Simple Beams Lab Report

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Abstract

This report presents a comprehensive comparison to assess the practicality of beam element theory (analytical solution), finite element analysis (computational solution) and real world trials (experimental solution) for modelling statically indeterminate beams, with a focus on predicting the displacements and strains of the beam vertically. The experimental procedure involves applying a variation of load along a statically indeterminate beam and measuring the displacement of the beam at the deflection gauges which work with the use of Wheatstone bridges at the 355mm position along the beam, providing precise data but subject to experimental uncertainties. FEA methods offer detailed simulations of beam behavior, providing displacement and strain predictions across various conditions with even different solvers working to improve accuracy in both 1D and 3D, albeit with computational resource demands. The software used in this report is ABAQUS by Dassault Systémes as it has both graphing and visual representational capabilities as shown in appendices A and B. Theoretical models, derived from beam element theory, offers analytical expressions for the deflection and consequently strain based on assumptions on the beam (following of Hooke's law), providing quick estimates but lacking accuracy in complex scenarios. By synthesising results from all three methods, a more comprehensive understanding of complex beam structures and deflections can be achieved, enhancing engineering design and analysis capabilities. The standards used within the experiment and FEA are BS EN ISO 4172:202 and BS EN 1993-1-8:2024 respectively. Additionally, the experimental data can be read through with appendix C and code used to generate graphs in MATLAB with appendix D as well as the FEA files which can be used through appendix E.

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Nomenclature

E	Young's Modulus (Pa)
v	Poisson Ratio (∘)
ε	Strain (∘)
I	Moment of Inertia (kgmm^2)
Δ	Deflection (mm)
x	Arbitrary distance (mm)
$M(x)$	Moment as a function of x (Nmm)
y	Perpendicular distance from the neutral axis (mm)
P	Load (N)
$U, U2$	Deflection in the Y direction (mm)

1 Introduction

Structural analysis is the prediction of the performance of a structure under prescribed loads and external effects such as support movements and temperature changes. The characteristics of performance which are of interest are the stresses on a beam, deflections on the nodes as well as the reactions of the supports.[1] Structural analysis emerged in the eighteenth century as a method to assess the safety of standing buildings and use the knowledge to design new buildings which are much more resourceful and structurally stronger to withstand decades of various condition. [2] The analysis of structures, however, introduces multiple engineering problems which range from assumptions used to simplify calculations and to make computational analysis tractable to reliability of a structure which relates to uncertainties and sensitivity studies to assess its safety. Additionally, problems can arise from computational solvers as more complex structures require significantly more memory and processing power as well as not converge at all making it problematic for highly nonlinear structures. Finally, if we were to assess the accuracy of a certain model to determine its reliability, building the model is costly and time-consuming endeavour which give rise to modeling limitations.

In this experiment the main question we aim to answer is how do displacements of the nodes and strain of the beam change when applying a fixed load to a statically indeterminate beam. Breaking this down, specifically, we aim to determine by how much a specific node displaces given constant material properties as well as accurately capture the beams behaviour along the length of the beam itself. Finally, investigate how these properties of structural analysis vary experimentally, analytically and computationally to assess the accuracy of all the three methods in predicting displacements and strains.

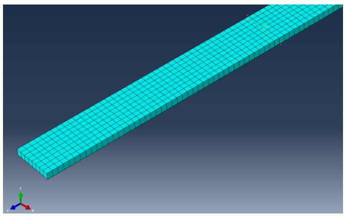
The study of structural beams plays a vital role in many aspects of technology used in everyday life, not just in building construction but also in aerospace applications for wing modelling as they are complex cantilevers, automotive engineering for the chassis of many cars and practically any application which uses a fixed surface with varying forces imposed upon it.[3] Beams can be split into two different categories based on if they are statically determinate and statically indeterminate. Statically determinate structures can be solved very easily as it involves the number of unknown reactions and the number of equilibrium equations available, ensuring a unique solution. This simplicity allows engineers to use easy mathematics to design statically determinate structures without the need for advanced techniques. However, such simplicity comes at a cost which is design flexibility and structural efficiency. Due to fixed support conditions they lack redundancy making them more susceptible to failure under certain loading conditions and do not always fully optimise the material's usage making them very inefficient.[4] On the other hand, statically indeterminate beams present significant challenges in analysis due to their inherent complexity and the lack of sufficient equilibrium equations to determine all internal forces and deformations directly. Indeterminate structures possess redundant supports and members, leading to an excess of unknowns, however, in contrast to this they redistribute internal forces and deformations incredibly effectively. Indeterminacy allows for localised failures and overloads by redistributing loads to other parts of the structure and the redundancy makes it more efficient use of materials, enabling engineers to design lighter and more economical structures while maintaining the required strength and stiffness.[5]

Experiments have been conducted in many ways with different materials and different number of supports and loads over the decades to accurately model indeterminate beams and some even using numerical solutions. The investigation of the beam problem started with Galileo in 1638 and the use of cantilevers but he could not make significant steps through indeterminacy due to the lack of knowledge about elasticity. The advancements of statically determinate beams continued until the late 19th century when Clerk Maxwell used Rankine's reciprocal diagrams to devise the theorem of reciprocal deflections which has led to the method of solving such problems.[6] Although many experiments have been conducted by Daniel Bernoulli, Charles Coulomb and Eaton Hodgkinson, the main experiment used to determine a way to solve indeterminate structures was by William H. Barlow who used a series of beams and cantilevers with joints introduced at flexure points and varies the forces incrementally, this knowledge was then used by Rankine to find the standard formulae to model the deflections. This was one of the first times the method of superposition was used to simplify measures and made huge contributions to beam theory which is now used in FEA.[7] In the analytical solutions to structural problems the standard related is BS EN ISO 4172:202, however, during the experiment the standards followed is BS EN 1993-1-8:2024 for joints using in the supports as well as analysis after is through uncertainties in the gradients of the graphs.

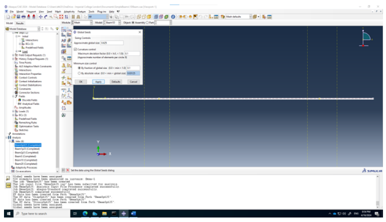
2 FE Model Setup and Mesh Convergence Studies

2.1 FE Model Setup

To model a beam with supports and loads in three dimensions is a computationally difficult task requiring lots of processing power. However, from it there is the ability to see the displacement and strain at different widths of the beam as opposed to its 1D counterpart in which you can only see the variation in length. To be able to model this all that is necessary is the CAD structure, in 3D - in this case a cuboid, and the modelling requirements of the part. The requirements allow the part to decompose such a difficult task into advantages we can use to make the problem easier and tractable. Initially, the material of the beam can be set to the appropriate Young's modulus, E, of 200GPa and Poisson ratio, v, of 0.3 affecting U and ε of the beam. Sections are created to pinpoint locations for adding loads and supports along the beam with the use of planes on datum points. By using planes set along the beam's length, the problem is simplified as only individual sections need consideration. Supports and loads are then initialised at specific points along the beam - 0mm, 135mm, 465mm and 575mm points - allowing the FEA tool to identify reaction forces as shown in figure 1b for the 1D model. Meshing can then occur with a predetermined size, in figure 1a the 3D mesh size used was 2.5mm, from here the processes used differs between 1D and 3D models. While the 1D model reduces complexity and time, it sacrifices accuracy regarding the model's width variation compared to the 3D model.



(a) Mesh Size 2.5mm

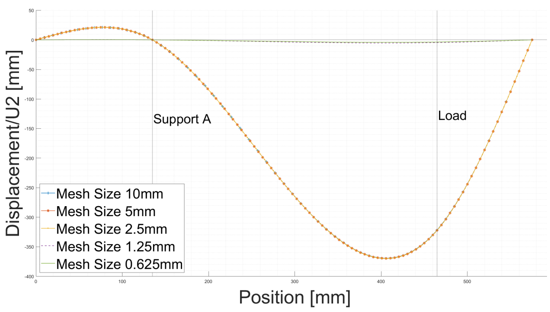


(b) 1D Model Set up

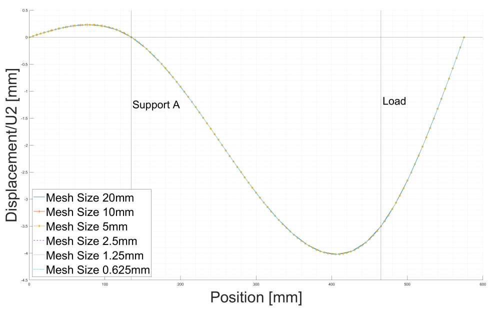
Figure 1: 3D and 1D Mesh model initialisation

2.2 Mesh Convergence

Conducting a convergence study in ABAQUS 2024 involves the refining of a mesh size to evaluate the sensitivity of our solution. By gradually adjusting these parameters, it is possible for engineers to observe how the solution approaches stability, ideally converging towards a consistent result. Convergence criteria are defined to determine when a solution is sufficiently converged, aiding in optimising computational resources while ensuring accurate results. These studies validate modeling approaches and ensure fidelity in capturing complex structural behaviors. As seen in appendix A, the gradual reduction in mesh size makes a significant difference along each dimension of the beam. Reducing the mesh size to 0.625mm yields the most optimal results in displacement and strain, as the accuracy achieved outweighs the processing power and rendering required compared to even smaller models possible. Figure 2a illustrates a significant reduction in mesh size beyond 2.5mm, stabilising thereafter, indicating 0.625mm as the most effective option without excessive resource consumption. Conversely, in the 1D model (Figure 2b), a mesh size of 10mm suffices for accurate solutions, aligning closely except for minor deviations at a microscopic scale with smaller mesh sizes. Visual representations are available in Appendix B within the ABAQUS software for both strain and displacement. In summary, the chosen mesh sizes are 0.625mm for 3D and 10mm for 1D FEA.



(a) 3D Displacement



(b) 1D Displacement

Figure 2: Determination of mesh size required for accurate yet resourceful solutions

3 Results and Discussions

3.1 Analytical Result

Deriving a solution to find the deflection and strain on a statically indeterminate beam can be accomplished in many ways. The approach I used involved the double integration method which calculates the deflection based on the moment distribution to solve for reactions, internal forces and compatibility equations to ensure continuity at connections as in equation 1. The strain can also be calculated by using a derivation of the strain formula involving the moment distribution of beam dimensions and material properties as shown in equation 2. The moment distribution can be formulated by using the temporary variation of length along with macaulay to recognise the points by which the specific moment changes.

$$EI\frac{d^2\triangle}{dx^2} = M(x) \tag{1}$$

$$\varepsilon = \frac{-M(x)}{EI}y \tag{2}$$

From these, after substituting in the necessary equations based on the material and the beam along with forming the moment equation for the indeterminate case the equations used for the analytical solution is shown below in equation 3 and 4.

$$\triangle = \frac{1}{EI}(\frac{-121}{2484}Px^3 + \frac{81675}{92}Px + \frac{91}{864}P < x - 135 >^3 - \frac{P}{6} < x - 465 >^3) \tag{3}$$

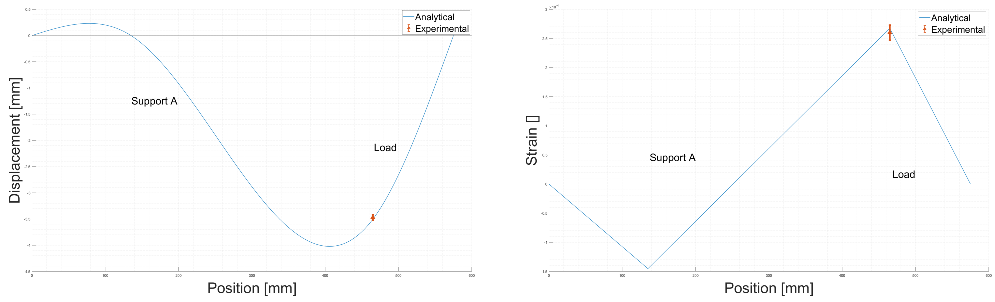
$$\varepsilon = \frac{y}{EI}(\frac{-121}{414}Px + \frac{91}{144}P < x - 135 > -P < x - 465 >) \tag{4}$$

3.2 Analytical comparison to experimental data

Comparing the analytical and experimental plots of the data for displacement. As seen in figure 3a, the general trend for the analytical is mostly decreasing and reaches a maximum positive displacement of 0.26mm at the point 85mm along the beam and the maximum negative displacement of -3.9mm at 410mm along the beam. This graph roughly resembles one of cubic origin with three roots as well as a maxima and a minima. Based on the experimental procedure undertaken under various loads at various positions and the change of the beam from statically determinate to indeterminate, there was only one situation in which a 40N load was applied at the 465mm position in the indeterminate case. In this position, the displacement reading was taken by the deflection gauge, calculates the deflection based on the Wheatstone bridge, which displayed -3.476mm and as the gauge is a reading, it comes with uncertainty equal to half the smallest number possible on the gauge giving a 0.82% error. This value lies closely with the experimental value and also lies within the error margins to the analytical value at the same point of -3.505mm. This close comparison at the particular point suggests to us that the analytical solution is the correct approach in to modelling a statically indeterminate beam as it approximately the same in the real world value. This is greatly important as it provides a much more mathematical way to approach such a problem without conducting the experiment every single time in various conditions which is both time intensive and inefficient as well as expensive for huge projects.

As seen in figure 3b, the general trend for the analytical strain is a series of linear curves changing only when a significant aspect of the beam is introduced, this is either supports (as seen by support A) and loads. The linearity comes from the assumptions made in beam theory, specifically Euler-Bernoulli beam theory.[8] Under these assumptions beams are assumed to deform only by bending and not by stretching or shearing as well as the material assuming to behave within the elastic limit. The graph reaches a maximum negative strain of -1.44E-4 at support A which is 135mm along the beam and the maximum positive strain of 2.67E-4 at the load which is 465mm along the beam. As the strain is calculated from the same experimental procedure and from the displacement value there is also only one experimental value demonstrable which is 2.624E-4 producing a 1.72% error. This value equally lies close to the value and within the error expected and gives us the same idea of being able to mathematically model the strain on an indeterminate beam.

Although, the values obtained experimentally are similar, they do not exactly match, however, this can be explained by various assumptions we have used. Analytically, there are various material properties idealised, as well as geometric imperfections in experimental setups. Additionally, differences in load application, support conditions, and measurement errors can contribute to discrepancies which have to be taken into consideration. Despite all of this, the solutions show how these two methods can both be used to determine important aspects of structures.



(a) Displacement

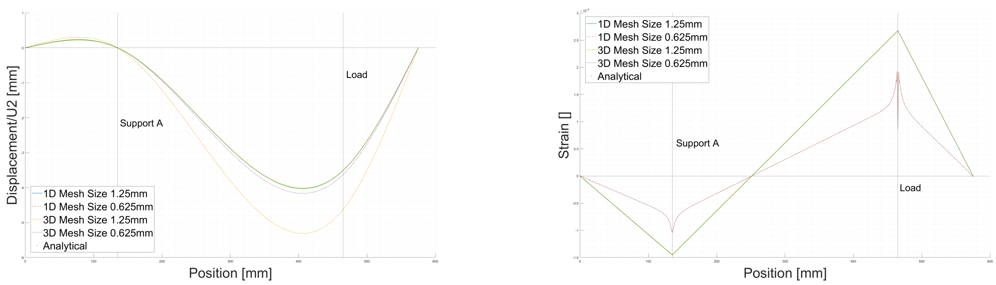
(b) Strain

Figure 3: Comparison of the experimental data to the analytical solution

3.3 Analytical comparison to FEA

As seen in figure 3, modelling using FEA software is an extremely consistent and accurate way of visualising the beams - even statically indeterminate ones - whilst being able to measure parameters along the beam without the use of many deflection gauges within the experiment. Looking at the displacement in figure 4a, the general trend of the FEA solution follows closely along with the trend of the analytical solution as it reaches maxima and minima at the same points along the beam however the values obtained are not exactly the same as the analytical but does converge towards it as the mesh size decreases (from -5.42mm to -4.19mm where the analytical is at -3.98mm at minima). This similarity trend follows the strain curve as well, the minima and maxima occur at the support and load points on the beam but the value associated converges towards the analytical. The 1D representation is very accurate because the beam is thin, and variations in stress and strain along its thickness are negligible compared to its length, making it more precise. Additionally, it's less affected by changes in mesh size along other dimensions, making the results more reliable and cost-effective.

From these 2 graphs as well as figure 2 in mesh convergence, it is possible to see that the number of elements required to accurately capture the beam's behaviour is at 1.25mm with the number of elements being 13800 using linear brick elements (C3D8R), similarly for 1D, on the microscopic scale, the one which accurately follows beam behaviour is 2.5mm with 2040 elements using B31. However, the choice of element type affects accuracy in the sense that different element types have different interpolation functions, schemes, and capabilities to represent different types of structures. These then impact the mathematics used and can drastically reduce the number of elements required and processing time as well as cost. In this, having used a quadratic solver (C3D20R), the accuracy of solutions is greatly increased as well as using B32 for 1D. The type of element used greatly affects the solution's accuracy and its alignment with analytical data. Some element types are better suited for specific geometries or loading conditions, such as shell or solid elements.



(a) Displacement

(b) Strain

Figure 4: Comparison of the experimental data to FEA at the load point

3.4 Experimental comparison to FEA & load variation

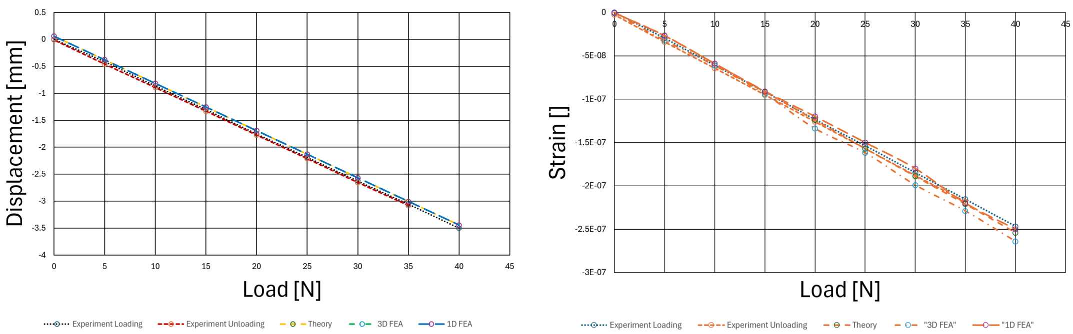
As seen in table 1, the comparison of experimental data with FEA results reveals several insights into the accuracy of the numerical simulations. The 1D FEA method tends to slightly overestimate both displacement and strain compared to the experimental data, although the differences are relatively small. Conversely, the 3D FEA method shows larger discrepancies, particularly in strain estimation, where it significantly underestimates the experimental value. The choice of mesh size also affects the results, with the finer mesh of the 3D FEA providing more accurate displacement but less accurate strain predictions compared to the coarser mesh of the 1D FEA. These comparisons underscore the importance of validating numerical simulations with experimental results to ensure the accuracy and reliability of FEA predictions.

Method	Displacement at Load [mm]	Strain at Load []
1D FEA mesh size 10mm	-3.505	2.458E-4
3D FEA mesh size 0.625mm	-3.622	1.929E-4
Experimental	-3.476	2.612E-4

Table 1: Comparison of the experimental data to FEA at the load point

Comparing the trend for variation of load to displacement and strain as shown in figure 5 below for all the different methods highlights the linear increase of the absolute value of displacement (figure 5a) and strain (figure 5b). The gradients vary at an uncertainty of 2.3% compared to the analytical and the y-intercept only differs on the unloading experiencing discrepancies due to hysteresis of minute plastic deformation of the beam, this could also just be an anomalous piece of data to discard as zero load is zero displacement hinting at a random error within the gauge. The main reason behind the linearity is the elastic behaviour of metals such as the steel used in conjunction with Hooke's law[9]. Also since the load is small in comparison to the strength of the material as load increases, so does deformation linearly. This close variation depicts clearly how all methods can be used for the modelling of beams to simplify the analysis of real world structures.

Although in this case, a finer mesh did improve accuracy for the variation of load, under greater stress it doesn't ensure the same level of accurate predictions. FEA's assumptions of linear material behavior and simplified boundary conditions may not reflect reality especially at high loads. Numerical solvers also struggle with nonlinear problems and modelling boundary conditions to reflect real constraints without experimental validation, so the behavior of structures under these loads involves complexities that FEA struggles to capture.



(a) Displacement

(b) Strain

Figure 5: Variation of loads with displacement and strain for all methods

4 Conclusions

In conclusion, the different approaches have own set of advantages, disadvantages and limitations and gave great insights when compared and shows how they are all valid ways for analysing complex structures in the real world. The FEA approach offers flexibility and the ability to model complex geometries and loading conditions. However, its accuracy depends on factors such as mesh density and model assumptions, and it may struggle with nonlinear behavior. Experimental testing provides direct and tangible data, offering insights into real world behavior. Yet, experiments can be costly, time-consuming, and subject to limitations such as test setup complexity and environmental factors. Analytical methods, including mathematical equations and theoretical models, offer simplicity and quick insights. However, they often rely on simplifying assumptions that may not accurately represent real-world behavior, especially for complex structures or loading conditions. The variation of results is very small, differing by only 2.3% in gradients for the variation of load for strain and displacement and the experimental only varies by 1.72% to the analytical, these findings also agree closely to the results of Garber and Gallardo showing the reproducibility of the experiments in producing the same conclusions.[10]. However, if repeated, I would conduct several more experiments to reduce uncertainty whilst going until the beam fails to determine the accuracy of FEA in the failure scenarios.

References

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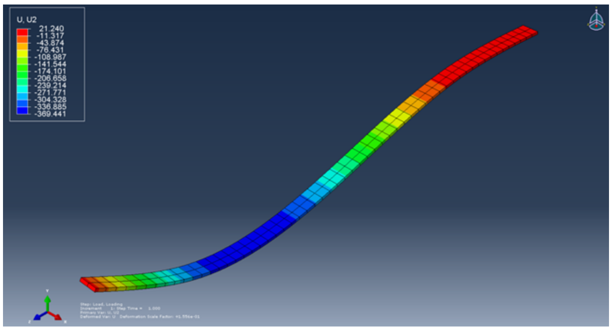
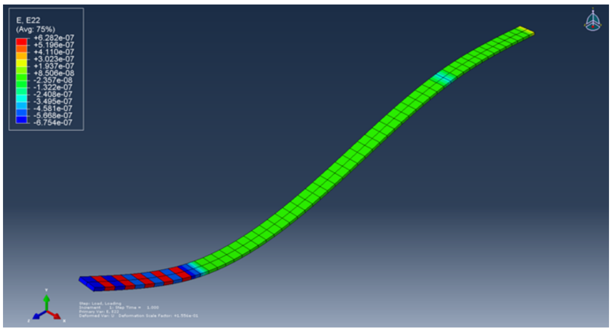
Appendix

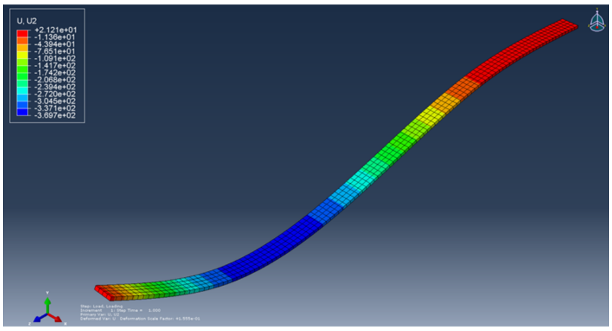
4.1 Appendix A: 3D FEA Full Mesh Analysis



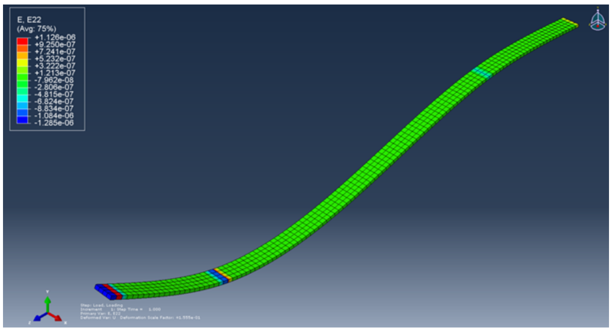
Displacement 10mm



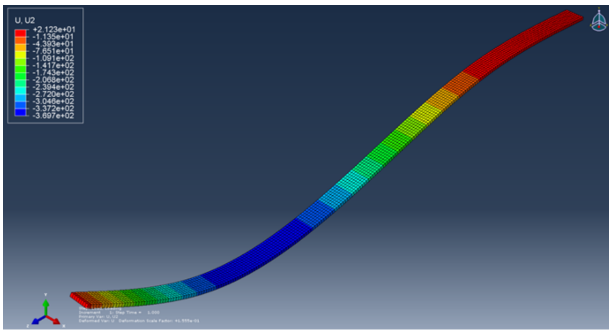
Strain 10mm



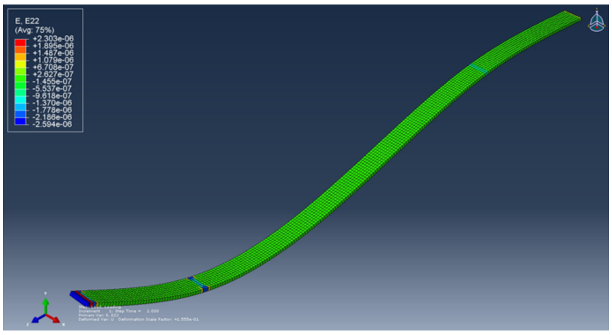
Displacement 5mm



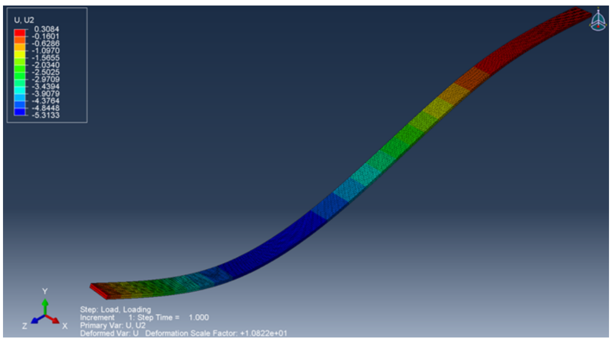
Strain 5mm



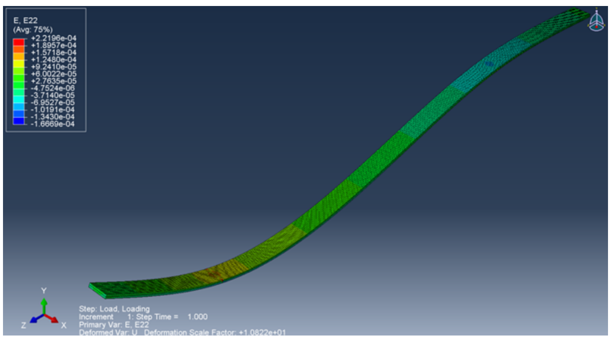
Displacement 2.5mm



Strain 2.5mm

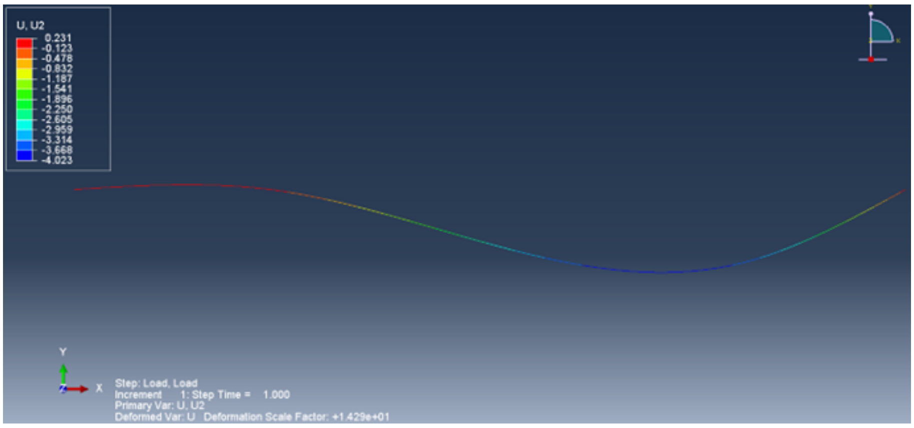


Displacement 1.25mm

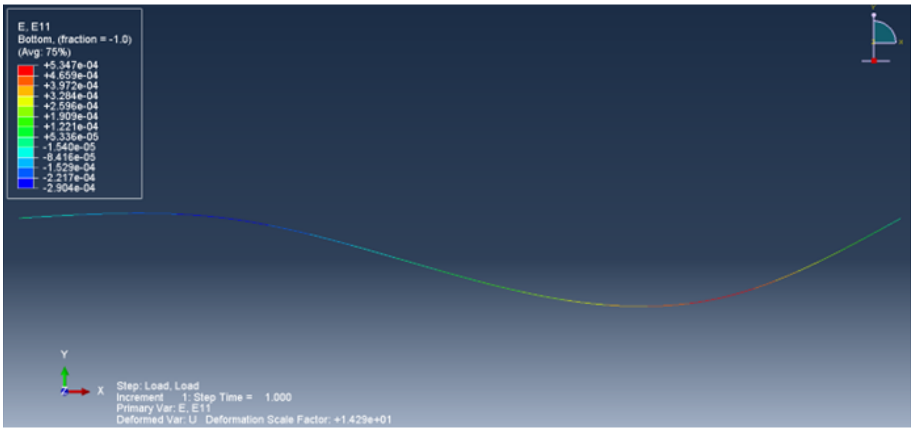


Strain 1.25mm

4.2 Appendix B: 1D FEA Full Mesh Analysis



Displacement



Strain

4.3 Appendix C: Link to Detailed Experimental Data

EXPERIMENTAL DATA

4.4 Appendix D: Link to MATLAB Graph Generation Program

MATLAB PROGRAMMING FILES

4.5 Appendix E: Link to ABAQUS Scripts

ABAQUS FILES