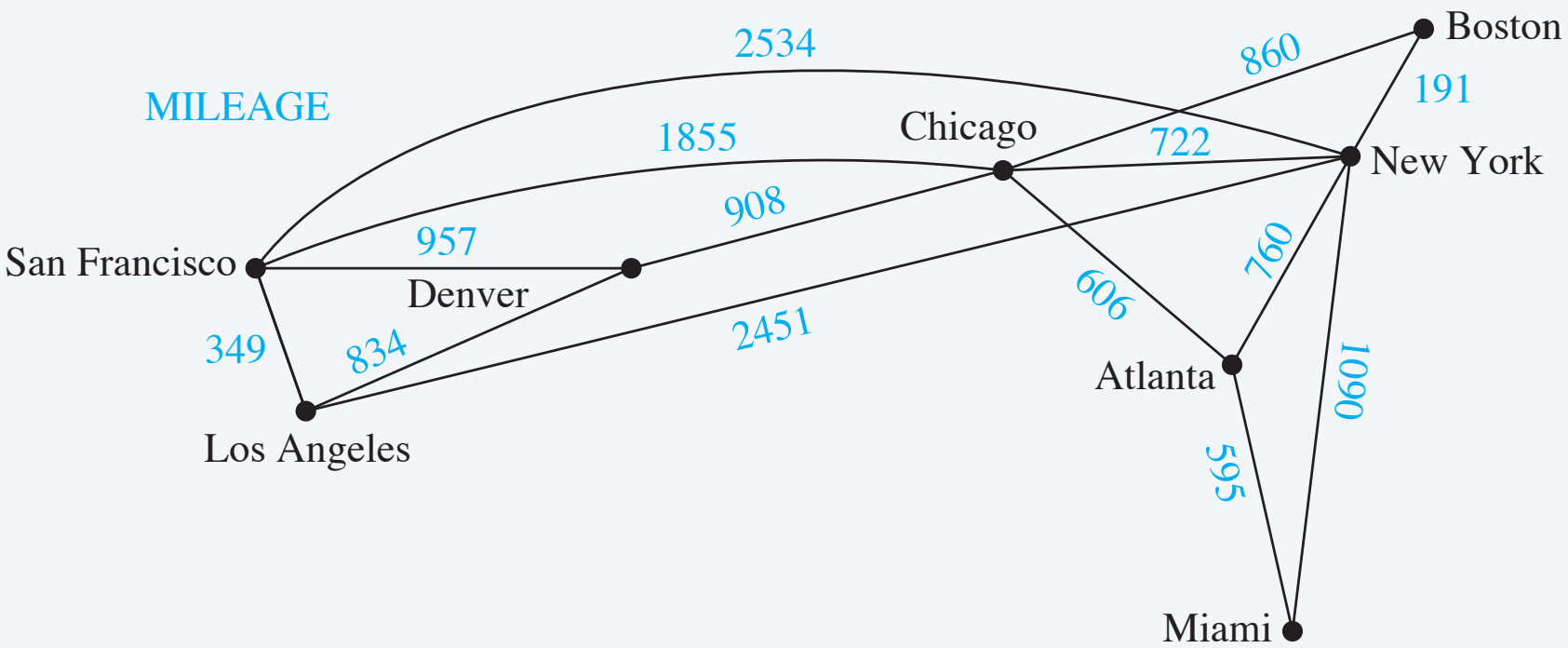
Đường đi ngắn nhất

Trần Vĩnh Đức

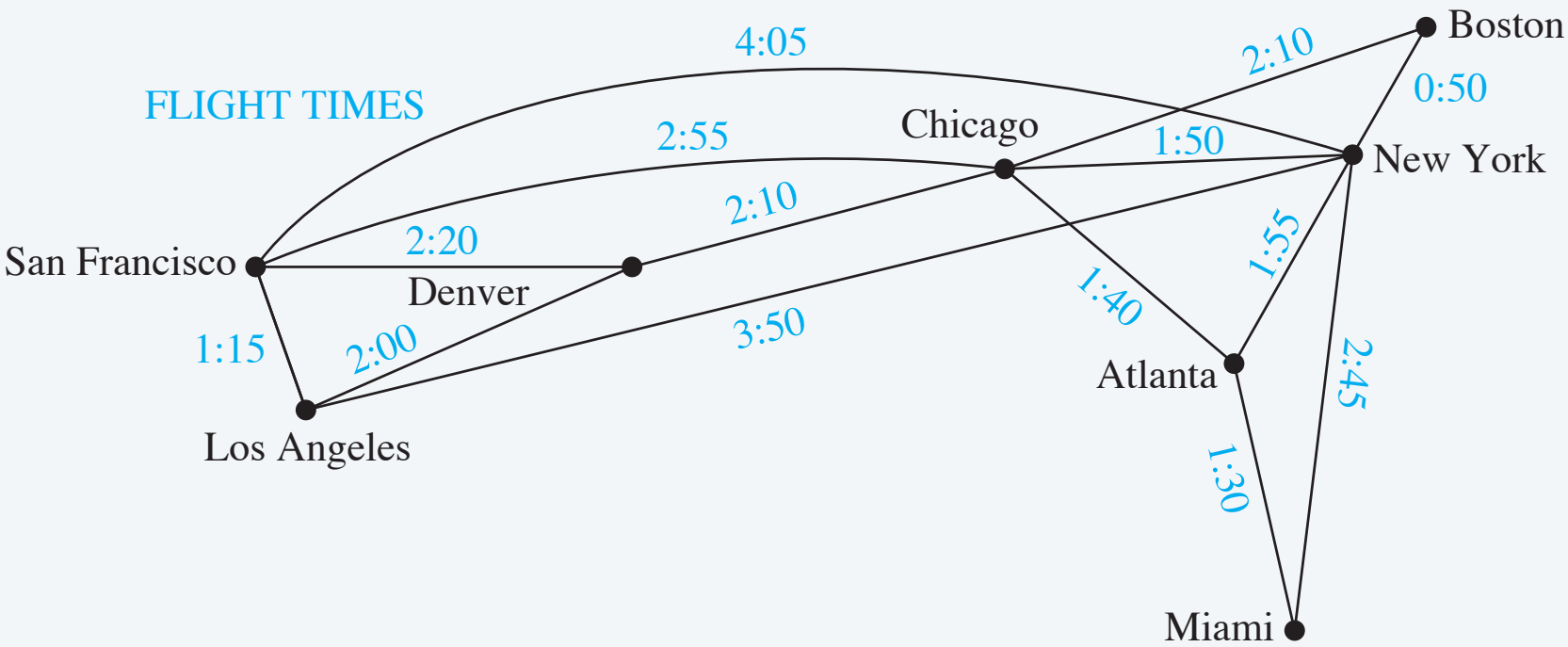
Ngày 14 tháng 2 năm 2017

Nội dung

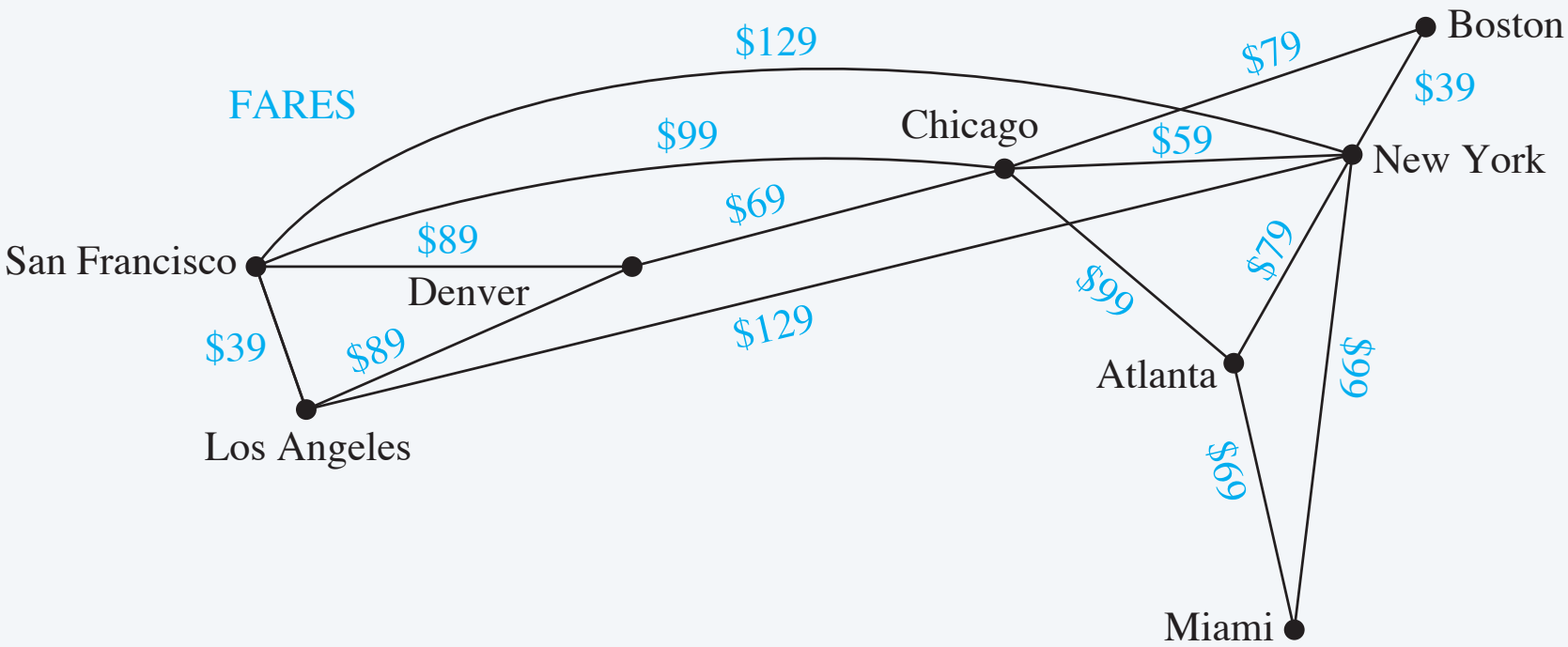
Mô hình đường bay với trọng số là khoảng cách



Mô hình đường bay với trọng số là thời gian bay



Mô hình hệ thống đường bay với trọng số là giá vé



Đồ thị có trọng số

- Đồ thị với mỗi cạnh được gán một trọng số.

Đồ thị có trọng số

- Đồ thị với mỗi cạnh được gán một trọng số.
- Ký hiệu (V, E, w) trong đó (V, E) là đồ thị và

$$w : E \longrightarrow \mathbb{R}^+$$

là một hàm từ tập cạnh E lên tập số dương (trọng số).

Đồ thị có trọng số

- Đồ thị với mỗi cạnh được gán một trọng số.
- Ký hiệu (V, E, w) trong đó (V, E) là đồ thị và

$$w : E \longrightarrow \mathbb{R}^+$$

là một hàm từ tập cạnh E lên tập số dương (trọng số).

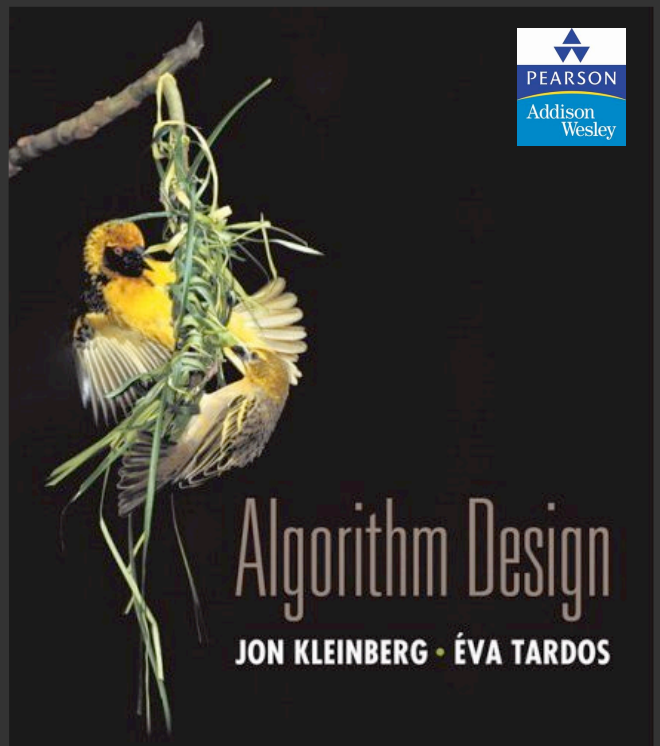
- Trọng số hay độ dài của đường đi

$$p : v_0 \to v_1 \to v_2 \to \cdots \to v_k$$

là tổng

$$w(p) = w(v_0 \to v_1) + w(v_1 \to v_2) + \cdots + w(v_{k-1} \to v_k).$$

Nội dung



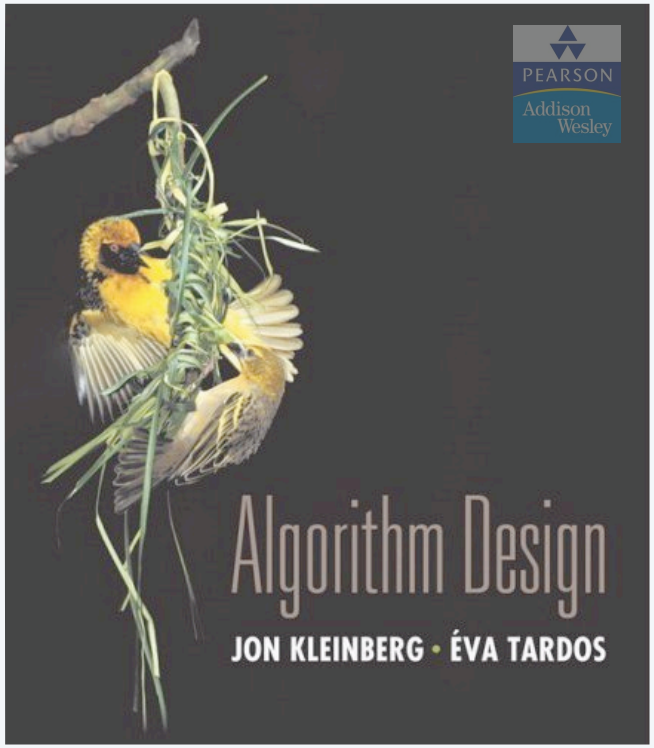
4. Greedy Algorithms II

- *Dijkstra's algorithm demo*
- *improved Dijkstra's algorithm demo*

Lecture slides by Kevin Wayne

http://www.cs.princeton.edu/~wayne/kleinberg-tardos

Last updated on Sep 8, 2013 6:19 AM



SECTION 4.4

4. GREEDY ALGORITHMS II

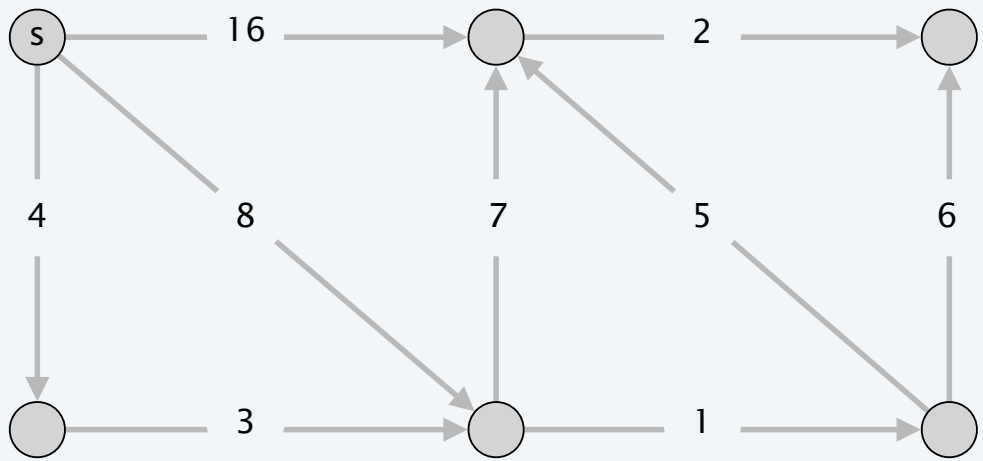
- *Dijkstra's algorithm demo*
- *improved Dijkstra's algorithm demo*

Dijkstra's algorithm demo

- Initialize $S = \{ s \}, \ d(s) = 0$.
- Repeatedly choose unexplored node v which minimizes

$$\pi(v) = \min_{e = (u,v) : u \in S} d(u) + \ell_e,$$

add v to S; set $d(v) = \pi(v)$.

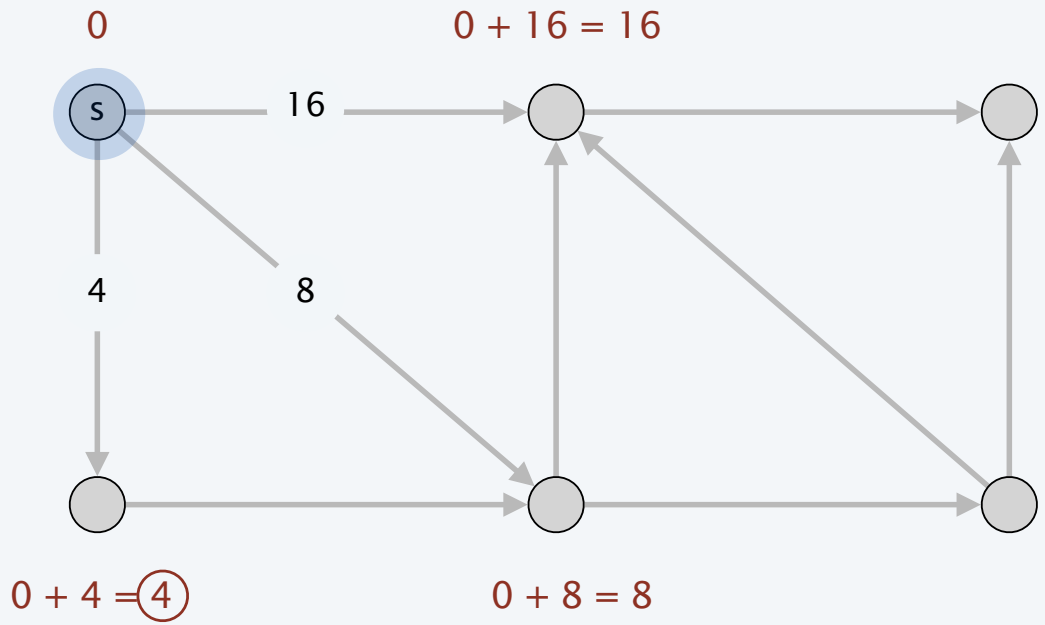


Dijkstra's algorithm demo

- Initialize $S = \{ s \}$, $d(s) = 0$.
- Repeatedly choose unexplored node v which minimizes

$$\pi(v) = \min_{e = (u,v) : u \in S} d(u) + \ell_e ,$$

add v to S; set $d(v) = \pi(v)$.

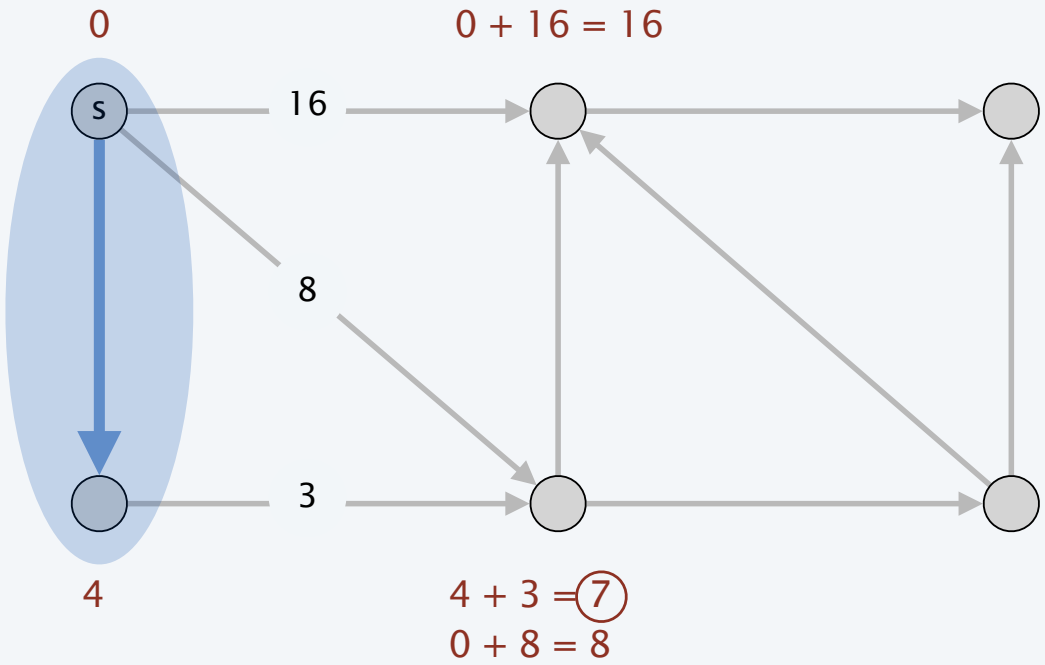


Dijkstra's algorithm demo

- Initialize $S = \{ s \}$, $d(s) = 0$.
- Repeatedly choose unexplored node v which minimizes

$$\pi(v) = \min_{e = (u,v) : u \in S} d(u) + \ell_e ,$$

add v to S; set $d(v) = \pi(v)$.

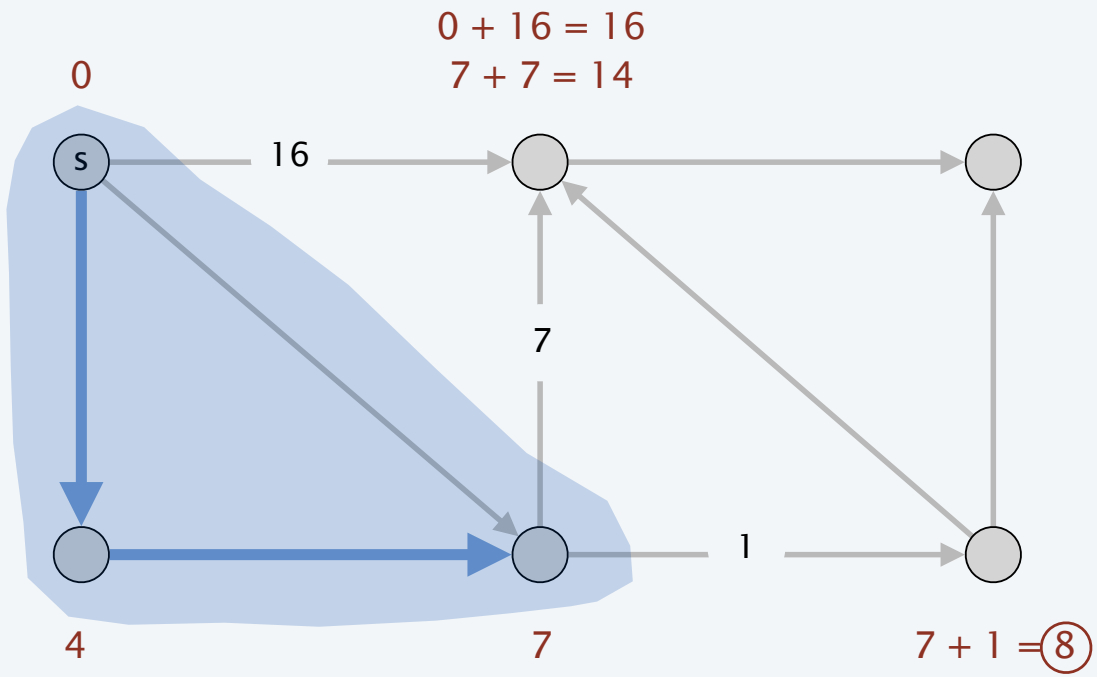


Dijkstra's algorithm demo

- Initialize $S = \{ s \}$, $d(s) = 0$.
- Repeatedly choose unexplored node v which minimizes

$$\pi(v) = \min_{e = (u,v) : u \in S} d(u) + \ell_e ,$$

add v to S; set $d(v) = \pi(v)$.

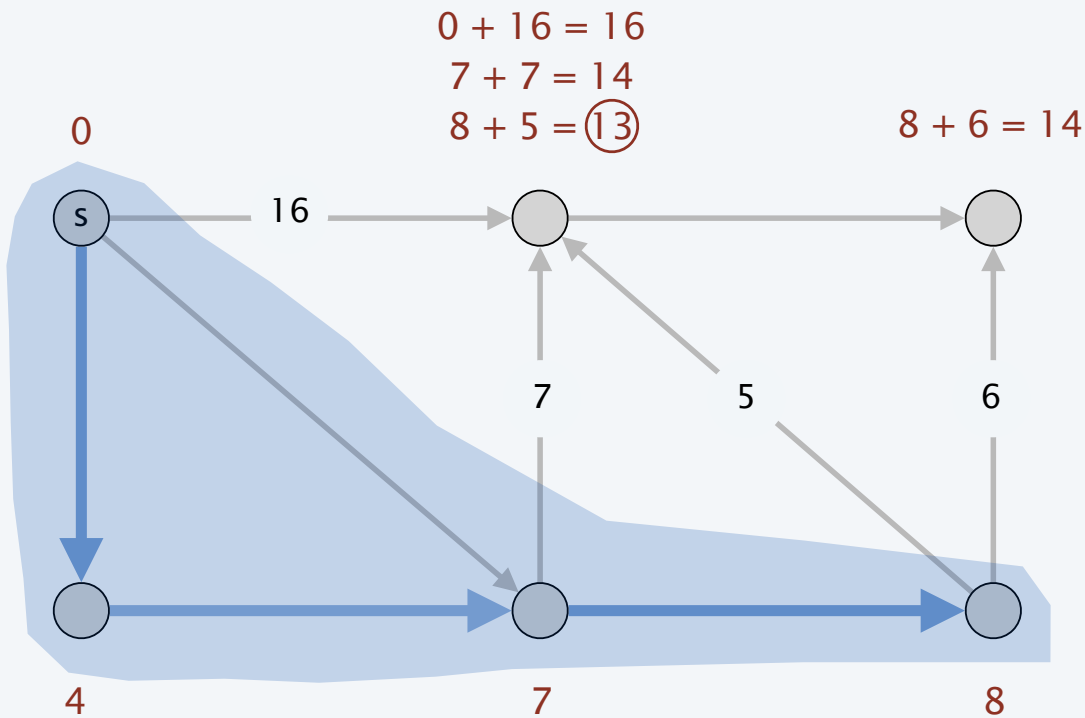


Dijkstra's algorithm demo

- Initialize $S = \{ s \}$, $d(s) = 0$.
- Repeatedly choose unexplored node v which minimizes

$$\pi(v) = \min_{e = (u,v) : u \in S} d(u) + \ell_e ,$$

add v to S; set $d(v) = \pi(v)$.

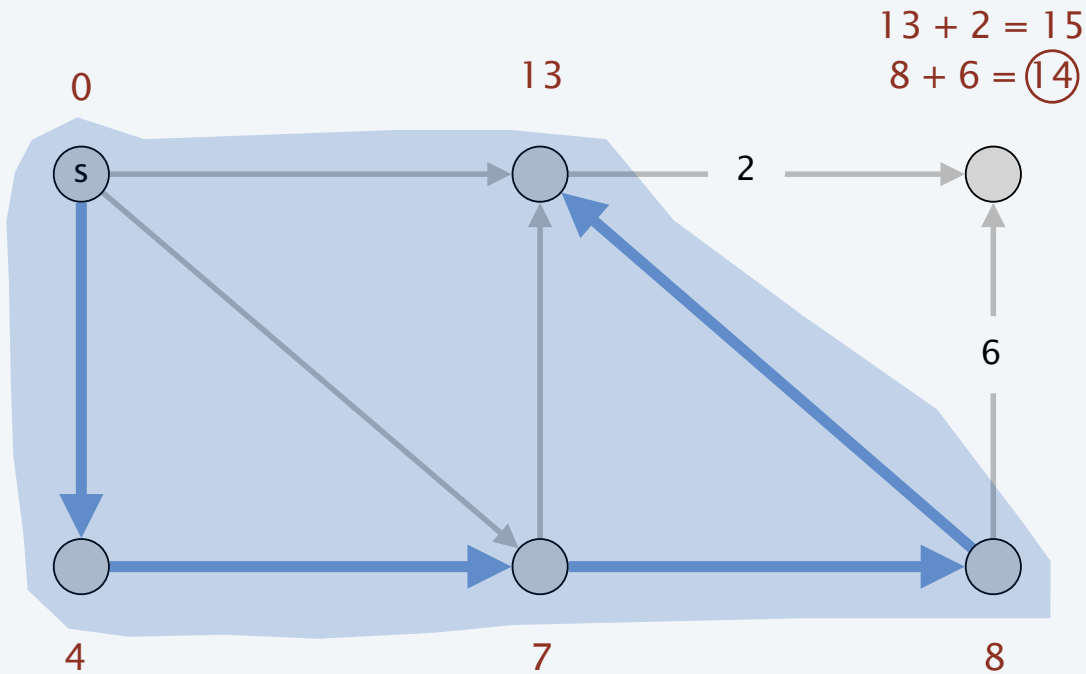


Dijkstra's algorithm demo

- Initialize $S = \{ s \}$, $d(s) = 0$.
- Repeatedly choose unexplored node v which minimizes

$$\pi(v) = \min_{e = (u,v) : u \in S} d(u) + \ell_e ,$$

add v to S; set $d(v) = \pi(v)$.

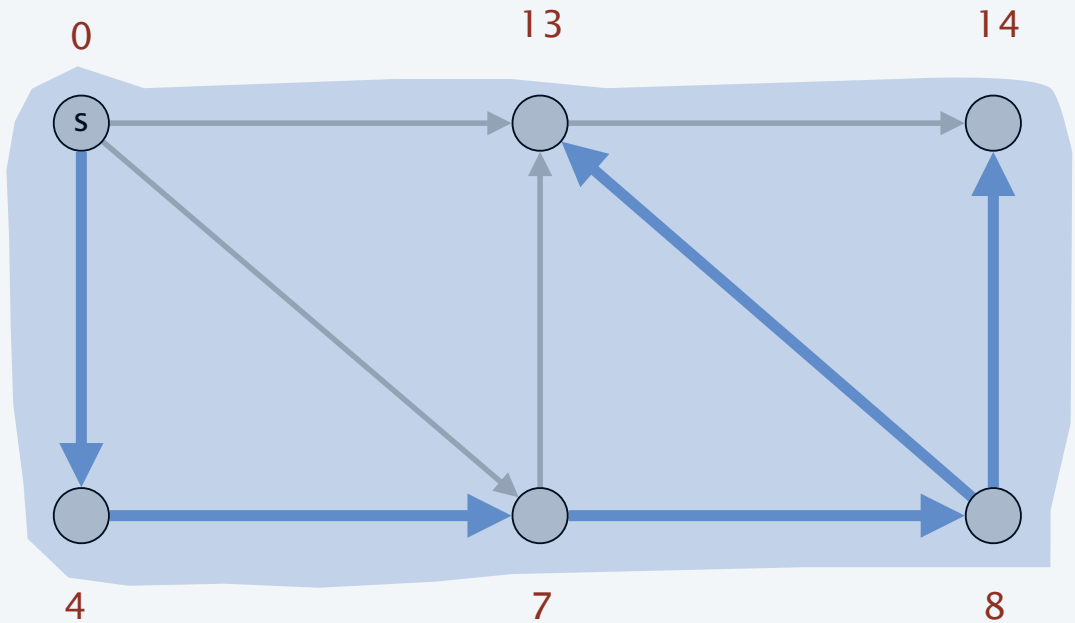


Dijkstra's algorithm demo

- Initialize $S = \{ s \}$, $d(s) = 0$.
- Repeatedly choose unexplored node v which minimizes

$$\pi(v) = \min_{e = (u,v) : u \in S} d(u) + \ell_e ,$$

add v to S; set $d(v) = \pi(v)$.

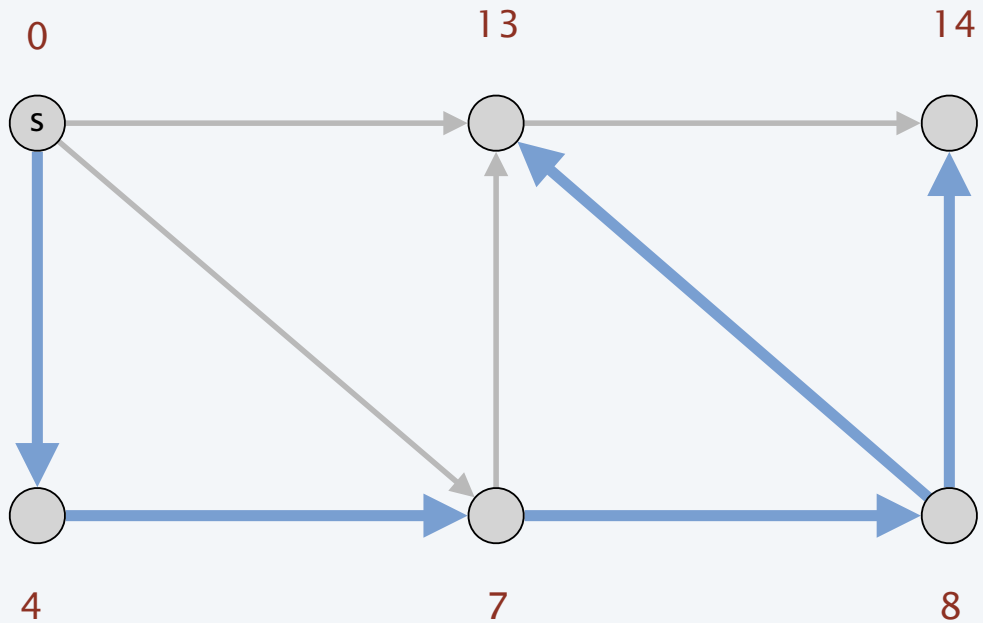


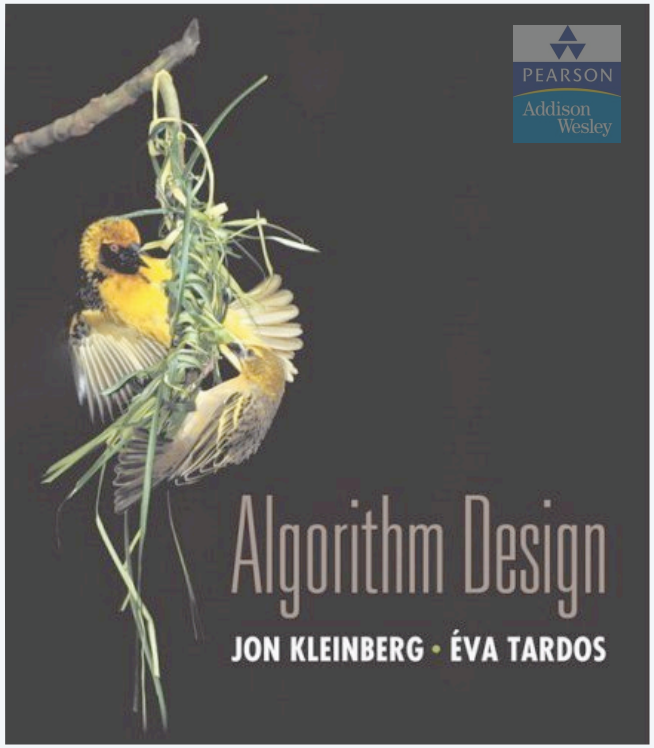
Dijkstra's algorithm demo

- Initialize $S = \{ s \}$, $d(s) = 0$.
- Repeatedly choose unexplored node v which minimizes

$$\pi(v) = \min_{e = (u,v) : u \in S} d(u) + \ell_e ,$$

add v to S; set $d(v) = \pi(v)$.





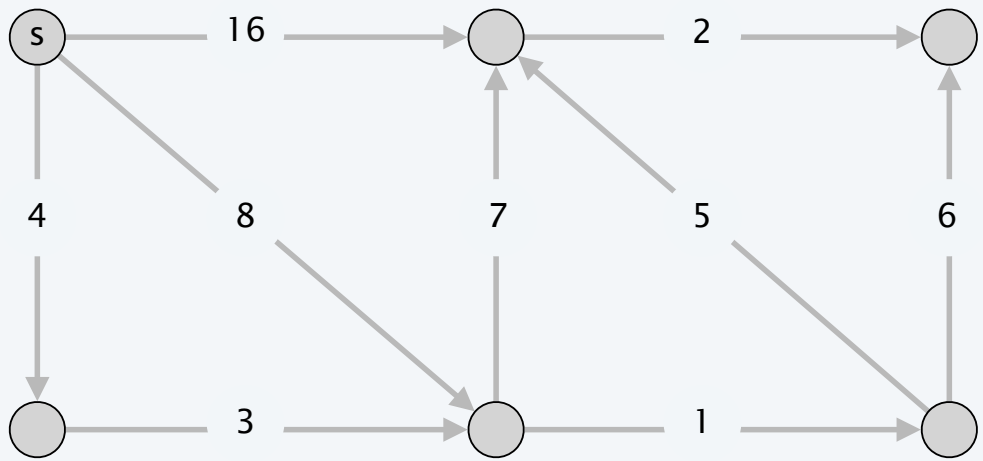
SECTION 4.4

4. GREEDY ALGORITHMS II

- *Dijkstra's algorithm demo*
- *improved Dijkstra's algorithm demo*

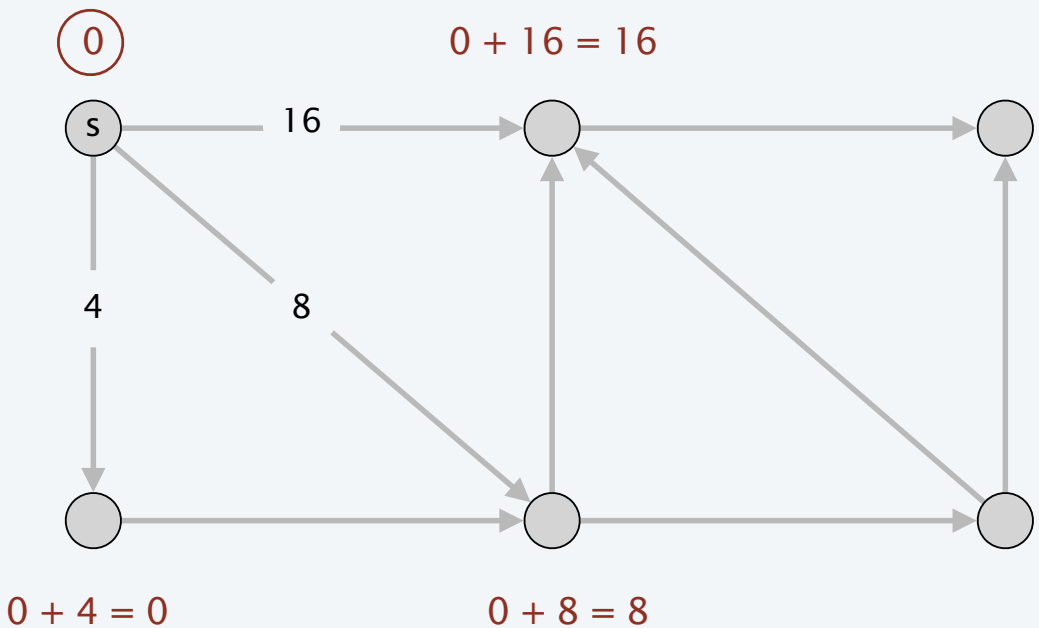
Improved Dijkstra's algorithm demo

- Initialize $\pi(s) = 0$.
- Repeatedly choose $u \notin S$ with minimum $\pi(v)$.
  - for each edge (u, v) leaving u, set $\pi(v) = \min \{ \pi(v), \pi(u) + \ell(u, v) \}$
  - add u to S



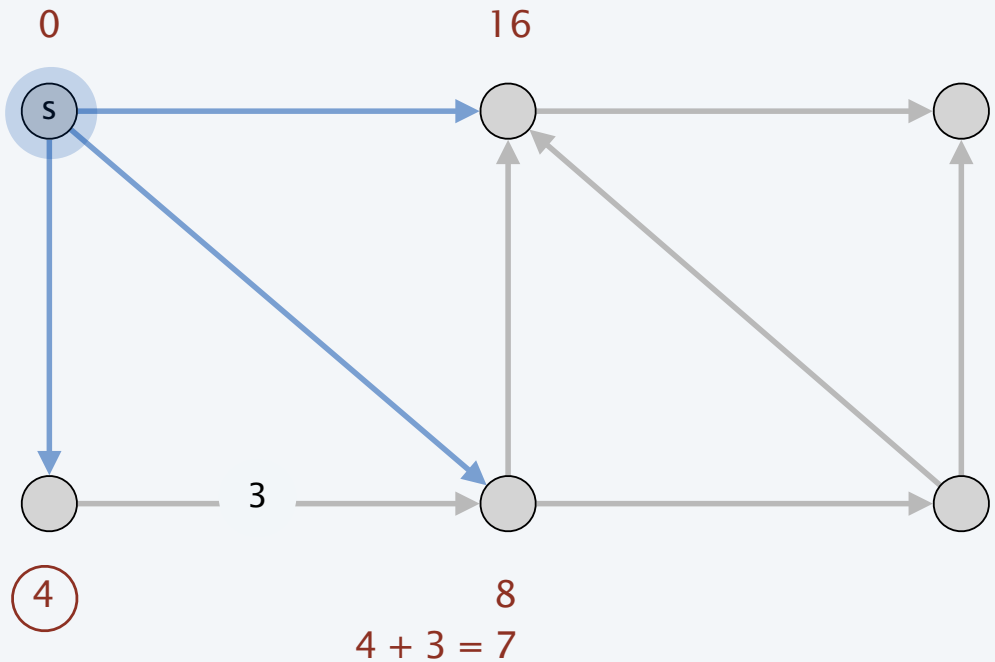
Improved Dijkstra's algorithm demo

- Initialize $\pi(s) = 0$.
- Repeatedly choose $u \notin S$ with minimum $\pi(v)$.
  - for each edge (u, v) leaving u, set $\pi(v) = \min \{ \pi(v), \pi(u) + \ell(u, v) \}$
  - add u to S



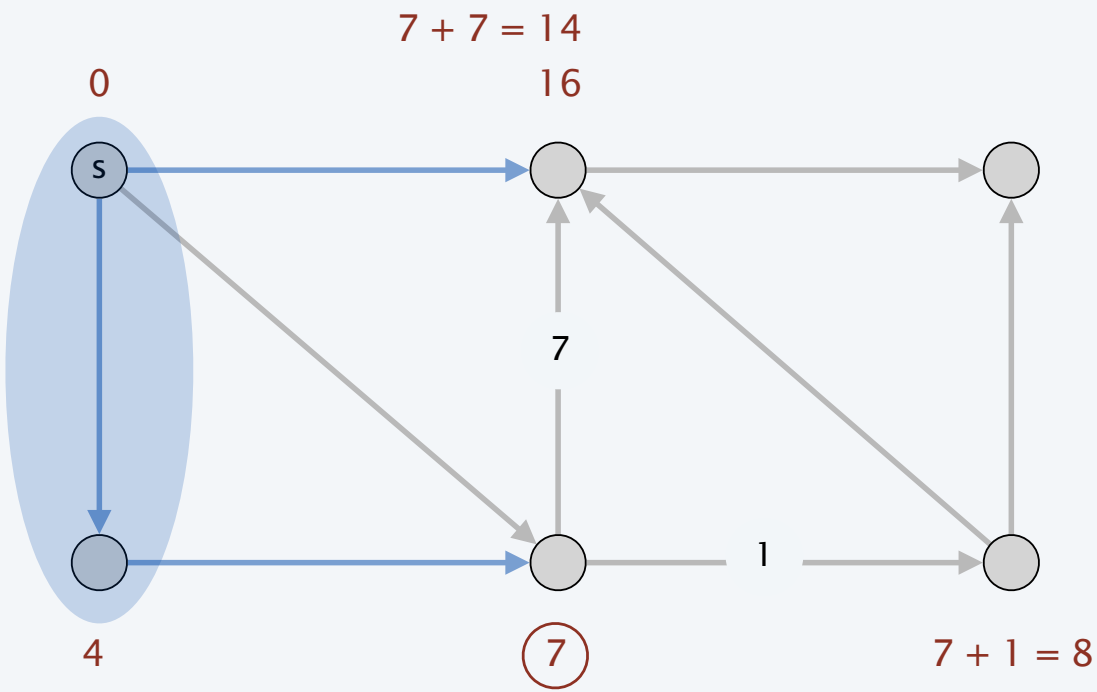
Improved Dijkstra's algorithm demo

- Initialize $\pi(s) = 0$.
- Repeatedly choose $u \notin S$ with minimum $\pi(v)$.
  - for each edge (u, v) leaving u, set $\pi(v) = \min \{ \pi(v), \pi(u) + \ell(u, v) \}$
  - add u to S



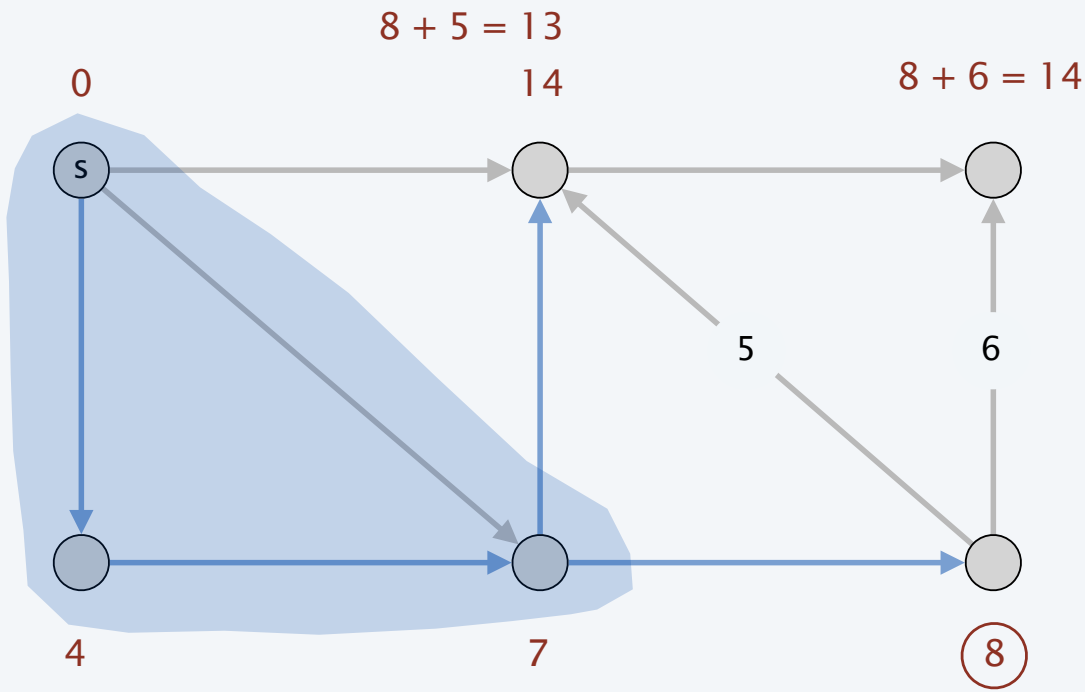
Improved Dijkstra's algorithm demo

- Initialize $\pi(s) = 0$.
- Repeatedly choose $u \notin S$ with minimum $\pi(v)$.
  - for each edge (u, v) leaving u, set $\pi(v) = \min \{ \pi(v), \pi(u) + \ell(u, v) \}$
  - add u to S



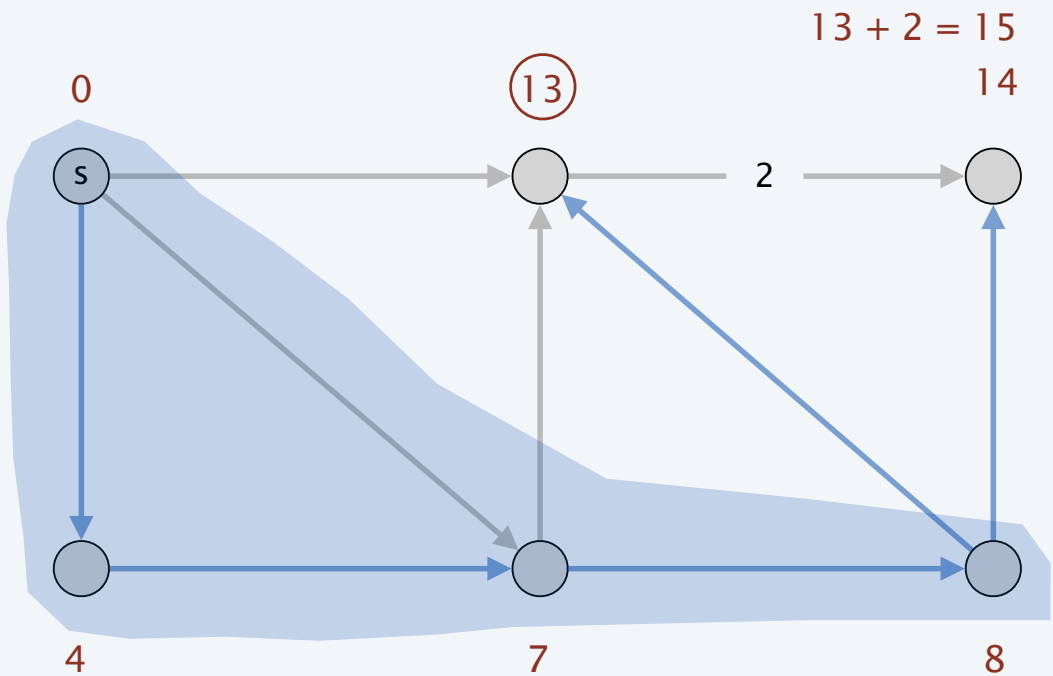
Improved Dijkstra's algorithm demo

- Initialize $\pi(s) = 0$.
- Repeatedly choose $u \notin S$ with minimum $\pi(v)$.
  - for each edge (u, v) leaving u, set $\pi(v) = \min \{ \pi(v), \pi(u) + \ell(u, v) \}$
  - add u to S



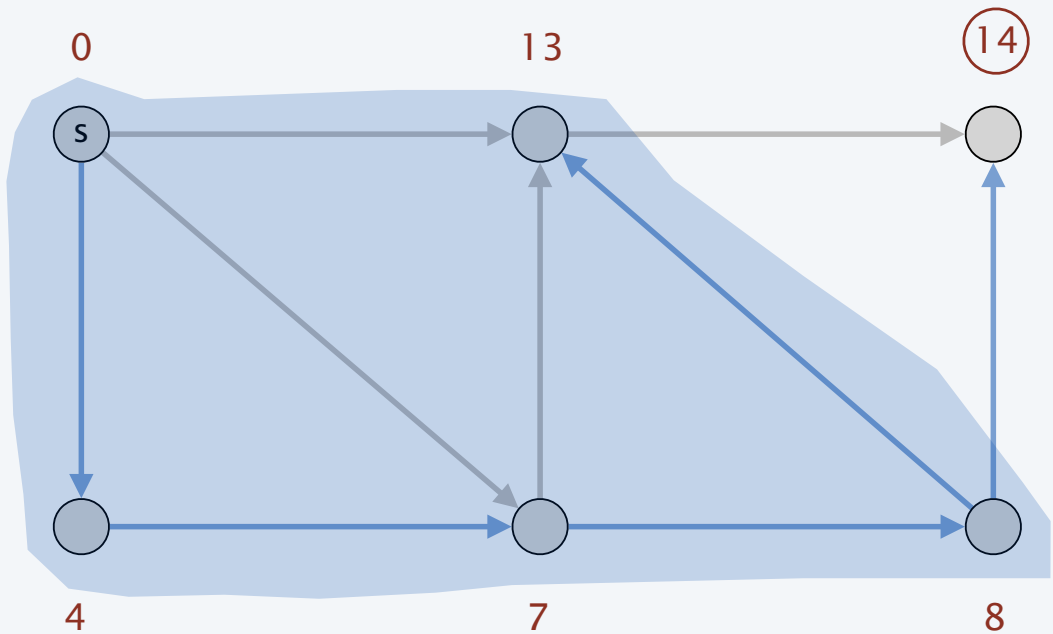
Improved Dijkstra's algorithm demo

- Initialize $\pi(s) = 0$.
- Repeatedly choose $u \notin S$ with minimum $\pi(v)$.
  - for each edge (u, v) leaving u, set $\pi(v) = \min \{ \pi(v), \pi(u) + \ell(u, v) \}$
  - add u to S



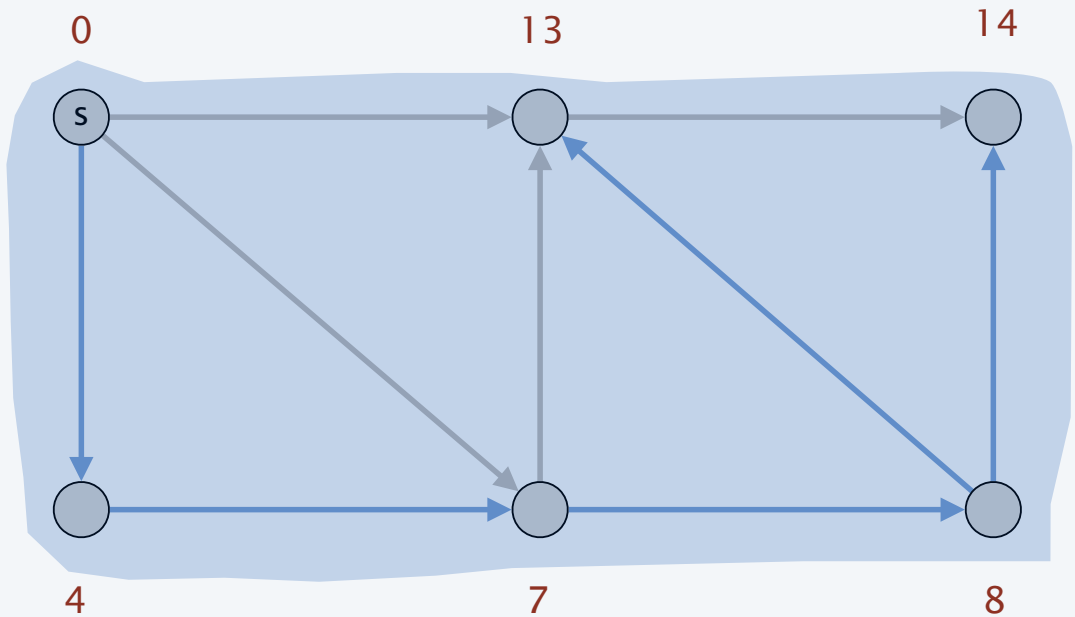
Improved Dijkstra's algorithm demo

- Initialize $\pi(s) = 0$.
- Repeatedly choose $u \notin S$ with minimum $\pi(v)$.
  - for each edge (u, v) leaving u, set $\pi(v) = \min \{ \pi(v), \pi(u) + \ell(u, v) \}$
  - add u to S



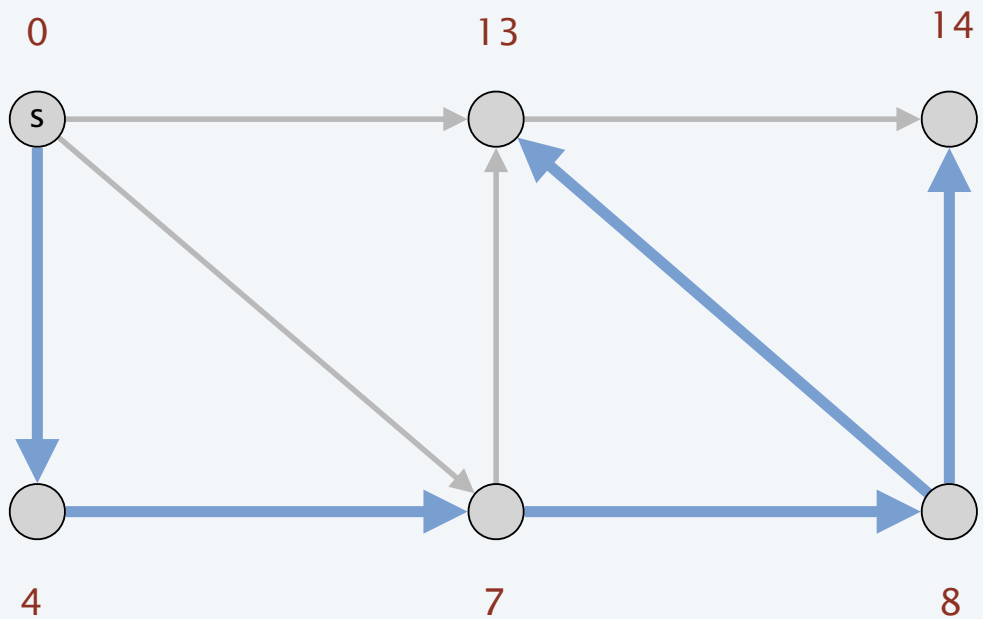
Improved Dijkstra's algorithm demo

- Initialize $\pi(s) = 0$.
- Repeatedly choose $u \notin S$ with minimum $\pi(v)$.
  - for each edge (u, v) leaving u, set $\pi(v) = \min \{ \pi(v), \pi(u) + \ell(u, v) \}$
  - add u to S

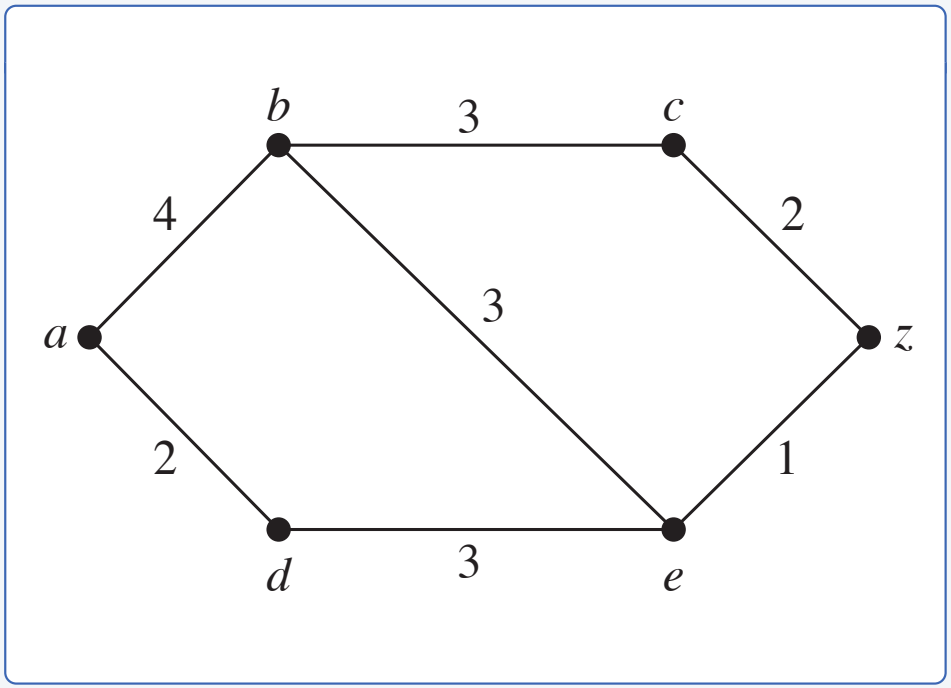


Improved Dijkstra's algorithm demo

- Initialize $\pi(s) = 0$.
- Repeatedly choose $u \notin S$ with minimum $\pi(v)$.
  - for each edge (u, v) leaving u, set $\pi(v) = \min \{ \pi(v), \pi(u) + \ell(u, v) \}$
  - add u to S

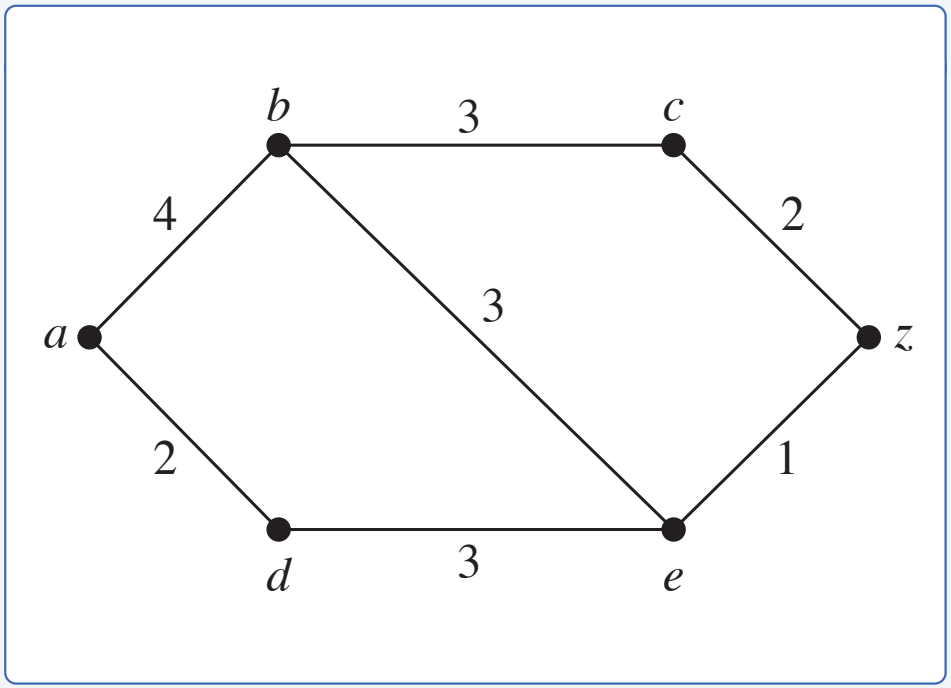


Thuật toán tìm đường đi ngắn nhất



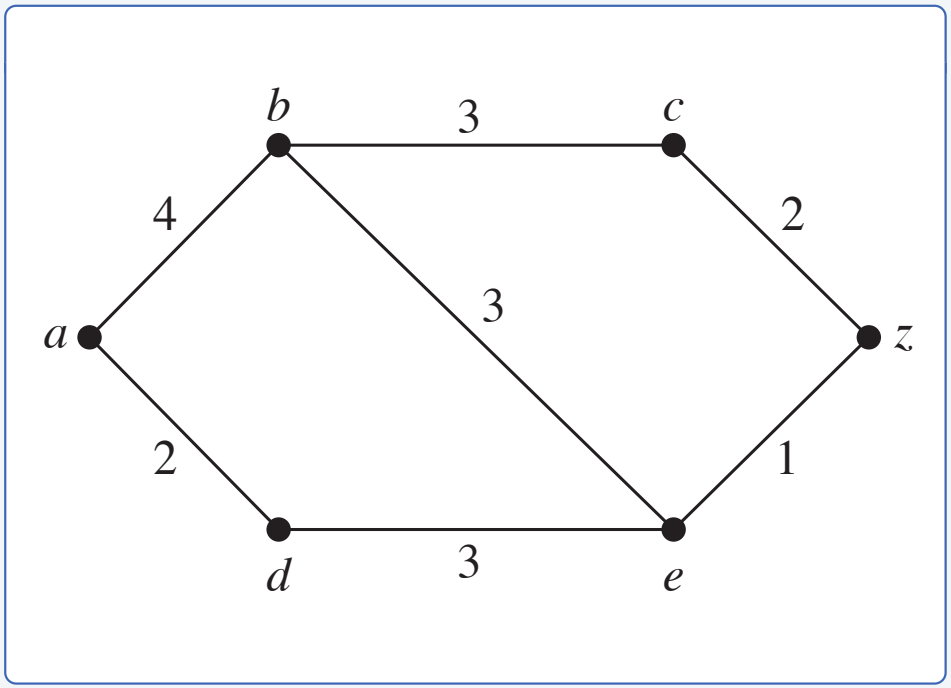
a	b	c	d	e	z
0	∞	∞	∞	∞	∞

Thuật toán tìm đường đi ngắn nhất



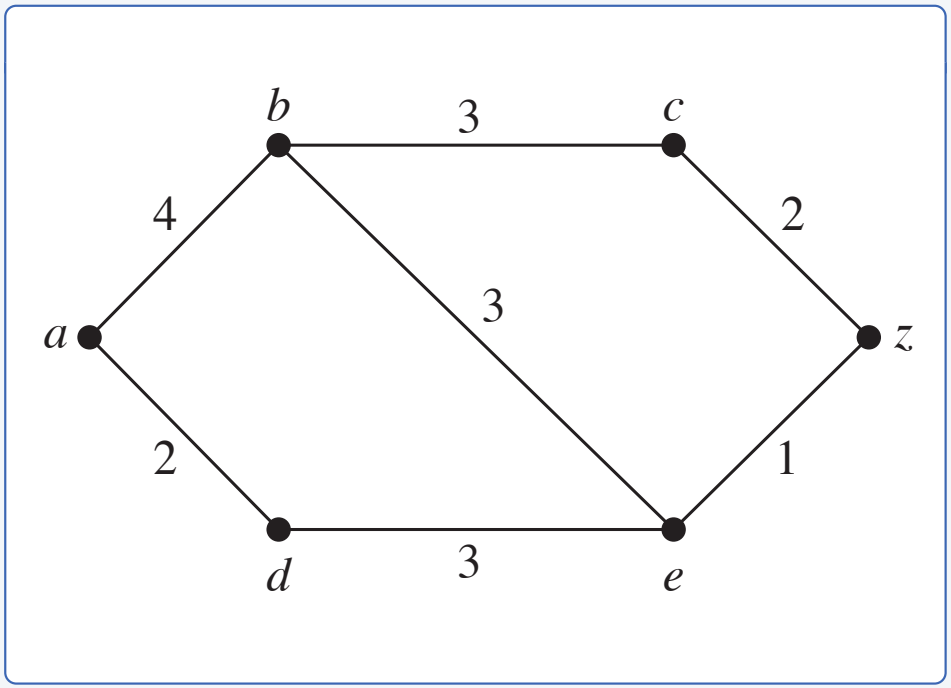
a	b	c	d	e	z
$\boxed{0}$	∞	∞	∞	∞	∞
	$4a$	∞	$\boxed{2}a$	∞	∞

Thuật toán tìm đường đi ngắn nhất



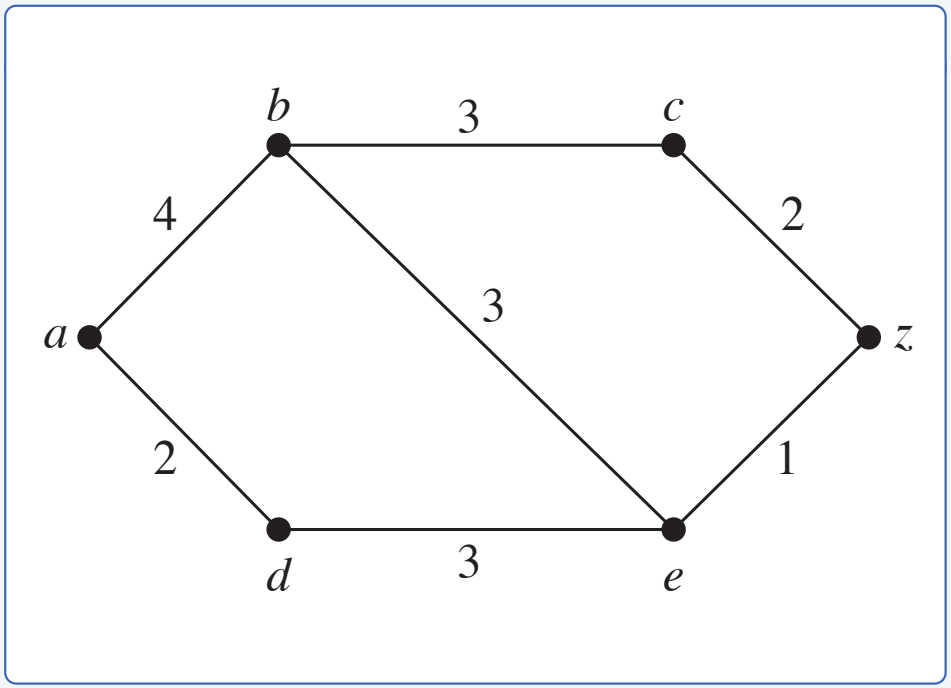
a	b	c	d	e	z
$\boxed{0}$	∞	∞	∞	∞	∞
	$4a$	∞	$\boxed{2}a$	∞	∞
	$\boxed{4}a$	∞		$5d$	∞

Thuật toán tìm đường đi ngắn nhất



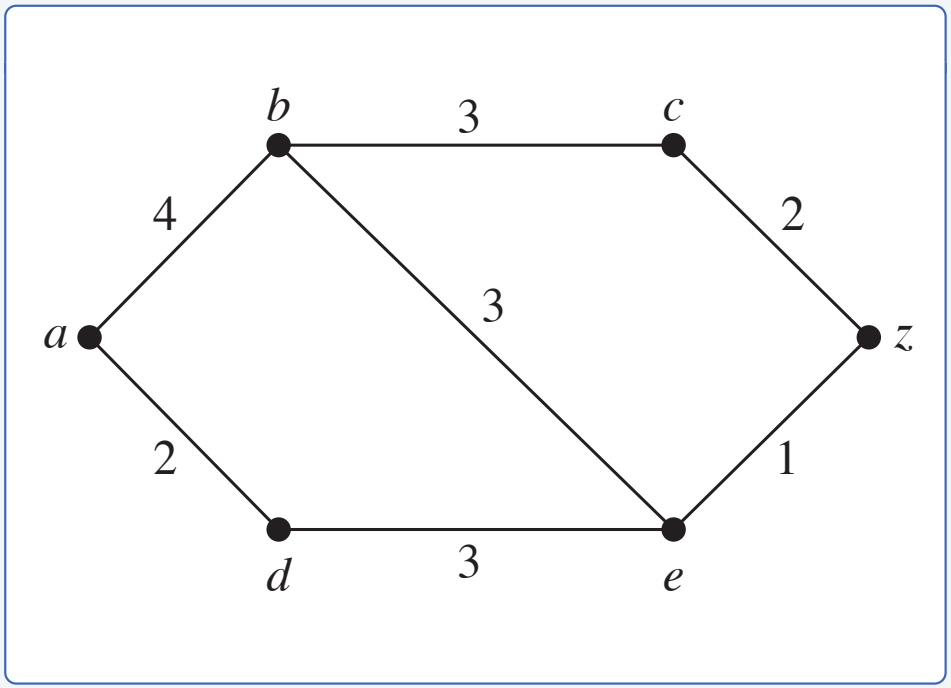
a	b	c	d	e	z
$\boxed{0}$	∞	∞	∞	∞	∞
	$4a$	∞	$\boxed{2}a$	∞	∞
	$\boxed{4}a$	∞		$5d$	∞
		$7b$		$\boxed{5}d$	∞

Thuật toán tìm đường đi ngắn nhất



a	b	c	d	e	z
$\boxed{0}$	∞	∞	∞	∞	∞
	$4a$	∞	$\boxed{2}a$	∞	∞
	$\boxed{4}a$	∞		$5d$	∞
		$7b$		$\boxed{5}d$	∞
		$7b$			$\boxed{6}e$

Thuật toán tìm đường đi ngắn nhất



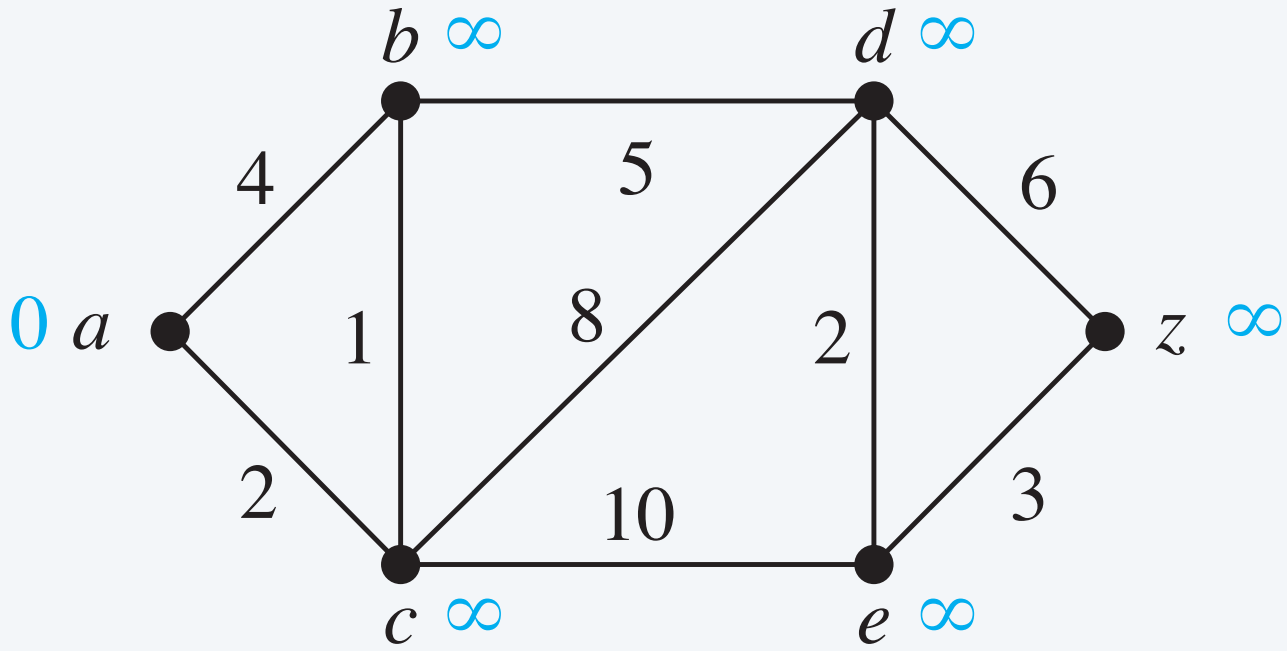
a	b	c	d	e	z
$\boxed{0}$	∞	∞	∞	∞	∞
	$4a$	∞	$\boxed{2}a$	∞	∞
	$\boxed{4}a$	∞		$5d$	∞
		$7b$		$\boxed{5}d$	∞
		$7b$			$\boxed{6}e$

- Đường đi ngắn nhất: $z \leftarrow e \leftarrow d \leftarrow a$.

Thuật toán `Dijkstra`(Đồ thị G có trọng số, một đỉnh a)

for $v \in V$ **do**
  $L[v] = \infty$
  $Truoc[v] = -1$
end
$L[a] = 0$
$S = \emptyset$
while $S \neq V$ **do**
  Tìm đỉnh $u \notin S$ có nhãn $L[u]$ nhỏ nhất
  $S = S \cup \{u\}$
  for $v \in V \setminus S$ **do**
    if $L[u] + w(u, v) < L[v]$ **then**
      $L[v] = L[u] + w(u, v)$
      $Truoc[v] = u$
    end
  end
end

Ví dụ



Ví dụ