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Fast, Scalable Phrase-Based SMT Decoding

Anonymous ACL submission

Abstract

The utilization of statistical machine translation (SMT) has grown enormously over the last decade, many using open-source software developed by the NLP community. As commercial utilization has increased, there has been a pressing need that is optimized for their requirements. Specifically, faster phrase-based decoding, and more efficient utilization of modern multicore servers.

We present in this paper a re-assessment of the major components of phrase-based decoding and decoder implementation with particular emphasis on speed and scalability to multicore machines. The result is a drop-in replacement for the Moses decoder which is up to fifteen times faster and scales almost linearly with the number of cores. Furthermore, the decoder makes less search errors than the current Moses decoder.

1 Introduction

SMT has been one of the outstanding success story from the NLP community in the last decade, progressing from a mostly research discipline to public useability via services such as Google Translate, Microsoft Translator Hub, as well as services and products built around offline products such as Language Weaver and the open-source Moses toolkit. The latter has spawned a cottage industry encompassing a range of organizations and services from small language service providers seeking to reduce translation cost, to large intergovermental organizations such as the EU and the UN that require high volume, high quality translation.

For high volume users, decoding is a largest and most critical part of the translation process which needs to be fast and efficient. However, it has been noticed that the Moses decoder, amongst others, is unable to efficiently use multiple CPU cores that are now common on modern servers (reviewed paper, github discussion). That is, the time taken to decode a test set does not substantial decrease when more cores are used, in fact, decoding time may increase when more cores are added. The problem will continue to grow as the commercial use of SMT increases and the number of CPU cores increases.

There have be speculation on the causes of the inefficiency as well as potential remedies. This paper is the first we know of that seeks to tackle this problem head on. We present an phrase-based decoder that is not only significantly faster than the Moses baseline for single-threaded operation, but is able to scale run multiple threads on multicore machines with only a slightly loss in linear speed. Model scores and functionality are compatible with Moses to aid comparison and ease of transition for users. All source code will be made available under an open-source license.

1.1 Prior Work

There are a number of open-source SMT projects, most includes a decoder. The most well known is Moses, which supports phrase-based models, hierarchical phrase-based as well as various syntax-based models. Joshua also supports hiearchical and syntax models and has recently supported phrase-based models. Phrasal supports a number of variants of the phrase-based model. CDEC supports hierarchical and syntactic models.

A number of the decoders support multithreading whilst others use alternative methods such as Hadoop or external scripts to parallelize decoding. We shall investigate the efficiency of using

parallelizing decoding using the multi-processor approach. None of the decoder focus on multi-threads decoding.

(Recently reviewed) describes running multiple processes of the Moses decoder for increased speed.

Other prior work look to optimizing specific components of decoding. (Liang and Chiang) describes the cube-pruning and cube-growing algorithm for decoding which allows the tradeoff between speed and translation quality to the adjusted with a simgle parameter. (KenLM) and (DALM) describes fast, efficient datastructures for language models. (Zen) describes an implementation of a phrase-table for an SMT decoder that is loaded on demand, reducing the initial loading time and memory requirements. (CompactPT) extends this by compressing the on-disk phrase table and lexicalized re-ordering model resulting in impressive speed gains over previous work.

(mtplz) is perhaps closest in intent to this work. This takes a wholistic approach to decoding, describing a novel decoding algorithm which is fis focused on better decoding speed. It also describes a number of implementation details for faster decoding. However, the decoding algorithm is only able to incorporate one stateful feature function which precludes some of the useful decoding configurations which contains multiple stateful feature functions. It does not include a load-on-demand phrase table, therefore, cannot be used in a commercial environment where phrase-table has not be filtered with a know test set for any realistic size phrase-table. Neither did this paper analyze the scalability of their work to multicore servers.

The rest of the paper will be broken up into the following sections. Next, we will describe the phrase-based model and the major implementation components, with particular emphasis on decoding time shortcomings. We will then describe modifications to improve decoding speed and present results. We conclude in the last section discuss suggested improvements and future work.

2 Phrase-Based Model

The objective of decoding is to find the target translation with the maximum probability, given a source sentence. That is, for a source sentence s, the objective is to find a target translation \hat{t} which has the highest conditional probabil-

ity p(t|s). Mathematically, this is written as:

$$\hat{t} = \arg\max_{t} p(t|s) \tag{1}$$

where the *arg max* function is the search. The log-linear model generalizes Equation 1 to include more component models and weighting each model according to the contribution of each model to the total probability.

$$p(t|s) = \frac{1}{Z} \exp(\sum_{m} \lambda_{m} h_{m}(t,s))$$
 (2)

where λ_m is the weight, and h_m is the feature function, or 'score', for model m. Z is the partition function which can be ignored for optimization.

2.1 Beam Search

A translation of a source sentence is created by applying a series of translation rules which together translate each source word once, and only once. Each partial translation is called a *hypothesis*, which is created by applying a rule to an existing hypothesis. This process is called *hypothesis expansion* and starts with a hypothesis that has translated no source word and ends with a completed hypothesis that has translated all source words. The highest-scoring completed hypothesis, according to the model score, is returned as most probable translation, \hat{t} . Incomplete hypotheses are referred to as partial hypotheses.

Each rule translates a contiguous sequence of source words but successive translation options do not have to be adjacent on the source side, depending on the distortion limit. However, the target output is constructed strictly left-to-right from the target string of successive translation options. Therefore, successive translation options which are not adjacent and monotonic in the source causes translation reordering.

A beam search algorithm is used to create the completed hypothesis set efficiently. Partial hypotheses are organized into stacks where each stack holds a number of comparable hypotheses. Hypotheses in the same stack have the same coverage cardinality |C|, where C is the coverage set, $C \subseteq \{1,2,...|s|\}$ of the number of source words translated. Therefore, |s|+1 number of stacks are created for the decoding of a sentence s.

There are three main optimization to the search that we shall investigate. Firstly, the search creates and destroy a large number of hypothesis objects in memory which puts a heavy burden on the operating system. We shall optimize the search algorithm to use memory pools and object pools, replacing the operating system's general purpose memory management with our own application-aware management.

The speed of memory access is dependent on whether the data is in the CPU cache which is a constrained resource compared to memory size, typically 20MB in the latest processors. We shall seek to re-use recently accessed information to increase likelihood of the data being in the CPU cache.

In multiprocessor servers, the CPU cache is attached to each processor and each core. If a sentences is being decoded on one CPU is switched to another, the CPU cache on the new CPU must be repopulated, slowing down decoding. We will therefore investigate binding threads to specific cores.

Lastly, we shall investigate different stack configurations other than coverage cardinality to see whether they can improve speed and translation quality.

2.2 Feature Functions

Features functions are the h_m in Equation 2, calculating a score for each hypothesis.

The standard feature functions in the phrasebased model include:

- 1. log transforms translation model probabilities, $p_{TM}(t|s)$ and $p_{TM}(s|t)$, and wordbased translation probabilities $p_w(t|s)$ and $p_w(s|t)$,
- 2. log transforms of the lexicalized re-ordering probabilities,
- 3. log transforms of the target language model probability p(t),
- 4. a distortion penalty
- 5. a phrase-penalty,
- 6. a word penalty,
- 7. an unknown word penalty.

The first three feature functions frequently trained on data and require the feature to read the model from files. The other feature functions do not require model files. We shall investigate the first two feature functions for optimization.

2.3 Translation Model

Load-on-demand 'binary' phrase-tables are often used for MT deployment due to their fast loading and querying speed, and because they can be used with large phrase-tables. We therefore focus on optimizing decoding speed with these phrase-tables, specifically the Probing PT.

We shall look at the caching strategies to reduce the number of phrase-table lookups. We shall also investigate the datastructures used by the phrasetable and their impact on decoding speed.

2.4 Lexicalized Reordering Model

The lexicalized reordering model is trained on parallel data, usually requiring random lookups of the model file during decoding. However, the key to the lookup are the source and target phrase of each translation rule. We shall investigate the advantages of storing the mode data within the translation rule.

3 Experimental Setup

We trained a phrase-based system using the Moses toolkit with standard settings. The training data consisted of most of the publicly available Arabic-English data from Opus (Jrg Tiedemann, 2012,) containing over 69 million parallel sentences, and tuned on a held out set. The phrase-table was then pruned, keeping only the top 100 entries per source phrase, according to p(t|s). All models files were then binarized; the language models were binaized using KenLM (???), the phrase table using Probing PT (???), lexicalized reordering model using the compact datastructure described in ???. These binary formats were choosen for their best-of-class multithreaded performance. Table 1 gives details of the resultant sizes of the model files. For verification, we also used a second system trained on the French-English Europarl corpus. For testing decoding speed, we used

	ar-en	fr-en
Phrase table	17	5.8
Language model	3.1	1.8
Lex-re model	2.3	637MB

Table 1: Model sizes in GB

a subset of the training data, Table 2. The two test set have differing characterics that we are interested in analyzing, ar-en have short sentences while fr-en have overly long sentences.

	ar-en	fr-en
# sentences	800k	200k
# words	5.8m	5.9m
Avg words/sent	7.3	29.7

Table 2: Test sets

Where we need to compare the model score of the algorithms, we used a held out set; ??? for ar-en and ??? for fr-en.

Standard Moses phrase-based configurations are used, except that we use the cube-pruning algorithm (???) with a pop-limit of 400, rather than the basic phrase-based algorithm. The cube-pruning algorithm is often employ by users who require fast decoding as it gives them the ability to trade speed with translation quality with a simple pop-limit parameter.

As a baseline, we use the latest version of the Moses decoder taken from the github repository.

For all experiments, we used a Dell PowerEdge R620 server with 16 cores, 32 hyper-threads, split over 2 physical processors (Intel Xeon E5-2650 @ 2.00GHz). The server has 380GB RAM. The operating system was Ubuntu 14.04, the code was compiled with gcc 4.8.4 and Boost library 1.59.

4 Results

4.1 Optimizing Memory

We create a dynamic memory pool which can grow as more memory is requested. The memory is not released, instead the pool can be reset in order for the memory to be re-used. We instantiate two pools for each thread, one which is never reset and another which is reset after the decoding each sentence. Objects are created in either pool according to their life cycle.

For critical objects with high churn such as the hypotheses, thread-specific LIFO queues are used to recycle objects which are no longer used. This not only reduces memory wastage but re-uses recent objects which are likely to be in the CPU cache.

Over 24% of the Moses decoder running time is spent on memory management and this increases when more threads are used, Table 3, braking the scaling of the decoder to many cores. By contrast, our decoder spends 11% on memory management and does not significantly increase with more cores.

	Moses		Our Work	
# threads	1	32	1	32
Memory	24	39	11	13
LM	12	2	47	38
Phrase-table	9	5	2	4
Lex RO	8	2	2	2
Search	2	0	14	19
Misc/Unknown	45	39	24	29

Table 3: Profile of %age decoding time

4.2 Stack Configuration

The most popular stack configuration for phrase-based model, as implemented in Pharaoh, Moses and Joshua, has been by coverage cardinality, ie. hypotheses that have translated the same number of source words are stored in the same stack. There have been research into other stack layouts such as (Ortiz-Martínez et al., 2006), and it has been noted that the decoder in (Brown et al., 1993) uses coverage stacks, as opposed to coverage cardinality.

We also note that distortion limit which constrains hypothesis extension is dependent on the hypothesis' coverage vector, C and the end position of most recent source word that has been translated, e. The distortion limit must be checked for every instance of a hypothesis and translation rule, Figure 1. However, by separating hypothe-

```
for all hypo in stack_{|C|} do

for all translation rules do

if can-expand(C(hypo), e(hypo), translation rule range) then

expand hypo with translation rule \rightarrow new hypo

add new hypo into next stack

end if

end for

end for
```

Figure 1: Hypothesis Expansion with Unsorted Stack

ses into 'ministacks' according to coverage and end position, the distortion limit only needs to be checked for each ministack, Figure 2. We therefore looked at the effects of three stack configurations.

- 1. coverage cardinality,
- 2. coverage,

```
for all ministack_{C,e} in stack_{|C|} do

for all translation rules do

if can-expand(C, e, translation rule range)

then

for all hypo in ministack_{C,e} do

expand hypo with translation rule \rightarrow

new hypo

add new hypo into next stack

end for

end if

end for

end for
```

Figure 2: Hypothesis Expansion with Sorted Stack

3. coverage and end position of most recent translated source word.

5 BLAH BLAH

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Command	Output
{\ " a}	ä
{\^e}	ê
{\ ` i}	ì
{\.I}	İ
{\0}	ø
{\'u}	ø ú
{\aa}	å

Command	Output
{\c c}	ç
{\u g}	ç ğ
{\1}	ł
{\~n}	ñ
{\H 0}	ő
{\v r}	ř
{\ss}	ß

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abstract text	10 pt	
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Table 5: Font guide.

breviations for conferences¹ and journals².

Appendices: Appendices, if any, directly follow the text and the references (but see above). Letter them in sequence and provide an informative title: **Appendix A. Title of Appendix**.

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