Coordinates and Datums

Both topics have to do with locating things on maps, or globes. Globe locations are given in degrees, i.e. in terms of angles, since the alternative is stretching a cloth tape on the globe surface. Map coordinates can be given in degrees, or in xy measurements like miles or meters.

A **datum** is an elaborate and difficult to acquire specification of coordinates for a region. Most countries have their own datum as a national treasure, periodically revised. Involved in a datum are both a chosen ellipsoid or geoid, and a chosen projection. The goals are consistency and accuracy.

At a technical level, a **geoid** is the earth's shape if it were all ocean, but with the existing densities and gravitational forces. People thus discuss a virtual ocean as extending under the current land masses. A region lies within multiple datums. Canada can have its own, but so can North America. To accurately calculate the distance between two points, it is best to find a datum for a region encompassing both points, and probably the smallest such datum.

Geographic coordinates refer to giving map location using the familiar **latitude and longitude**. Lines of longitude are called 'meridians', and the zero or prime meridian runs through Greenwich, England. Using the usual notion of 360° to go around a sphere, from the prime meridian one can go West 180°, and East 180°. So 112° west is denoted by 112W or by -112. Zero degrees latitude is the equator, and one proceeds north to the North Pole from 1 to 90. Latitude lines progress to the South with negative values from -1 to -90. The **'grid'** given by drawing lines of latitude and longitude on a map is called a **'graticule'**. Grid lines can be in local coordinates, and meet at right angles. The graticule can appear as arcs, in many projections. So grid is a more general term than graticule.

By long division, it can be seen that the length of one degree of latitude is 111km, on average. Because of elliptical flattening, the distance is a little less at the equator, a little more near the poles. If you did the math right, this is distance measured along the earth's surface. People who claim they took the real geodetic between two points probably were good at tunneling. Since that can be difficult, shortest paths are considered to be those taken along the earth surface following a great circle. It should be clear that the distance of a degree of longitude is 111km at the equator, and converges to zero as you approach a pole.

The shortest path distance between two points can be computed by the formula: **3D law of cosines**

* cos(D) = sin a \* sin b + cos a \* cos b \* cos(P)
* D = inverse cos(sin a \* sin b + cos a \* cos b \* cos(P)) \* 111.1111
  + D: the angular distance between points A and B
  + a: the latitude of A
  + b: the latitude of B
  + P: the longitudinal angle difference between A and B

These angles will be machine calculated as radians or degrees. So you calculate cos(D), find D using arcos (aka cosine inverse), and then find the real d as D\*111km if D is in degrees. For radians, divide by 2\*pi, then multiply.

NOTE: always indicate that you want to go the 'short way' between two points. So given longitudes of 100W and 50E, let P be 150 and not 210. Think of the formula as 'correct' for P of 180 degrees or less.

Serious navigators, like airplane and ship captains, need these distances, but this is not a hard program feature to write. Another example of a need for a universal system is GPS recorders.

**Map scale** is determined by the scale ratio between earth and the mathematical globe used for the projection. This is a 'nominal' scale, and because maps are nowhere perfectly equidistant, the real scale varies from place to place on a map. Most maps use a rectangular grid coordinate system where grid lines meet at right angles. Much care is taken to minimize the resultant distortion.

One popular system is **UTM (Universal Transverse Mercator)**. This is really a transverse Mercator, so it produces a rectangular grid via the projections, and it works well for most countries, and sees use in the U.S. and Canada. With UTM, a new 'central meridian' is used every 6°, starting from longitude 180, in order to minimize distortion, so each projection covers 6°, west to east. To further reduce distortion, a secant cylinder is used instead of a tangent one. With most UTM versions, the 60 zones are numbered from 1 to 60, starting with 180W to 174W as zone 1, and working east.

To simplify reference, 6° by 8° quadrangles are used, and subdivided into little squares, accessed by letter indices. The 6x8 large quads are indexed from 1 to 60, going from 180W to 180E, and the latitude belts (or designators) are indexed using [C-X] - {O,I}, since ‘O’, and ‘I’ look like numbers. Doing the math, we have 20\*8 = 160°, which is OK since not much happens at the poles, and planar projections are used up there. The effort is to make all positions have positive coordinates. Within the little squares into which the 6x8 quad is divided we use 'easting' and 'northing' from the lower left hand corner of the square.

NOTE: several sources state that UTM is used from -80° to 84°, and it is not clear where the extra 4° north come from. In fact, zone X is simply elongated to 12°.

NOTE: San Francisco is in quadrangle 10S, San Diego is in 11S; the U.S. (contiguous) is spanned by zones 10 through 19, and belts R, S, T, and U.

NOTE: some UTM systems do not use the alphabetic belts, but measure everything from the equator.

To be more specific, 8° latitude is 888km, so if we want 100km tall little squares, we want roughly nine divisions north to south. The indexing starts at the equator, and uses A-V, cycling as often as needed, so there will be a run of about 9 letters in the breakup of a cell into 100km by 100km squares. This actually applies to odd indexed zones. Even indexed zones are offset by 5 and run through the 20 letters starting with F. From West to East the letters A to Z, less I and O, are used, starting at 180W, and recycling every

18 degrees. Coordinates will then be specified something like ‘11S, DJ, 44, 82’ (or 11SDJ4482) which means zone 11, north-to-south belt S, the 100kmx100km square DJ within that cell. If there are four digits, then in this example 44 is an easting of 44km, and a northing of 82 km. This narrows it to the nearest kilometer. If we say 445829 then within that square kilometer we go east 0.5 km, and north 0.9 km. This is the metric system at work.

NOTE: some UTM systems do not use this alphabetic breakdown.

NOTE: the coordinate scheme described about is known as the 'military grid' version of the UTM coordinate system.

NOTE: 'zone' is consistently used to describe the UTM band of 6° longitude.

NOTE: the UTM coordinate system has a horizontal emphasis because it starts with those 60 zones. Good locations and measurements result by using a UTM cylinder tangent to the meridian in the center of each zone, i.e. we use 60 UTM projections, along with the 60 zones.

Surveyors and others in the US use the SPCS (State Plane Coordinate System). This is based on the NAD-83 datum (North America Datum – 1983). Another datum is SAD-69, and yes, you did guess what it stands for. Two other commonly used datums are NAD-27 and WGS-84 (World Geodetic System).

Often small scale maps use geographical coordinates given in terms of degrees of longitude and latitude. In this case the scheme can be in one of two forms:

1. DMS (Degrees, Minutes, and Seconds)
   1. dd.mm.ss.xx
   2. There are 60 minutes in a degree and there are 60 seconds in a minute, and the xx is a decimal fraction of a second if you want to be more particular
2. Decimal Degrees
   1. here the measurements are given in the form: dd.xxxx or usually dd.xxx
   2. So 1.001, 0.000 would be a point on the equator which is 1° east of the prime meridian, plus 1 thousandth of a degree.
   3. Since a degree is 111km at the equator, more or less, 0.001 is about 111 meters, or 111 healthy strides. By going to dd.xxxx we can distinguish objects 11 strides apart.

NOTE: since both forms are in heavy use, little Java functions to convert between the two are nice to have. Call the methods dmsToDd and dDToDms. Prototype and write them!

In closing we give one more formula, this one for specifying the distance between two consecutive lines of longitude (two meridians) at a latitude of n degrees.

111.11 \* cos(n)