Week 07

Duong Thu Phuong

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1 EX1:

$$a, f(x) = log(x^4)sin(x^3)$$

$$\Rightarrow f'(x) = \frac{4x^3}{x^4}sin(x^3) + log(x^4).3x^2.cos(x^3) = \frac{4sin(x^3)}{x} + 3x^2log(x^4)cos(x^3)$$

$$b, f(x) = \frac{1}{1+e^{-x}}$$

$$\Rightarrow f'(x) = \frac{e^{-x}}{(1+e^{-x})^2}$$

$$c, f(x) = e^{-\frac{1}{2\delta^2}(x-\mu)^2}$$

$$\Rightarrow f'(x) = -\frac{x-\mu}{2\delta^2}f(x)$$

2 EX2:

a, f(x) = a$^T Xb$

We have:

$$a^T Xb = \sum_{j=1}^{n}\sum_{i=1}^{n} a_j x_{ij} b_i$$

$$\Rightarrow \frac{\delta f}{\delta x} = a_j b_i$$

b, f(x) = x$^T Ax$

We have:

$$x^t Ax = \begin{bmatrix} x_1 & x_2 & x_3 & .... & x_n \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & ... & a_{1n} \\ ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ ... \\ x_n \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=1}^{n} a_{i1}x_i ...... \sum_{i=1}^{n} a_{in}x_i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ ... \\ x_n \end{bmatrix}$$

$$= \sum_{j=1}^{n} x_j \sum_{i=1}^{n} a_{ij}x_i$$

$$= \sum_{j=1}^{n} \sum_{i=1}^{n} a_{ij}x_i x_j$$

$$\frac{\delta f}{\delta x} = \begin{bmatrix} \frac{\delta f}{\delta x_1} & \frac{\delta f}{x_2} & ... & \frac{\delta f}{\delta x_n} \end{bmatrix}$$

$$= 2x_1a_{11} + a_{21}x_2 + ... + a_{n1}x_n + a_{12}x_2 + ... + a_{1n}x_n = \sum_{i=1}^{n} x_i a_{i1} + \sum_{i=1}^{n} x_i a_{1i}$$

$$= x^T A + x^T A^T = x^T (A + A^T)$$

c, f(x) = (Ax-b)$^T(Ax - b)$

We have:

$$(Ax - b)^T (Ax - b) = ((Ax)^T - b^T)(Ax - b) = (x^T A^T - b^T)(Ax - b)$$

$$= x^T A^T Ax - x^T A^T b - b^T Ax + b^T b = x^T A^T Ax - 2b^T Ax + b^T b$$

$$\frac{\delta f}{\delta x} = x^T (A^T A + A^T A) - 2b^T A = 2x^T A^T A - 2b^T A$$

3 Ex3:

$$a, f(x) = e^x \ \ with \ \ x = 0$$

$$f^{(1)}(x) = e^x \Rightarrow f^{(1)}(0) = 1$$

$$f^{(2)}(x) = e^x \Rightarrow f^{(2)}(0) = 1$$

$$f^{(3)}(x) = e^x \Rightarrow f^{(3)}(0) = 1$$

....

$$f^{(n)}(x) = e^x \Rightarrow f^{(n)}(0) = 1$$
$$\Rightarrow f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

$$b, f(x) = \frac{1}{1+x} \ \ with \ \ x = -2$$

We have:

$$f^{(1)}(x) = \frac{-1}{(1+x)^2} \Rightarrow f^{(1)}(-2) = -1$$
$$f^{(2)}(x) = \frac{2}{(1+x)^3} \Rightarrow f^{(2)}(-2) = -2$$
$$f^{(3)}(x) = \frac{-2.3}{(1+x)^4} \Rightarrow f^{(3)}(-2) = -2.3$$
$$f^{(4)}(x) = \frac{2.3.4}{(1+x)^5} \Rightarrow f^{(4)}(-2) = 2.3.4$$
$$.............$$
$$f^{(n)}(x) = \frac{n!(-1)^n}{(1+x)^{n+1}} \Rightarrow f^{(n)}(-2) = \frac{n!(-1)^n}{(-1)^{n+1}}$$
$$\Rightarrow f(x) = \sum_{n=0}^{infty} (-1)^n (x+2)^n$$

$$c, f(x) = tanh(x) with x = 0$$

We have:

$$tanh(x) = \frac{sinh(x)}{cosh(x)}$$
$$(sinh(x))' = cosh(x) \Rightarrow cosh(0) = 1$$
$$(cosh(x))' = sinh(x) \Rightarrow sinh(x) = 0$$

*) sinh(x)

$$f^{(1)}(x) = cosh(x) \Rightarrow f^1(0) = 1$$
$$f^{(2)}(x) = sinh(x) \Rightarrow f^2(0) = 0$$
$$f^{(3)}(x) = cosh(x) \Rightarrow f^3(0) = 1$$
$$.............$$

$$\Rightarrow sinh(X) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + ...$$

Similar, we have:

$$cosh(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + ...$$

$$\Rightarrow tanh(x) = \frac{x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + ...}{1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + ...}$$

$$= x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{135} + ...$$

4 Ex4:

$$a, \frac{df}{dx} = \frac{df}{dz}.\frac{dz}{dy}.\frac{dy}{dx}$$

We have:

$$\frac{df}{dz} \ has \ dimension \ 1x1$$

$$\frac{dz}{dy} \ has \ dimension \ 1xD$$

$$\frac{dy}{dx} \ has \ dimension \ DxD$$

$$\Rightarrow \frac{df}{dx} \ has \ demension \ 1xD$$

$$b, We \ have:$$

$$xx^T + \delta^2 I = \begin{bmatrix} x_1 \\ X_2 \\ ... \\ x_D \end{bmatrix} \begin{bmatrix} x_1 & x_2 & ... & x_D \end{bmatrix} + \begin{bmatrix} \delta^2 & 0 & 0 & ... \\ 0 & \delta^2 & 0 & ... \\ 0 & 0 & \delta^2 & ... \\ ... & ... & ... & ... \\ 0 & 0 & ... & \delta^2 \end{bmatrix}$$

$$= \begin{bmatrix} x_1^2 + \delta^2 & x_1 x_2 & ... & x_1 x_D \\ x_2 x_x 1 & x_2^2 + \delta^2 & ... & x_2 x_D \\ ... & ... & ... & ... \\ x_D x_1 & ... & ... & x_D^2 + \delta^2 \end{bmatrix}$$

$$\Rightarrow tr(xx^T + \delta^2 I) = x_1^2 + x_2^2 + ...x_D^2 + D\delta^2$$

$$\Rightarrow \frac{df}{dx} = \begin{bmatrix} 2x_1 \\ 2x_2 \\ ... \\ 2x_D \end{bmatrix} = 2x^T$$

$$c, \frac{df}{dx} = \frac{df}{dz}.\frac{dz}{dx}$$

$$\frac{df}{dz} \ has \ dimension \ 1xM$$

Because:

$$A \ has \ dimension \ MxN$$

$$x \ has \ dimension \ Nx1$$

$$b \ has \ dimension \ 1xM$$

$$\Rightarrow Ax + b \ has \ dimension \ MxM$$

$$\Rightarrow \frac{dz}{dx} \ has \ dimension \ MxM$$

$$\Rightarrow \frac{df}{dx} \ has \ demension \ 1xM$$