Week 08

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1 Ex1:

We have:

$$y = x^3 + 6x^2 - 3x - 5$$

$$\frac{dy}{dx} = 3x^2 + 12x - 3$$

At stationary points: $\frac{dy}{dx} = 0$
Factorising gives: $3x^2 + 12x - 3 = 0$
With solutions: $x = -2 - \sqrt{5}$ and $x = -2 + \sqrt{5}$
We have:

$$\frac{d^2y}{dx^2} = y'' = 6x + 12$$

With $x = -2 - \sqrt{5}$ ⇒y"=-6$\sqrt{5}$ $\Rightarrow Negative$
$With$x = -2+$\sqrt{5}$ ⇒y"=6$\sqrt{5}$ $\Rightarrow Positive$
$Thus,$x$_1 = -2 - \sqrt{5}$ is maximum and $x_2 = -2 + \sqrt{5}$ is minimum

2 Ex2:

We have:

$$L(x, y) = 81x^2 + y^2 + \lambda(4x^2 + y^2)$$

$$\begin{cases} L'(x) = 162x + 81\lambda x = 0 \\ L'(y) = 2y + 2\lambda y = 0 \\ 4x^2 + y^2 = 9 \end{cases}$$

Solution x=0 or $\lambda = \frac{-81}{4}$ and y=0 or $\lambda = -1$
With $\lambda = -20,25$ and y=0 $\Rightarrow x = 1,5 \Rightarrow f(1,5;0) = 9 \Rightarrow Minimum$
$With \lambda = -1$ and x=0 $\Rightarrow y = 3 \Rightarrow f(0,3) = 182,25 \Rightarrow Maximum$

3 Ex3:

We have:

$$f(p,q) = -\sum_{i=1}^{c} p_i log(q_i)$$

$$L(p,q)_{\lambda_1,\lambda_2} = -\sum_{i=1}^{c} p_i log(q_i) + \lambda_1(1 - \sum_{i=1}^{c} p_i) + \lambda_2(1 - \sum_{i=1}^{c} q_i)$$

We have:

$$\frac{d_L}{d_p} = [-log(q_1) - \lambda_1, ..., -log(q_c) - \lambda_1] = [0, ..., 0]$$

$$\frac{d_L}{d_q} = [\frac{-p_1}{q_1} - \lambda_2, ..., \frac{-p_c}{q_c} - \lambda_2] = [0, ..., 0]$$

$$\frac{d_L}{d_{_1}} = -\sum_{i=1}^{c} p_i + 1 = 0$$

$$\frac{d_L}{d_{_2}} = -\sum_{i=1}^{c} q_i + 1 = 0$$

$$\Rightarrow p_1 = p_2 = p_3 = ... = p_c = \frac{1}{c} \;\; and \;\; q_1 = q_2 = q_3 = ... = q_c = \frac{1}{c}$$

$$minf(p,q) = -c.\frac{1}{c}log(\frac{1}{c}) = -log(\frac{1}{c})$$

4 Ex4:

We have:

$$\frac{\| Ax \|_2}{\| x \|_2} = \lambda$$

Surpose $\| x \|_2 = 1$

$$L(x,\lambda) = \| Ax \|_2 + \lambda(1 - \| x \|_2)$$

$$\frac{d_L}{d_x} = A^T Ax.\frac{1}{\| Ax \|_2} - \lambda x = 0$$

$$\frac{d_L}{d_\lambda} = 1 - \|_2 = 0$$

$$\Rightarrow A^T Ax.\frac{1}{\| Ax \|_2} = \lambda x$$

$$\Leftrightarrow x^T A^T Ax.\frac{1}{\| Ax \|_2} = \lambda x^T x$$

$$\Leftrightarrow \frac{\| Ax \|_2^2}{\| Ax \|_2} = \lambda \| x \|_2^2 = \lambda$$

$$\Leftrightarrow \| Ax \| = \lambda$$

We also have:

$$\frac{A^T Ax}{\| Ax \|_2} = \lambda x \Leftrightarrow A^T Ax = \lambda^2 x$$

=¿ λ is singular value of A