Chapter 7 Section 3 Translation Theorems and Derivatives of a Transform - Solutions by Dr. Sam Narimetla, Tennessee Tech

1. $\mathscr{L}\{te^{10t}\}$

   Solution: $\mathscr{L}\{te^{10t}\} = \mathscr{L}\{e^{10t}t\} = [\mathscr{L}\{t\}]_{s\to(s-10)} = \left[\frac{1}{s^2}\right]_{s\to(s-10)} = \boxed{\frac{1}{(s-10)^2}}$

2. $\mathscr{L}\{te^{-6t}\}$

   Solution: $\mathscr{L}\{te^{-6t}\} = \mathscr{L}\{e^{-6t}t\} = [\mathscr{L}\{t\}]_{s\to(s+6)} = \left[\frac{1}{s^2}\right]_{s\to(s+6)} = \boxed{\frac{1}{(s+6)^2}}$

3. $\mathscr{L}\{t^3e^{-2t}\}$

   Solution: $\mathscr{L}\{t^3e^{-2t}\} = \mathscr{L}\{e^{-2t}t^3\} = \left[\mathscr{L}\{t^3\}\right]_{s\to(s+2)} = \left[\frac{3!}{s^4}\right]_{s\to(s+2)} = \boxed{\frac{6}{(s+2)^4}}$

4. $\mathscr{L}\{t^{10}e^{-7t}\}$

   Solution: $\mathscr{L}\{t^{10}e^{-7t}\} = \mathscr{L}\{e^{-7t}t^{10}\} = \left[\mathscr{L}\{t^{10}\}\right]_{s\to(s+7)} = \left[\frac{10!}{s^{11}}\right]_{s\to(s+7)} = \boxed{\frac{10!}{(s+7)^{11}}}$

5. $\mathscr{L}\{e^t \sin 3t\}$

   Solution: $\mathscr{L}\{e^t \sin 3t\} = [\mathscr{L}\{\sin 3t\}]_{s\to(s-1)} = \left[\frac{3}{s^2+9}\right]_{s\to(s-1)} = \boxed{\frac{3}{(s-1)^2+9}}$

6. $\mathscr{L}\{e^{-2t} \cos 4t\}$

   Solution: $\mathscr{L}\{e^{-2t} \cos 4t\} = [\mathscr{L}\{\cos 4t\}]_{s\to(s+2)} = \left[\frac{s}{s^2+16}\right]_{s\to(s+2)} = \boxed{\frac{s+2}{(s+2)^2+16}}$

9. $\mathscr{L}\{t\,(e^t+e^{2t})^2\}$

   Solution: $\mathscr{L}\{t\left(e^t+e^{2t}\right)^2\} = \mathscr{L}\{t\left(e^{2t}+2e^{3t}+e^{4t}\right)\}$

   $= [\mathscr{L}\{t\}]_{s\to(s-2)} + 2\,[\mathscr{L}\{t\}]_{s\to(s-3)} + [\mathscr{L}\{t\}]_{s\to(s-4)} = \left[\frac{1}{s^2}\right]_{s\to(s-2)} + 2\left[\frac{1}{s^2}\right]_{s\to(s-3)} + \left[\frac{1}{s^2}\right]_{s\to(s-4)}$

   $= \boxed{\frac{1}{(s-2)^2} + \frac{2}{(s-3)^2} + \frac{1}{(s-4)^2}}$

10. $\mathscr{L}\{e^{2t}(t-1)^2\}$

    Solution: $\mathscr{L}\{e^{2t}(t-1)^2\} = \mathscr{L}\{e^{2t}(t^2-2t+1)\}$

    $= \left[\mathscr{L}\{t^2\}\right]_{s\to(s-2)} - 2\,[\mathscr{L}\{t\}]_{s\to(s-2)} + [\mathscr{L}\{1\}]_{s\to(s-2)} = \boxed{\frac{2}{(s-2)^3} - \frac{2}{(s-2)^2} + \frac{1}{s-2}}$

11. $\mathscr{L}\{e^{-t}\sin^2 t\}$

    Solution: $\mathscr{L}\{e^{-t}\sin^2 t\} = \mathscr{L}\{e^{-t} \cdot \tfrac{1}{2}(1-\cos 2t)\} = \frac{1}{2}\,[\mathscr{L}\{1-\cos 2t\}]_{s\to(s+1)}$

    $= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+4}\right]_{s\to(s+1)} = \boxed{\frac{1}{2}\left[\frac{1}{(s+1)} - \frac{s+1}{(s+1)^2+4}\right]}$

12. $\mathscr{L}\{e^t \cos^2 3t\}$

Solution: $\mathscr{L}\{e^t \cos^2 3t\} = \mathscr{L}\{e^t \cdot \frac{1}{2}(1+\cos 6t)\} = \dfrac{1}{2}\left[\mathscr{L}\{1+\cos 6t\}\right]_{s\to(s-1)}$

$$= \frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2+36}\right]_{s\to(s-1)} = \boxed{\frac{1}{2}\left[\frac{1}{(s-1)} + \frac{s-1}{(s-1)^2+36}\right]}$$

13. $\mathscr{L}^{-1}\left\{\dfrac{1}{(s+2)^3}\right\}$

Solution: $\mathscr{L}^{-1}\left\{\dfrac{1}{(s+2)^3}\right\} = \mathscr{L}^{-1}\left\{\left[\dfrac{1}{s^3}\right]_{s\to(s+2)}\right\} = e^{-2t}\mathscr{L}^{-1}\left\{\left[\dfrac{1}{s^3}\right]\right\} = \boxed{e^{-2t}\ \dfrac{t^2}{2}}$

14. $\mathscr{L}^{-1}\left\{\dfrac{1}{(s-1)^4}\right\}$

Solution: $\mathscr{L}^{-1}\left\{\dfrac{1}{(s-1)^4}\right\} = \mathscr{L}^{-1}\left\{\left[\dfrac{1}{s^4}\right]_{s\to(s-1)}\right\} = e^{t}\mathscr{L}^{-1}\left\{\left[\dfrac{1}{s^4}\right]\right\} = \boxed{e^{t}\ \dfrac{t^3}{3!}}$

15. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^2-6s+10}\right\}$

Solution: Observe that the denominator cannot be factored in terms of real numbers because the discriminant $b^2-4ac = 36-40 = -4 < 0$. So, we complete the square.

$$\mathscr{L}^{-1}\left\{\frac{1}{s^2-6s+10}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{s^2-6s+\left(\frac{-6}{2}\right)^2-\left(\frac{-6}{2}\right)^2+10}\right\} = \mathscr{L}^{-1}\left\{\left[\frac{1}{(s-3)^2+1^2}\right]\right\}$$

$$= \mathscr{L}^{-1}\left\{\left[\frac{1}{s^2+1^2}\right]_{s\to(s-3)}\right\} = e^{3t}\mathscr{L}^{-1}\left\{\left[\frac{1}{s^2+1^2}\right]\right\} = \boxed{e^{3t}\ \sin t}$$

16. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^2+2s+5}\right\}$

Solution: Observe that the denominator cannot be factored in terms of real numbers because the discriminant $b^2-4ac = 4-20 = -16 < 0$. So, we complete the square.

$$\mathscr{L}^{-1}\left\{\frac{1}{s^2+2s+5}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{s^2+2s+\left(\frac{2}{2}\right)^2-\left(\frac{2}{2}\right)^2+5}\right\} = \mathscr{L}^{-1}\left\{\left[\frac{1}{(s+1)^2+2^2}\right]\right\}$$

$$= \mathscr{L}^{-1}\left\{\left[\frac{1}{s^2+2^2}\right]_{s\to(s+1)}\right\} = e^{-t}\mathscr{L}^{-1}\left\{\left[\frac{1}{s^2+2^2}\right]\right\} = \boxed{e^{-t}\ \frac{1}{2}\ \sin 2t}$$

17. $\mathscr{L}^{-1}\left\{\dfrac{s}{s^2+4s+5}\right\}$

Solution: Observe that the denominator cannot be factored in terms of real numbers because the discriminant $b^2-4ac = 16-20 = -4 < 0$. So, we complete the square.

$$\mathscr{L}^{-1}\left\{\frac{s}{s^2+4s+5}\right\} = \mathscr{L}^{-1}\left\{\frac{s}{s^2+4s+\left(\frac{4}{2}\right)^2-\left(\frac{4}{2}\right)^2+5}\right\} = \mathscr{L}^{-1}\left\{\left[\frac{s}{(s+2)^2+1^2}\right]\right\}$$

$$= \mathscr{L}^{-1}\left\{\left[\frac{(s+2)-2}{(s+2)^2+1^2}\right]\right\} = \mathscr{L}^{-1}\left\{\left[\frac{s}{s^2+1^2}-\frac{2}{s^2+1^2}\right]_{s\to(s+2)}\right\} = e^{-2t}\mathscr{L}^{-1}\left\{\left[\frac{s}{s^2+1^2}-\frac{2}{s^2+1^2}\right]\right\}$$

$$= \boxed{e^{-2t}\ (\cos t - 2\sin t)}$$

18. $\mathscr{L}^{-1}\left\{\dfrac{2s+5}{s^2+6s+34}\right\}$

Solution: Observe that the denominator cannot be factored in terms of real numbers because the discriminant $b^2-4ac = 36-136 = -100 < 0$. So, we complete the square.

$$\mathscr{L}^{-1}\left\{\frac{2s+5}{s^2+6s+34}\right\} = \mathscr{L}^{-1}\left\{\frac{2s+5}{s^2+6s+\left(\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2+34}\right\} = \mathscr{L}^{-1}\left\{\left[\frac{2s+5}{(s+3)^2+5^2}\right]\right\}$$

$$= \mathscr{L}^{-1}\left\{\left[\frac{2(s+3)-1}{(s+3)^2+5^2}\right]\right\} = \mathscr{L}^{-1}\left\{\left[\frac{2s}{s^2+5^2}-\frac{1}{s^2+5^2}\right]_{s\to(s+3)}\right\} = e^{-3t}\mathscr{L}^{-1}\left\{\left[\frac{2s}{s^2+5^2}-\frac{1}{s^2+5^2}\right]\right\}$$

$$= \boxed{e^{-3t}\ \left(2\cos 5t - \frac{1}{5}\sin 5t\right)}$$

19. $\mathscr{L}^{-1}\left\{\dfrac{s}{(s+1)^2}\right\}$

Solution: $\mathscr{L}^{-1}\left\{\dfrac{s}{(s+1)^2}\right\} = \mathscr{L}^{-1}\left\{\dfrac{(s+1)-1}{(s+1)^2}\right\} = \mathscr{L}^{-1}\left\{\left[\dfrac{s}{s^2}-\dfrac{1}{s^2}\right]_{s\to(s+1)}\right\} = e^{-t}\mathscr{L}^{-1}\left\{\left[\dfrac{1}{s}-\dfrac{1}{s^2}\right]\right\}$

$$= \boxed{e^{-t}\ (1-t)}$$

20. $\mathscr{L}^{-1}\left\{\dfrac{5s}{(s-2)^2}\right\}$

Solution: $\mathscr{L}^{-1}\left\{\dfrac{5s}{(s-2)^2}\right\} = \mathscr{L}^{-1}\left\{\dfrac{5(s-2)+10}{(s-2)^2}\right\} = \mathscr{L}^{-1}\left\{\left[\dfrac{5s}{s^2}+\dfrac{10}{s^2}\right]_{s\to(s-2)}\right\} = e^{2t}\mathscr{L}^{-1}\left\{\left[\dfrac{5}{s}+\dfrac{10}{s^2}\right]\right\}$

$$= \boxed{e^{2t}\ (5+10t)}$$

21. $\mathscr{L}^{-1}\left\{\dfrac{2s-1}{s^2(s+1)^3}\right\}$

Solution: There is no simple way to do this. Just do partial fraction decomposition and work it.

$$\mathscr{L}^{-1}\left\{\frac{2s-1}{s^2(s+1)^3}\right\} = \mathscr{L}^{-1}\left\{\frac{5}{s} - \frac{1}{s^2} - \frac{3}{(s+1)^3} - \frac{4}{(s+1)^2} - \frac{5}{(s+1)}\right\} = \boxed{5 - t - e^{-t}\left(\frac{3}{2}\,t^2 + 4t + 5\right)}$$

22. $\mathscr{L}^{-1}\left\{\dfrac{(s+1)^2}{(s+2)^4}\right\}$

Solution: $\mathscr{L}^{-1}\left\{\dfrac{(s+1)^2}{(s+2)^4}\right\} = \mathscr{L}^{-1}\left\{\dfrac{[(s+2)-1]^2}{(s+2)^4}\right\} = \mathscr{L}^{-1}\left\{\dfrac{[(s+2)^2-2(s+2)+1]}{(s+2)^4}\right\}$

$$= \mathscr{L}^{-1}\left\{\frac{1}{(s+2)^2} - \frac{2}{(s+2)^3} + \frac{1}{(s+2)^4}\right\} = \mathscr{L}^{-1}\left\{\left[\frac{1}{s^2} - \frac{2}{s^3} + \frac{1}{s^4}\right]_{s\to(s+2)}\right\}$$

$$= \boxed{e^{-2t}\left(t - t^2 + \frac{t^3}{6}\right)}$$

23. $\mathscr{L}\{(t-1)\mathscr{U}(t-1)\}$

Solution: $\mathscr{L}\{(t-1)\mathscr{U}(t-1)\} = e^{-s}\mathscr{L}\left\{[t-1]_{t\to(t+1)}\right\} = e^{-s}\mathscr{L}\left\{[(t+1)-1]\right\} = e^{-s}\mathscr{L}\left\{t\right\}$

$$= \boxed{e^{-s}\,\frac{1}{s^2}}$$

24. $\mathscr{L}\{e^{2-t}\mathscr{U}(t-2)\}$

Solution: $\mathscr{L}\{e^{2-t}\mathscr{U}(t-2)\} = e^{-2s}\mathscr{L}\left\{\left[e^{2-t}\right]_{t\to(t+2)}\right\} = e^{-2s}\mathscr{L}\left\{\left[e^{2-(t+2)}\right]\right\} = e^{-2s}\mathscr{L}\left\{e^{-t}\right\}$

$$= \boxed{e^{-2s}\,\frac{1}{s+1}}$$

25. $\mathscr{L}\{t\ \mathscr{U}(t-2)\}$

Solution: $\mathscr{L}\{t\ \mathscr{U}(t-2)\} = e^{-2s}\mathscr{L}\left\{[t]_{t\to(t+2)}\right\} = e^{-2s}\mathscr{L}\left\{[t+2]\right\} = \boxed{e^{-2s}\left[\dfrac{1}{s^2} + \dfrac{2}{s}\right]}$

26. $\mathscr{L}\{(3t+1)\mathscr{U}(t-3)\}$

Solution: $\mathscr{L}\{(3t+1)\mathscr{U}(t-3)\} = e^{-3s}\mathscr{L}\left\{[3t+1]_{t\to(t+3)}\right\} = e^{-3s}\mathscr{L}\left\{[3(t+3)+1]\right\} = e^{-3s}\mathscr{L}\left\{3t+10\right\}$

$$= \boxed{e^{-3s}\left[\frac{3}{s^2} + \frac{10}{s}\right]}$$

27. $\mathscr{L}\{\cos 2t\ \mathscr{U}(t-\pi)\}$

Solution: $\mathscr{L}\{\cos 2t\ \ \mathscr{U}(t-\pi)\} = e^{-\pi s}\mathscr{L}\left\{[\cos 2t]_{t\to(t+\pi)}\right\} = e^{-\pi s}\mathscr{L}\left\{[\cos(2t+2\pi)]\right\} = e^{-\pi s}\mathscr{L}\left\{\cos 2t\right\}$

$$= \boxed{e^{-\pi s}\left[\frac{s}{s^2+4}\right]}$$

28. $\mathscr{L}\{\sin t\ \mathscr{U}\left(t-\frac{\pi}{2}\right)\}$

Solution: $\mathscr{L}\{\sin t\ \mathscr{U}\left(t-\frac{\pi}{2}\right)\} = e^{-\pi s/2}\mathscr{L}\left\{[\sin t]_{t\to(t+\pi/2)}\right\} = e^{-\pi s/2}\mathscr{L}\left\{[\sin(t+\pi/2)]\right\}$

$= e^{-\pi s/2}\mathscr{L}\left\{[\sin t\cos\pi/2+\cos t\sin\pi/2]\right\} = e^{-\pi s/2}\mathscr{L}\left\{[\sin t\ (0)+\cos t\ (1)]\right\} = e^{-\pi s/2}\mathscr{L}\left\{[\cos t]\right\}$

$= \boxed{e^{-\pi s/2}\ \left[\frac{s}{s^2+1}\right]}$

29. $\mathscr{L}\{(t-1)^3e^{t-1}\mathscr{U}(t-1)\}$

Solution: $\mathscr{L}\{(t-1)^3e^{t-1}\mathscr{U}(t-1)\} = e^{-s}\mathscr{L}\left\{\left[(t-1)^3e^{t-1}\right]_{t\to(t+1)}\right\} = e^{-s}\mathscr{L}\left\{\left[t^3\ e^t\right]\right\}$

$= e^{-s}\left[\mathscr{L}\left\{t^3\right\}\right]_{s\to(s-1)} = e^{-s}\left[\frac{3!}{s^4}\right]_{s\to(s-1)} = \boxed{e^{-s}\ \left[\frac{6}{(s-1)^4}\right]}$

30. $\mathscr{L}\{te^{t-5}\mathscr{U}(t-5)\}$

Solution: $\mathscr{L}\{te^{t-5}\mathscr{U}(t-5)\} = e^{-5s}\mathscr{L}\left\{\left[te^{t-5}\right]_{t\to(t+5)}\right\} = e^{-5s}\mathscr{L}\left\{\left[(t+5)\ e^t\right]\right\}$

$= e^{-5s}\left[\mathscr{L}\left\{t+5\right\}\right]_{s\to(s-1)} = e^{-5s}\left[\frac{1}{s^2}+\frac{5}{s}\right]_{s\to(s-1)} = \boxed{e^{-5s}\ \left[\frac{1}{(s-1)^2}+\frac{5}{s-1}\right]}$

31. $\mathscr{L}^{-1}\left\{\frac{e^{-2s}}{s^3}\right\}$

Solution: Recall that $\mathscr{L}^{-1}\{e^{-as}\ F(s)\} = [\mathscr{L}^{-1}\{F(s)\}]_{t\to(t-a)}\mathscr{U}(t-a)$

Do not forget to multiply by $\mathscr{U}(t-a)$ at the end

$$\mathscr{L}^{-1}\left\{\frac{e^{-2s}}{s^3}\right\} = \left[\mathscr{L}^{-1}\left\{\frac{1}{s^3}\right\}\right]_{t\to(t-2)}\mathscr{U}(t-2) = \left[\frac{t^2}{2}\right]_{t\to(t-2)}\mathscr{U}(t-2) = \boxed{\left[\frac{(t-2)^2}{2}\right]\mathscr{U}(t-2)}$$

32. $\mathscr{L}^{-1}\left\{\frac{(1+e^{-2s})^2}{s+2}\right\}$

Solution: Recall that $\mathscr{L}^{-1}\{e^{-as}\ F(s)\} = [\mathscr{L}^{-1}\{F(s)\}]_{t\to(t-a)}\mathscr{U}(t-a)$

Do not forget to multiply by $\mathscr{U}(t-a)$ at the end

$$\mathscr{L}^{-1}\left\{\frac{(1+e^{-2s})^2}{s+2}\right\} = \mathscr{L}^{-1}\left\{\frac{1+2e^{-2s}+e^{-4s}}{s+2}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{s+2}+e^{-2s}\frac{2}{s+2}+e^{-4s}\frac{1}{s+2}\right\}$$

$$= e^{-2t}+\left[\mathscr{L}^{-1}\left\{\frac{2}{s+2}\right\}\right]_{t\to(t-2)}\mathscr{U}(t-2)+\left[\mathscr{L}^{-1}\left\{\frac{1}{s+2}\right\}\right]_{t\to(t-4)}\mathscr{U}(t-4)$$

$$= e^{-2t}+\left[2e^{-2t}\right]_{t\to(t-2)}\mathscr{U}(t-2)+\left[e^{-2t}\right]_{t\to(t-4)}\mathscr{U}(t-4)$$

$$= \boxed{e^{-2t}+\left[2e^{-2(t-2)}\right]\mathscr{U}(t-2)+\left[e^{-2(t-4)}\right]\mathscr{U}(t-4)}$$

33. $\mathscr{L}^{-1}\left\{\dfrac{e^{-\pi s}}{s^2+1}\right\}$

Solution: Recall that $\mathscr{L}^{-1}\{e^{-as}\ F(s)\} = [\mathscr{L}^{-1}\{F(s)\}]_{t\to(t-a)}\mathscr{U}(t-a)$

Do not forget to multiply by $\mathscr{U}(t-a)$ at the end

$$\mathscr{L}^{-1}\left\{\frac{e^{-\pi s}}{s^2+1}\right\} = \left[\mathscr{L}^{-1}\left\{\frac{1}{s^2+1}\right\}\right]_{t\to(t-\pi)}\mathscr{U}(t-\pi) \quad = \quad [\sin t]_{t\to(t-\pi)}\,\mathscr{U}(t-\pi)$$

$= \boxed{[\sin(t-\pi)]\,\mathscr{U}(t-\pi) = [-\sin t]\,\mathscr{U}(t-\pi)}$ since $\sin(t-\pi) = \sin t\cos\pi - \cos t\sin\pi = -\sin t$

34. $\mathscr{L}^{-1}\left\{\dfrac{se^{-\pi s/2}}{s^2+4}\right\}$

Solution: Recall that $\mathscr{L}^{-1}\{e^{-as}\ F(s)\} = [\mathscr{L}^{-1}\{F(s)\}]_{t\to(t-a)}\mathscr{U}(t-a)$

Do not forget to multiply by $\mathscr{U}(t-a)$ at the end

$$\mathscr{L}^{-1}\left\{\frac{se^{-\pi s/2}}{s^2+4}\right\} = \left[\mathscr{L}^{-1}\left\{\frac{s}{s^2+4}\right\}\right]_{t\to(t-\pi/2)}\mathscr{U}(t-\pi/2) \quad = \quad [\cos 2t]_{t\to(t-\pi/2)}\,\mathscr{U}(t-\pi/2)$$

$$= \boxed{[\cos(2t-\pi)]\,\mathscr{U}(t-\pi/2) = [-\cos 2t]\,\mathscr{U}(t-\pi/2)}$$

35. $\mathscr{L}^{-1}\left\{\dfrac{e^{-s}}{s(s+1)}\right\}$

Solution: Recall that $\mathscr{L}^{-1}\{e^{-as}\ F(s)\} = [\mathscr{L}^{-1}\{F(s)\}]_{t\to(t-a)}\mathscr{U}(t-a)$

Do not forget to multiply by $\mathscr{U}(t-a)$ at the end

$$\mathscr{L}^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\} = \left[\mathscr{L}^{-1}\left\{\frac{1}{s(s+1)}\right\}\right]_{t\to(t-1)}\mathscr{U}(t-1) = \left[\mathscr{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+1}\right\}\right]_{t\to(t-1)}\mathscr{U}(t-1)$$

$$= \quad \left[1-e^{-t}\right]_{t\to(t-1)}\,\mathscr{U}(t-1) \quad = \quad \boxed{\left[1-e^{-(t-1)}\right]\mathscr{U}(t-1)}$$

36. $\mathscr{L}^{-1}\left\{\dfrac{e^{-2s}}{s^2(s-1)}\right\}$

Solution: Recall that $\mathscr{L}^{-1}\{e^{-as}\ F(s)\} = [\mathscr{L}^{-1}\{F(s)\}]_{t\to(t-a)}\mathscr{U}(t-a)$

Do not forget to multiply by $\mathscr{U}(t-a)$ at the end

$$\mathscr{L}^{-1}\left\{\frac{e^{-2s}}{s^2(s-1)}\right\} = \left[\mathscr{L}^{-1}\left\{\frac{1}{s^2(s-1)}\right\}\right]_{t\to(t-2)}\mathscr{U}(t-2)$$

$$= \left[\mathscr{L}^{-1}\left\{\frac{1}{s-1} - \frac{1}{s} - \frac{1}{s^2}\right\}\right]_{t\to(t-2)}\mathscr{U}(t-2)$$

$$= \quad \left[e^t - 1 - t\right]_{t\to(t-2)}\mathscr{U}(t-2) \quad = \quad \boxed{\left[e^{(t-2)} - 1 - (t-2)\right]\mathscr{U}(t-2)}$$

37. $\mathscr{L}\{t\cos 2t\}$

Solution: $\mathscr{L}\{t\cos 2t\} = -\dfrac{d}{ds}\left[\mathscr{L}\{\cos 2t\}\right] = -\dfrac{d}{ds}\left[\dfrac{s}{s^2+4}\right] = -\left[\dfrac{(s^2+4)(1)-s(2s)}{(s^2+4)^2}\right] =$

$$\boxed{\frac{s^2-4}{(s^2+4)^2}}$$

40. $\mathscr{L}\{t^2\cos t\}$

Solution: $\mathscr{L}\{t^2\cos t\} = \dfrac{d^2}{ds^2}\left[\mathscr{L}\{\cos t\}\right] = \dfrac{d^2}{ds^2}\left[\dfrac{s}{s^2+1}\right] = \dfrac{d}{ds}\left[\dfrac{(s^2+1)(1)-s(2s)}{(s^2+1)^2}\right]$

$$= \frac{d}{ds}\left[\frac{1-s^2}{(s^2+1)^2}\right] = \frac{(s^2+1)^2(-2s)-(1-s^2)(2)(s^2+1)(2s)}{(s^2+1)^4} = \frac{(s^2+1)[-2s(s^2+1)-4s(1-s^2)]}{(s^2+1)^4}$$

$$= \frac{-2s^3-2s-4s+4s^3}{(s^2+1)^3} = \frac{2s^3-6s}{(s^2+1)^3} = \boxed{\frac{2s(s^2-3)}{(s^2+1)^3}}$$

41. $\mathscr{L}\{te^{2t}\sin 6t\}$

Solution: $\mathscr{L}\{te^{2t}\sin 6t\} = \mathscr{L}\{e^{2t}(t\ \sin 6t)\} = \left[-\dfrac{d}{ds}\mathscr{L}\{\sin 6t\}\right]_{s\to(s-2)} = \left[-\dfrac{d}{ds}\left(\dfrac{6}{s^2+36}\right)\right]_{s\to(s-2)}$

$$= \left[(-6)\cdot\frac{-1}{(s^2+36)^2}\cdot(2s)\right]_{s\to(s-2)} = \left[\frac{12s}{(s^2+36)^2}\right]_{s\to(s-2)} = \boxed{\frac{12(s-2)}{[(s-2)^2+36]^2}}$$

42. $\mathscr{L}\{te^{-3t}\cos 3t\}$

Solution: $\mathscr{L}\{te^{-3t}\cos 3t\} = \mathscr{L}\{e^{-3t}(t\ \cos 3t)\} = \left[-\dfrac{d}{ds}\mathscr{L}\{\cos 3t\}\right]_{s\to(s+3)} = \left[-\dfrac{d}{ds}\left(\dfrac{s}{s^2+9}\right)\right]_{s\to(s+3)}$

$$= -\left[\frac{(s^2+9)(1)-s(2s)}{(s^2+9)^2}\right]_{s\to(s+3)} = \left[\frac{s^2-9}{(s^2+9)^2}\right]_{s\to(s+3)} = \boxed{\frac{(s+3)^2-9}{[(s+3)^2+9]^2}}$$

51. Write the function in terms of unit step functions. Then find the Laplace Transform.

$$f(t) = \begin{cases} 2, & 0 \le t < 3 \\ -2, & t \ge 3 \end{cases}$$

Solution: $f(t) = (2)\ [\underbrace{\mathscr{U}(t-0)}_{=1} - \mathscr{U}(t-3)] + (-2)\ [\mathscr{U}(t-3) - \underbrace{\mathscr{U}(t-\infty)}_{=0}]$

$$= (2)\ [1-\mathscr{U}(t-3)] - 2\ [\mathscr{U}(t-3)] = 2 - 4\mathscr{U}(t-3)$$

$$\Rightarrow F(s) = \mathscr{L}\{2-4\mathscr{U}(t-3)\} = \boxed{\frac{2}{s} - e^{-3s}\,\frac{4}{s}}$$

52. Write the function in terms of unit step functions. Then find the Laplace Transform.

$$f(t) = \begin{cases} 1, & 0 \le t < 4 \\ 0, & 4 \le t < 5 \\ 1, & t \ge 5 \end{cases}$$

Solution: $f(t) = (1)\ [\mathscr{U}(t-0)-\mathscr{U}(t-4)]+(0)\ [\mathscr{U}(t-4)-\mathscr{U}(t-5)]+(1)\ [\mathscr{U}(t-5)-\mathscr{U}(t-\infty)]$

$= (1)\ [1-\mathscr{U}(t-4)] + (1)\ [\mathscr{U}(t-5)] = 1 - \mathscr{U}(t-4) + \mathscr{U}(t-5)$

$\Rightarrow F(s) = \mathscr{L}\{1-\mathscr{U}(t-4)+\mathscr{U}(t-5)\} = \boxed{\dfrac{1}{s} - e^{-4s}\ \dfrac{1}{s} + e^{-5s}\ \dfrac{1}{s}}$

53. Write the function in terms of unit step functions. Then find the Laplace Transform.

$$f(t) = \begin{cases} 0, & 0 \le t < 1 \\ t^2, & t \ge 1 \end{cases}$$

Solution: $f(t) = (0)\ [\mathscr{U}(t-0) - \mathscr{U}(t-1)] + (t^2)\ [\mathscr{U}(t-1) - \mathscr{U}(t-\infty)] = t^2\ \mathscr{U}(t-1)$

$\Rightarrow F(s) = \mathscr{L}\{t^2\ \mathscr{U}(t-1)\} = e^{-s}\mathscr{L}\{(t+1)^2\} = e^{-s}\mathscr{L}\{t^2+2t+1\} = \boxed{e^{-s}\left[\dfrac{2}{s^3} + \dfrac{2}{s^2} + \dfrac{1}{s}\right]}$

54. Write the function in terms of unit step functions. Then find the Laplace Transform.

$$f(t) = \begin{cases} 0, & 0 \le t < \frac{3\pi}{2} \\ \sin t, & t \ge \frac{3\pi}{2} \end{cases}$$

Solution: $f(t) = (0)\ [\mathscr{U}(t-0) - \mathscr{U}(t-3\pi/2)] + (\sin t)\ [\mathscr{U}(t-3\pi/2) - \mathscr{U}(t-\infty)]$

$= \sin t\ \mathscr{U}(t-3\pi/2)$

$\Rightarrow F(s) = \mathscr{L}\{\sin t\ \mathscr{U}(t-3\pi/2)\} = e^{-3\pi s/2}\mathscr{L}\{\sin(t+3\pi/2)\} = e^{-3\pi s/2}\mathscr{L}\{-\cos t\}$

$= \boxed{-e^{-3\pi s/2}\left[\dfrac{s}{s^2+1}\right]}$

55. Write the function in terms of unit step functions. Then find the Laplace Transform.

$$f(t) = \begin{cases} t, & 0 \le t < 2 \\ 0, & t \ge 2 \end{cases}$$

Solution: $f(t) = (t)\ [\mathscr{U}(t-0) - \mathscr{U}(t-2)] + (0)\ [\mathscr{U}(t-2) - \mathscr{U}(t-\infty)]$

$= (t)\ [1 - \mathscr{U}(t-2)] = t - t\mathscr{U}(t-2)$

$\Rightarrow F(s) = \mathscr{L}\{t - t\mathscr{U}(t-2)\} = \dfrac{1}{s^2} - e^{-2s}\ \mathscr{L}\{t+2\} = \boxed{\dfrac{1}{s^2} - e^{-2s}\left[\dfrac{1}{s^2} + \dfrac{2}{s}\right]}$

56. Write the function in terms of unit step functions. Then find the Laplace Transform.

$$f(t) = \begin{cases} \sin t, & 0 \le t < 2\pi \\ 0, & t \ge 2\pi \end{cases}$$

Solution: $f(t) = (\sin t)\ [\mathscr{U}(t-0) - \mathscr{U}(t-2\pi)] + (0)\ [\mathscr{U}(t-2\pi) - \mathscr{U}(t-\infty)]$

$$= (\sin t)\ [1 - \mathscr{U}(t-2\pi)] = \sin t - \sin t\ \mathscr{U}(t-2\pi)$$

$$\Rightarrow F(s) = \mathscr{L}\{\sin t - \sin t \mathscr{U}(t-2\pi)\} = \frac{1}{s^2+1} - e^{-2\pi s}\ \mathscr{L}\{\sin(t+2\pi)\}$$

$$= \frac{1}{s^2+1} - e^{-2\pi s}\ \mathscr{L}\{\sin t\} = \boxed{\frac{1}{s^2+1} - e^{-2\pi s}\left[\frac{1}{s^2+1}\right]}$$