

Week - 06

Ex 03:

$$A = \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix} \rightarrow A \cdot A' = \begin{bmatrix} 8 & 0 \\ 0 & 2 \end{bmatrix}$$

We have: $|AA' - \lambda I| = 0$

$$\Rightarrow \begin{cases} \lambda = 2 \\ \lambda = 8 \end{cases}$$

With $\lambda = 2$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow \text{length} = 1$

$$\rightarrow u_1 = (0, 1)$$

$$\lambda = 8 \quad \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow u_2 = (1, 0)$$

$$\Sigma = \begin{bmatrix} 2\sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$V_i = \frac{1}{6} A^T u_i$$

$$\rightarrow V = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

Ex 04:

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$

$$A_1 = 6 \cdot u_1 \cdot v_1^T = 5 \cdot \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$= \frac{5}{2} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$A_2 = 6 \cdot u_2 \cdot v_2^T = 3 \cdot \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{3\sqrt{2}} & \frac{1}{3\sqrt{2}} & \frac{4}{3\sqrt{2}} \end{bmatrix}$$

$$= 3 \begin{pmatrix} 1/6 & -1/6 & -2/3 \\ -1/6 & 1/6 & 2/3 \end{pmatrix}$$

$$\rightarrow \text{Rank } A_1 = \text{Rank } A_2 = 1 \Rightarrow A = A_1 + A_2$$

Ex 02:

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 17 & 8 \\ 8 & 17 \end{bmatrix}$$

We have $(AA' - \lambda I) = 0$

$$\rightarrow \begin{cases} \lambda = 9 \\ \lambda = 25 \end{cases}$$

With $\lambda = 9$

$$v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \rightarrow \text{length} = \sqrt{2}$$

$$\rightarrow u_1 = \left(-\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \right)$$

$$\lambda = 25 \rightarrow v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow \text{length} = \sqrt{2}$$

$$\rightarrow u_2 = \left(\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \right)$$

$$A = U \Sigma V^T$$

$$\Sigma = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix} \quad U = [u_1 \ u_2] = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$v_j = \frac{1}{\sigma_j} A^T u \rightarrow V = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{18}} & -\frac{2}{3} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{18}} & \frac{2}{3} \\ 0 & \frac{4}{3\sqrt{2}} & \frac{1}{3} \end{bmatrix}$$