

A Mass Ordering Sum Rule for the Neutrino Disappearance Channels in T2K, NOvA and JUNO

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(Dated: April 12, 2024)

We revisit a method for determining the neutrino mass ordering by using precision measurements of the atmospheric Δm^2 's in both electron neutrino and muon neutrino disappearance channels, proposed by the authors in 2005 [1]. The mass ordering is a very important outstanding question for our understanding of the elusive neutrino and determination of the mass ordering has consequences for other neutrino experiments. The JUNO reactor experiment will start data taking this year, and the precision of the atmospheric Δm^2 's from electron anti-neutrino measurements will improve by a factor of three from Daya Bay's 2.4% to 0.8% within a year. This measurement, when combined with the atmospheric Δm^2 's measurements from T2K and NOvA for muon neutrino disappearance, will contribute substantially to the $\Delta\chi^2$ between the two remaining neutrino mass orderings. In this paper we derive a mass ordering sum rule that can be used to address the possibility that JUNO's atmospheric Δm^2 's measurement, when combined with other experiments in particular T2K and NOvA, can determine the neutrino mass ordering at the 3σ confidence level within one year of operation. For a confidence level of 5σ in a single experiment we will have to wait until the middle of the next decade when the DUNE experiment is operating.

Introduction — We have known for more than a quarter of century that neutrinos are massive [2] but we still do not know whether the neutrino with the least amount of ν_e , usually labelled ν_3 , is at the top or bottom of the neutrino mass spectrum. This is the neutrino mass ordering question. The SNO experiment [3] determined that the mass ordering of the other two neutrino mass eigenstates was such that the neutrino with the most ν_e , usually labelled ν_1 , was lighter than the other mass state, ν_2 which has a smaller ν_e fraction than ν_1 but a larger ν_e fraction than ν_3 . Thus, the remaining possible mass ordering for the neutrino mass states is, either $m_1 < m_2 < m_3$ which is known as the normal ordering (NO) or $m_3 < m_1 < m_2$ which is known as the inverted ordering (IO), see Fig. 1. The mass squared splitting between ν_2 and ν_1 was measured with good precision by the KamLAND experiment to be [4]

$$\Delta m_{21}^2 \equiv m_2^2 - m_1^2 \approx +7.5 \times 10^{-5} \text{ eV}^2. \quad (1)$$

Whereas the magnitude of the mass-squared-splitting between ν_3 and ν_1 has been determined by a number of experiments to be 30 times larger, i.e.

$$\Delta m_{31}^2 \equiv m_3^2 - m_1^2 \approx \pm 2.5 \times 10^{-3} \text{ eV}^2, \quad (2)$$

where the ambiguity in the sign comes from the undetermined mass ordering[5]. There exist numerous ways to determine the mass ordering and hence the above sign in the literature [1, 6–18]. Nevertheless, the use of matter

effects in neutrino oscillations have been guiding most of the experimental efforts. The long-baseline accelerator neutrino experiments NOvA [20] and T2K [21] as well as the atmospheric neutrinos experiments Super-Kamiokande [22] and Ice-Cube [23] operate in a regime where neutrino oscillations are mostly driven by the higher mass-squared-splitting and matter effects are significant in the $\nu_\mu \rightarrow \nu_e$ (and antineutrino) appearance probabilities. They are responsible for the current sta-

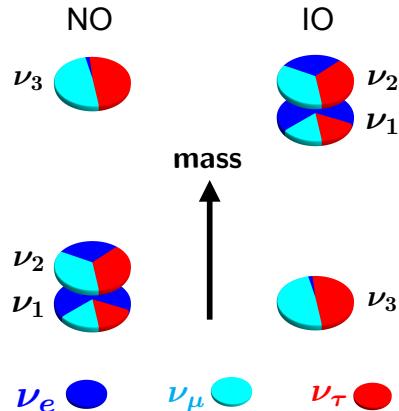


FIG. 1. The two remaining mass orderings for the 3 neutrino mass states, from [19]. Dark blue is the ν_e fraction, cyan the ν_μ fraction and red the ν_τ fraction. If the mass state with the least fraction of ν_e , labelled ν_3 , is at the top of the spectrum this is called the normal ordering (labelled NO) whereas if it is at the bottom of the spectrum it is called the inverted ordering (IO). SNO [3] determined the mass ordering of the other two mass states using solar neutrinos. The set (ν_1, ν_2) , usually called the solar pair, has the state with most ν_e , labelled ν_1 , below the other member of the pair, labelled ν_2 .

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tus of our understanding of the mass ordering question. At present T2K and NOvA, individually, have a slight preference for NO, while the combined fit, flips this preference to IO [24, 25], although the preference is weak. Ice-Cube has no preference for either NO or IO. On the other hand, Super-Kamiokande data seems to favor NO by 92.3% CL even though their Monte Carlo simulation indicates they should not be able to discriminate the ordering better than $\sim 80\%$ CL, see [22]. It is the statistical weight of their data, corresponding to an exposure of 364.8 kiloton-years, that makes the final global data fit prefer NO [25]. So, at the current time, we do not have 3σ or more preference for one ordering over the other.

In this article we revisit one way proposed by the authors of this paper [1] which requires precision measurements of $|\Delta m_{31}^2|$ or $|\Delta m_{32}^2|$ by both $\nu_e/\bar{\nu}_e$ and $\nu_\mu/\bar{\nu}_\mu$ disappearance experiments. The reason for writing this paper now is that within the next year the JUNO experiment [26, 27] is expected to improve the precision of these measurements for $\bar{\nu}_e$ disappearance by a factor of approximately 3, i.e. from Daya Bay's 2.4% to better than 0.8%, a very significant improvement.

There are a number of papers, such as [28–30], that use the method first presented in [1] to address the mass ordering question. Here, we revisit the combination of long baseline experiments and reactor experiments by simplifying and extend all of these analyses. First we derive a mass ordering sum rule, eq. 7, for the electron and muon neutrino disappearance channels. Then we use NuFIT's combined T2K and NOvA analysis to generate a contour plot of the $\Delta\chi^2$ between the mass orderings in the plane of JUNO's measurement of Δm_{31}^2 [NO] and its precision, Fig. 2. This plot allows the reader to immediately estimate the $\Delta\chi^2$ between the two mass orderings as soon as JUNO presents their $\Delta m_{31}^2|^{NO}$ measurement with its corresponding precision. Neither the sum rule or the contour plot has appeared elsewhere.

In [1], with more details available in [31], we have shown that the effective atmospheric Δm^2 (Δm_{atm}^2) for ν_e and $\bar{\nu}_e$ disappearance at a baseline divided by neutrino energy of 0.5 km/GeV, in vacuum, is given by[32]

$$\Delta m_{ee}^2 = \Delta m_{31}^2 \cos^2 \theta_{12} + \Delta m_{32}^2 \sin^2 \theta_{12}. \quad (3)$$

This is the only Δm^2 measured by Daya Bay [33] or RENO [34] without additional information from other

experiments. The final precision obtained by Daya Bay (DB) was 2.4%, see [35]:

$$|\Delta m_{ee}^2|_{DB} = 2.519 \pm 0.060 \text{ eV}^2. \quad (4)$$

Since the Daya Bay experiment is insensitive to the neutrino mass ordering, the magnitude of Δm_{ee}^2 is the same for both orderings and the sign is undetermined. Matter effects on the magnitude of Δm_{ee}^2 are smaller than one tenth of one per cent level, see [36], and are therefore much smaller than the measurement uncertainties.

Similarly, for ν_μ (and $\bar{\nu}_\mu$) disappearance, in vacuum, $\Delta m_{\mu\mu}^2$ is given by

$$\begin{aligned} \Delta m_{\mu\mu}^2 \approx & \Delta m_{31}^2 \sin^2 \theta_{12} + \Delta m_{32}^2 \cos^2 \theta_{12} \\ & + \sin \theta_{13} \cos \delta \Delta m_{21}^2, \end{aligned} \quad (5)$$

where δ is the CP phase and we have set $\sin 2\theta_{12} \tan \theta_{23} \approx 1$ for simplicity in the coefficient of the $\cos \delta$ term. For the current long baseline experiments such as T2K and NOvA, even though matter effects are significant for the appearance channels, it was recently shown by Denton and Parke, in [37], that for the ν_μ disappearance channels matter effects are at the tenths of one per cent level. Therefore the ν_μ disappearance channels are effectively in vacuum as there is a cancellation between the matter effects in $\nu_\mu \rightarrow \nu_e$ and in $\nu_\mu \rightarrow \nu_\tau$ for the long baseline experiments T2K and NOvA. As a result these disappearance channels are also insensitive to the neutrino mass ordering. Similar to Daya Bay and RENO, $|\Delta m_{\mu\mu}^2|$ is the only Δm^2 that is measurable in T2K and NOvA without information from other experiments.

For NO $|\Delta m_{31}^2| > |\Delta m_{32}^2|$ whereas for IO $|\Delta m_{31}^2| < |\Delta m_{32}^2|$, therefore, since $\sin^2 \theta_{12} \approx 0.3$ we can determine the mass ordering by comparing the magnitude of $|\Delta m_{ee}^2|$ to $|\Delta m_{\mu\mu}^2|$:

$$\begin{aligned} |\Delta m_{ee}^2| &> |\Delta m_{\mu\mu}^2| \text{ for NO} \\ \text{whereas } |\Delta m_{ee}^2| &< |\Delta m_{\mu\mu}^2| \text{ for IO}, \end{aligned} \quad (6)$$

although the size of the difference between $|\Delta m_{ee}^2|$ and $|\Delta m_{\mu\mu}^2|$ is at the couple of percent level and therefore precise measurements are required [38]. Fortunately we are about to enter an era where very precise measurements will be made for $\bar{\nu}_e$ disappearance by the JUNO experiment.

The above observations can be converted into a “mass ordering sum rule for the neutrino disappearance channels” :

$$(\Delta m_{31}^2|_{\mu \text{ disp}}^{NO} - \Delta m_{31}^2|_{e \text{ disp}}^{NO}) + (|\Delta m_{32}^2|_{e \text{ disp}}^{IO} - |\Delta m_{32}^2|_{\mu \text{ disp}}^{IO}) = (2 \cos 2\theta_{12} - 2 \sin \theta_{13} \widehat{\cos \delta}) \Delta m_{21}^2, \quad (7)$$

where the RHS can also be written as $(2.4 - 0.9 \widehat{\cos \delta})\% |\Delta m_{atm}^2|$. The subscript “ μ disp” means the results from ν_μ disappearance measurements in T2K and NOvA, whereas “ e disp” means the result from $\bar{\nu}_e$ disappearance experiments such as Daya Bay and RENO. The symbol $\widehat{\cos \delta}$ is the average $\cos \delta$ for the NO and IO fits. If one changes which Δm^2 one uses for both experiments for a given mass ordering, the RHS of this sum rule is unchanged. For detailed derivation of this sum rule see the appendix/supplemental material.

From this sum rule it is clear that if NO is Nature's choice then the first term will be zero, within measurement uncertainties, and if IO is Nature's choice the second term will be zero but in both scenarios the sum of the two must add up to the RHS, independent of the mass ordering. Consequently, if NO is Nature's ordering, the measurements of the Δm_{32}^2 's assuming IO will not align between ν_e and ν_μ disappearance within $\approx 2.4\%$ and similar for the IO ordering. The measurement uncertainties of the experiments Daya Bay, T2K and NOvA are now small enough that this method is already contributing to the global fits on the neutrino mass ordering. This can be perceived in the latest [NuFIT figure on the synergies for the \$\Delta m_{3i}^2\$'s](#), see [39], one can observe a preference for NO although the precision of the current measurements is not sufficient for a 3σ determination of the neutrino mass ordering.

The question of immediate current interest is how will the precision measurements of the Δm_{3i}^2 's by JUNO affect the determination of the mass ordering as JUNO measurements are expected to have an uncertainty smaller than one third of Daya Bay's. This measurement is expected to come very quickly after JUNO turns on, most likely in the first year of operation.

Reactor Measurement ($\bar{\nu}_e$ Disappearance) — JUNO is a medium baseline (~ 50 km) high precision reactor antineutrino oscillation experiment aiming to determine the neutrino mass ordering by a careful measurement of the $\bar{\nu}_e$ energy spectrum using an idea first proposed in [12] and further investigated in [40, 41]. It was shown in [42] that medium baseline reactor experiments can, in principle, determine the ordering by precisely measuring the effective combination Δm_{ee}^2 and the sign of a phase ($\pm\Phi_\odot$; + for NO, - for IO). This is a very challenging measurement due to various systematic effects (energy resolution, non-linear detector response etc.) that have to be tamed and understood to a very high level. The JUNO collaboration claimed, in their 2015 paper [26], that it will take 6 years to determine the mass ordering at 3σ , although more recent papers, see e.g. [43], have questioned that claim[44]. JUNO's recent update [27] does not contain an update on their expected mass ordering sensitivity.

On the other hand, JUNO is expected to reach, after a few months of operation, and much sooner than they can start to be sensitive to the mass ordering, unprecedented sub-percent precision on the determination of $|\Delta m_{atm}^2|$ where $|\Delta m_{atm}^2|$ could be any one of $|\Delta m_{ee}^2|$, or $|\Delta m_{31}^2|$, or $|\Delta m_{32}^2|$ depending on the experiment's analysis choice for both mass orderings but all are related to one another. JUNO claims that after 100 days of data taking they will be able to determine $|\Delta m_{atm}^2|$ at 0.8% precision and will continue to improve ultimately reaching 0.2%

$\Delta(\Delta m^2)$	NO ee	NO 31	NO 32
IO ee	1.8	-0.5	6.9
IO 31	-0.5	-2.7	4.7
IO 32	6.9	4.7	12.1

TABLE I. This Table relates the different Δm_{atm}^2 's for the best fits of the JUNO experiment, such that each entry gives the difference between $\Delta(\Delta m^2) \equiv |\Delta m_{ij}^2|^{IO} - |\Delta m_{kl}^2|^{NO}$ in units of 10^{-5} eV 2 where (ij) and (kl) are (ee, 31, 32). A value of 2.5 in this table represents a 1.0% difference. The difference between $|\Delta m_{32}^2|^{IO}$ and $|\Delta m_{32}^2|^{NO}$ is $\sim 5\%$, much larger than the measurement error expected in JUNO, whereas the difference between $|\Delta m_{31}^2|^{IO}$ and $|\Delta m_{31}^2|^{NO}$ is $\sim 1\%$.

precision, see [27]. In contrast, the expected $\Delta\chi^2$ between the NO and IO in JUNO's fits is expected to grow quite slowly, at no more than 1.5 units per year.

It was shown in [43], that due to the phase advance (NO) or retardation (IO) the best fits to the spectrum at the far JUNO (JU) detector will give a $|\Delta m_{ee}^2|$ for IO which is 0.7% larger than $|\Delta m_{ee}^2|$ fit for NO, i.e.

$$|\Delta m_{ee}^2|_{JU}^{IO} = |\Delta m_{ee}^2|_{JU}^{NO} + 1.8 \times 10^{-5} \text{ eV}^2. \quad (8)$$

Note this shift is a fraction of Δm_{21}^2 and comes from the fact that in the IO the atmospheric oscillations are retarded with respect to the NO and therefore Δm_{ee}^2 for IO is slightly larger than that for NO to compensate for this phase shift. We have used that $0.007 \times |\Delta m_{ee}^2| \approx 0.018 \times 10^{-3} \text{ eV}^2$ and the value of Δm_{21}^2 is given by eq. 1. Using eq. 3 for each mass ordering we can relate this result for any of the other possible $|\Delta m_{atm}^2|$ as given in Table I.

This modifies the “mass ordering sum rule for the neutrino disappearance channels”, given in eq. 7, by increasing the RHS by 0.7%, so that for T2K, NOvA (LBL) and JUNO (JU) we have the following sum rule

$$\begin{aligned} &(|\Delta m_{31}^2|_{LBL}^{NO} - |\Delta m_{31}^2|_{JU}^{NO}) + (|\Delta m_{32}^2|_{JU}^{IO} - |\Delta m_{32}^2|_{LBL}^{IO}) \\ &\approx (3.1 - 0.9 \cos \delta)\% |\Delta m_{atm}^2|. \end{aligned} \quad (9)$$

With enough precision on the measurements of the Δm_{atm}^2 's between ν_e and ν_μ disappearance, the $\Delta\chi^2$ between the two mass ordering fits, can contribute significantly to the determination of the mass ordering.

We can describe for JUNO the χ^2 fit to data to determine $|\Delta m_{atm}^2|$, as the parabola which will depend on the assumed best fit value $|\Delta m_{atm}^2|^{NO}$ or $|\Delta m_{atm}^2|^{IO}$ and σ_{JU} , the precision of the measurement, which does not

depend on the ordering

$$\chi_{\text{NO}}^2(\Delta m^2, \sigma_{\text{JU}})_{\text{JU}} = \left(\frac{\Delta m^2 - \Delta m_{\text{atm}}^2|_{\text{NO}}}{\sigma_{\text{JU}}} \right)^2 \quad (10)$$

$$\chi_{\text{IO}}^2(|\Delta m^2|, \sigma_{\text{JU}})_{\text{JU}} = \left(\frac{|\Delta m^2| - |\Delta m_{\text{atm}}^2|_{\text{IO}}}{\sigma_{\text{JU}}} \right)^2 \quad (11)$$

Note that the subscript ‘atm’ can be any one of (ee, 31, 32) and they do not need to be the same for NO and IO. However, the best fit points must be related by the numbers given in Table I. Different experiments and different global fit groups may make different choices. Here, we will use the choice used by the NuFIT collaboration, that is Δm_{31}^2 for NO and Δm_{32}^2 for IO and the relationship that for the JUNO experiment

$$|\Delta m_{32}^2|_{\text{JU}}^{\text{IO}} = \Delta m_{31}^2|_{\text{JU}}^{\text{NO}} + 4.7 \times 10^{-5} \text{ eV}^2. \quad (12)$$

The physics conclusions will be independent of the (31, 32) arbitrary choices.

Long-baseline Accelerator Measurement ($\nu_\mu/\bar{\nu}_\mu$ Disappearance) — T2K is a long-baseline (~ 295 km) accelerator neutrino oscillation experiment in Japan that has collected a total of 1.97×10^{21} and 1.63×10^{21} protons on target in neutrino and antineutrino modes, respectively. Similarly, NOvA is a long-baseline (~ 810 km) accelerator neutrino oscillation experiment in the US that also has collected 1.36×10^{21} and 1.25×10^{21} protons on target of data in neutrino and antineutrino modes, respectively. Both experiments operate as a $\nu_\mu \rightarrow \nu_\mu/\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$ disappearance experiment as well as a $\nu_\mu \rightarrow \nu_e/\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ appearance experiment. Their disappearance measurements have no sensitivity to the mass ordering but are the responsible for the precise determination of $|\Delta m_{32}^2|$ (or equivalently $|\Delta m_{31}^2|$). T2K results given in [45] are $\Delta m_{32}^2|^{\text{NO}} = (2.49 \pm 0.05) \times 10^{-3} \text{ eV}^2$ and $|\Delta m_{31}^2|^{\text{IO}} = (2.46 \pm 0.05) \times 10^{-3} \text{ eV}^2$, note that uncertainty is $\sim 2\%$. NOvA’s results are given by $\Delta m_{32}^2|^{\text{NO}} = (2.39 \pm 0.06) \times 10^{-3} \text{ eV}^2$ and $|\Delta m_{32}^2|^{\text{IO}} = (2.44 \pm 0.06) \times 10^{-3} \text{ eV}^2$, see [20]. Given the consistency of T2K and NOvA disappearance measurements, they can be combined, as in [25], for both a NO and a IO fit. Using the update of these fits, given in [39], we have for NO

$$\chi_{\text{NO}}^2(\Delta m_{31}^2)_{\text{LBL}} = \left(\frac{\Delta m_{31}^2 - \Delta m_{31}^2|_{\text{NO}}}{\sigma_{\text{LBL}}^{\text{NO}}} \right)^2 \quad (13)$$

with $(\Delta m_{31}^2|_{\text{NO}}, \sigma_{\text{LBL}}^{\text{NO}}) = (2.516, 0.031) \times 10^{-3} \text{ eV}^2$. Similarly for the IO fit,

$$\chi_{\text{IO}}^2(|\Delta m_{32}^2|)_{\text{LBL}} = \left(\frac{|\Delta m_{32}^2| - |\Delta m_{32}^2|_{\text{IO}}}{\sigma_{\text{LBL}}^{\text{IO}}} \right)^2 \quad (14)$$

with $(|\Delta m_{32}^2|_{\text{IO}}, \sigma_{\text{LBL}}^{\text{IO}}) = (2.485, 0.031) \times 10^{-3} \text{ eV}^2$. Note since the uncertainty for both NO and IO are the same, $\sigma_{\text{LBL}}^{\text{IO}} = \sigma_{\text{LBL}}^{\text{NO}}$, we can drop the mass ordering on

this symbol. Also it is important that this combined uncertainty is $\sim 1.2\%$, much less than the RHS of eq. 7 and 9. T2K’s and NOvA’s results are not expected to change significantly in the next few years due to the large statistics already collected by these experiments.

Combining JUNO with T2K and NOvA — We combine the results of T2K, NOvA and JUNO by just adding the χ^2 for the combine LBL results to that of JUNO, using the best fit for NO and the measurement precision of JUNO as variables as follows:

$$\begin{aligned} \chi_{\text{NO}}^2(\Delta m_{31}^2, \sigma_{\text{JU}})_{\text{JU}} &= \chi_{\text{NO}}^2(\Delta m_{31}^2, \sigma_{\text{JU}})_{\text{JU}} + \chi_{\text{NO}}^2(\Delta m_{31}^2)_{\text{LBL}} \\ \chi_{\text{IO}}^2(|\Delta m_{32}^2|, \sigma_{\text{JU}})_{\text{JU}} &= \chi_{\text{IO}}^2(|\Delta m_{32}^2|, \sigma_{\text{JU}})_{\text{JU}} + \chi_{\text{IO}}^2(|\Delta m_{32}^2|)_{\text{LBL}} \end{aligned}$$

where for JUNO we use the relationship given in eq. 12 for the best fit values. Then the difference in the $\Delta\chi^2$ is given by

$$\Delta\chi^2(\Delta m_{31}^2|_{\text{JU}}^{\text{NO}}, \sigma_{\text{JU}})_{\text{min}} = \chi_{\text{IO}}^2|_{\text{min}} - \chi_{\text{NO}}^2|_{\text{min}}. \quad (15)$$

Now everything is determined except for the best fit value $\Delta m_{31}^2|_{\text{NO}}^{\text{NO}}$ and precision of this measurement σ_{JU}

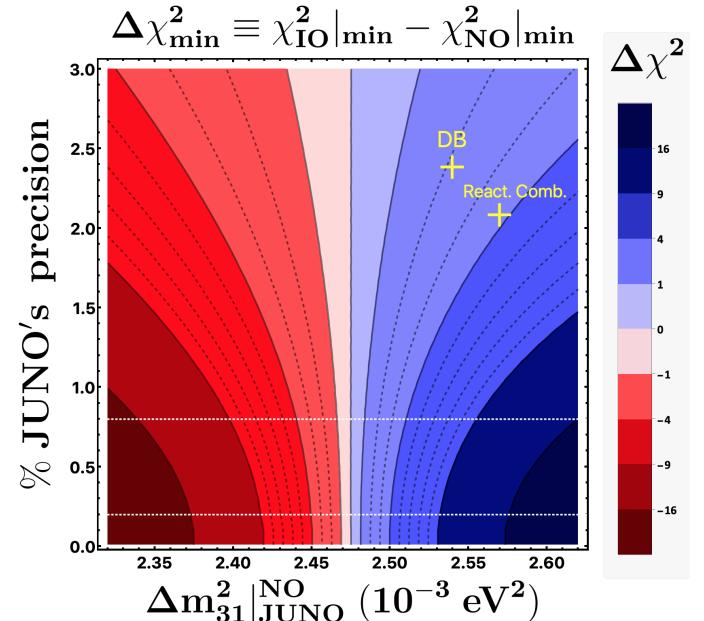


FIG. 2. Iso-contours of $\Delta\chi^2_{\text{min}} \equiv \chi_{\text{IO}}^2|_{\text{min}} - \chi_{\text{NO}}^2|_{\text{min}}$ in the plane of JUNO’s $(\Delta m_{31}^2|_{\text{NO}}^{\text{NO}} \otimes \sigma)$ fit to the far detector spectrum assuming normal ordering (NO). The precision is expressed as a % of $\Delta m_{31}^2|_{\text{NO}}^{\text{NO}}$. The blue region favors NO whereas the red region favors IO. The current values from the Daya Bay (DB) experiment and from Daya Bay and RENO combined according to NuFIT (React. Comb.) are also shown. Table I can be used to translate the horizontal axis to any other Δm_{atm}^2 . The white dashed lines mark the precision achievable by JUNO after 100 days (0.8%) and 6 years (0.2%).

by the JUNO experiment. In Fig. 2 we have plotted the $\Delta\chi^2_{\min}$ defined as the minimum for IO minus the minimum for NO as a function of JUNO’s $|\Delta m^2_{31}|^{\text{NO}}$ and the fractional precision of the measurement by JUNO[46]. On this fan shaped figure we have shown for reference Daya Bay’s result as well as the one from the combined fit of Daya Bay and RENO data given by the NuFIT group. Clearly as JUNO’s precision on the measurement gets better and better one moves down this plot which increases the $\Delta\chi^2$, however the central value could also move right (left) thereby increasing (decreasing) the $\Delta\chi^2$. This plot assumes that the disappearance Δm^2_{atm} results from T2K and NOvA will not significantly change from what they are now. So, as soon as JUNO presents results on the $|\Delta m^2_{\text{atm}}|$ fit to the spectrum at the far detector, one can read off from this plot the contribution that $\nu_\mu/\bar{\nu}_\mu$ and $\bar{\nu}_e$ disappearance measurements from T2K, NOvA and JUNO make to the $\Delta\chi^2$ for the mass ordering determination. If JUNO has the same central value as the NuFIT (React. Comb.) value but with a precision below 1%, then the contribution from these disappearance measurements will be greater than 9 units of $\Delta\chi^2$, i.e. greater than 3σ .

Conclusions — We re-examine an idea we have had on how to determine the neutrino mass ordering, almost 20 years ago, using only neutrino disappearance data in vacuum. This is auspicious today in the light of the current precision on $|\Delta m^2_{\text{atm}}|$ (31 for NO and 32 for IO) achieved by long-baseline experiments ($\sim 2\%$ individually and $\sim 1.2\%$ combined) and of the imminent few per mil determination of $|\Delta m^2_{\text{atm}}|$ by JUNO. This kind of unprecedented accuracy allows one to discuss a *mass ordering sum rule for the neutrino disappearance channels* (see eq. 9) which may be used to determine the mass ordering in the near future solely using data from these disappearance experiments. To show this with more clarity, we combined in a χ^2 function the present

results from T2K and NOvA ν_μ and $\bar{\nu}_\mu$ disappearance measurements with that expected from $\bar{\nu}_e$ disappearance at JUNO, as a function of the assumed best fit value ($|\Delta m^2_{31}|^{\text{NO}}$) and fractional accuracy of the JUNO measurement (σ_{JU}). In Fig. 2 one can see the main result, the values of $\Delta\chi^2_{\min} = \chi^2_{\text{IO}}|_{\min} - \chi^2_{\text{NO}}|_{\min}$ in the plane of JUNO’s ($|\Delta m^2_{31}|^{\text{NO}} \otimes \sigma$). This is a very useful figure because as soon as JUNO presents its first result on their $|\Delta m^2_{\text{atm}}|$ measurement, one can read from it if NO is preferred, and if so, how much it is preferred in terms of how many units of $\Delta\chi^2$. In this manner it is conceivable, if JUNO measures $|\Delta m^2_{\text{atm}}|$ close to the one given by combining Daya Bay and RENO data, that NO could be soon (in a year or so) determined, by the combined (T2K, NOvA and JUNO) disappearance measurements alone, to better than 3σ i.e. a confidence level of 99.73%.

ACKNOWLEDGMENTS

SJP would like to thank Peter Denton for many useful discussions on matter effects for ν_μ disappearance. SJP acknowledges support by the United States Department of Energy under Grant Contract No. DE-AC02-07CH11359. R. Z. F. is partially supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) under Contract No. 2019/04837-9, and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). We would both like to thank our long time collaborator, Hiroshi Nunokawa, for many enlightening discussions on this topic over many years. Its a pity he chose not to join us for this follow on paper of [1].

This project has also received support from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 860881-HIDDeN as well as under the Marie Skłodowska-Curie Staff Exchange grant agreement No 101086085 - ASYMMETRY.

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Appendix A: Supplemental Material

Here we derive the “mass ordering sum rule for neutrino disappearance channels” from the observations of [1]. Daya Bay and Reno measure the same $|\Delta m_{ee}^2|$ for both mass orderings. Therefore from eq. 3 we have the following:

$$\begin{aligned} \Delta m_{31}^2|_{e \text{ disp}}^{\text{NO}} &= |\Delta m_{ee}^2| + \sin^2 \theta_{12} \Delta m_{21}^2 \\ |\Delta m_{32}^2|_{e \text{ disp}}^{\text{IO}} &= |\Delta m_{ee}^2| + \cos^2 \theta_{12} \Delta m_{21}^2 \\ |\Delta m_{32}^2|_{e \text{ disp}}^{\text{IO}} - \Delta m_{31}^2|_{e \text{ disp}}^{\text{NO}} &= \cos 2\theta_{12} \Delta m_{21}^2. \end{aligned} \quad (\text{A1})$$

Daya Bay’s reported measurements given in [35] of $|\Delta m_{32}^2|_{\text{DB}}^{\text{NO}} = 2.466 \pm 0.060 \times 10^{-3}$ eV² and $|\Delta m_{32}^2|_{\text{DB}}^{\text{IO}} = 2.571 \pm 0.060 \times 10^{-3}$ eV² satisfy eq. A1, after adding Δm_{21}^2 to $\Delta m_{32}^2|_{\text{NO}}$ to obtain $\Delta m_{31}^2|_{\text{NO}}$. The agreement is much, much smaller than the quoted uncertainty, demonstrating that these two results are connected via $|\Delta m_{ee}^2|$.

Similarly for the disappearance channels in T2K and NOvA, they both measure the same $|\Delta m_{\mu\mu}^2|$ for both mass orderings and matter effects are negligible in this channel for both T2K and NOvA, see [37]. Therefore from eq. 5 we have the following:

$$\begin{aligned}
\Delta m_{31}^2|_{\mu \text{ disp}}^{\text{NO}} &= |\Delta m_{\mu\mu}^2| + (\cos^2 \theta_{12} - \sin \theta_{13} \cos \delta^{\text{NO}}) \Delta m_{21}^2 \\
|\Delta m_{32}^2|_{\mu \text{ disp}}^{\text{IO}} &= |\Delta m_{\mu\mu}^2| + (\sin^2 \theta_{12} + \sin \theta_{13} \cos \delta^{\text{IO}}) \Delta m_{21}^2 \\
\Delta m_{31}^2|_{\mu \text{ disp}}^{\text{NO}} - |\Delta m_{32}^2|_{\mu \text{ disp}}^{\text{IO}} &= (\cos 2\theta_{12} - 2 \sin \theta_{13} \widehat{\cos \delta}) \Delta m_{21}^2.
\end{aligned} \tag{A2}$$

where $\widehat{\cos \delta} \equiv \frac{1}{2}(\cos \delta^{\text{NO}} + \cos \delta^{\text{IO}})$. T2K provides enough significant figures such that again the identity eq. A2 can be checked. T2K results given in Table 13 of [45] are reported as $\Delta m_{32}^2|_{\text{T2K}}^{\text{NO}} = 2.494 \pm 0.054 \times 10^{-3} \text{ eV}^2$ and $|\Delta m_{32}^2|_{\text{T2K}}^{\text{IO}} = 2.463 \pm 0.049 \times 10^{-3} \text{ eV}^2$ again after correcting for the $31 \leftrightarrow 32$ for both mass ordering, we get excellent agreement when using $\widehat{\cos \delta} \approx 0$. The agreement is much, much smaller than the quoted uncertainty, demonstrating that these two results are connected via $|\Delta m_{\mu\mu}^2|$. NOvA, unfortunately, does not provide enough significant figures for their measurements to demonstrate the $|\Delta m_{\mu\mu}^2|$ connection as convincingly as T2K, but their measurements are in agreement with A2.

Now, adding and rearranging eq. A1 and A2 we have the “*mass ordering sum rule for neutrino disappearance channels*” given in eq. 7:

$$\begin{aligned}
(\Delta m_{31}^2|_{\mu \text{ disp}}^{\text{NO}} - \Delta m_{31}^2|_{e \text{ disp}}^{\text{NO}}) + (|\Delta m_{32}^2|_{e \text{ disp}}^{\text{IO}} - |\Delta m_{32}^2|_{\mu \text{ disp}}^{\text{IO}}) &= (2 \cos 2\theta_{12} - 2 \sin \theta_{13} \widehat{\cos \delta}) \Delta m_{21}^2, \\
&= (2.4 - 0.9 \widehat{\cos \delta}) \% |\Delta m_{\text{atm}}^2|,
\end{aligned} \tag{A3}$$

where for the last line we have used $\Delta m_{21}^2/|\Delta m_{\text{atm}}^2| = 0.03$, $\sin^2 \theta_{12} = 0.3$ and $\sin \theta_{13} = 0.15$. Note the interchange between “ μ disp” and “ e disp” when going from NO to IO. Also this sum is invariant if we replace Δm_{31}^2 with Δm_{32}^2 for NO or/and Δm_{32}^2 with Δm_{31}^2 for IO. We use the above choice because that is the choice made by NuFIT, but our physics conclusions are independent of this choice. In passing it is worth noting that the NuFIT results given in [39] suggest that $\widehat{\cos \delta} \leq 0$.

Due to the phase advance (NO) or retardation (IO) of the atmospheric oscillations, JUNO does not measure exactly the same $|\Delta m_{ee}^2|$ for both mass orderings, see [43], infact

$$|\Delta m_{ee}^2|^{\text{IO}} \approx 1.007 \times \Delta m_{ee}^2|^{\text{NO}} = \Delta m_{ee}^2|^{\text{NO}} + 1.8 \times 10^{-5} \text{ eV}^2. \tag{A4}$$

This changes eq. A1 and also the “*mass ordering sum rule for neutrino disappearance channels*” to what is given in eq. 9 by adding 0.7 % times $|\Delta m_{\text{atm}}^2|$ to the RHS:

$$(\Delta m_{31}^2|_{\text{LBL}}^{\text{NO}} - \Delta m_{31}^2|_{\text{JU}}^{\text{NO}}) + (|\Delta m_{32}^2|_{\text{JU}}^{\text{IO}} - |\Delta m_{32}^2|_{\text{LBL}}^{\text{IO}}) \approx (3.1 - 0.9 \widehat{\cos \delta}) \% |\Delta m_{\text{atm}}^2|. \tag{A5}$$

Here the label “e disp” (“ μ disp”) has been replaced with the label “JU” (“LBL”) as this sum rule is specific for T2K, NOvA (LBL) and JUNO (JU). The additional 0.7 % has an impact on Fig. 2 because of the precision of the JUNO’s measurements on Δm_{atm}^2 ’s.

If Nature’s choice for the mass ordering is NO, then

$$(\Delta m_{31}^2|_{\text{LBL}}^{\text{NO}} - \Delta m_{31}^2|_{\text{JU}}^{\text{NO}}) \approx 0 \quad \text{and} \quad (|\Delta m_{32}^2|_{\text{JU}}^{\text{IO}} - |\Delta m_{32}^2|_{\text{LBL}}^{\text{IO}}) \approx (3.1 - 0.9 \widehat{\cos \delta}) \% |\Delta m_{\text{atm}}^2| \tag{A6}$$

whereas for IO

$$(\Delta m_{31}^2|_{\text{LBL}}^{\text{NO}} - \Delta m_{31}^2|_{\text{JU}}^{\text{NO}}) \approx (3.1 - 0.9 \widehat{\cos \delta}) \% |\Delta m_{\text{atm}}^2| \quad \text{and} \quad (|\Delta m_{32}^2|_{\text{JU}}^{\text{IO}} - |\Delta m_{32}^2|_{\text{LBL}}^{\text{IO}}) \approx 0, \tag{A7}$$

where “ \approx ” should be interpreted to mean “within measurement uncertainties”. The current measurement uncertainty for the combined T2K and NOvA measurement is $\sim 1.2\%$. JUNO’s measurement uncertainty will reach 0.8 % within one year of data taking and reach 0.2 % within 6 years. If T2K and NOvA can improve their measurement uncertainties on the Δm_{atm}^2 ’s even modestly, in the future, this can impact the $\Delta\chi^2$ between the two mass orderings substantially.