



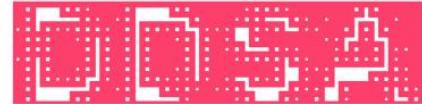
Seminar series : Mathematics of Data Science

Organizers:

Assoc Prof. Martin Andersen (SC)
Assoc Prof. Jakob Lemvig (MAT)
Assoc Prof. Allan Peter Engsig-Karup (SC)

DTU Compute

Department of Applied Mathematics and
Computer Science



Executable Digital Twin

*Reimagining industrial
operations through
Scientific Machine Learning*



MoDS Seminar, DTU | February 6th, 2024



F. Schnös



F. Sievers



S. Gavranovic



B. Peherstorfer



M. Schulz



E. Uy



G. Jouan



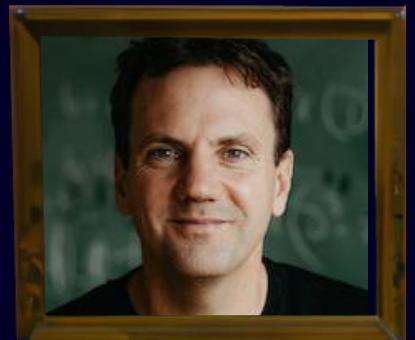
D. Berger



C. Lessing



Q. Zhuang



T. Richter



F. Dietrich



H. Van der Auweraer



B. Obst



?



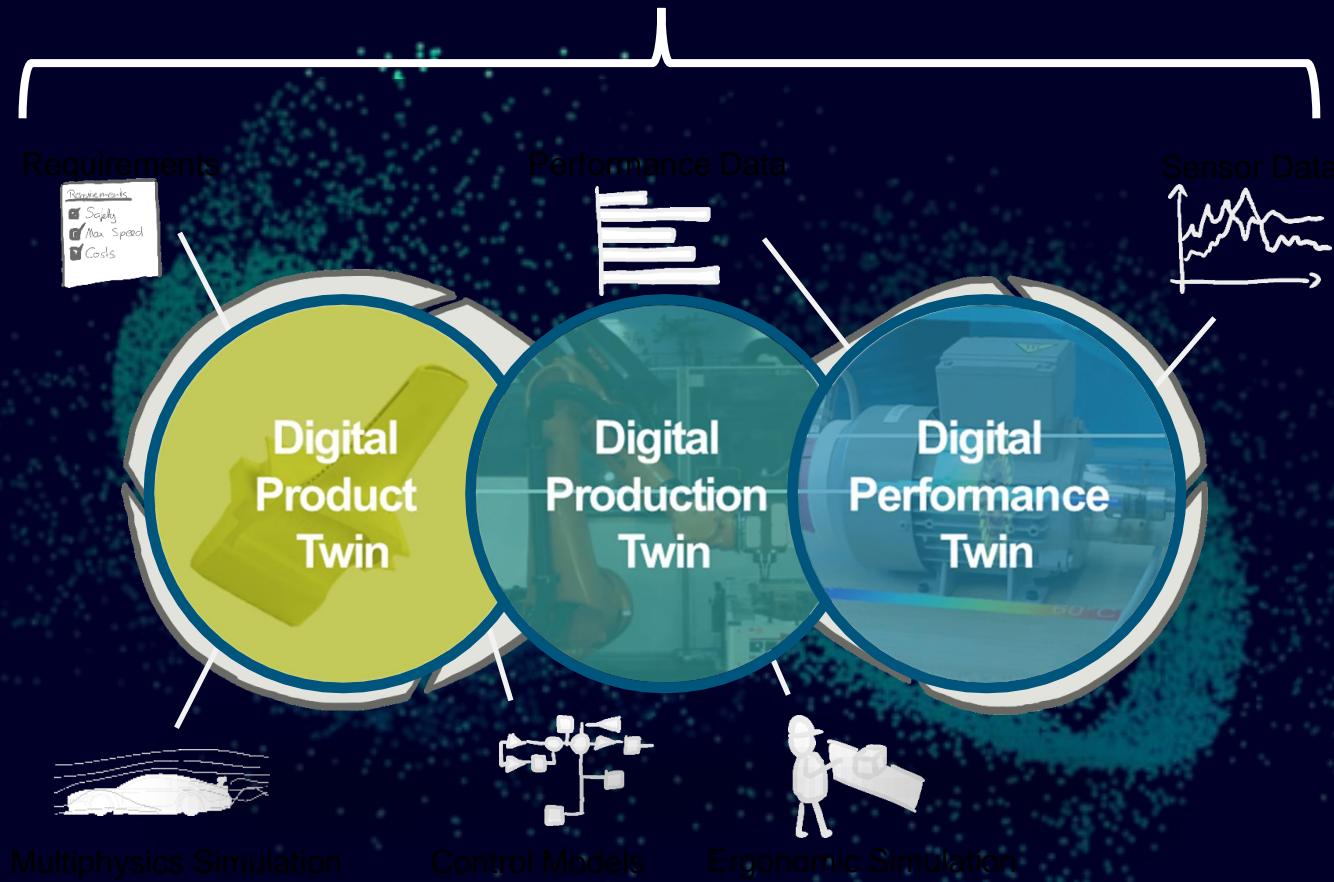
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We are able to combine
the real and digital
worlds like no other
company!

Dr. Roland Busch,
President and CEO of Siemens AG

The comprehensive Digital Twin

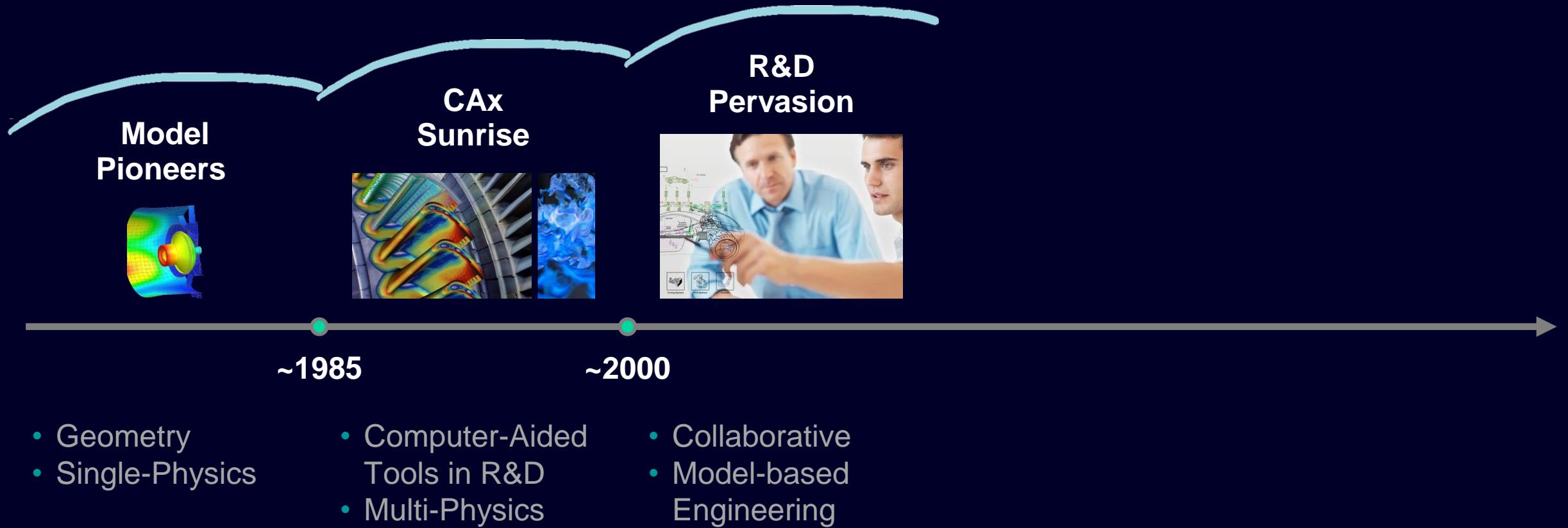
Digital world



Real world

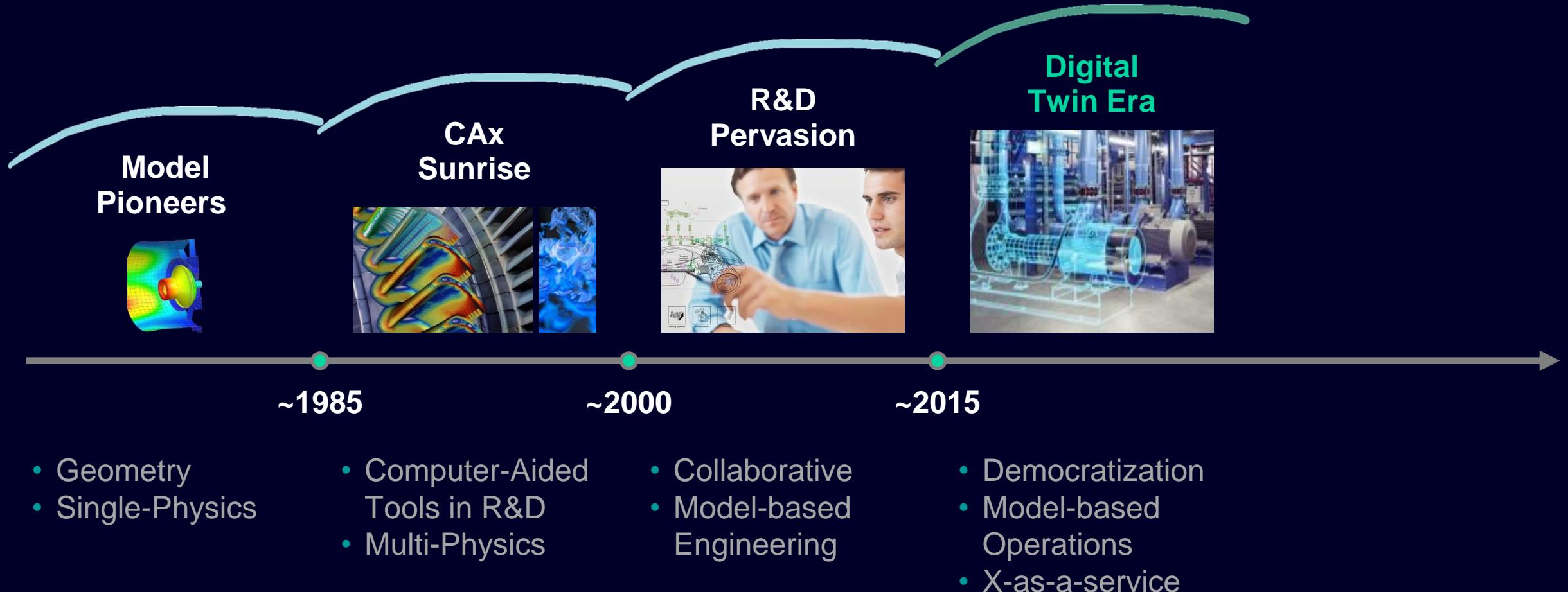
Digital Twin

Digital Twin - *An old story!*



CAx: Computer Aided Design, Engineering, & Manufacturing

Digital Twin - A new age of computational paradigms



CAx: Computer Aided Design, Engineering, & Manufacturing

Why now? Drivers & Enablers

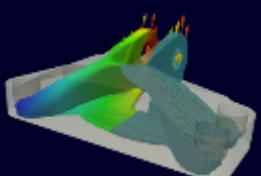


Challenged by increasingly complex systems and system requirements:

Mechanics – electronics – control – software... get tightly interconnected.
Performance demands become increasingly complex



"Moore's Law" – More than Moore - Cloud: Exploding computing capacity beyond scaling of chip performance and cloud power, e.g. GPUs, Reconfigurable Computing, ...



Algorithmic improvements: Creating breakthroughs will contribute significantly to efficiency of engineering process as well as open new ways of working and business propositions



Integrating Heterogeneous Models: different physics, different formulations, different scales: Multiphysics simulation – Co-simulation – FMI/FMU - Model Order Reduction - ...



Internet of Things: performance data everywhere and readily accessible
Data analytics – Data driven performance monitoring and modeling



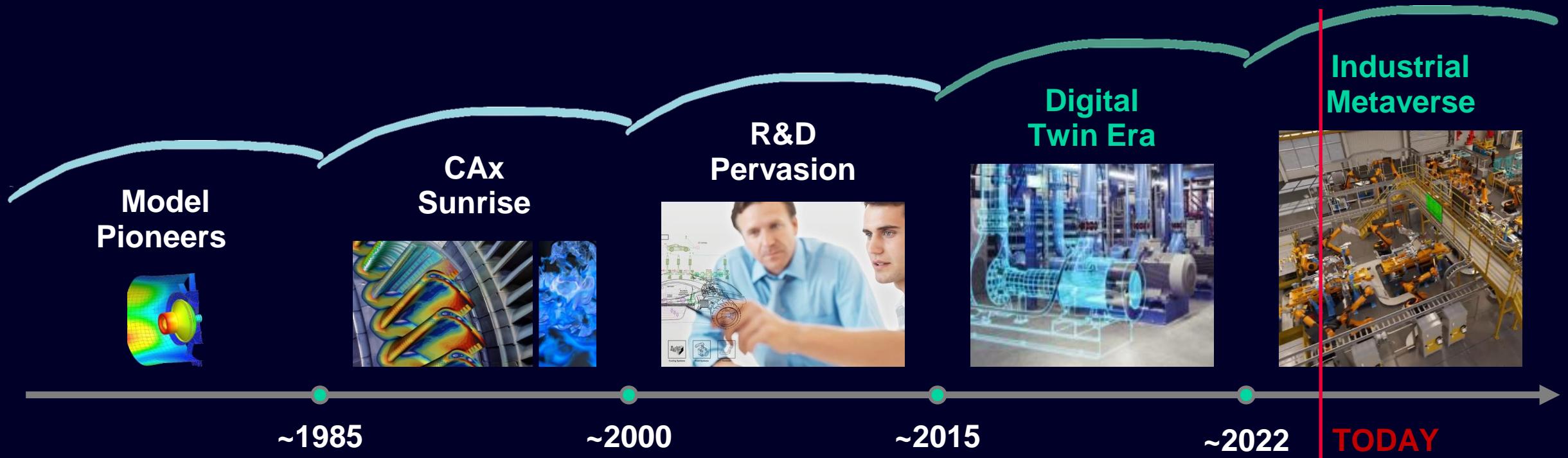
The Digital Twin Paradigm

“

A comprehensive set of digital models accepted as full substitutes for reality to understand, predict, and optimize the physical counterpart's performance characteristics for specific purposes.

Dirk Hartmann (2023)

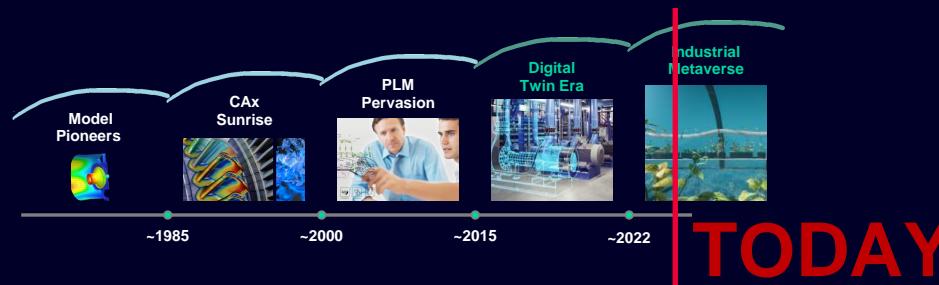
Digital Twin - A new age of computational paradigms



- Geometry
- Single-Physics
- Computer-Aided Tools in R&D
- Multi-Physics
- Collaborative
- Model-based Engineering
- Model-based Operations
- X-as-a-service
- Democratization
- Realtime++ Predictions

CAx: Computer Aided Design, Engineering, & Manufacturing

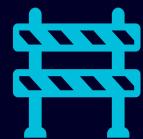
Digital Twin – *State of Industrial Adoption Today*



The Digital Twin market **grows** with annual **CAGRs of 40-60%** in maintenance, business optimization, performance monitoring, ...



Many companies struggle to implement Digital Twins: “Digital Twins are slow and bespoke!”



Road-blocks include company organizations change of business and processes, IT, ...

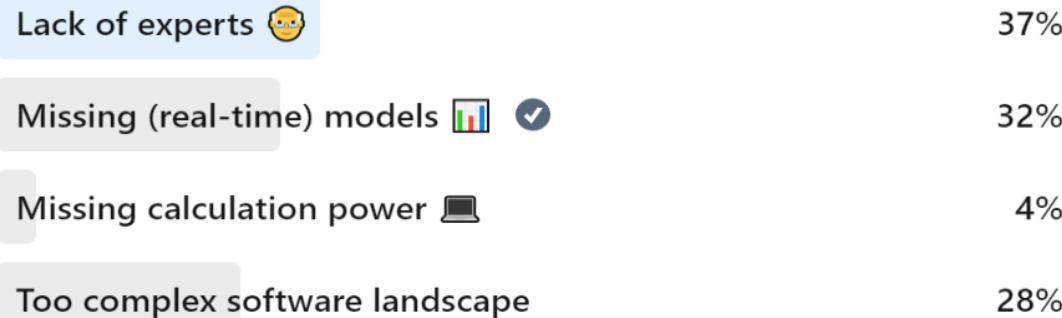
Sources: Digital Twin Market by Technology, Type, Application, Industry, and Geography – Global Forecast to 2026, Markets and Markets
Implementation Model in the Context of Use of Digital Twins, Digital Twin Readiness Assessment
Major Challenges in Digital Twin based Operations, LinkedIn Survey

Executable¹ Digital Twin

1) Scientific ML enabled

Major Challenges in Digital Twin based Operations

What are the major pain 😤 points for more and broader adoption of #DigitalTwins for #IndustrialOperation and #Service?



249 votes



“ Digital Twins are slow and bespoke!

The Executable Digital Twin

Digital

Real



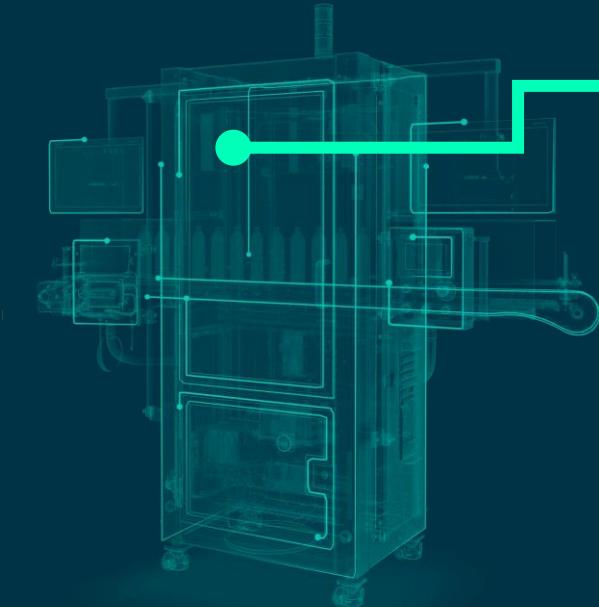
**Self-contained executable
digital behavior of an asset**



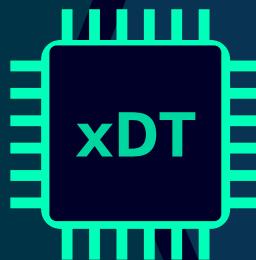
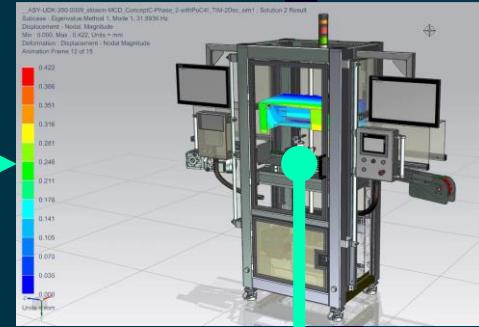
**Leveraged by anyone
at any point in lifecycle**

Use xDT to bring virtual and real worlds together

Virtual

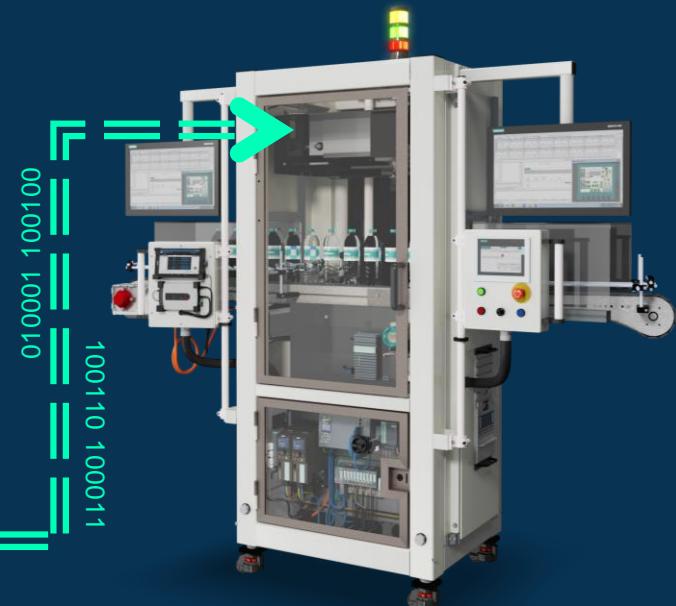


Digital twin models deliver insights



Real

Deploy xDT within or alongside the real asset to make insights actionable



SIEMENS

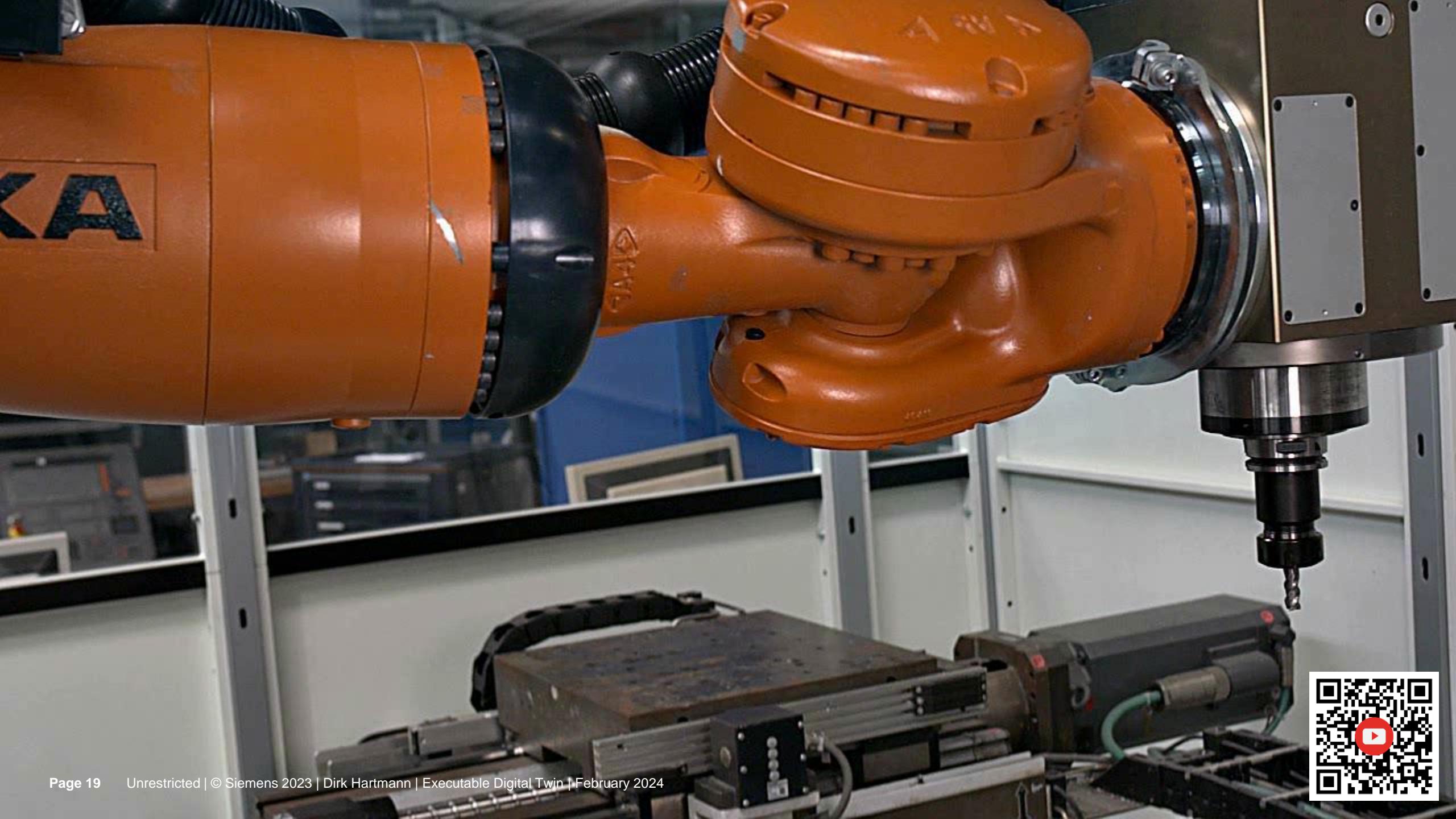
The “first” math paper quoted in an Industry Analyst paper

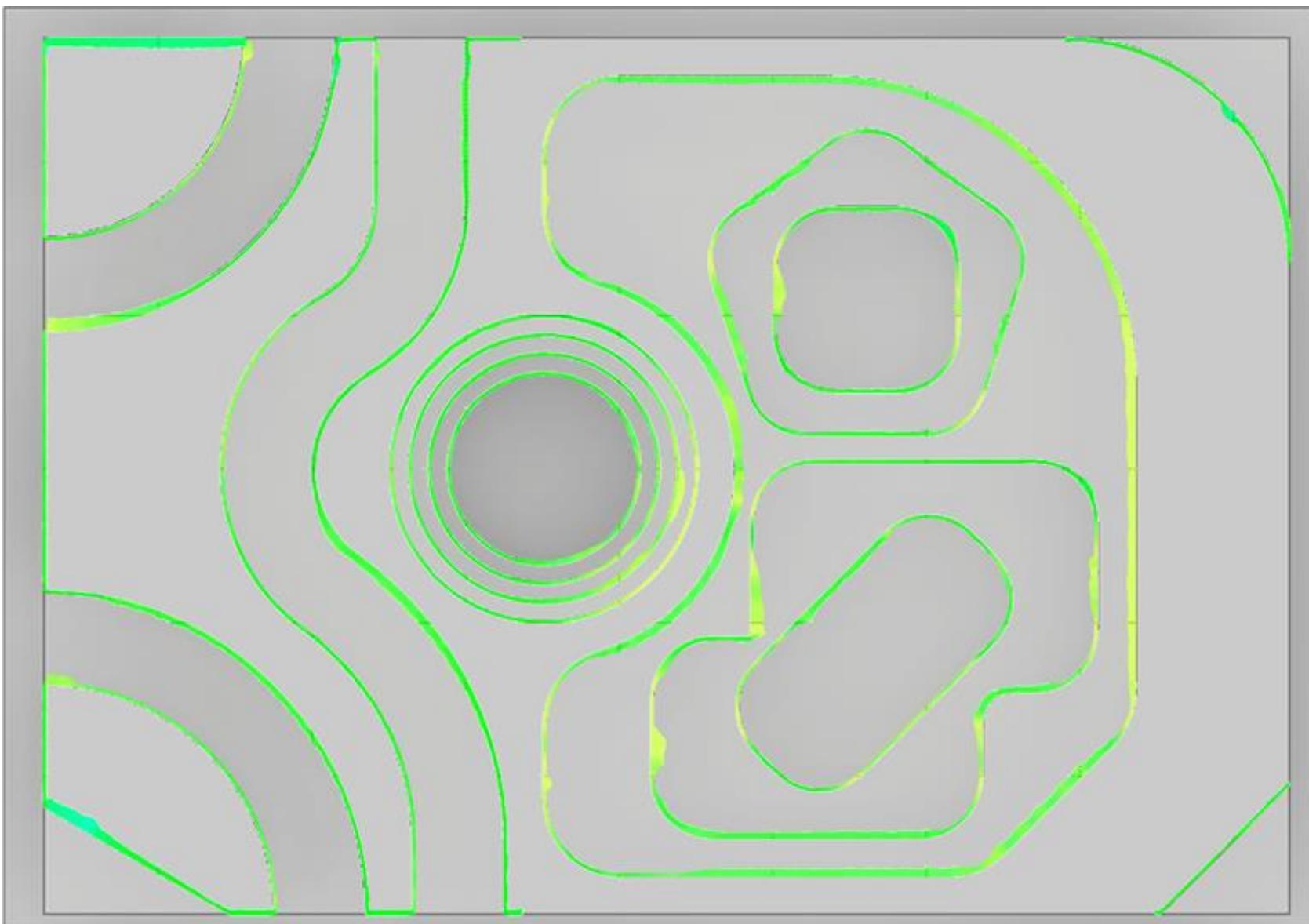
The image consists of four panels arranged in a grid-like fashion, connected by yellow lines.

- Top Left Panel:** A screenshot of the ARC Strategies report titled "THE DIGITAL TWIN IN INDUSTRY AND INFRASTRUCTURE". It features a diagram showing data flow from Edge to Model, then to Cloud Analytics, with Control and Orchestration layers. A callout box highlights the "Continuous Machine Learning And Deployment" section.
- Top Right Panel:** The cover of the book "Progress in Industrial Mathematics: Success Stories - The Industry and the Academia Points of View". The cover is blue with yellow and white text. It includes the Springer logo and the ICIAM 2019 Valencia logo.
- Bottom Left Panel:** A screenshot of the same ARC report, focusing on the "Continuous Machine Learning And Deployment" section. A callout box highlights the quote from Hartman and Van der Auweraer (2020).
- Bottom Right Panel:** A screenshot of a page from the book. The page is titled "Digital Twins" and shows a photograph of a man wearing a VR headset and a complex industrial setup with pipes and sensors. A caption below the photo reads: "Fig. 6 Mixed reality setup allowing to measure spatial temperature distributions parallel to operations by means of online simulations. Reproduced with permission. Copyright © Siemens AG".

Definition (Executable Digital Twin) An Executable Digital Twin is a specific encapsulated realization of a Digital Twin with its execution engines.¹¹ As such they enable the reuse of simulation models outside R&D. In order to do so, the ...

Sources: V. De Leeuw, D. Slansky (2023): [The Digital Twin in Industry and Infrastructure](#), ARC Advisory Group Industry Report
D. Hartmann, H. van der Auweraer (2020); [Digital Twins](#), Progress in Industrial Mathematics: Success Stories



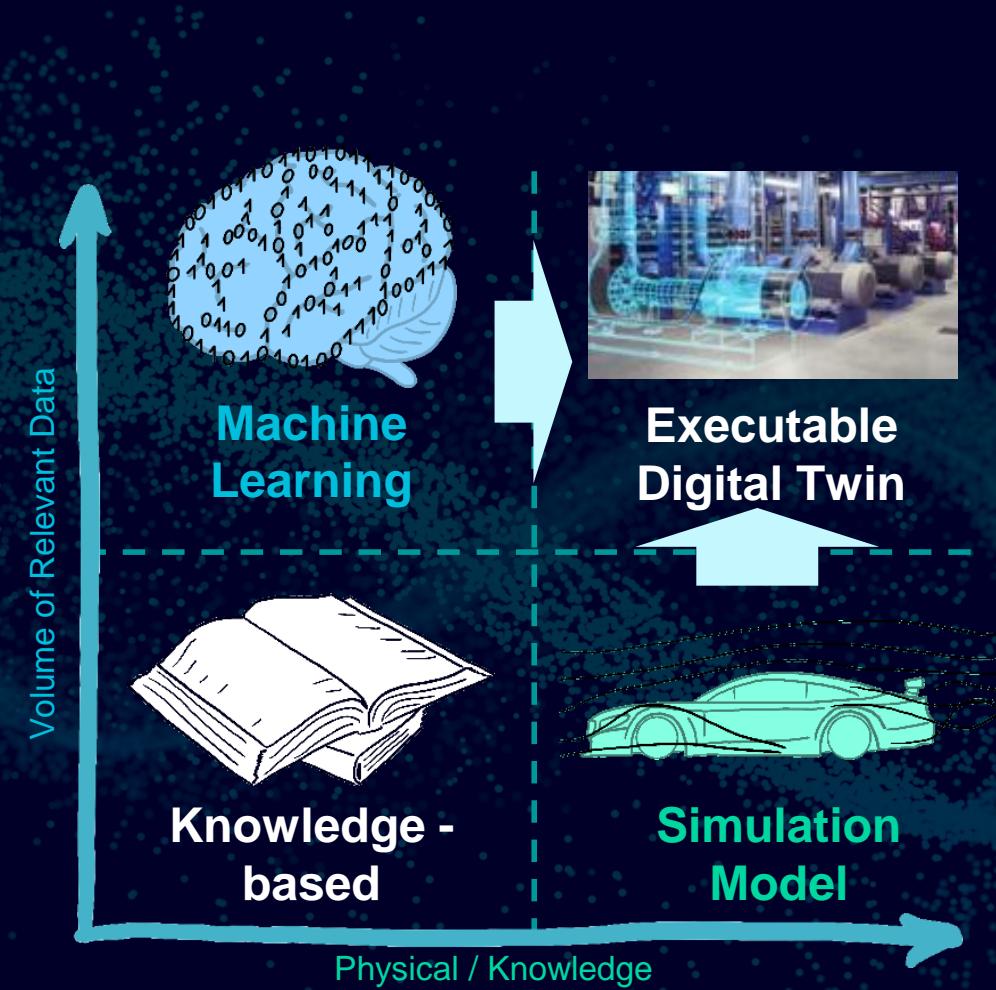




3

Math Deep Dive

ML combined with Simulation enable the Executable Digital Twins at scale



Courtesy to L. Horesh (2016): [Should you derive? Or let the data derive - Towards a first-principles data-driven symbiosis](#)

AI and ML boost Decisions in Engineering and Operation

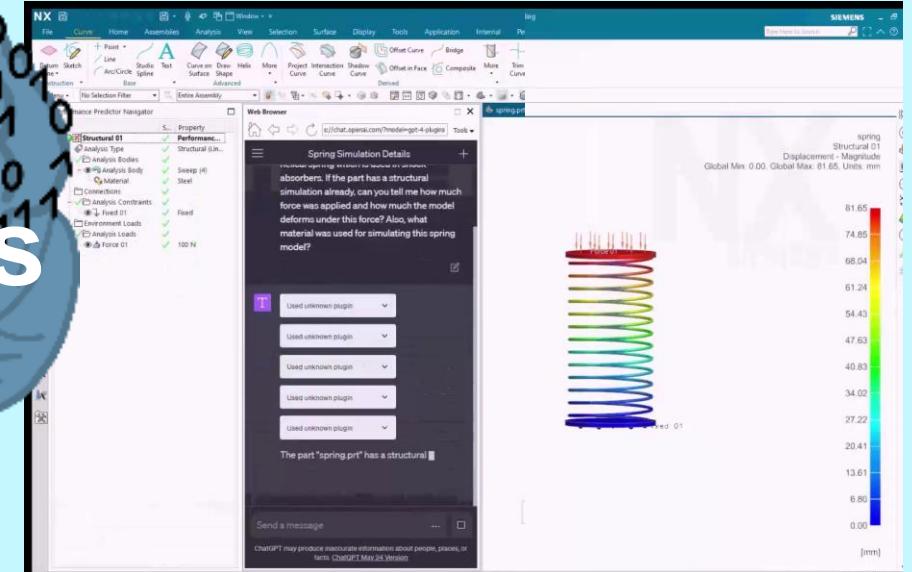
Accelerate Predictions



High End CFD simulation of a car

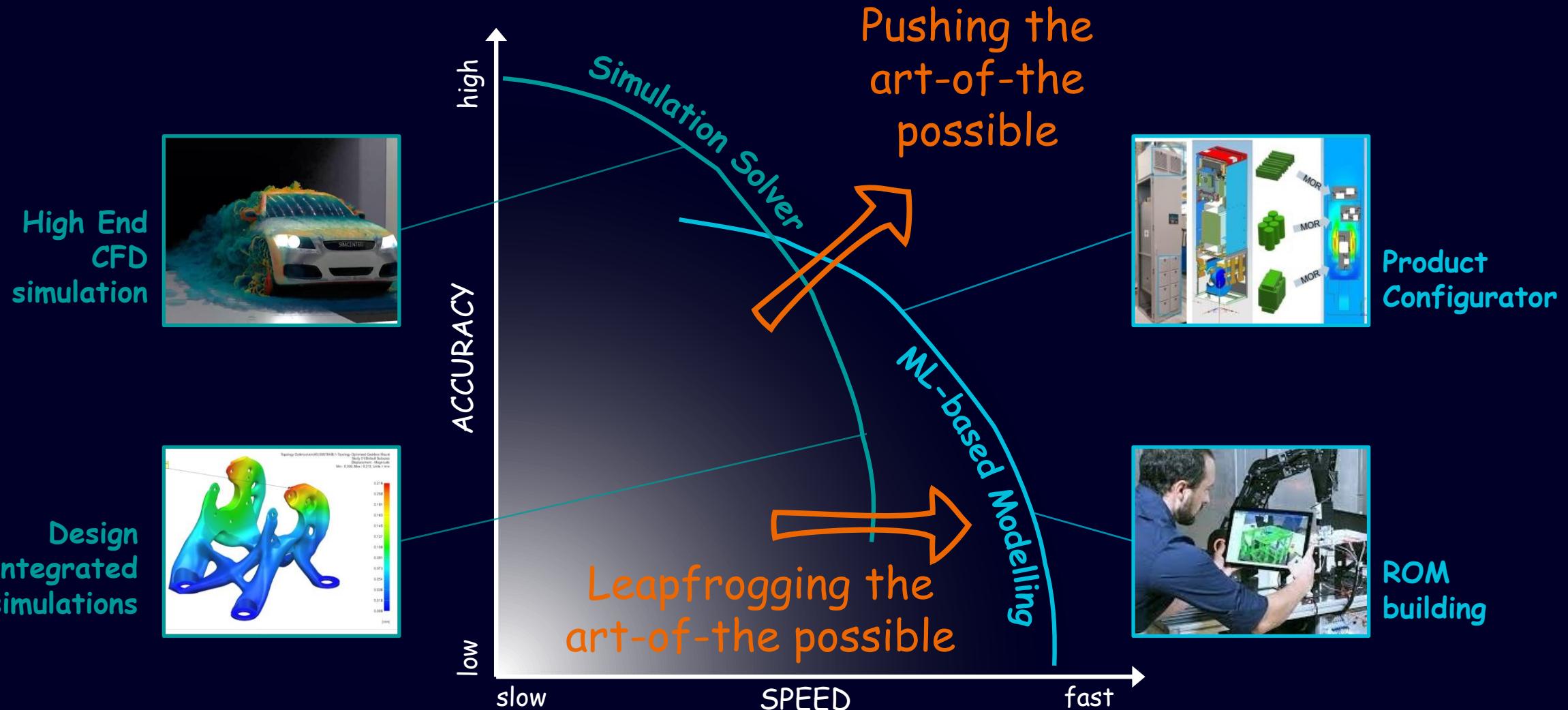
Faster
Decisions

Improve User Efficiency

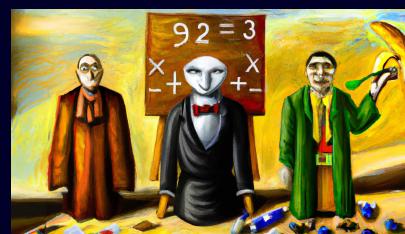
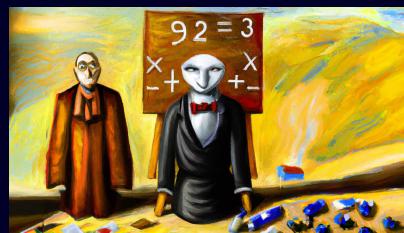
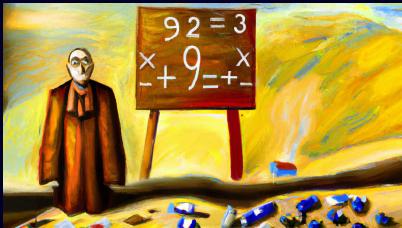
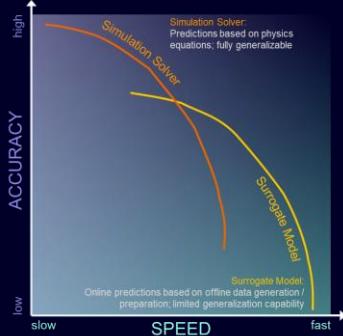


PoC

ML is challenging the Art of the Possible



ML-accelerated Prediction Use Cases



The Good
Acceleration of classical solvers

The Bad
Regression-based Model Order Reduction

The Ugly
Sampling strategies for industrial MOR workflows

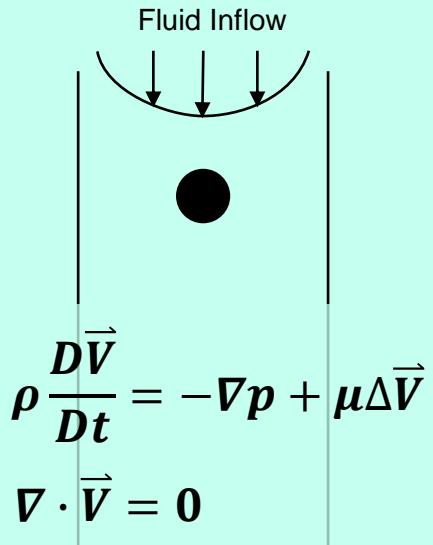


The Good

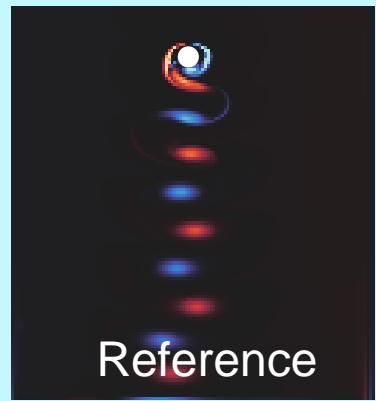
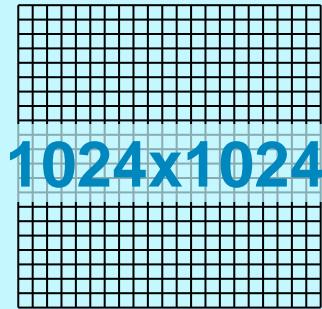
Acceleration of classical solvers

ML augmented CFD solver

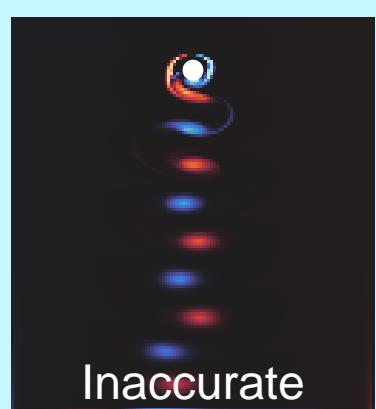
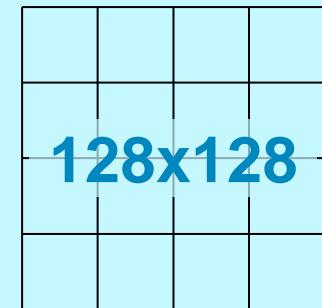
Simulation Task


$$\rho \frac{D\vec{V}}{Dt} = -\nabla p + \mu \Delta \vec{V}$$
$$\nabla \cdot \vec{V} = 0$$

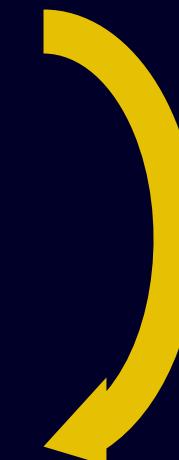
HiFi Simulation



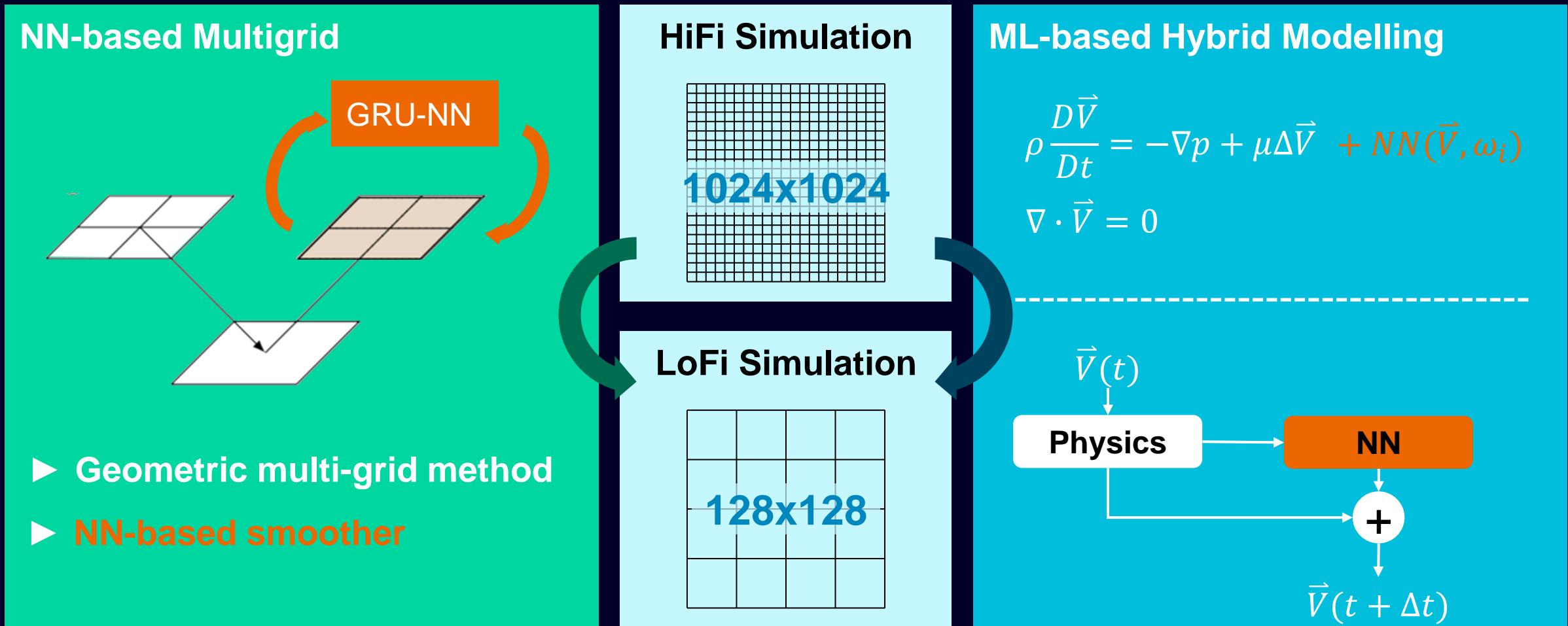
LoFi Simulation



Using a coarser
mesh allows a
significant
acceleration

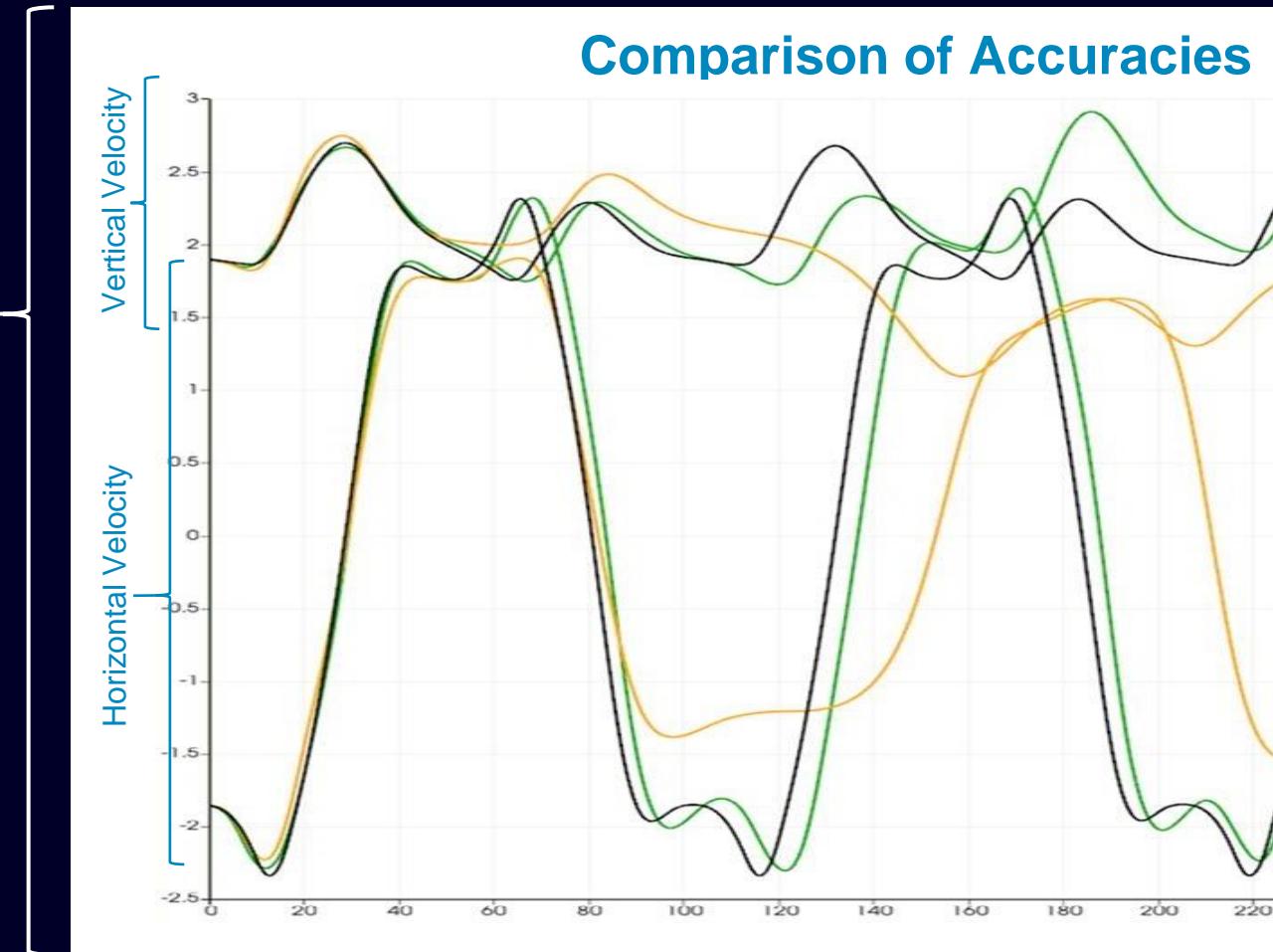


ML augmented CFD solver



Source: N Margenberg, D Hartmann, C Lessig, T Richter (2020): [A neural network multigrid solver for the Navier-Stokes equations](#); J. Comp. Phys.
D Kochkov, JA Smith, A Alieva, S Hoyer (2021): [Machine learning-accelerated computational fluid dynamics](#). PNAS

ML-based hybrid Modelling - Accuracy



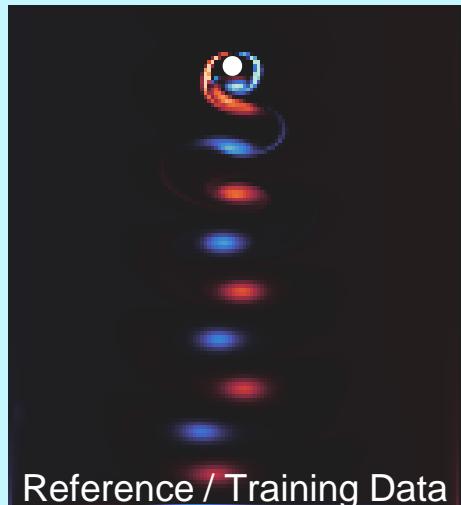
HiFi Simulation:
Reference & Training Data

LoFi Simulation:
128 x 128 grid
40x speedup

ML-augmented Simulation:
128 x 128 grid
+ NN augmentation
18x speedup

ML-based hybrid Modelling - Generalization

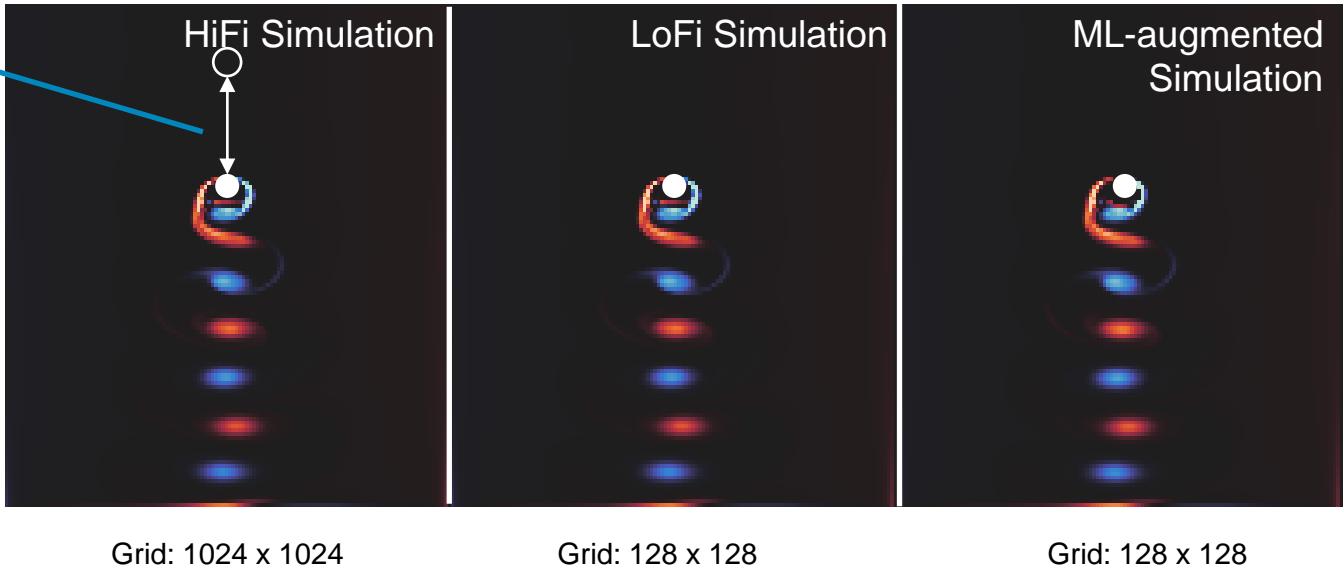
HiFi Simulation



Grid: 1024 x 1024
Solver: Industry-grade solver

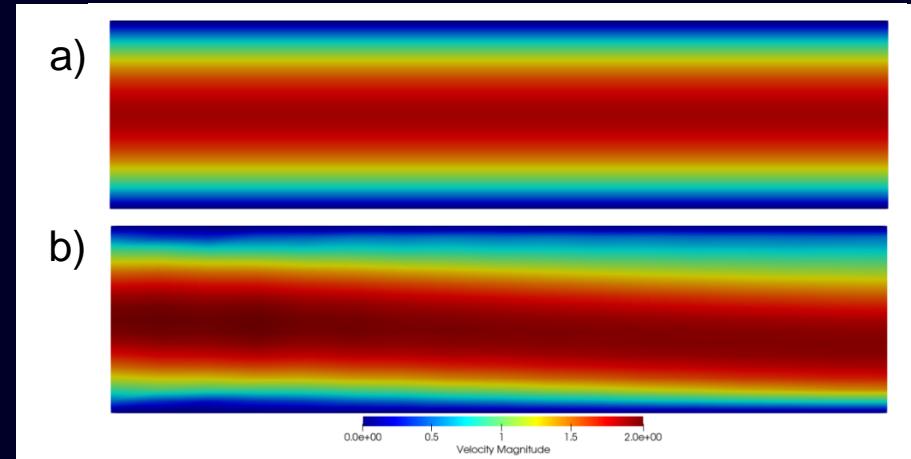
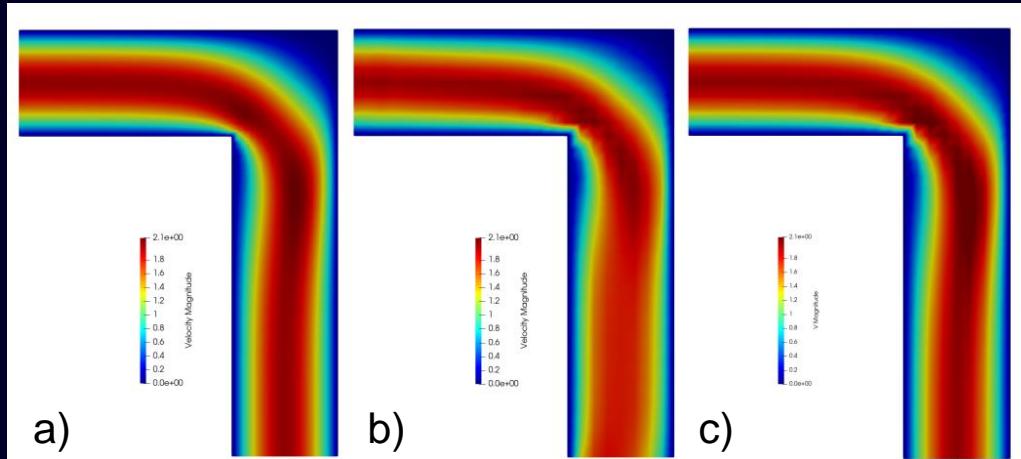
Exploration of Generalization Accuracies

Cylinder moved compared to Training Data

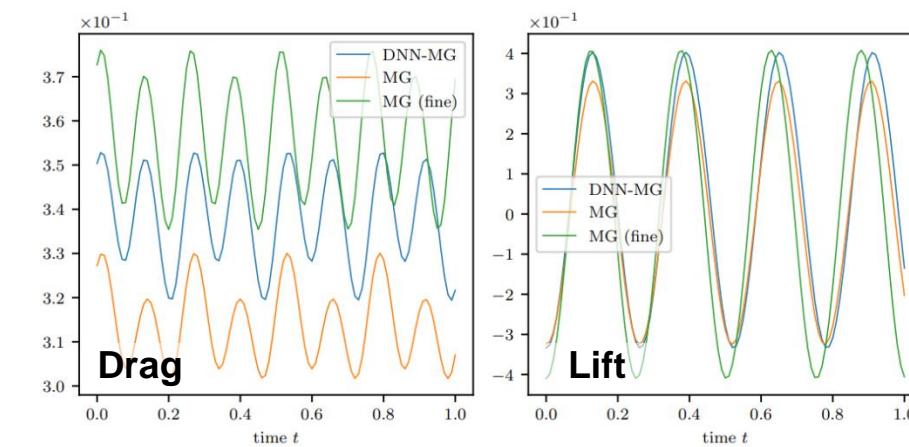
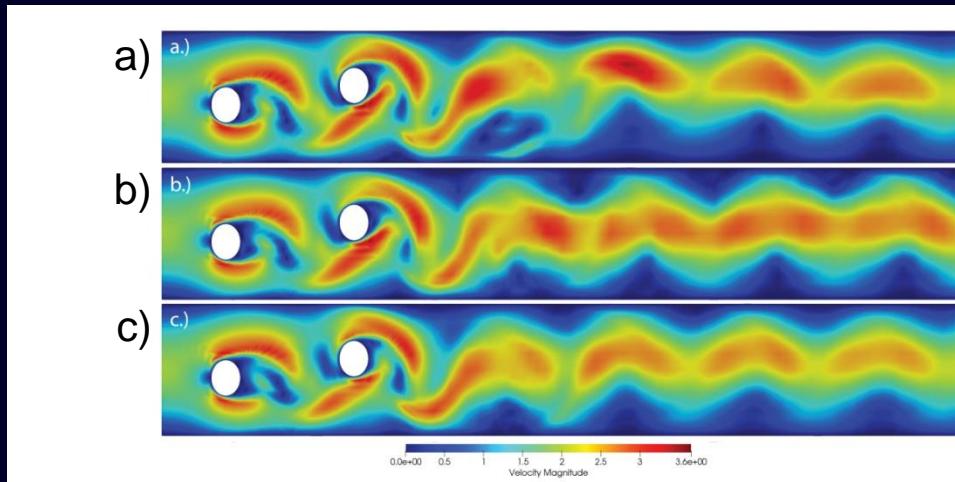


The model **extrapolates well into scenarios not seen in the training data**, something where classical ML methods fail

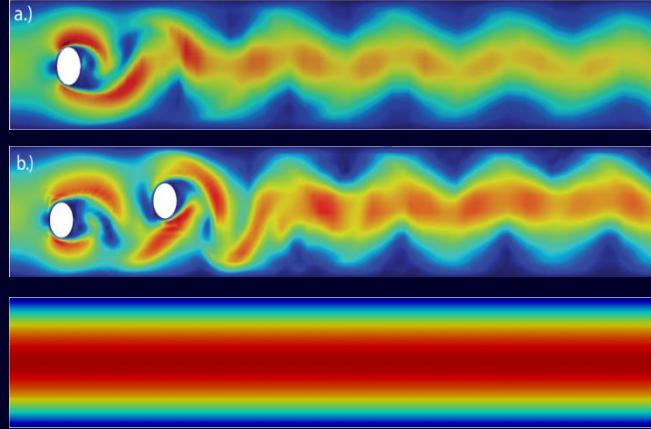
NN-based Multigrid Method - Generalization



a) MG ($L + 1$)
b) DNN-MG ($L + 1$)
c) MG (L)



Intrusive Solver Acceleration



- ✓ Local super-resolution approach (with the Model or Solver) ensure accessibility to training data
- ✓ Local structure provides impressive generalization capabilities
- ✓ Allows to build / extend classical well proven solver technology
- ? Further research and development required

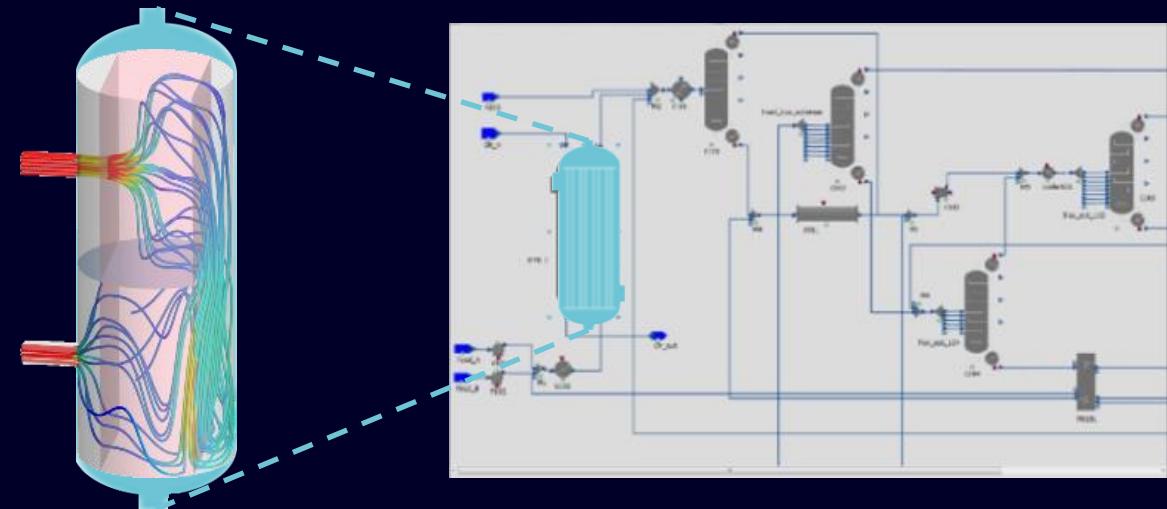


The Bad Regression-based MOR

Real-time capable model – a building block of future industrial solutions

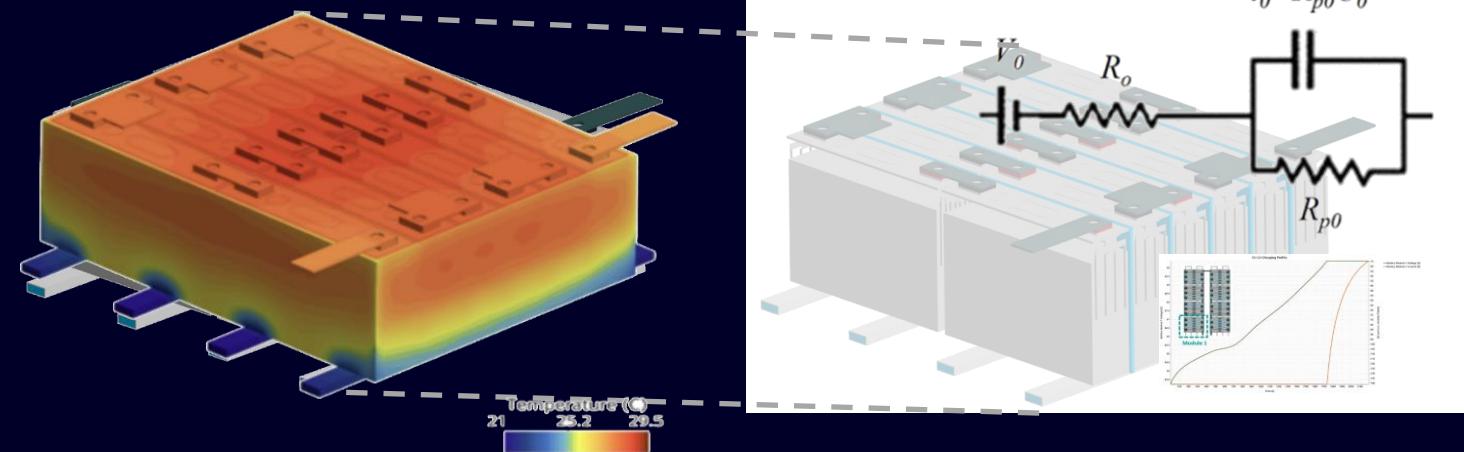
Use Case A:

Detailed resolution of flows in Process Engineering



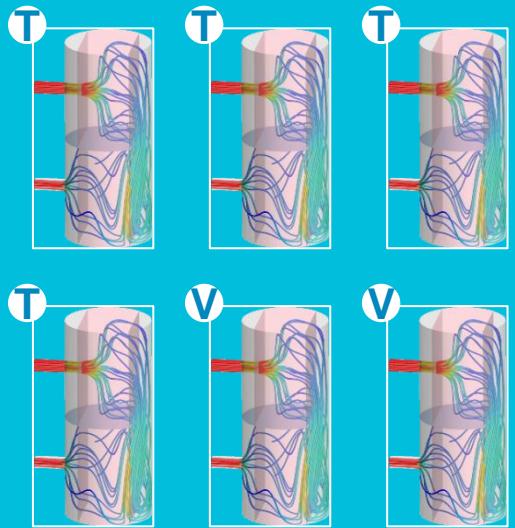
Use Case B:

Accurate prediction of Thermal management in Electrification



Non-linear Model Order Reduction in a nut-shell

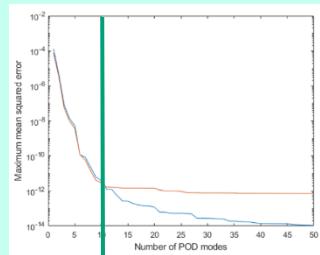
Full Model Snapshots



n is very large,
typically $n \gg 10^6$

Latent Dimension Identification

- ▶ Autoencoder
- ▶ Diffusion Maps
- ▶ Dynamic Mode Decomposition
- ▶ Proper Orthogonal Decomposition



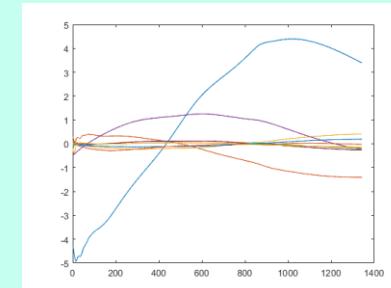
$$\hat{x} \in \mathbb{R}^m$$

with $m \sim 10 - 100$

Reduced Model Operator Discovery

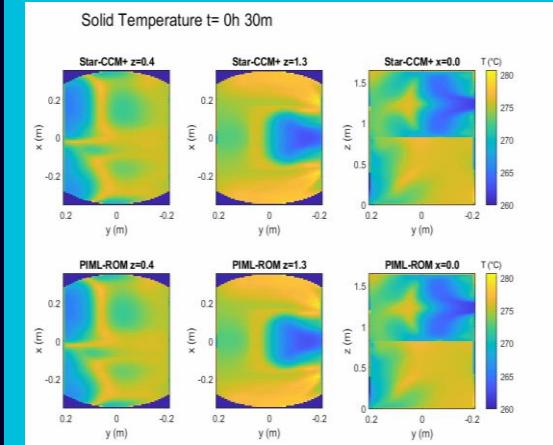
$$\partial_t \hat{x} = \hat{f}(\hat{x}, \mu)$$

- ▶ Discrete Empirical Interpolation
- ▶ Neural Networks
- ▶ Operator Inference



Reduced Coordinate
trajectories

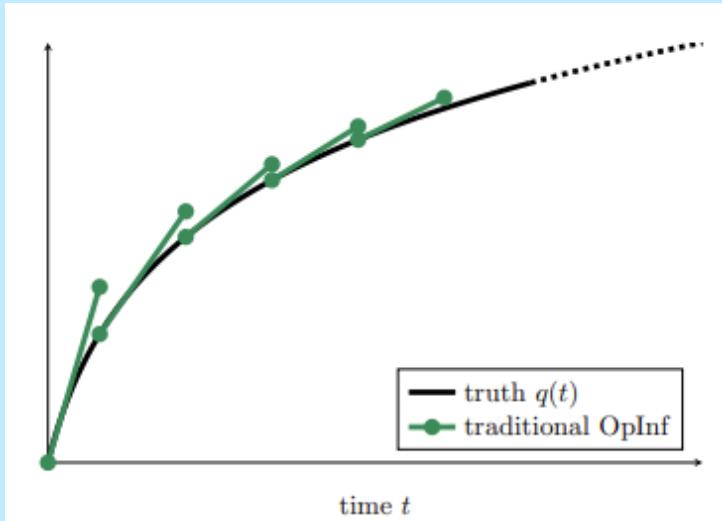
Reduced Model



Real-time
capable model,
predicting the full
field

Solver-in-the-loop Model Order Reduction

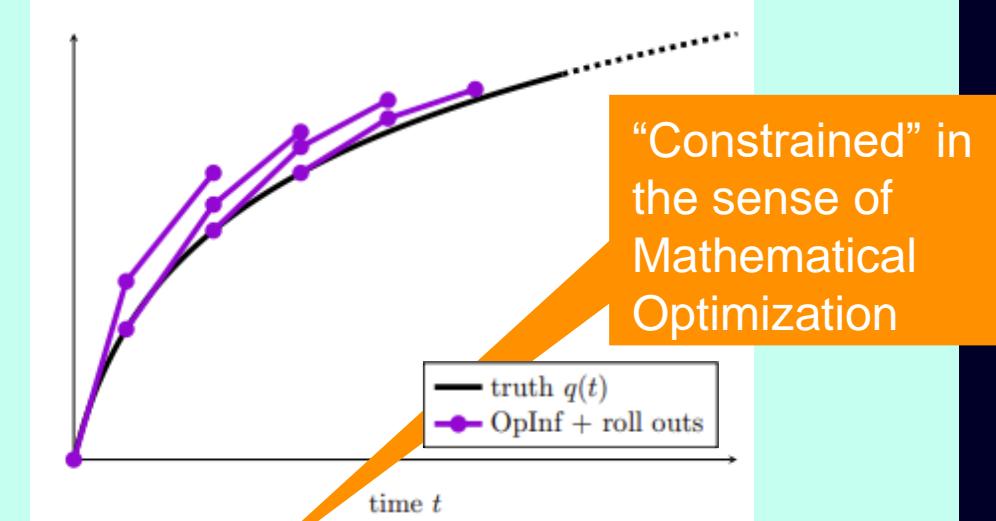
Classic Operator Inference



Least-Square Optimization Problem:

$$\arg \min_A \sum_i (\partial_t x_i - Ax_i)^2 + \lambda x_i^2$$

Solver-in-the-loop Operator Inference



Constraint Optimization Problem:

$$\arg \min_A \sum_i (x_i - \tilde{x}_i)^2$$

such that $\partial_t \tilde{x} = A\tilde{x}$

Source: D. Hartmann, L. Failer (2021): [A Differentiable Solver Approach to Operator Inference](#); arXiv

W Uy, D Hartmann, B Peherstorfer(2023): [Operator inference with roll outs for learning reduced models from scarce and low-quality data](#); Comput. Math. Appl..

Example: Complex Cooling Flow

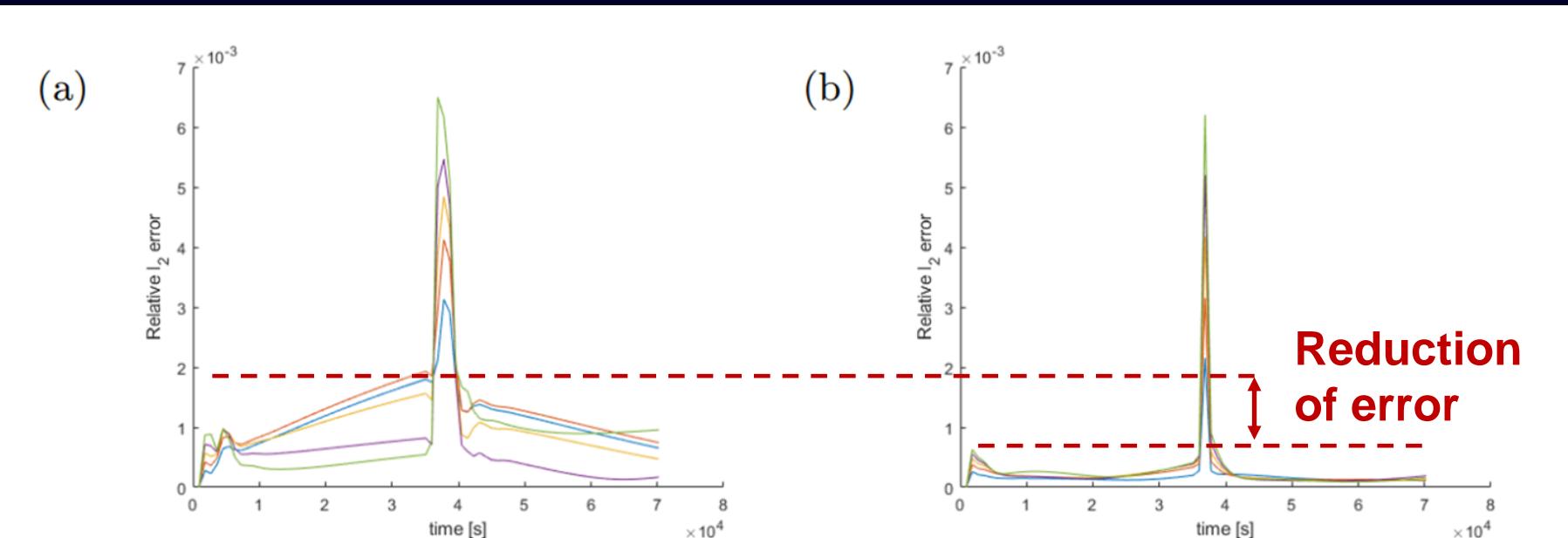
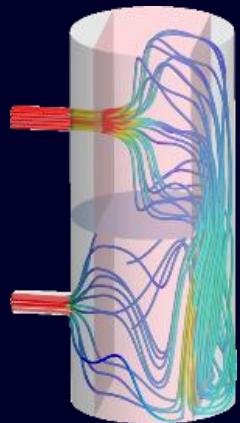
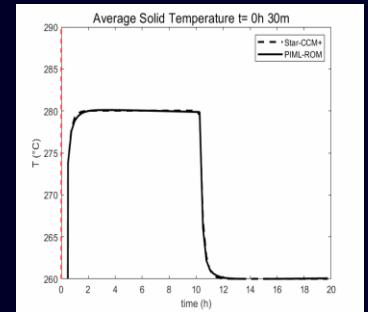


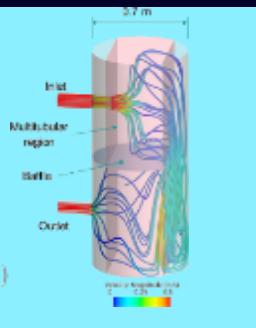
Figure: Operator Inference plus DEIM using 8 modes each: (a) Relative mean squared error of the dynamics predicted using stabilized operator inference (with stabilization parameter $\lambda = 1.0$) and (b) the same error after additional operator calibration (all 5 data sets, encoded in different color).

Example: Complex Cooling Flow

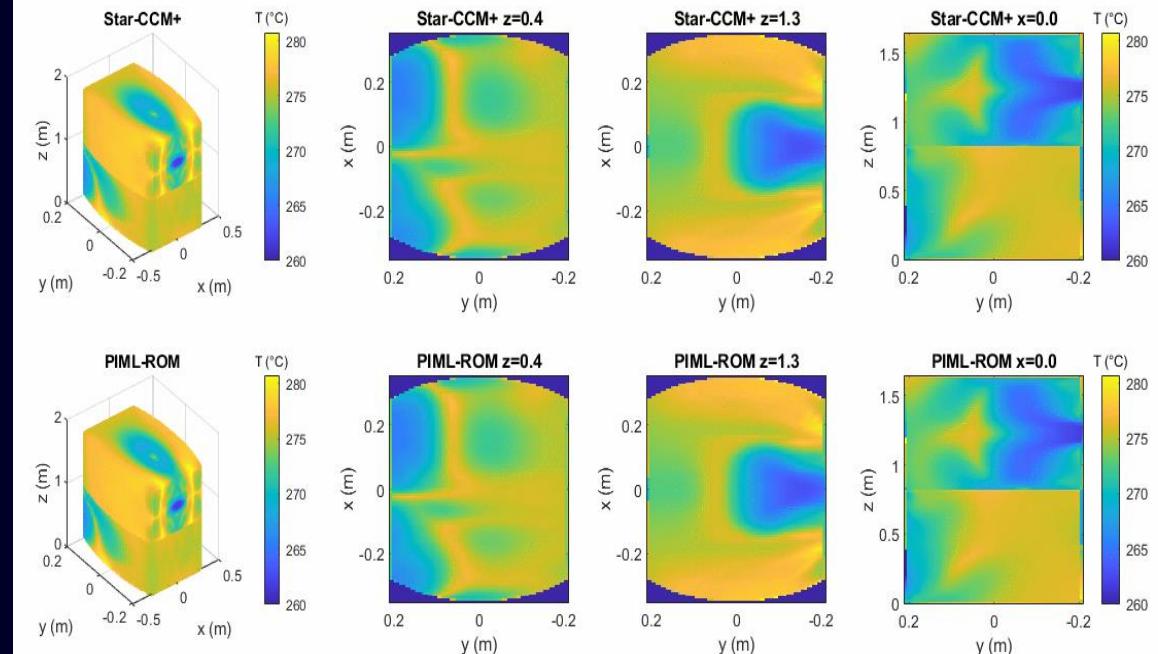


3D Physics with 400k DoF

$$\begin{aligned} \rho_e \left(\frac{\partial v}{\partial t} + (v \cdot \nabla) v \right) &= -\nabla p + \mu \Delta v + \alpha(x)v \\ \nabla \cdot v &= 0 \\ \rho_e c_{p,e} \left(\frac{\partial T_e}{\partial t} + (v \cdot \nabla) T_e \right) &= \nabla \cdot (\mathcal{K}_e \nabla T_e) + \chi_{\Omega_e} q(T_s - T_e) \\ \rho_s c_{p,s} \frac{\partial T_s}{\partial t} &= \nabla \cdot (\mathcal{K}_s \nabla T_s) - q(T_s - T_e) + \mathcal{P}(t, T_s) \end{aligned}$$



Solid Temperature t= 0h 30m

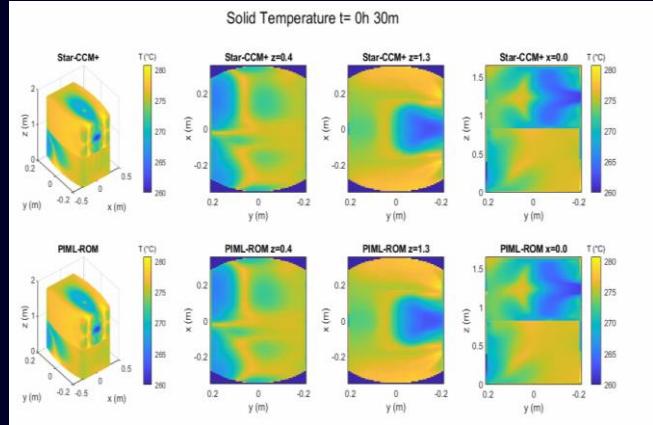


ODE with 8 DoF

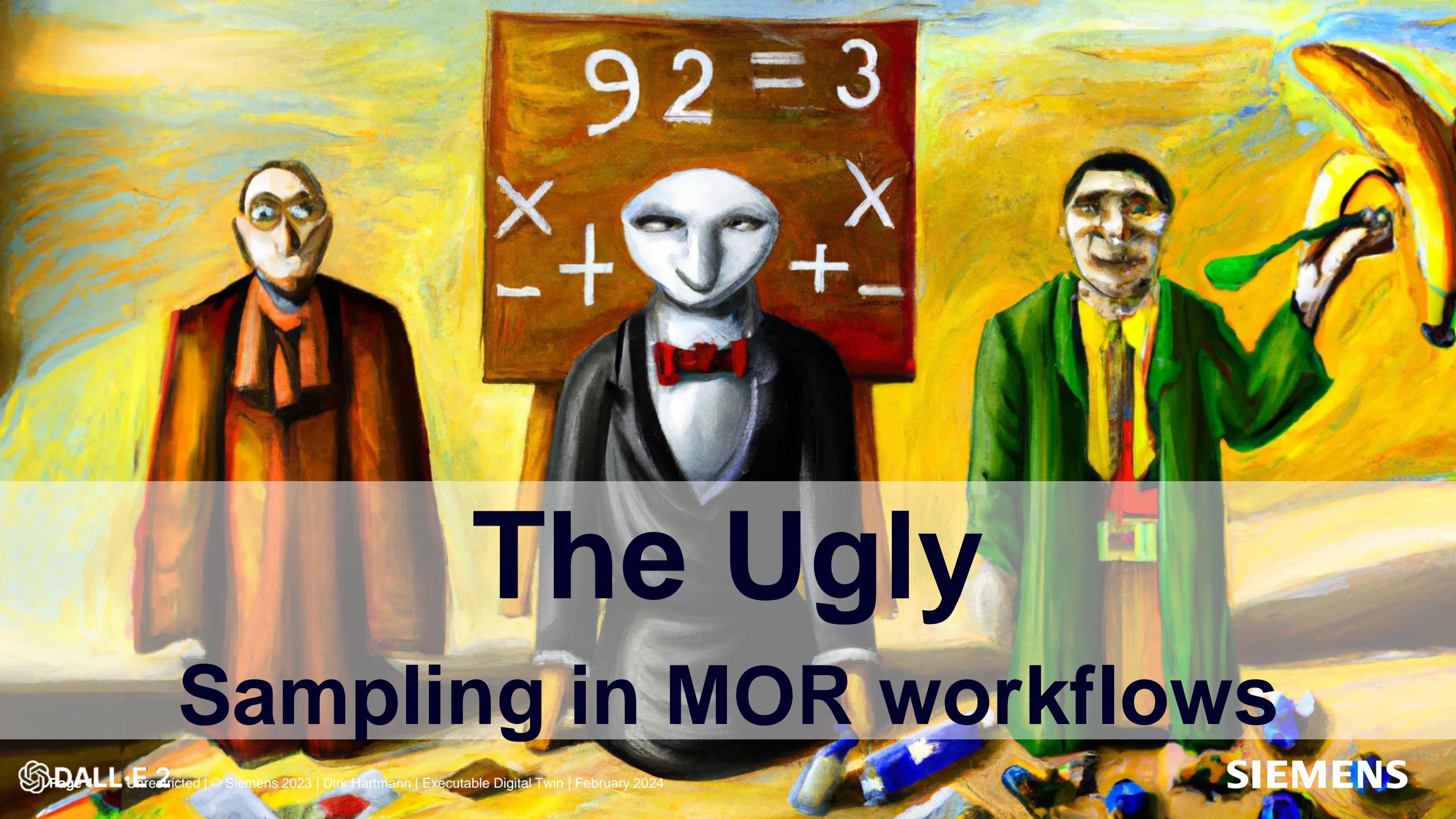
$$\dot{s} = As + R(t) A P_1 \exp(B/(P_2 s))$$

Source: D. Hartmann, L. Failer (2021): A Differentiable Solver Approach to Operator Inference; arXiv

Operator Inference

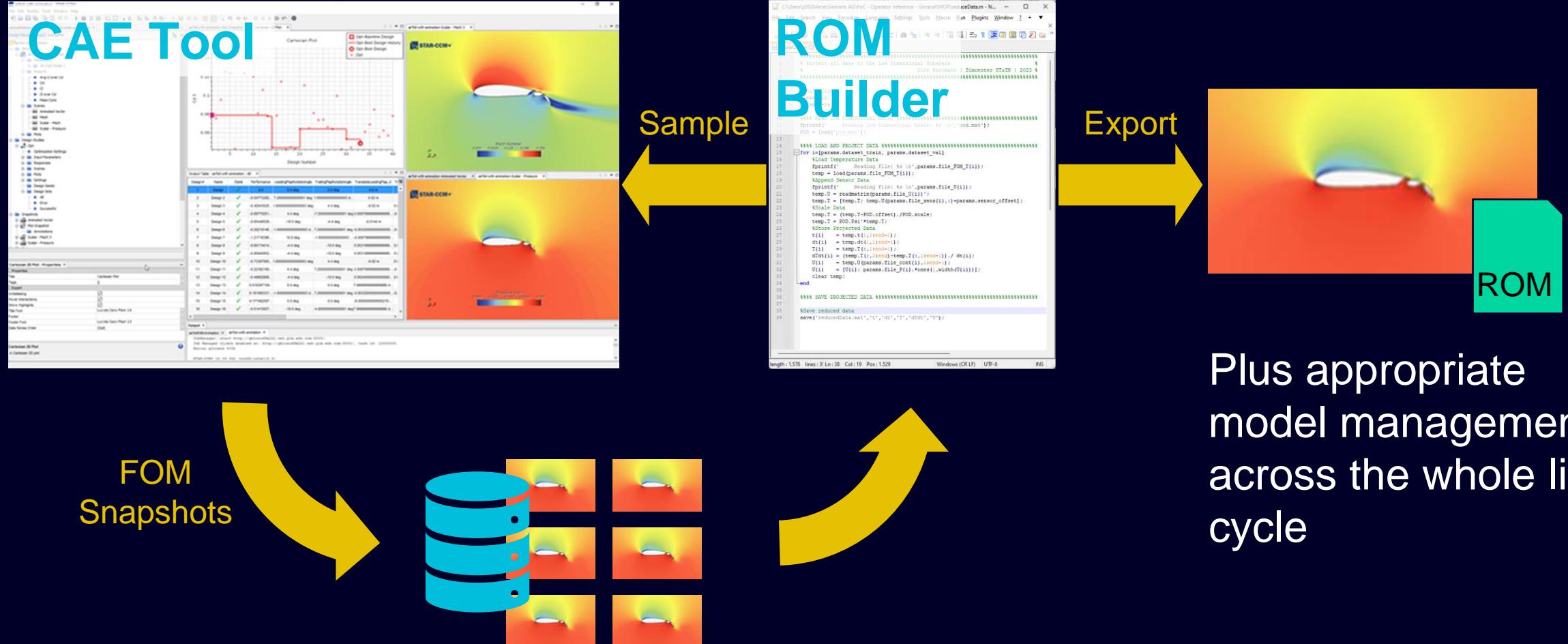


- ✓ Acceleration of prediction by orders of magnitude not loosing accuracy
- ✓ Explicit form of equations allows to be reused in many tools / systems
- ✓ Differentiable solver technology is not only key for machine learning applications
- ✗ Data generation can be quite cumbersome



The Ugly Sampling in MOR workflows

Industrial Model Order Reduction Workflows



How to sample effectively

► Static vs. Dynamic Parameter Sampling

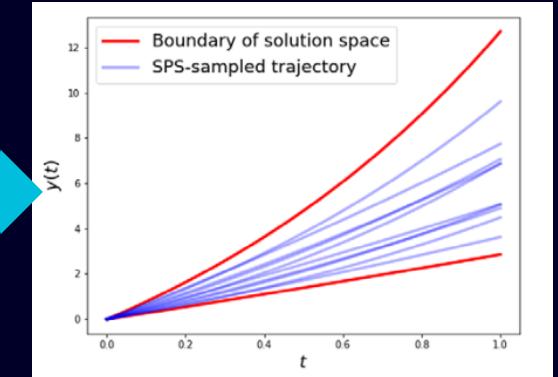
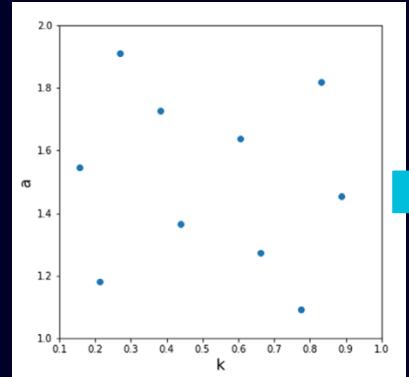
► One long trajectory vs many small trajectories

► ...

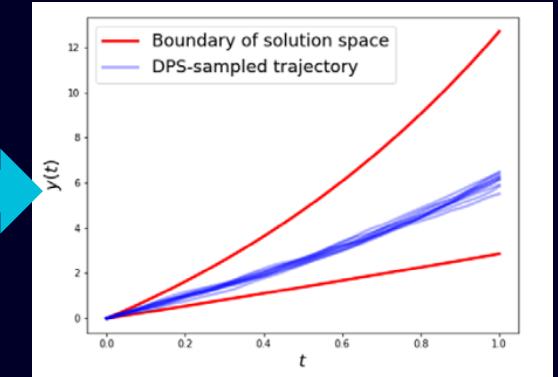
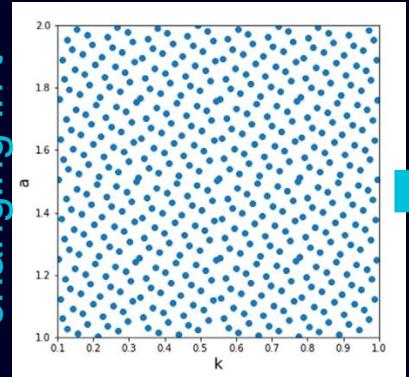
Example:

$$\dot{y} = k(t)y + e^{a(t)}$$

Static Sampling

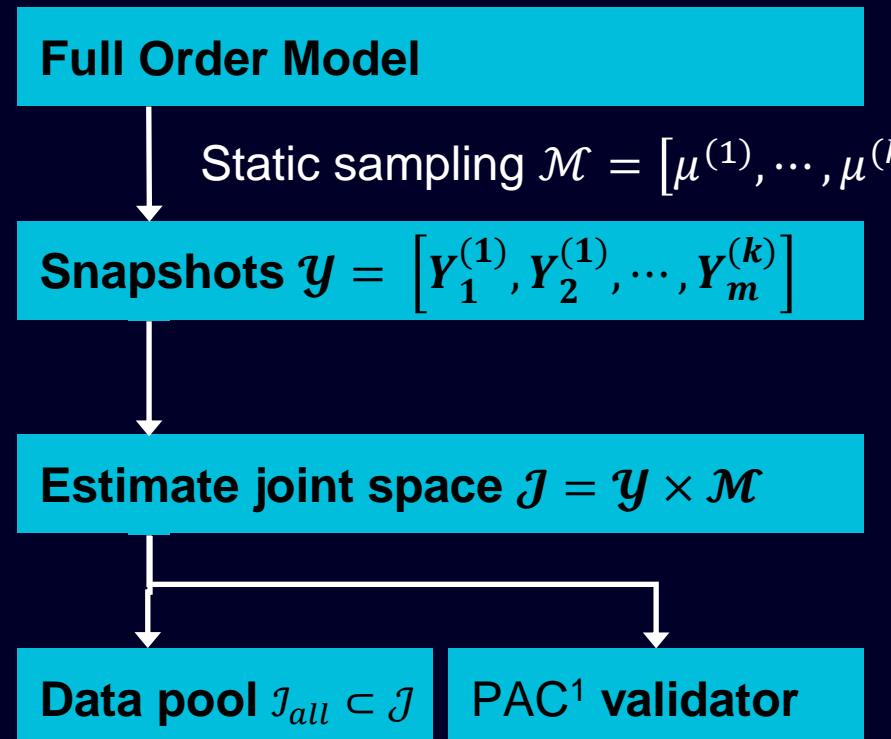


Dynamic Sampling changing in t



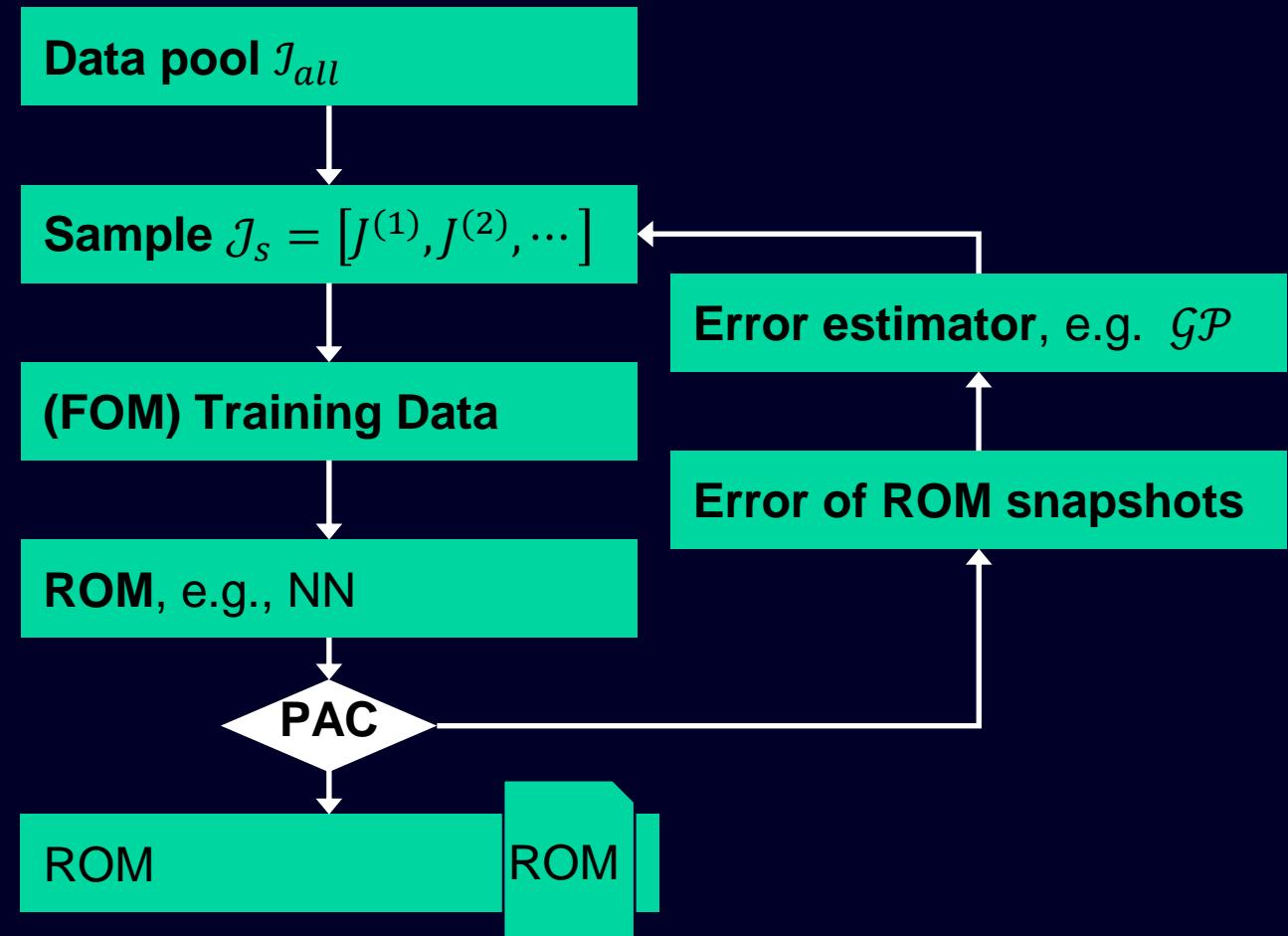
Active Learning Heuristic for industrial ROM

Preparation



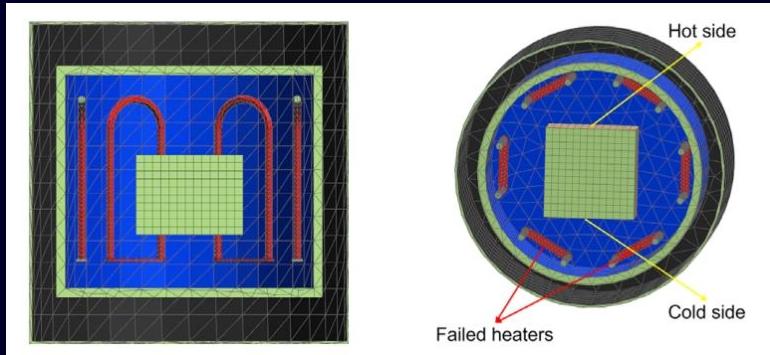
1) Probably approximately correct

Active Learning / Sampling



Source: Q Zhuang, D Hartmann, HJ Bungartz, JM Lorenzi,(2021): Active-learning-based nonintrusive model order reduction; Data-centric Eng

Active Learning Heuristic for industrial ROM

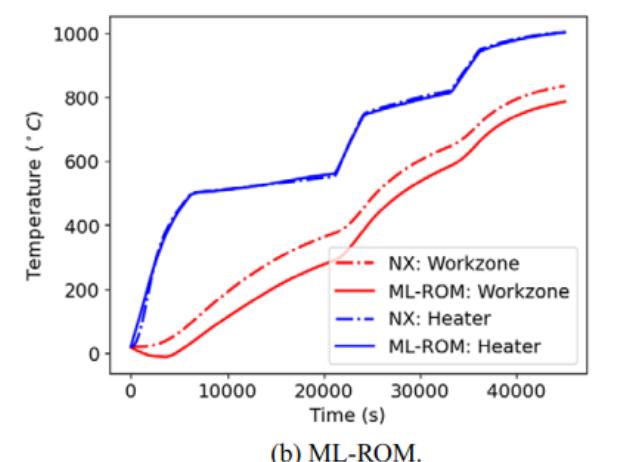
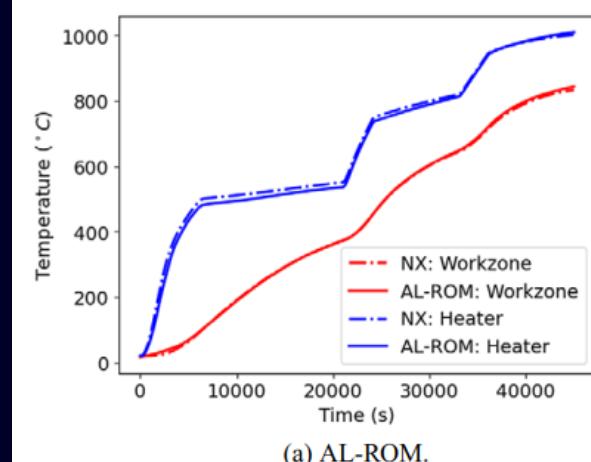


Test Case: Vacuum furnace

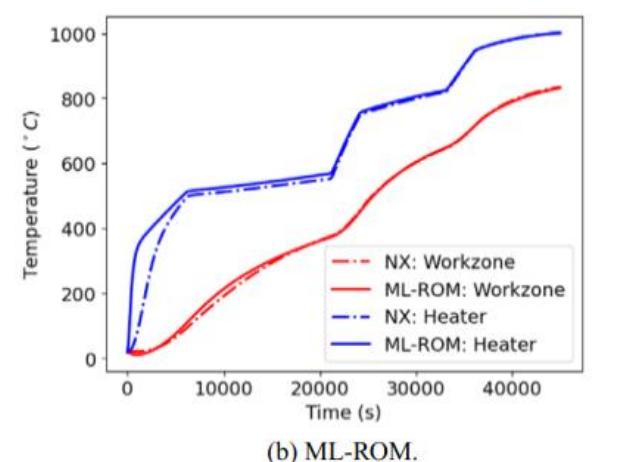
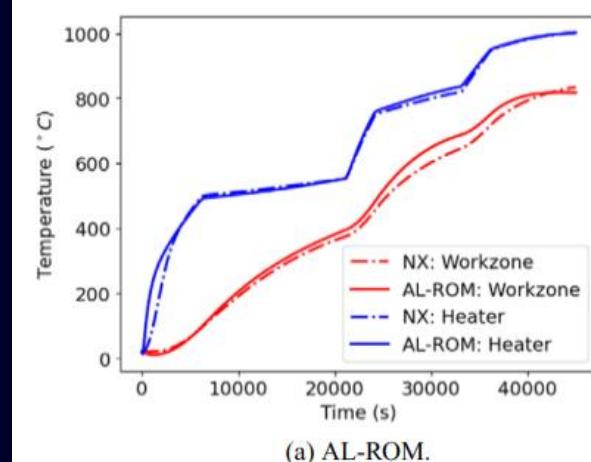
97%-confident ROM Error (PAC)

	#Samples	AL-ROM	ML-ROM
NN	20 000	1,00%	13,07%
OI	5 000	1,00%	7,54%

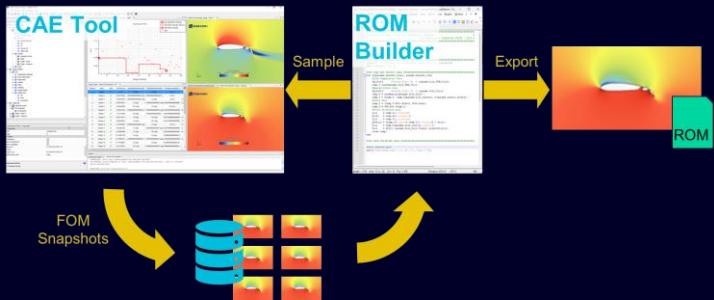
Euler Neural Network



Operator Inference



Industrial Model Order Reduction Workflows

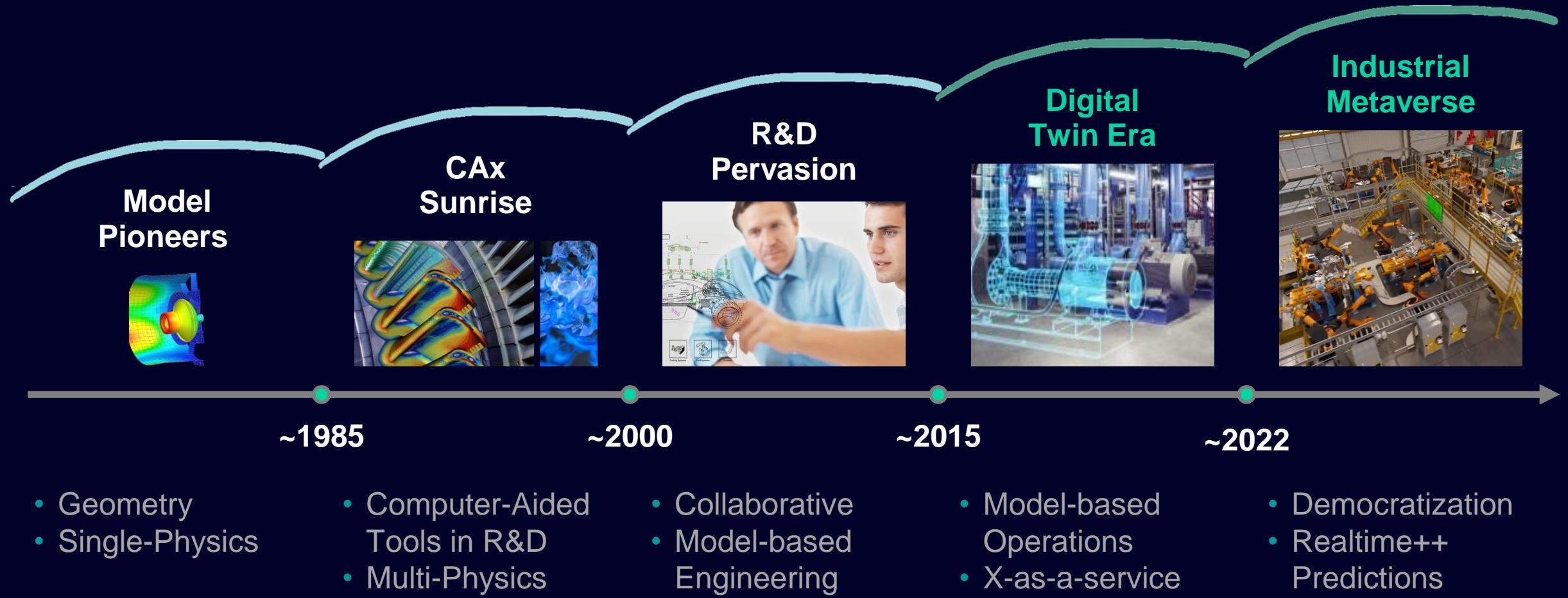


- ✓ Industrial workflows require a high degree of automation.
- ✓ Active learning strategies allow to achieve “optimal” ROMs.
- ✓ First heuristic strategies are available
- ✗ Analytic guarantees

4

Wrap Up

Digital Twin - A new age of computational paradigms



CAx: Computer Aided Design, Engineering, & Manufacturing

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