Artificial Intelligence

ADVERSARIAL SEARCH

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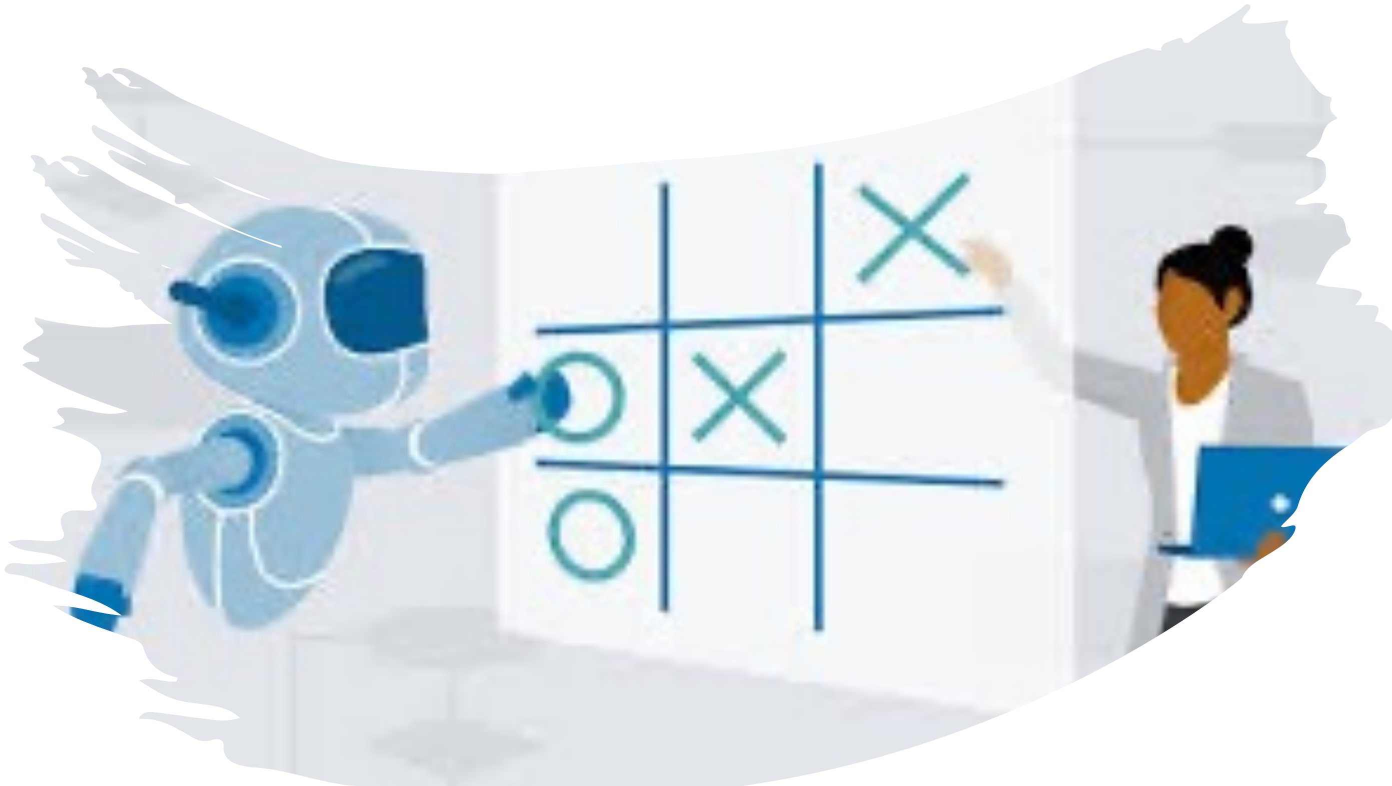
Outline

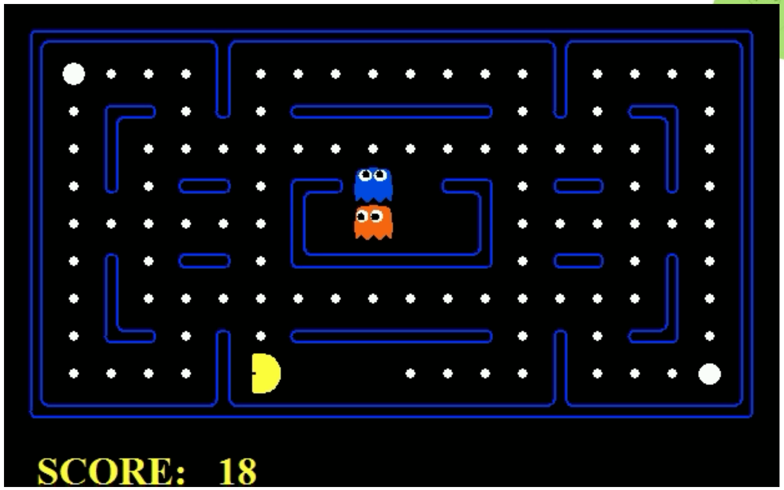
- The concept of games in AI
- Optimal decisions in games
- α-β Pruning
- Imperfect, real-time decisions
- Stochastic games



The concept of games in AI

Search in multiagent environments

- Each agent needs to consider the actions of other agents and how they affect its own welfare.
- The unpredictability of other agents introduce contingencies into the agent's problem-solving process

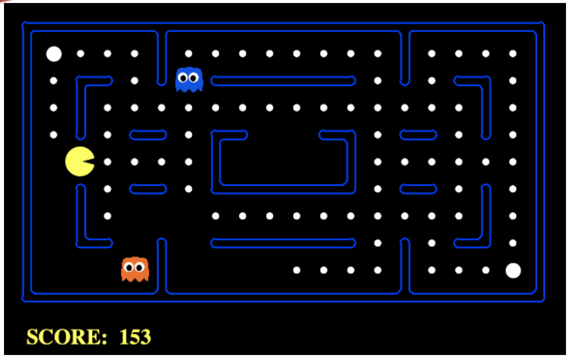


Game theory

- Game theory views any multiagent environment as a game.
  - The impact of each agent on the others is “significant,” regardless of whether the agents are cooperative or competitive.
- Types of games

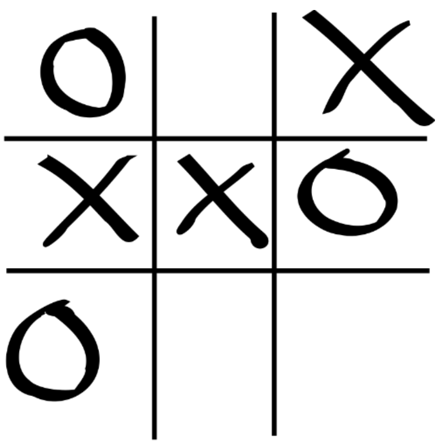
	Deterministic	Chance
Perfect information	Chess, Checkers, Go, Othello	Backgammon, Monopoly
Imperfect information		Bridge, poker, scrabble nuclear war

Types of Games



Adversarial search

- **Adversarial search** (known as **games**) covers competitive environments in which the agents' goals are in conflict.
- **Zero-sum games** of perfect information
  - Deterministic, fully observable environments, turn-taking, two-player
  - The utility values at the end are always equal and opposite.



Games vs. Search problems

- Complexity: games are too hard to be solved
  - Chess: $b \approx 35$, $d \approx 100$ (50 moves/player) $\rightarrow$ graph of 10^{40} nodes, search tree of 35^{100} or 10^{154} nodes
  - Go: $b \approx 1000$ (!)
- Time limits: make some decision even when calculating the optimal decision is infeasible
- Efficiency: penalize inefficiency severely
  - Several interesting ideas on how to make the best possible use of time are spawn in game-playing research.

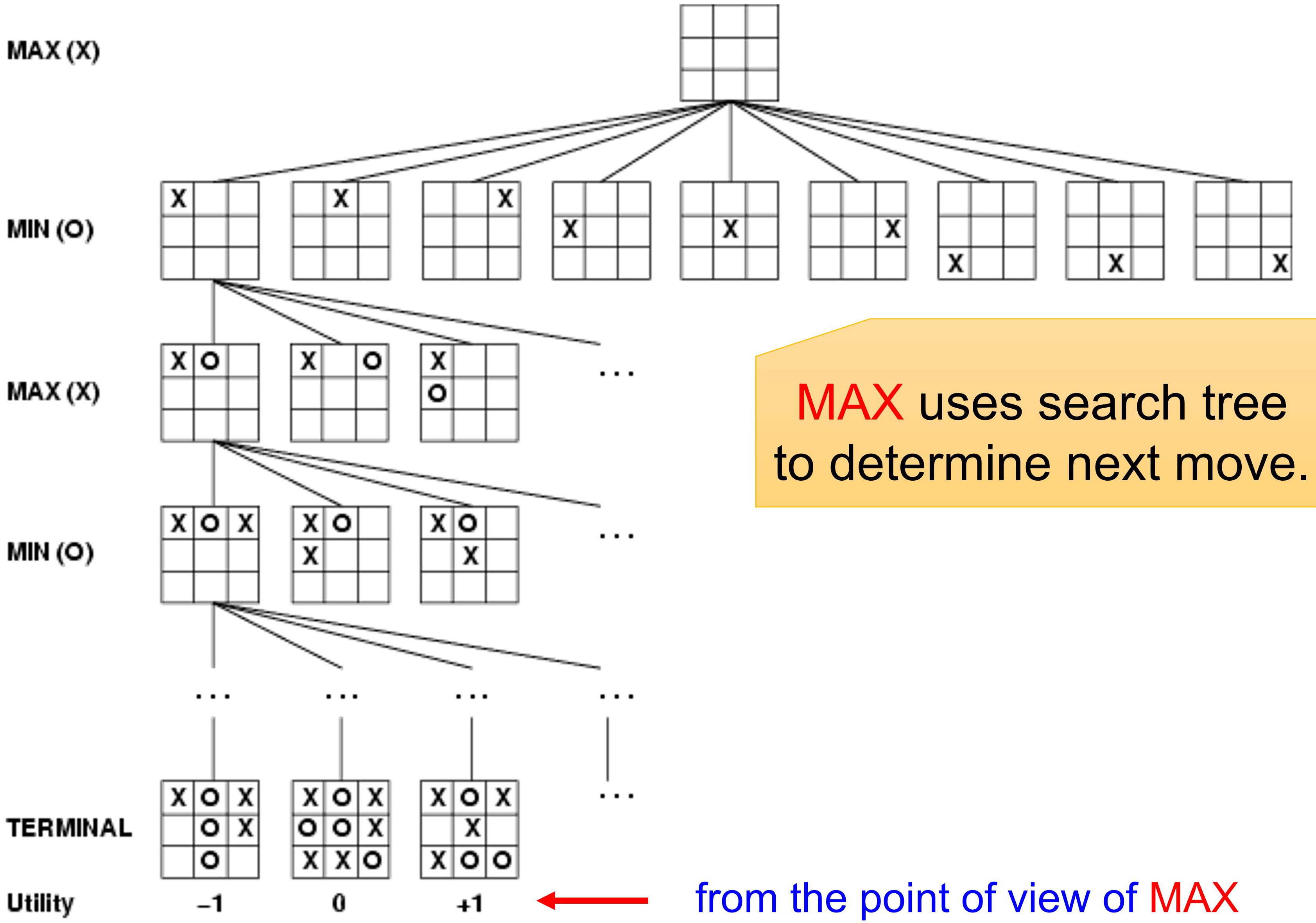
Primary assumptions

- Two players only, called MAX and MIN.
  - MAX moves first, and then they take turns moving until the game ends
  - Winner gets reward, loser gets penalty.
- Both players have complete knowledge of the game's state
  - E.g., chess, checkers and Go, etc. Counter examples: poker
- No element of chance
  - No dice thrown, no cards drawn, etc.
- Zero-sum games
  - The total payoff to all players is the same for every game instance.
- Rational players
  - Each player always tries to maximize his/her utility

Games as search

- S_0 – Initial state: How the game is set up at the start
  - E.g., board configuration of chess
- $\mathit{PLAYER}(s)$: Which player has the move in a state, MAX/MIN?
- $\mathit{ACTIONS}(s)$ – Successor function: A list of (move, state) pairs specifying legal moves.
- $\mathit{RESULT}(s, a)$ – Transition model: Result of move a on state s
- $\mathit{TERMINAL} - \mathit{TEST}(s)$: Is the game finished?
  - States where the game has ended are called **terminal states**
- $\mathit{UTILITY}(s, p)$ – Utility function: A numerical value of a terminal state s for a player p
  - E.g., chess: win (**+1**), lose (**-1**) and draw (**0**), backgammon: [0, 192]

The game tree of Tic-Tac-Toe



MAX uses search tree to determine next move.

Examples of game: Checkers

- Complexity
  - ~ 10^{18} nodes, which may require 100k years with 106 positions/sec
- **Chinook** (1989-2007)
  - The first computer program that won the world champion title in a competition against humans
  - 1990: won 2 games in competition with world champion Tinsley (final score: 2-4, 33 draws). 1994: 6 draws
- Chinook's search
  - Ran on regular PCs, played perfectly by using alpha-beta search combining with a database of 39 trillion endgame positions

Examples of game: Chess

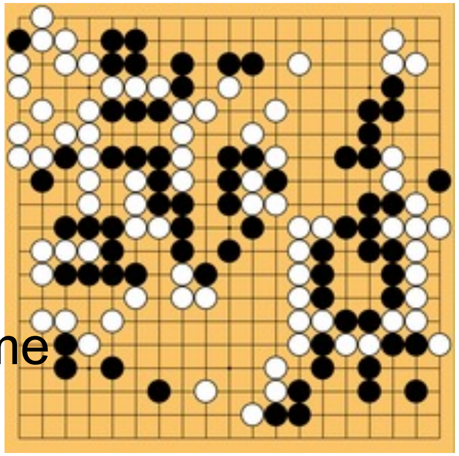
- Complexity
  - $b \approx 35$, $d \approx 100$, 10^{154} nodes (!!)
  - Completely impractical to search this
- **Deep Blue** (May 11, 1997)
  - Kasparov lost a 6-game match against IBM's Deep Blue (1 win Kasp – 2 wins DB) and 3 ties.
- In the future, focus will be to allow computers to **LEARN** to play chess rather than being **TOLD** how it should play

Deep Blue

- Ran on a parallel computer with 30 IBM RS/6000 processors doing alpha–beta search
- Searched up to 30 billion positions/move, average depth 14 (be able to reach to 40 plies)
- Evaluation function: 8000 features
  - highly specific patterns of pieces (~4000 positions)
  - 700,000 grandmaster games in database
- Working at 200 million positions/sec, even Deep Blue would require $\mathbf{10^{100}}$ years to evaluate all possible games.
  - (The universe is only 10^{10} years old.)
- Now: algorithmic improvements have allowed programs running on standard PCs to win World Computer Chess Championships.
  - Pruning heuristics reduce the effective branching factor to less than 3

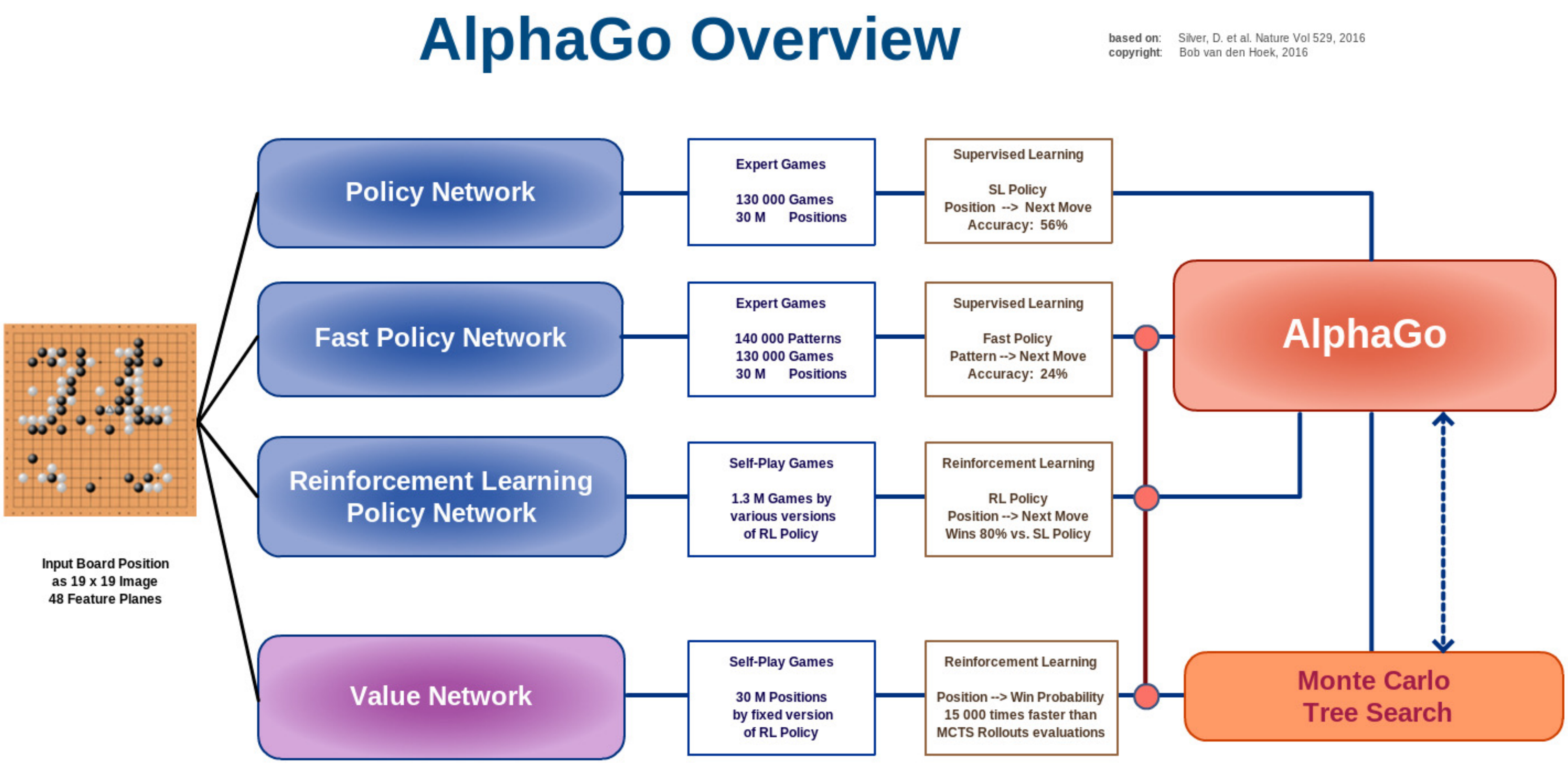
GO

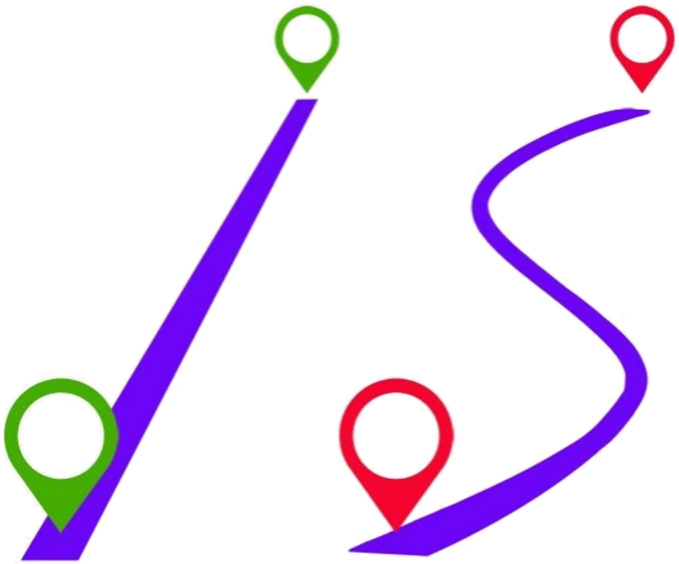
1 million trillion trillion trillion trillion more configurations than chess!



- Complexity
  - Board of 19x19, $b \approx 361$, average depth ≈ 200
  - 10^{174} possible board configuration.
  - Control of territory is unpredictable until the endgame.
- **AlphaGo** (2016) by Google
  - Beat 9-dan professional Lee Sedol (4-1)
  - Machine learning + Monte Carlo search guided by a "value network" and a "policy network" (implemented using *deep neural network* technology)
  - Learn from human + Learn by itself (self-play games)

An overview of AlphaGo





Optimal decisions in games

- *Minimax algorithm*
- *Optimal decisions in multiplayer games*

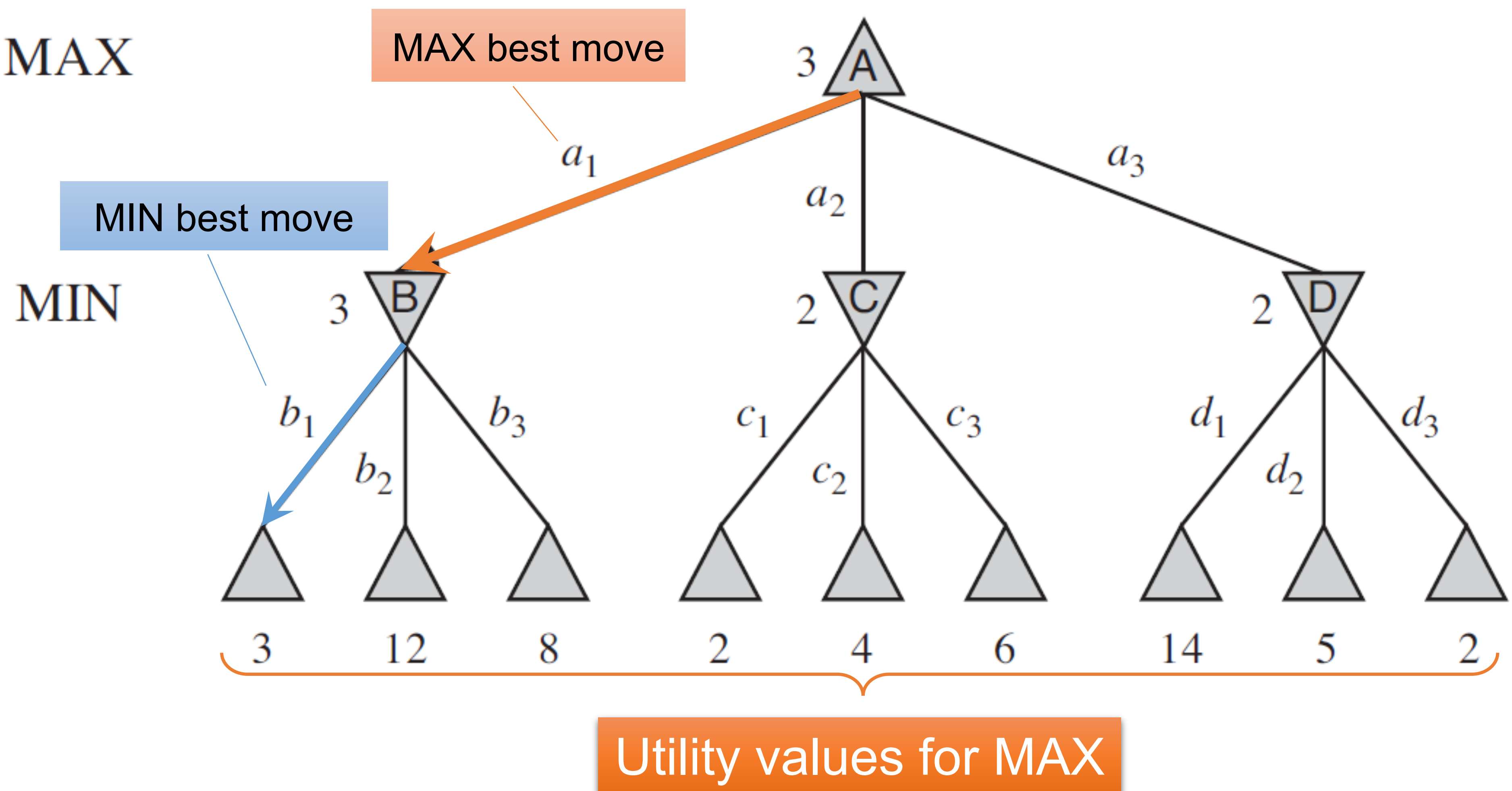
Optimal decision in games

- Normal search problem
  - The optimal solution is a sequence of action leading to a goal state.
- Games
  - The optimal strategy is a search path that guarantee win for a player
  - This can be determined from the **minimax value** of each node.

$$\textsc{Minimax}(s) = \begin{cases} \boxed{\textsc{Utility}(s)} \ \text{For MAX} & \text{if } \textsc{Terminal-Test}(s) \\ \max_{a \in Actions(s)} \textsc{Minimax}(\textsc{Result}(s,a)) & \text{if } \textsc{Player}(s) = \textsc{max} \\ \min_{a \in Actions(s)} \textsc{Minimax}(\textsc{Result}(s,a)) & \text{if } \textsc{Player}(s) = \textsc{min} \end{cases}$$

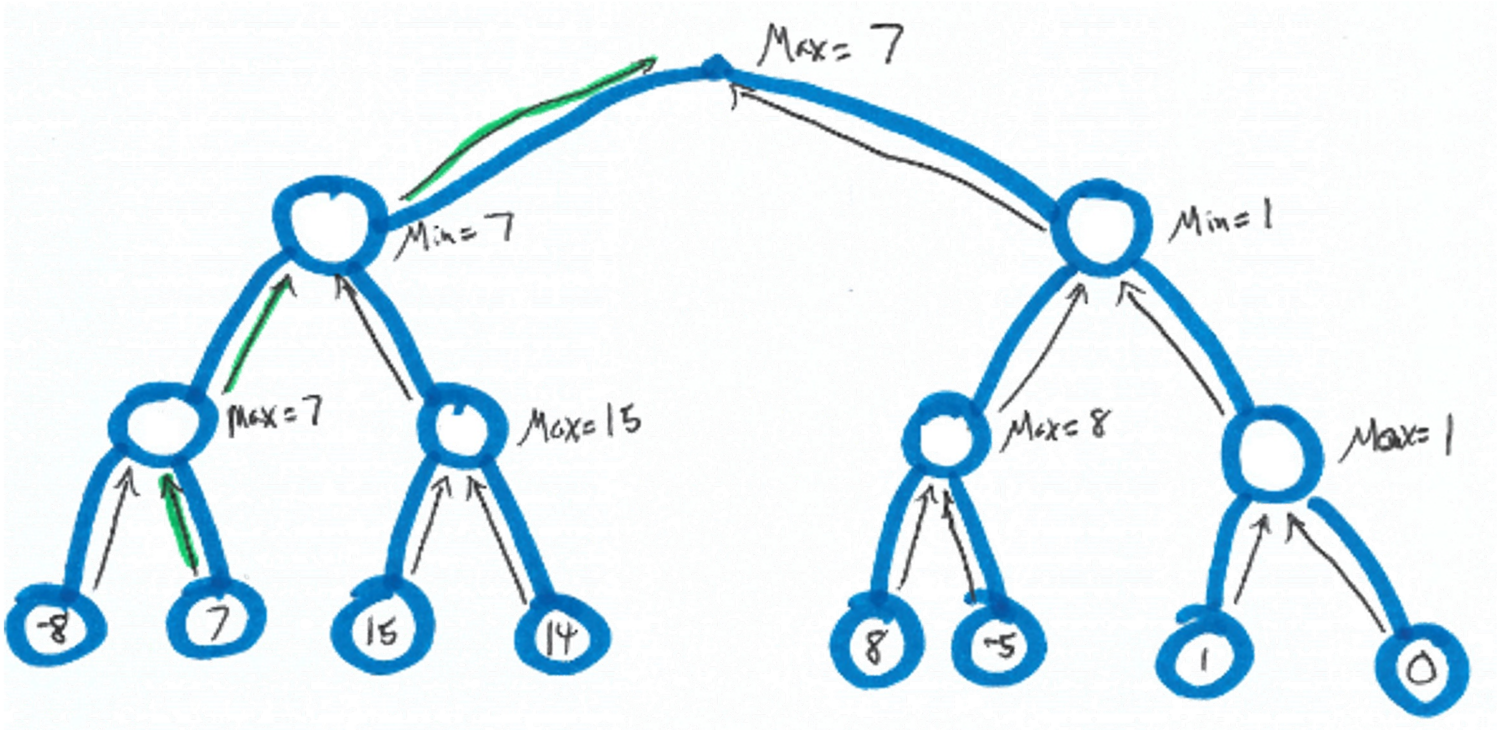
Assume that both players play optimally from there to the end of the game

An example of two-ply game tree



Minimax algorithm

- Make a minimax decision from the current state, using a recursive computation of minimax values at each successor
  - The recursion proceeds all the way down to the leaves, and then back up the minimax values through the tree as it unwinds.



Minimax algorithm

function MINIMAX-DECISION(*state*) **returns** an action
  return arg $\max_{a \in \text{ACTIONS}(s)}$ MIN-VALUE(RESULT(*state, a*))

---

function MAX-VALUE(*state*) **returns** a utility value
  if TERMINAL-TEST(*state*) **then return** UTILITY(state)
  $v \leftarrow -\infty$
  for each *a* **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(RESULT(*s, a*)))
  return v

---

function MIN-VALUE(*state*) **returns** a utility value
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow \infty$
  for each *a* **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(RESULT(*s, a*)))
  return v

Properties of Minimax algorithm

- A complete depth-first exploration of the game tree
- Completeness
  - Yes (if tree is finite)
- Optimality
  - Yes (against an optimal opponent)
- Time complexity
  - $O(b^m)$
- Space complexity
  - $O(bm)$ (depth-first exploration)

Note:

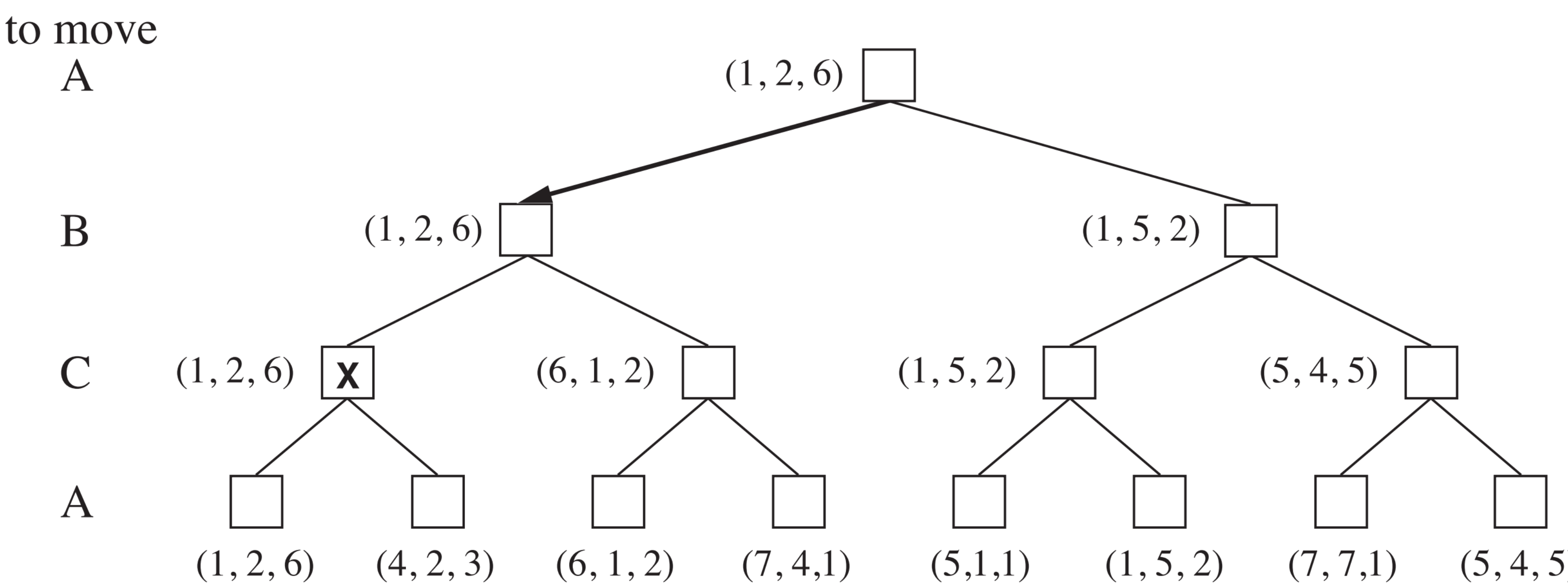
m: the maximum depth of the tree

b: the legal moves at each point

For chess, $b \approx 35, m \approx 100$ for "reasonable" games
$\rightarrow$ exact solution completely infeasible

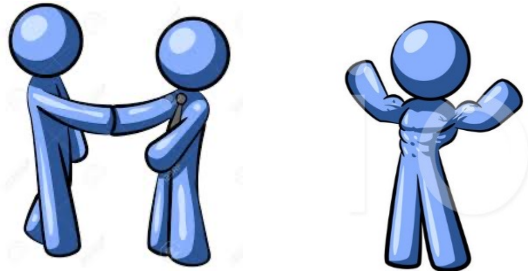
Optimality in multiplayer games

- A single value is replaced with a vector of values.
  - → the UTILITY function returns a vector of utilities
- For terminal states, this vector gives the utility of the state from each player's viewpoint.

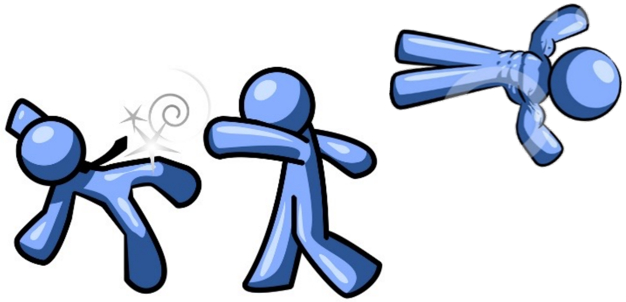


Optimality in multiplayer games

- Multiplayer games usually involve alliances, which are made and broken as the game proceeds.



A and B are weak while C is strong.
A forms an alliance with B.



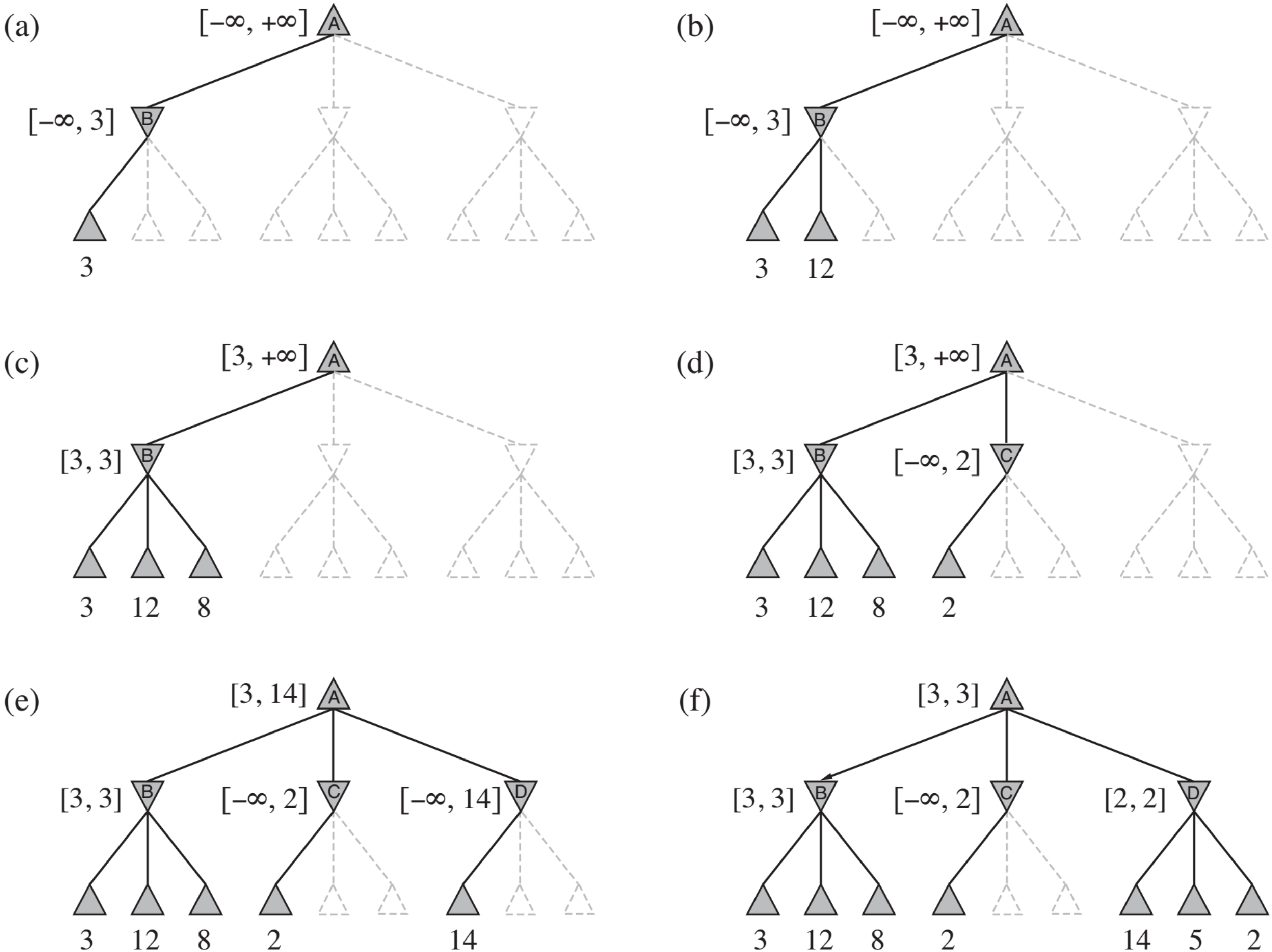
C becomes weak.
A or B could violate the agreement

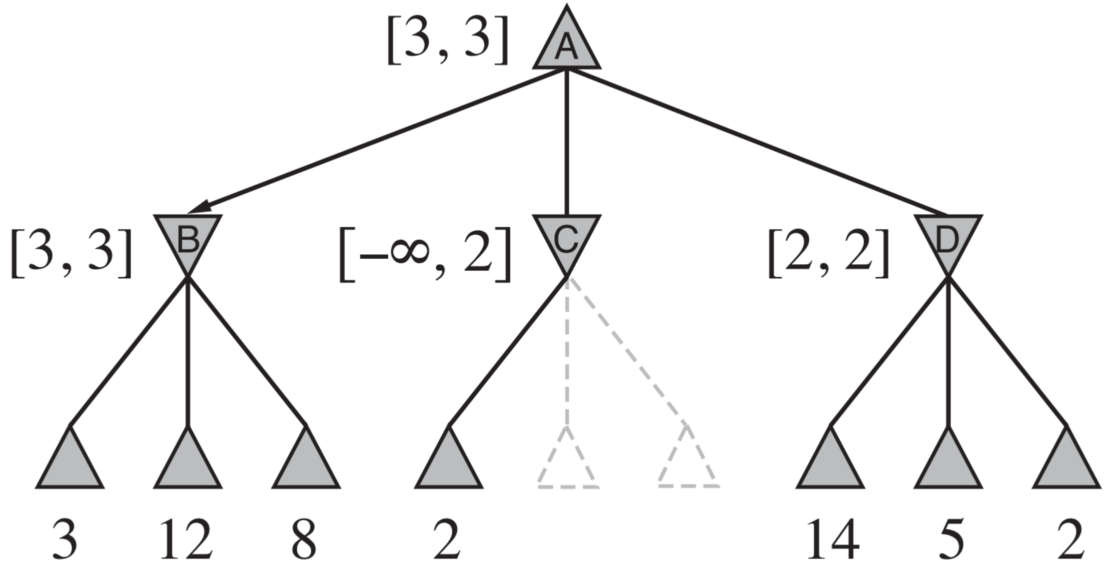
- If the game is not zero-sum, then collaboration can also occur with just two players.

Alpha-beta pruning

Problem with minimax search

- The number of game states is exponential in the tree's depth
  $\rightarrow$ Do not examine every node
- Alpha-beta pruning: Prune away branches that cannot possibly influence the final decision
- Bounded lookahead
  - Limit depth for each search
  - This is what chess players do: look ahead for a few moves and see what looks best


(a)
[−∞, +∞]
A
[−∞, 3]
B
3
(b)
[−∞, +∞]
A
[−∞, 3]
B
3
12
(c)
[3, +∞]
A
[3, 3]
B
3
12
8
(d)
[3, +∞]
A
[3, 3]
B
[−∞, 2]
C
3
12
8
2
(e)
[3, 14]
A
[3, 3]
B
[−∞, 2]
C
[−∞, 14]
D
3
12
8
2
14
(f)
[3, 3]
A
[3, 3]
B
[−∞, 2]
C
[2, 2]
D
3
12
8
2
14
5
2



Another way to look at this is as a simplification of the formula for MINIMAX.

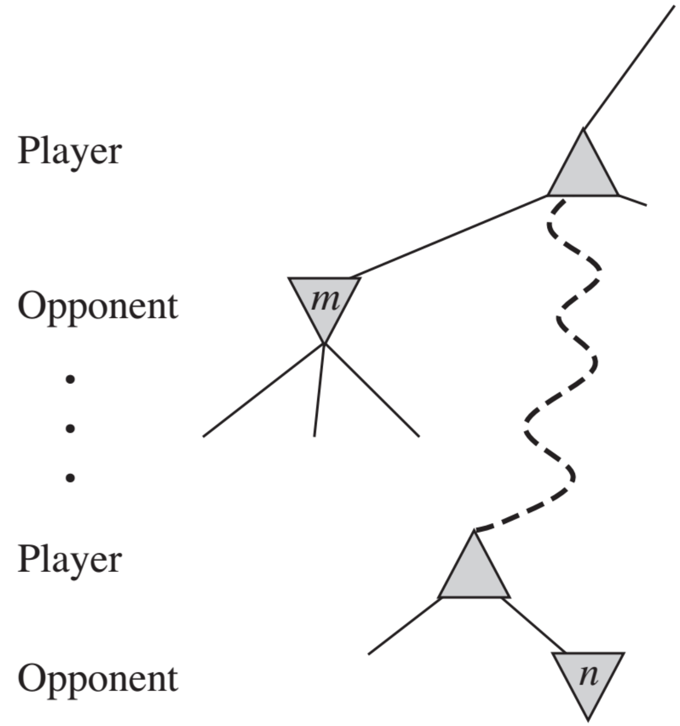
Let the two unevaluated successors of node C have values x and y.

Then the value of the root node is given by

$$\begin{aligned}
\textsc{Minimax}(root) &= \max(\min(3, 12, 8), \min(2, x, y), \min(14, 5, 2)) \\
&= \max(3, \min(2, x, y), 2) \\
&= \max(3, z, 2) \qquad \text{where } z = \min(2, x, y) \leq 2 \\
&= 3.
\end{aligned}$$

Alpha-beta pruning

- If a move n is determined to be worse than move m that has already been examined and discarded, then examining move n once again is pointless.



α = the value of the best (i.e., highest-value) choice we have found so far at any choice point along the path for MAX.

β = the value of the best (i.e., lowest-value) choice we have found so far at any choice point along the path for MIN.

Alpha-beta search algorithm

function ALPHA-BETA-SEARCH(*state*) **returns** an action
    $v \leftarrow$ MAX-VALUE(state,-∞,+∞)
    return the action in ACTIONS(*state*) with value v

---

function MAX-VALUE(*state*,α,β) **returns** a utility value
    if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
    $v \leftarrow -\infty$
    for each a **in** ACTIONS(*state*) **do**
        $v \leftarrow$ MAX(v, MIN-VALUE(RESULT(s,a),α,β))
        if $v \geq \beta$ **then return** v
        $\alpha \leftarrow$ MAX(α, v)
return v

Alpha-beta search algorithm

function MIN-VALUE(*state*,α,β) **returns** a utility value
  if TERMINAL-TEST(*state*) **then return** UTILITY(state)
  $v \leftarrow +\infty$
  for each *a* **in** ACTIONS(state) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(RESULT(s,a) ,α,β))
    if $v \leq \alpha$ **then return** v
    $\beta \leftarrow$ MIN(β, v)
  return v

Properties of alpha-beta pruning

- Pruning does not affect the result
  - Its worst case is as good as the minimax algorithm
- Good move ordering improves effectiveness of pruning
  - With "perfect ordering“: time complexity $O(b^{m/2}) \rightarrow$ x2 search depth
  - The effective branching factor becomes $\sqrt{b}$ instead of b.
    - E.g., for chess, about 6 instead of 35.

Imperfect real-time decisions

- *Evaluation functions*
- *Cutting off search*
- *Forward pruning*
- *Search versus Lookup*

Heuristic minimax

- Both minimax and alpha-beta pruning search all the way to terminal states.
  - This depth is usually **impractical** because moves must be made in a reasonable amount of time (~ minutes).
- Cut off the search earlier with some depth limit
- Use an evaluation function
  - An estimation for the desirability of position (win, lose, tie?)

$$\text{H-MINIMAX}(s,d) = \begin{cases} \boxed{\text{EVAL}(s) \qquad \text{if CUTOFF-TEST}(s,d)} \\ \max_{a \in Actions(s)} \text{H-MINIMAX}(\text{RESULT}(s,a), d+1) & \text{if PLAYER}(s) = \text{MAX} \\ \min_{a \in Actions(s)} \text{H-MINIMAX}(\text{RESULT}(s,a), d+1) & \text{if PLAYER}(s) = \text{MIN}. \end{cases}$$

Evaluation functions

- These evaluation function should order the terminal states in the same way as the true utility function does
  - States that are wins must evaluate better than draws, which in turn must be better than losses.
- The computation must not take too long!
- For nonterminal states, their orders should be strongly correlated with the actual chances of winning.

Evaluation functions

- For chess, typically linear weighted sum of features

$$\boldsymbol{Eval(s) = w_1 f_1 (s) + w_2 f_2(s) + \ldots + w_n f_n(s)}$$

  - where f_i could be the numbers of each kind of piece on the board, and w_i could be the values of the pieces
  - E.g., $Eval(s) = 9q + 5r + 3b + 3n + p$
- Implicit strong assumption: the contribution of each feature is independent of the values of the other features.
  - E.g., assign the value 3 to a bishop ignores the fact that bishops are more powerful in the endgame → **Nonlinear combination**

Cutting off search

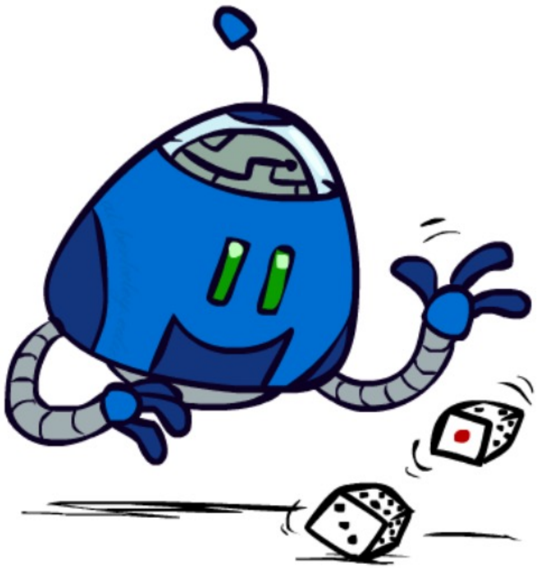
- *Minimax Cutoff* is identical to *Minimax Value* except
  1. $Terminal?$ is replaced by $Cutoff?$
  2. $Utility$ is replaced by $Eval$

if CUTOFF-TEST(state, depth) **then return** EVAL(state)

- Does it work in practice?
  - $b^m = 10^6, b = 35 \rightarrow m = 4$
  - 4-ply lookahead is a hopeless chess player!
  - 4-ply ≈ human novice, 8-ply ≈ typical PC, human master, 12-ply ≈ Deep Blue, Kasparov

A more sophisticated cutoff test

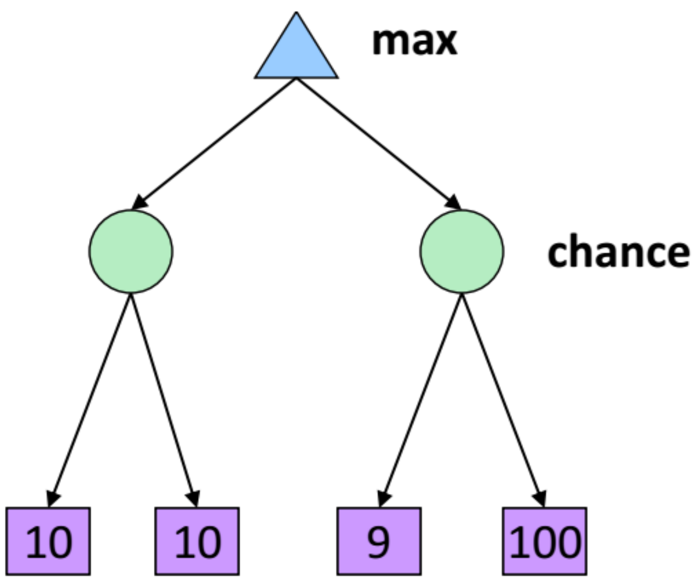
- Quiescent positions are those unlikely to exhibit wild swings in value in the near future.
    - E.g., in chess, positions in which favorable captures can be made are not quiescent for an evaluation function counting material only
- **Quiescence search:** expand nonquiescent positions until quiescent positions are reached.

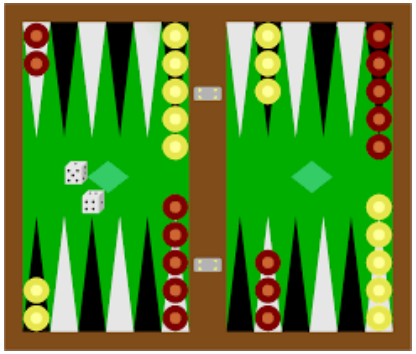


Stochastic games

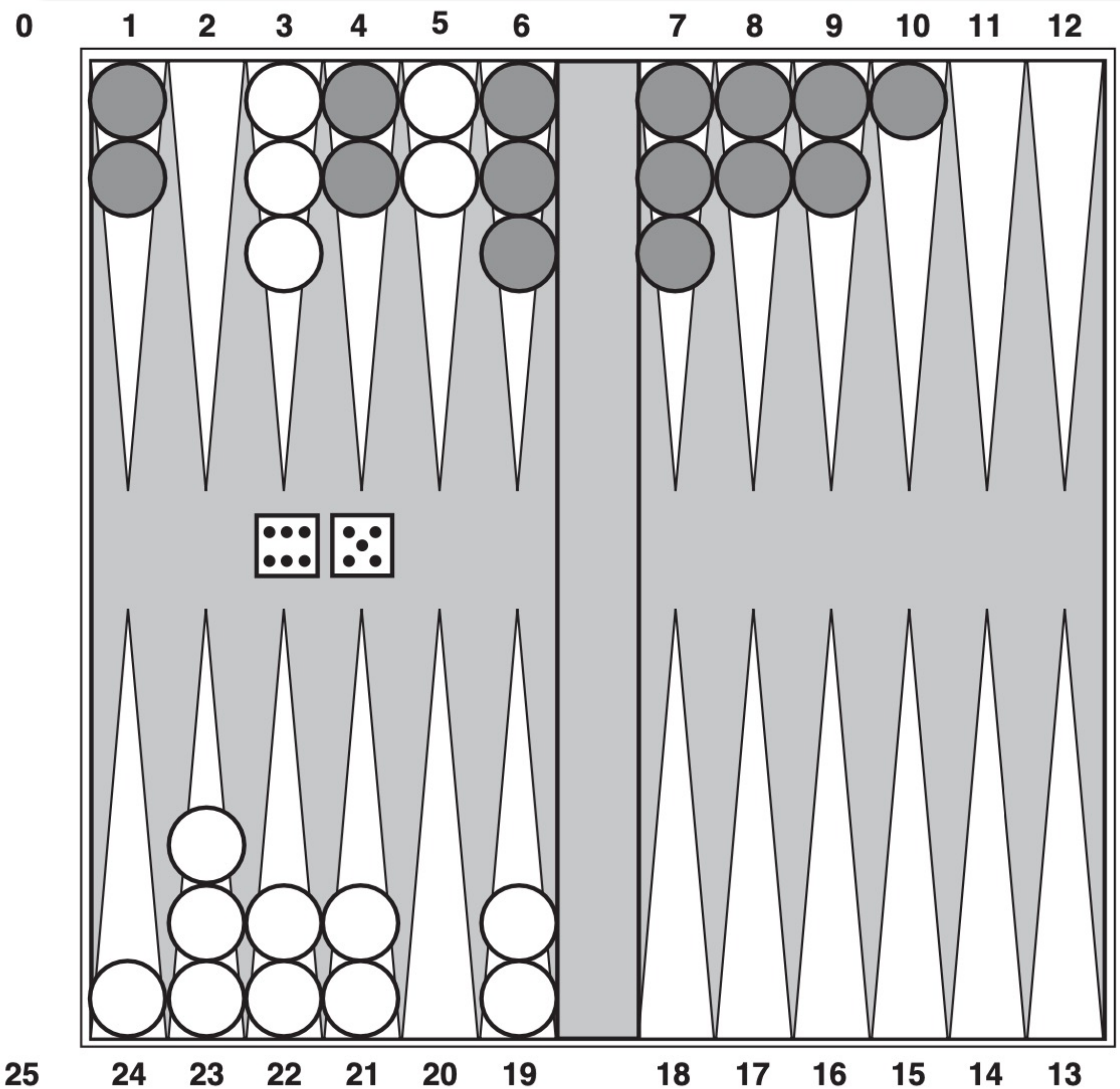
Stochastic behaviors

- Uncertain outcomes controlled by chance, not an adversary!
- *Why wouldn't we know what the result of an action will be?*
  - Explicit randomness: rolling dice
  - Unpredictable opponents: the ghosts respond randomly
  - Actions can fail: when a robot is moving, wheels might slip



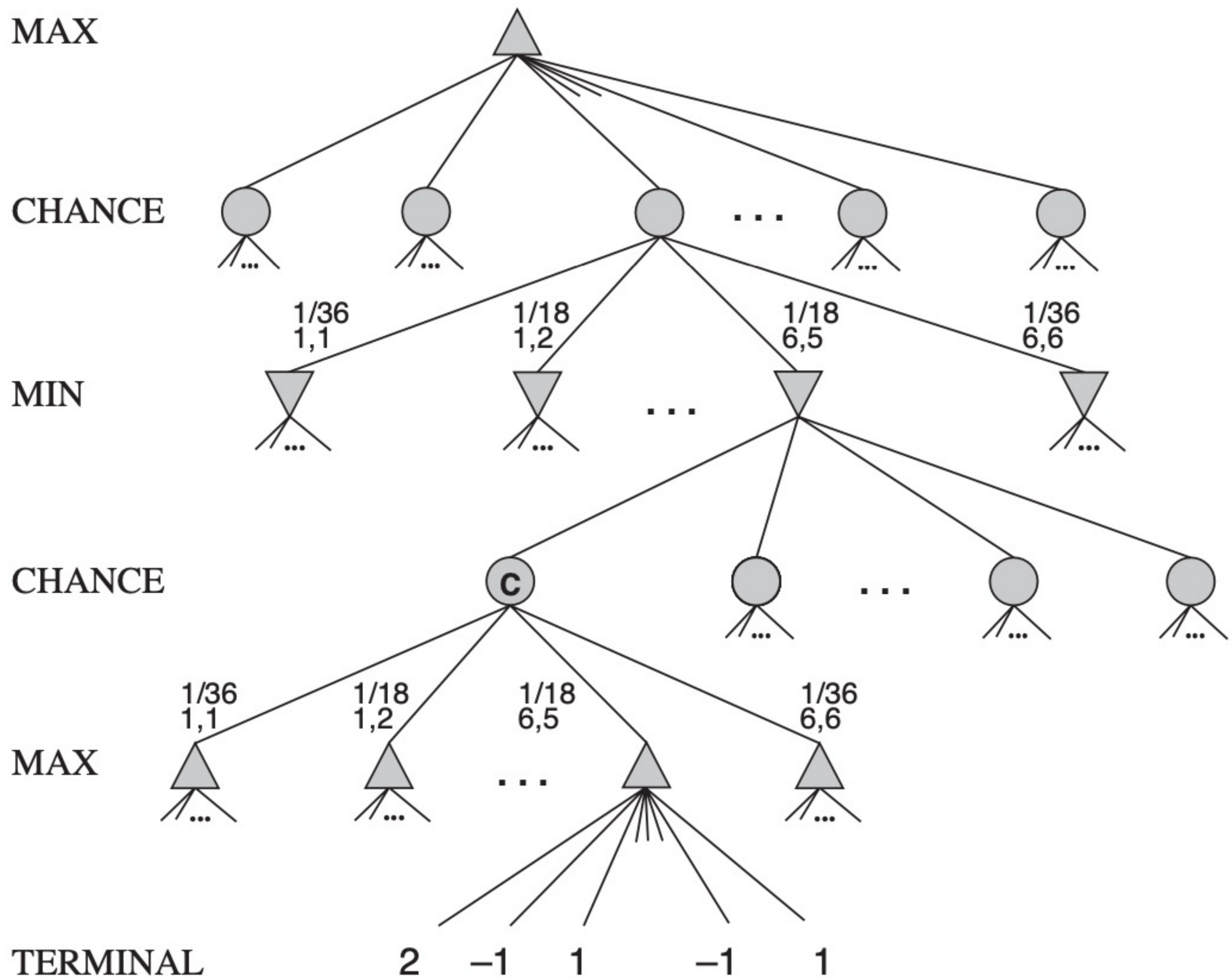


A stochastic game - Backgammon



A typical backgammon position. The goal of the game is to move all one's pieces off the board. White moves clockwise toward 25, and Black moves counterclockwise toward 0. A piece can move to any position unless multiple opponent pieces are there; if there is one opponent, it is captured and must start over. In the position shown, White has rolled 6–5 and must choose among four legal moves: (5–10,5–11), (5–11,19–24), (5–10,10–16), and (5–11,11–16), where the notation (5–11,11–16) means move one piece from position 5 to 11, and then move a piece from 11 to 16.

A game tree for a backgammon position



Epectiminimax

$$
\textsc{Expectiminimax}(s) = \begin{cases} \textsc{Utility}(s) & \text{if } \textsc{Terminal-Test}(s) \\ \max_a \textsc{Expectiminimax}(\textsc{Result}(s,a)) & \text{if } \textsc{Player}(s) = \textsc{max} \\ \min_a \textsc{Expectiminimax}(\textsc{Result}(s,a)) & \text{if } \textsc{Player}(s) = \textsc{min} \\ \sum_r P(r)\textsc{Expectiminimax}(\textsc{Result}(s,r)) & \text{if } \textsc{Player}(s) = \textsc{chance} \end{cases}
$$

THE END