A **potential field** in geophysics refers to a field that can be described as the gradient of a scalar potential function. This means that the field is derived from a potential energy function and satisfies certain mathematical conditions, such as Laplace's or Poisson's equation. Potential fields are typically conservative, meaning the work done moving a particle in the field depends only on the starting and ending points, not the path taken.

Key Characteristics of Potential Fields:

1. **Scalar Potential**: The potential field is associated with a scalar potential function Φ, such that the field $\mathbf{F}$ can be expressed as the gradient of Φ:

$$\mathbf{F} = -\nabla\Phi$$

   This implies that the field is conservative.
2. **Conservative Field**: In a conservative field, the circulation of the field along a closed path is zero, meaning there is no net energy gain or loss when moving in a loop within the field.
3. **Laplace's and Poisson's Equations**: Potential fields often satisfy Laplace's equation ($\nabla^2\Phi = 0$) in free space, or Poisson's equation ($\nabla^2\Phi = \rho$) when sources are present. Here, ρ represents a distribution of sources, such as mass in gravitational fields or charge in electric fields.
4. **Inverse-Square Law**: In many cases, potential fields (like gravitational and magnetic fields) follow an inverse-square law, meaning the field strength decreases with the square of the distance from the source.

Types of Potential Fields in Geophysics:

- **Gravitational Field**: The Earth's gravitational field is a potential field governed by Newton's law of gravitation. The potential function for gravity, Φ_g, relates to the gravitational acceleration $\mathbf{g}$ through $\mathbf{g} = -\nabla\Phi_g$.
- **Magnetic Field**: The Earth's magnetic field can be considered a potential field in regions where no electric currents are present. The magnetic potential Φ_m describes the magnetic field $\mathbf{B}$ in such regions, with $\mathbf{B} = -\nabla\Phi_m$.
- **Electric Field**: In electrostatics, the electric field $\mathbf{E}$ is a potential field, derived from an electric potential Φ_e as $\mathbf{E} = -\nabla\Phi_e$.

Potential Fields in Geophysical Surveys:

- In **gravity surveys**, the measured quantity is related to the gradient of the gravitational potential field, helping to map variations in subsurface density.

* In **magnetic surveys**, the magnetic potential field is measured to infer the distribution of magnetic minerals and geological structures.

Example:

In gravitational potential theory, if we consider a point mass M, the gravitational potential at a distance r from the mass is:

$$\Phi_g = -\frac{GM}{r}$$

where G is the gravitational constant. The gravitational field is the gradient of this potential:

$$\mathbf{g} = -\nabla \Phi_g = -\frac{GM}{r^2}\hat{\mathbf{r}}$$

This is a typical inverse-square law potential field, common in many geophysical applications.

The magnetic force of a dipole decreases as you move away from the dipole, and its dependence on distance follows an inverse power law. The expression for the magnetic field of a dipole at a distance r in a vacuum can be written as:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi r^3} \left[3(\mathbf{m} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m} \right]$$

where:

* $\mathbf{B}(\mathbf{r})$ is the magnetic field at position $\mathbf{r}$,
* μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ T $\cdot$ m/A),
* $\mathbf{m}$ is the magnetic dipole moment,
* $\hat{\mathbf{r}}$ is the unit vector in the direction of $\mathbf{r}$,
* r is the distance from the dipole.

Simplified form for magnetic field along the axis of the dipole:

If you're considering the field along the dipole's axis (assuming $\theta = 0$, where θ is the angle between $\mathbf{m}$ and $\mathbf{r}$):

$$B_{\text{axis}}(r) = \frac{\mu_0}{2\pi} \frac{m}{r^3}$$

This expression shows that the magnetic field decreases as the cube of the distance r.

To determine whether a function of time and/or space is a **potential** (i.e., a scalar potential function from which a vector field can be derived), it must satisfy certain mathematical conditions, which depend on whether the function is static or dynamic. Here are the key criteria and steps to test whether a given function can be considered a potential:

1. For Static Fields (Time-independent):

In cases where the field does not vary with time (e.g., gravitational, electrostatic, or magnetostatic fields), we typically use the following tests:

a. Conservative Field Condition

- A vector field $\mathbf{F}$ is conservative if it can be expressed as the gradient of a scalar potential function Φ, such that $\mathbf{F} = -\nabla\Phi$.
- The field is conservative (and hence has a potential) if its **curl is zero**:

$$\nabla \times \mathbf{F} = 0$$

This is equivalent to the statement that the field has no rotational or circulatory components. In this case, the function Φ is the scalar potential.
If $\nabla \times \mathbf{F} \neq 0$, the field is not conservative and does not have a scalar potential function in a simple form.

b. Path Independence

- If the work done by a vector field $\mathbf{F}$ along any path between two points is independent of the path taken, the field is conservative, meaning it can be derived from a potential function.
- Mathematically, for any two points $\mathbf{r}_1$ and $\mathbf{r}_2$, the work done is:

$$W = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F} \cdot d\mathbf{r}$$

If this integral depends only on the endpoints (and not the path), $\mathbf{F}$ is a potential field.

c. Laplace's or Poisson's Equation

- If the potential function Φ satisfies **Laplace's equation**:

$$\nabla^2 \Phi = 0$$

(in regions without sources) or **Poisson's equation**:

$$\nabla^2 \Phi = \rho$$

(in regions with a source distribution ρ, such as charge or mass density), then Φ is a valid potential function.

d. Single-Valuedness

- For the function Φ to be a potential, it must be **single-valued**, meaning it returns the same value for the same spatial coordinates. Functions that are multi-valued (e.g., θ in cylindrical coordinates) are not valid scalar potentials.

2. For Dynamic Fields (Time-dependent):

In cases where the field varies with time, such as electromagnetic fields or other dynamic systems, the concept of a potential becomes more complex. Time-varying fields might still have potentials, but additional conditions are needed.

a. Gauge Potentials in Electromagnetism

- For time-varying electromagnetic fields, a scalar potential Φ and a vector potential $\mathbf{A}$ can be used. The electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ are derived from these potentials as:

$$\mathbf{E} = -\nabla \Phi - \frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

These potentials are subject to gauge conditions (such as the Lorenz or Coulomb gauge), but they do not strictly define a scalar potential in the same way as in static fields.

b. Helmholtz Decomposition

- Any vector field can be decomposed into a gradient of a scalar potential and a curl of a vector potential:

$$\mathbf{F} = -\nabla \Phi + \nabla \times \mathbf{A}$$

The scalar part $-\nabla\Phi$ represents the conservative (irrotational) part of the field, while the curl term $\nabla \times \mathbf{A}$ accounts for the rotational components.

3. Example: Gravitational and Electrostatic Fields

- **Gravitational potential**: For the gravitational field $\mathbf{g}$, the potential Φ_g is given by:

$$\mathbf{g} = -\nabla\Phi_g$$

If $\mathbf{g}$ satisfies $\nabla \times \mathbf{g} = 0$, then it can be expressed as the gradient of a gravitational potential.
- **Electrostatic potential**: In electrostatics, the electric field $\mathbf{E}$ can be derived from the electrostatic potential Φ_e:

$$\mathbf{E} = -\nabla\Phi_e$$

If $\mathbf{E}$ satisfies $\nabla \times \mathbf{E} = 0$, then Φ_e is a valid scalar potential.

Summary of Conditions for a Potential:

- **Static Fields**:
  - The field must be conservative ($\nabla \times \mathbf{F} = 0$).
  - Work must be path-independent.
  - The potential must satisfy Laplace's or Poisson's equation.
  - The potential must be single-valued.
- **Dynamic Fields**:
  - For time-varying fields, potentials can exist but may require additional vector potentials and gauge conditions (e.g., in electromagnetism).

For practical geophysical applications, this formula is useful in interpreting magnetic anomalies and understanding the behavior of magnetic sources at different distances, such as in magnetic surveying. In geophysical magnetic methods, the primary property measured is **the Earth's magnetic field and its variations**. Specifically, these methods are used to measure the **magnetic field intensity**, which can provide information about the subsurface distribution of magnetized materials (such as rocks and minerals). The key quantities typically measured include:

1. Magnetic Field Intensity (Total Magnetic Field)

- This is the strength of the magnetic field at a specific location, often measured in **nanoteslas (nT)**.
- Variations in the magnetic field intensity can indicate the presence of magnetized rocks, such as iron-bearing minerals, which cause local anomalies in the Earth's magnetic field.

2. Magnetic Anomalies

- These are deviations from the normal (background) magnetic field of the Earth. Magnetic anomalies are caused by differences in the magnetization of subsurface rocks.
- By studying these anomalies, geophysicists can infer the presence of different rock types, structures, and even mineral deposits.

3. Magnetic Susceptibility

- Although not directly measured in the field, magnetic susceptibility is a key parameter that can be derived from magnetic data. It refers to the degree to which a material can be magnetized in an external magnetic field.
- Rocks with higher magnetic susceptibility (such as basalt or iron-rich rocks) will have a stronger influence on the local magnetic field compared to rocks with low susceptibility (such as limestone or granite).

4. Magnetic Declination and Inclination

- **Declination** is the angle between the geographic north and the direction to which a compass needle points (magnetic north).
- **Inclination** is the angle of the magnetic field relative to the horizontal plane. These components help in interpreting the orientation of magnetic sources.

Uses of Magnetic Methods:

- **Mineral Exploration**: Detecting ferrous minerals, such as iron ore, nickel, or even gold in some cases.
- **Geological Mapping**: Understanding the distribution and structure of different rock units.
- **Archaeological Surveys**: Identifying buried structures through magnetic anomalies.
- **Environmental and Engineering Investigations**: Locating buried ferrous objects or determining soil magnetization.

The magnetic methods are non-invasive, widely used, and effective in providing clues about the Earth's subsurface structure and composition.

In the Earth's subsurface, different types of magnetism arise due to the presence of various minerals and materials. The types of magnetism commonly found in the Earth are:

1. Diamagnetism

- **Description**: Diamagnetic materials create an induced magnetic field that is weak and in the opposite direction of the applied magnetic field.
- **Properties**: These materials have a very weak negative susceptibility to magnetic fields, meaning they are repelled by both poles of a magnet.
- **Example minerals**: Quartz, calcite, and water.
- **Geophysical importance**: Diamagnetic materials generally have little influence on geophysical magnetic surveys because their magnetic susceptibility is so small.

2. Paramagnetism

- **Description**: Paramagnetic materials have unpaired electrons, which align with an external magnetic field, causing a weak attraction.
- **Properties**: These materials have a small positive magnetic susceptibility, and they enhance the local magnetic field slightly.
- **Example minerals**: Olivine, biotite, and pyroxene.
- **Geophysical importance**: Paramagnetic materials contribute weakly to magnetic anomalies and usually do not dominate geophysical magnetic signals.

3. Ferromagnetism

- **Description**: Ferromagnetic materials exhibit strong magnetization in the presence of a magnetic field and can retain magnetization even after the external field is removed.
- **Properties**: These materials have very high positive magnetic susceptibility and can form permanent magnets.
- **Example minerals**: Pure iron, nickel, cobalt (though these are rare in nature in their pure forms).
- **Geophysical importance**: Ferromagnetic materials are rare in Earth's crust, so they are not the primary contributors to magnetic anomalies in geological settings. However, small amounts may be found in some rocks.

4. Ferrimagnetism

- **Description**: Ferrimagnetic materials are similar to ferromagnetic materials but have unequal magnetic moments aligned in opposite directions, leading to a net magnetization.
- **Properties**: Strong magnetic susceptibility, and these materials can retain magnetization after the external field is removed.
- **Example minerals**: Magnetite and titanomagnetite.
- **Geophysical importance**: Ferrimagnetic minerals like magnetite are among the primary sources of magnetic anomalies in the Earth's crust, and they are commonly found in igneous and metamorphic rocks.

5. Antiferromagnetism

- **Description**: In antiferromagnetic materials, magnetic moments of atoms or ions align in opposite directions and cancel each other out, resulting in no net magnetization.
- **Properties**: These materials exhibit weak magnetic susceptibility, and they do not retain magnetization after the external magnetic field is removed.
- **Example minerals**: Hematite, ilmenite.
- **Geophysical importance**: While hematite is antiferromagnetic, it can still contribute to magnetic anomalies in certain situations, especially when present in high concentrations.

6. Superparamagnetism

- **Description**: Superparamagnetism occurs in very small particles (typically less than 30 nm) of ferromagnetic or ferrimagnetic materials. These particles exhibit magnetic properties similar to paramagnetic materials, but their magnetization can align with an external field.
- **Properties**: These particles do not retain magnetization after the external field is removed, but they show a strong response in the presence of a field.
- **Example minerals**: Small particles of magnetite or hematite.
- **Geophysical importance**: Superparamagnetic particles can affect the interpretation of magnetic data in certain geological settings, especially in sediments or soils.

Geophysical Implications:

- **Diamagnetism and Paramagnetism**: Typically, these materials contribute little to magnetic anomalies due to their weak magnetic properties.
- **Ferrimagnetism**: Minerals like magnetite are key to understanding the Earth's magnetic field and are often the primary sources of magnetic anomalies in geophysical surveys.

- **Antiferromagnetism**: Minerals like hematite can contribute to magnetic anomalies but are generally less significant than ferrimagnetic minerals.
- **Superparamagnetism**: Important in certain contexts, particularly in fine-grained sediments and soils, where it can complicate magnetic survey interpretations.

The type and distribution of these magnetic properties help geophysicists interpret subsurface geology and locate magnetic anomalies, often associated with specific rock types or mineral deposits.

In magnetic surveys, **magnetic susceptibility** is not directly measured, but instead inferred from the measurements of the total magnetic field and magnetic anomalies. Here's how the process works:

1. Magnetic Anomalies and Susceptibility

Magnetic susceptibility (χ) describes how strongly a material can be magnetized in the presence of an external magnetic field. It contributes to the overall magnetic anomaly, which is the deviation of the measured magnetic field from the Earth's ambient magnetic field.

- **Total Magnetic Field**: In magnetic surveys, the total magnetic field $\mathbf{B}$ at the surface is measured, typically using magnetometers. This field includes contributions from the Earth's main magnetic field, local geological sources, and any induced magnetization in the subsurface.
- **Magnetic Anomalies**: Magnetic susceptibility influences the intensity of magnetic anomalies. These anomalies arise because subsurface materials with different susceptibilities distort the Earth's magnetic field. The anomaly is defined as:

$$\Delta B = B_{\mathrm{measured}} - B_{\mathrm{regional}}$$

  where B_{measured} is the observed total magnetic field, and B_{regional} is the background magnetic field (Earth's main field).

2. Deriving Susceptibility from Magnetic Data

To estimate magnetic susceptibility from the total field measurements, several steps are involved:

a. Magnetic Survey

- A magnetic survey is conducted using **magnetometers** to map the variations in the total magnetic field over an area of interest.
- This data contains information about the magnetic field anomalies caused by subsurface structures and materials.

b. Data Processing

- **Removal of the Regional Field**: The background regional magnetic field (Earth's main magnetic field) is removed using models like the International Geomagnetic Reference Field (IGRF). This isolates the **magnetic anomaly** attributable to local sources.

c. Inverse Modeling

- The next step is to perform **inversion** of the magnetic anomaly data. This process involves using mathematical models to link the observed anomalies to the distribution of magnetic susceptibility in the subsurface.
- Common modeling techniques include:
  - **Forward Modeling**: A hypothetical subsurface model with varying susceptibility is created, and the magnetic field is calculated for that model. The model is adjusted iteratively until it matches the observed anomalies.
  - **Inverse Modeling**: Mathematical techniques are used to directly infer the susceptibility distribution from the observed magnetic anomalies. This involves solving an inverse problem where the susceptibility of rocks at different depths is calculated based on the data.

d. Magnetic Susceptibility Mapping

- The result of inversion modeling is a map of **magnetic susceptibility** in the subsurface. Higher susceptibility corresponds to regions with ferromagnetic or ferrimagnetic minerals (e.g., magnetite), while lower susceptibility corresponds to less magnetized rocks (e.g., quartz or limestone).

3. Measuring Susceptibility in the Field

Although susceptibility itself is inferred from the total magnetic field measurements, there are also direct ways to measure it:

- **Laboratory Measurements**: Rock samples collected from the field can be measured for their magnetic susceptibility in the laboratory using **magnetic susceptibility meters**.
- **In Situ Measurements**: In some cases, magnetic susceptibility can be measured directly in the field using handheld **magnetic susceptibility meters**. These are used on rock outcrops or boreholes to obtain a direct measurement of the magnetic properties of the rocks.

4. Mathematical Relationship

For materials with induced magnetization (which is proportional to the external magnetic field), the relationship between magnetization $\mathbf{M}$ and susceptibility χ is:

$$\mathbf{M} = \chi\mathbf{H}$$

where $\mathbf{H}$ is the external magnetic field strength.

The total magnetic field $\mathbf{B}$ in the vicinity of a magnetic material is then:

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(1 + \chi)\mathbf{H}$$

where μ_0 is the magnetic permeability of free space.

By analyzing the distortion in the magnetic field caused by $\mathbf{M}$, the susceptibility χ can be inferred.

Summary of Measuring Susceptibility:

- **Indirect measurement**: Susceptibility is derived by analyzing magnetic anomalies and performing inverse modeling on the data collected during a magnetic survey.
- **Direct measurement**: Susceptibility can be measured directly using susceptibility meters in the field or laboratory.

The process of estimating susceptibility from magnetic surveys involves both field data collection and computational modeling.

In geophysics, **electric potential methods** are used to study the subsurface by measuring the electrical properties of the Earth. These methods primarily involve injecting current into the ground and measuring the resulting electric potentials at the surface to infer information about subsurface resistivity, conductivity, and structure. Here are the most common electric potential methods:

1. Resistivity Method (Electrical Resistivity Tomography, ERT)

The **resistivity method** is one of the most widely used techniques in geophysical exploration. It involves injecting an electrical current into the ground using a pair of electrodes and measuring the resulting potential difference between other pairs of electrodes.

- **How It Works**: A current is injected into the ground through two current electrodes (C1 and C2), and the resulting voltage difference is measured between two potential electrodes (P1 and P2). The apparent resistivity is calculated using Ohm's law, and the data can be inverted to produce a resistivity model of the subsurface.
- **Applications**:
  - Groundwater exploration.
  - Mapping soil contamination.

- Archaeological investigations.
  - Investigating bedrock depth and soil properties.
- **Types**:
  - **Electrical Resistivity Tomography (ERT)**: A more advanced form of resistivity method that uses multiple electrodes to create a detailed 2D or 3D image of subsurface resistivity.

2. Self-Potential (SP) Method

The **self-potential method** measures naturally occurring electric potentials (voltages) in the Earth, without any external current being injected. These potentials arise due to various natural electrochemical, geothermal, or hydrological processes.

- **How It Works**: Electrodes are placed on the ground surface, and the natural voltage differences between them are measured. The sources of these voltages are typically related to fluid flow, chemical gradients, or mineralization.
- **Applications**:
  - Monitoring groundwater flow and contamination.
  - Locating ore bodies (especially sulfide deposits).
  - Detecting geothermal activity.
  - Tracking fluid movement in volcanic areas.

3. Induced Polarization (IP) Method

The **induced polarization method** is an extension of the resistivity method and measures how the ground "polarizes" after an electrical current is injected. This technique is sensitive to the chargeability of the subsurface materials, which is influenced by the presence of minerals, especially metallic or clayey materials.

- **How It Works**: A current is injected into the ground, and when the current is stopped, the decay of the voltage over time is measured. This decay is related to how the subsurface materials store and release electrical charges.
- **Applications**:
  - Mineral exploration, especially for disseminated sulfide deposits.
  - Environmental studies, such as identifying clays or contaminated areas.
  - Hydrogeological investigations.
- **Types**:
  - **Time-Domain IP (TDIP)**: Measures the decay of the voltage after the current is turned off.
  - **Frequency-Domain IP (FDIP)**: Measures the impedance of the ground at different frequencies.

4. Magnetotellurics (MT)

Magnetotellurics is a passive method that measures the Earth's natural electric and magnetic fields to determine the electrical conductivity of the subsurface.

- **How It Works**: Natural electromagnetic waves, generated by lightning and solar activity, induce electric currents in the Earth. These currents create electric and magnetic fields that can be measured at the surface. The resistivity of the subsurface can be inferred from these measurements.
- **Applications**:
  - Investigating deep Earth structure (e.g., lithosphere and mantle).
  - Geothermal exploration.
  - Oil and gas exploration.
  - Mapping regional tectonic features.
- **Types**:
  - **Audio-Magnetotellurics (AMT)**: Focuses on higher frequencies to study shallow features.
  - **Broadband MT**: Covers a wide frequency range to provide more detailed resistivity models.

5. Transient Electromagnetic (TEM) Method

Also called **time-domain electromagnetics (TDEM)**, this method uses electromagnetic induction to measure the conductivity of the subsurface.

- **How It Works**: A current is passed through a loop of wire, generating a primary magnetic field. When the current is turned off, the changing magnetic field induces secondary currents (eddy currents) in the subsurface. The decay of these currents is measured, providing information about subsurface resistivity.
- **Applications**:
  - Groundwater exploration.
  - Mineral exploration.
  - Environmental contamination studies.
  - Mapping permafrost and ice thickness.

6. Controlled Source Audio-Magnetotellurics (CSAMT)

CSAMT is similar to magnetotellurics but uses an artificial source of electromagnetic waves to measure the resistivity of the subsurface.

- **How It Works**: An artificial current source (typically a grounded wire) generates electromagnetic fields, which induce currents in the ground. By measuring the resulting electric and magnetic

fields at various distances from the source, the resistivity of the subsurface can be inferred.

- **Applications**:
  - Mineral exploration.
  - Groundwater studies.
  - Mapping deep geological structures.

7. Dipole-Dipole Array Method

A specific electrode configuration used in resistivity and IP surveys is the **dipole-dipole array**. It involves using pairs of current and potential electrodes that are moved along the survey line to increase resolution.

- **How It Works**: A current is injected into the ground through two electrodes (current dipole), and the voltage difference is measured between another pair of electrodes (potential dipole). The distance between the pairs can be varied to probe different depths.
- **Applications**: This method is commonly used in **ERT** and **IP** surveys to provide detailed 2D or 3D images of the subsurface resistivity and chargeability.

Summary of Common Electric Potential Methods:

- **Resistivity**: Investigates subsurface resistivity by injecting current and measuring potentials.
- **Self-Potential (SP)**: Measures natural electric potentials generated by electrochemical, geothermal, or fluid flow processes.
- **Induced Polarization (IP)**: Measures how the ground polarizes after current injection, useful for identifying minerals.
- **Magnetotellurics (MT)**: Uses natural electromagnetic fields to study deep structures.
- **Transient Electromagnetics (TEM)**: Measures the decay of secondary currents induced in the ground by a changing magnetic field.
- **Controlled Source AMT (CSAMT)**: Similar to MT but uses an artificial source of electromagnetic fields.

Each method provides insights into different aspects of the subsurface, depending on the specific properties (e.g., resistivity, conductivity, chargeability) they are sensitive to.