

Dale Smith, Ph.D.  
Nexidia, Inc.

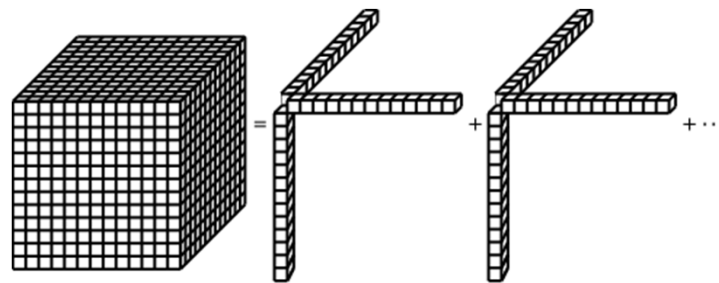
# Tensor Decompositions and Machine Learning

# What is a Tensor?

- We're moving from linear algebra to multilinear algebra...
- We know about vectors and matrices (linear transformations) from Linear Algebra. But tensors are not so familiar. A Rubik's Cube can help with visualization...and so can [What's a Tensor?](#) via YouTube!
- Think of a hypercube in your data warehouse.

# What is a Tensor Decomposition?

- Think of a hypercube in your data warehouse – can you do a decomposition into lower-rank objects that reveal hidden features or hierarchies?
- A rank-2 tensor decomposition – the Singular Value Decomposition – computes  $A = UDV^T$ . Principal Components Analysis approximates the original matrix by reducing the rank.
- For rank higher than 2 there are two types of decompositions: a sum of rank-1 tensors and a decomposition into orthonormal subspaces. Oh, and there is a multilinear PCA as well...



- Feature discovery via hierarchical structures in high-dimensional data sets.
- Recommender systems – similarity measures between users and items.
- Time-evolving network expressed as a 3-tensor.
- Topic modeling to extract hidden topics in a document corpus.
- Multilinear subspace learning – a technique of dimensionality reduction.
- Tensor methods are fast, scalable, and in many cases are embarrassingly parallel.

- Anima Anandkumar, et. al.: [\*Fast Detection of Overlapping Communities via Online Tensor Methods\*](#), JMLR 2014 and [\*Tensor Methods for Large-Scale Machine Learning\*](#), Strata+Hadoop, February 2015.
- Computational issues with estimating latent variable models, e.g. topic models.
  - Local optima.
  - Slow convergence.
  - Markov Chain Monte Carlo can fail.
- Tensor decomposition approaches *guarantee* finding the global optimum.
- Observed speedups of 100x over the Expectation-Maximization and other estimation methods using data from Facebook and Yelp restaurant reviews.

# Tensor Decomposition Software



MATLAB	C++	R	Python
<a href="#"><u>Tensor Toolbox</u></a>	<a href="#"><u>TensorDecomposition4Topic Modeling</u></a>	<a href="#"><u>rTensor</u></a>	<a href="#"><u>TensorToolbox</u></a>
<a href="#"><u>Tensorlab</u></a>	<a href="#"><u>TH++</u></a>		<a href="#"><u>PyTensor</u></a>
			<a href="#"><u>scikit-tensor</u></a>

- Survey papers with excellent visualizations of tensor decompositions
  - *Tensor Decompositions for Signal Processing Applications*, Cichocki, De Lathauwer, et. al., IEEE Signal Processing Magazine, March 2015 ([arXiv.org version](#)).
  - *Tensor Decompositions and Applications*, Tamara G. Kolda and Brett W. Bader, SIAM Review, 2009 ([Sandia pdf report](#)).
- 2009 Workshop Report: [\*Future Directions in Tensor-Based Computation and Modeling\*](#).
- Tutorials
  - [\*Tutorial on Spectral and Tensor Methods for Guaranteed Learning\*](#).
  - [\*Mining Large Time-evolving Data using Matrix and Tensor Tools\*](#).
  - [\*Mining and Forecasting of Big Time-Series Data\*](#).

- Blog Posts and Podcasts
  - Anima Anandkumar, [Tensor Methods for Large-Scale Machine Learning](#), Strata+Hadoop, February 2015.
  - Ben Lorica, [Let's build open source tensor libraries for data science](#), O'Reilly Radar, March 17, 2015.
  - [The tensor renaissance in data science](#) and associated podcast, O'Reilly Radar, May 7, 2015.
- [Tensor Decompositions in Smart Patient Monitoring](#), SIAM News, September 2015.
- SIAM Conference on Applied Linear Algebra, October 26<sup>th</sup> – 30<sup>th</sup>, Atlanta.
  - Sessions on Tuesday the 27<sup>th</sup>.
  - Keynote on Friday the 30<sup>th</sup>.
- [Workshop on Tensor Decompositions and Applications](#), January 2016.



- Do you have...
  - Time series of matrices or cubes?
  - Multiple aspects: time, location, type?
  - Time-evolving graphs?
  - Primary source data with multiple metadata elements?
- ...then you may want to consider tensor decomposition methods to estimate models.