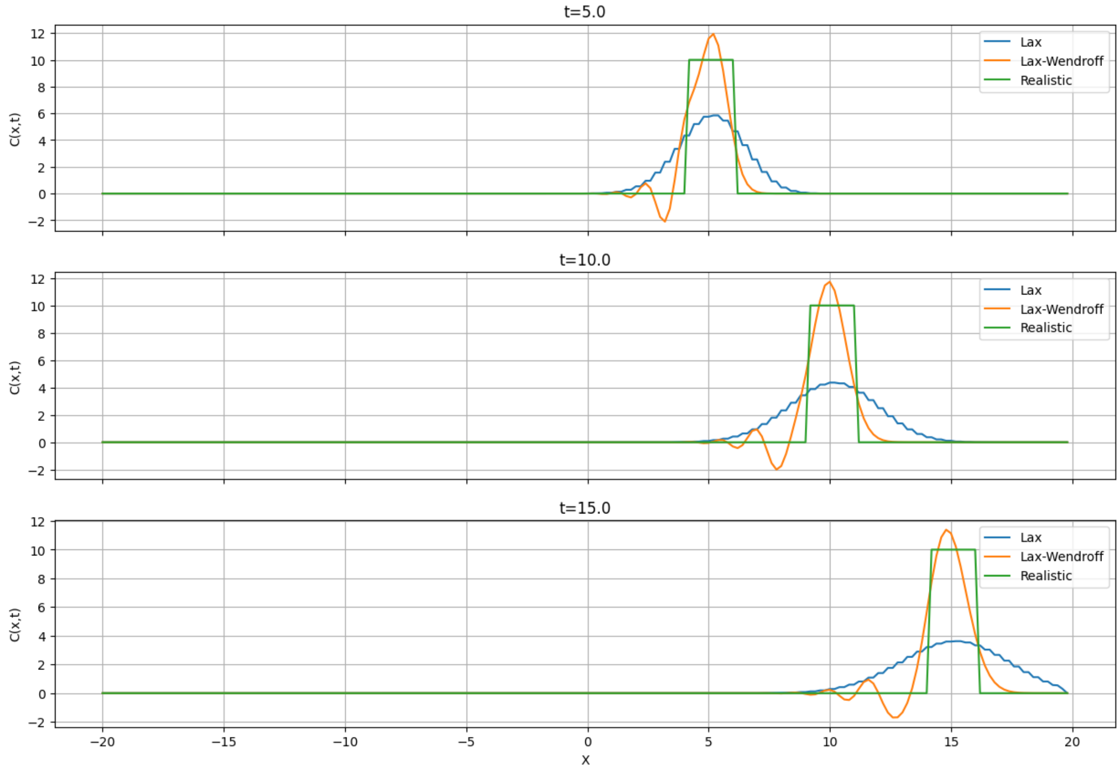
[14]: Text(0.5, 0, 'X')



The FTUS curve can remove most of the fluctuations, and the range of oscillation values is also narrower. I continue to remove this.

[15]:
```
fig, ax = plt.subplots(3, 1, figsize=(15, 10), sharex=True)

for i in range(0, 3):
    ax[i].set_title(r't=' + str(t[tpoint[i]]))
    ax[i].plot(x, C_L[:, tpoint[i]], label='Lax')
    ax[i].plot(x, C_LW[:, tpoint[i]], label='Lax-Wendroff')
    ax[i].plot(x, C_real[:, tpoint[i]], label='Realistic')
    ax[i].grid(True)
    ax[i].legend()
    ax[i].set_ylabel('C(x,t)')

ax[2].set_xlabel('X')
```

[15]: Text(0.5, 0, 'X')



Compared to others above, these are the two most stable schemes.

4 Conclusion

After breaking the CFL criterion, among all schemes mentioned:

- CTCS and FTCS are the two most unstable.
- FTUS can remove most of the fluctuations.
- Lax and Lax-Wendroff are the two most stable.

[ ]: