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Course: Applied Mathematics for Artificial Intelligence

Assignment 4
This latex only cover part 3 of Assignment, for other parts please refer to Assignment4.ipynb file on google colab.
Hyper link to google colab notebook: Assignment 4.ipynb
Hyper link to github repository: Assignment 4

Part 3

Problem 1

We define that:

A is "Test positive"
B is "Has Ebola"

According to the problem, we have:
P(A|B) = 84%
P($A|\overline{B}$) = 11%
P(B) = 0.9%

If a random person is tested positive, the probability that he/she actually has Ebola is:

$$\begin{aligned} P(B|A) &= \frac{P(A|B)\cdot P(B)}{P(A)} \\ &= \frac{P(A|B)\cdot P(B)}{P(A|B)\cdot P(B)+P(A|\overline{B})\cdot P(\overline{B})} \\ &= \frac{P(A|B)\cdot P(B)}{P(A|B)\cdot P(B)+P(A|\overline{B})\cdot (1-P(B))} \\ &= \frac{84\%\cdot 0{,}4\%}{84\%\cdot 0{,}4\%+11\%\cdot(1-0{,}4\%)} \\ &= 2,98\% \end{aligned}$$

Problem 2

We define that:

A is "players type 1"
B is "players type 2"
C is "players type 3"
W is "winning a game"

According to the problem, we have:
P(A) = $0,5$
P(B) = 0,25
P(B) = 0,25

P(W|A) = 0,3
P(W|B) = 0,4
P(W|C) = 0,5

(A) The probability of winning against a random player is:

$$
\begin{aligned}
P(W) &= P(W|A)\cdot P(A) + P(W|B)\cdot P(B) + P(W|C)\cdot P(C) \\
&= 0,3\cdot 0,5 + 0,4\cdot 0,25 + 0,5\cdot 0,25 \\
&= 0,375
\end{aligned}
$$

(B)The probability of winning against a player of type 1 is:

$$
\begin{aligned}
P(W|A) &= \tfrac{P(W|A)\cdot P(A)}{P(W)} \\
&= \tfrac{0,3\cdot 0,5}{0,375} \\
&= 0,4
\end{aligned}
$$

Problem 3

Sample space:
$\Omega = red, green, blue, orange, yellow$
(A) The smallest possible vaild event space A is:

$$\mathcal{A} = \{\emptyset, \{red, green, blue, orange, yellow\}\}$$

(B) Smallest possible event space that contains the event set {blue} is:

$$\mathcal{B} = \{\emptyset, \{blue\}, \{red, green, blue, orange, yellow\}\}$$

(C) Smallest possible event space that contains both the event set {blue} and {red, green} is:

$\mathcal{C} = \{\emptyset, \{blue\}, \{red, green\}, \{orange, yellow\}, \{blue, orange, yellow\}, \{red, green, blue\},$
$\{red, green, orange, yellow\}, \{red, green, blue, orange, yellow\}\}$

Problem 4

Y						
	y_1	0.01	0.02	0.03	0.1	0.1
	y_2	0.05	0.1	0.05	0.07	0.2
	y_3	0.1	0.05	0.03	0.05	0.04
		x_1	x_2	x_3	x_4	x_5
				X		

Figure 1: Problem 4

(A) Marginal distribution of X is:

X	1	2	3	4	5
P(X)	0.16	0.17	0.11	0.22	0.34

(B) Marginal distribution of Y is:

Y	1	2	3
P(Y)	0.26	0.47	0.27

(C) Conditional distribution of X given Y = y_1 is:
$P(X|Y=1) = \frac{P(X,y_1)}{P(y_1)}$,

X	1	2	3	4	5
P(X|Y=1)	$\frac{1}{26}$	$\frac{1}{13}$	$\frac{3}{26}$	$\frac{5}{13}$	$\frac{5}{13}$

(D) Conditional distribution of Y given X = x_3 is:
$P(Y|x_3) = \frac{P(x_3,Y)}{P(x_3)}$,

Y	1	2	3
P(Y|x_3)	$\frac{3}{11}$	$\frac{5}{11}$	$\frac{3}{11}$

(E) Conditional distribution of X given Y $\neq y_1$ is:
$P(X|Y \neq y_1) = \frac{P(X,Y=y_{23})}{P(Y=y_{23})}$,

X	1	2	3	4	5
P(X|Y$\neq y_1$)	$\frac{15}{74}$	$\frac{15}{74}$	$\frac{4}{37}$	$\frac{6}{37}$	$\frac{12}{37}$

Problem 5

The solution for this problem is in the attached ipynb file.