Week05

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1 EX1:

Using Laplace expansion, we have:

$$det(A) = \begin{vmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 0 & 2 & 4 \end{vmatrix} = 1(4.4 - 6.2) - 3(2.4 - 6.0) + 5(2.2 - 4.0) = 0$$

Using Sarrus Rule, we have:

$$det(A) = \begin{vmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 0 & 2 & 4 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 3 & 5 & 1 & 3 \\ 2 & 4 & 6 & 2 & 4 \\ 0 & 2 & 4 & 0 & 2 \end{vmatrix} \Rightarrow det(A) = 1.4.4 + 3.6.0 + 5.2.2 - 0.4.5 - 2.6.1 - 4.2.3 = 0$$

2 EX2:

$$\begin{bmatrix} 2 & 0 & 1 & 2 & 0 \\ 2 & -1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ -2 & 0 & 2 & -1 & 2 \\ 2 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 2 & 2 & 0 \\ 0 & -1 & -1 & -1 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ 0 & 0 & 3 & 1 & 2 \\ 0 & 0 & -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 2 & 2 & 0 \\ 0 & -1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 3 & 1 & 2 \\ 0 & 0 & -1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 2 & 2 & 0 \\ 0 & -1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 2 & 2 & 0 \\ 0 & -1 & -1 & -1 & 1 \\ 0 & 01 & 0 & 3 & \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 & -3 \end{bmatrix}$$

$$\Rightarrow det(A) = 2.(-1).1.1.(-3) = 6$$

3 EX3:

a, We have:

$$det(A - \lambda I) = \left| (\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}) \right|$$

$$\begin{vmatrix} 1-\lambda & 0 \\ 1 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)^2 = 0 \Rightarrow \lambda = 1$$

we have:

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\Rightarrow \begin{cases} x_1 = x_1 \\ x_1 + x_2 = x_2 \end{cases}$$

$$\Rightarrow Span \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

b, We have:

$$det(A - \lambda I) = \left| (\begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}) \right|$$

$$\Rightarrow \begin{vmatrix} -2-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (-2-\lambda)(1-\lambda) - 4 = 0$$

$$\Rightarrow \begin{cases} \lambda = 2 \\ \lambda = -3 \end{cases}$$

$$With \quad \lambda = 2 \quad we \quad have:$$

$$\begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\Rightarrow \begin{cases} -2x_1 + 2x_2 = 2x_1 \\ -2x_1 + x_2 = 2x_2 \end{cases}$$

$$\Rightarrow x_1 = 0 \quad and \quad x_2 = 0$$

$$\Rightarrow Span = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$With \quad \lambda = -3 \quad we \quad have:$$

$$\begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -3 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\Rightarrow \begin{cases} x_1 + 2x_2 = 0 \\ 2x_1 + 4x_2 = 0 \end{cases}$$

$$\Rightarrow Infinite \quad solution$$

$$\Rightarrow Span = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

4 EX4:

We have:

$$A = \begin{bmatrix} 0 & -1 & 1 & 1 \\ -1 & 1 & -2 & 3 \\ -2 & -1 & 0 & 0 \\ 2 & -1 & 1 & 0 \end{bmatrix}$$

$$det(A - \lambda I) = 0 \Rightarrow \begin{vmatrix} -\lambda & -1 & 1 & 1 \\ -1 & 1-\lambda & -2 & 3 \\ 2 & -1 & -\lambda & 0 \\ 2 & -1 & 1 & -\lambda \end{vmatrix} = 0$$