Monitoring Report Tables Section 5.3

27 September, 2021

Sensivity Analysis

Purpose

The Sensitivity Analysis produces a report which assesses the Total Effect Index of parameters for which uncertainty is included in the calculations. Not all parameters and sources of uncertainty are assessed.

- [] Todo: Reference Uncertainty list and list of assessed parameters.
- [] Todo: Reference (Word document with value with uncertainty defined) either in CG document (link) or coding table with parameter and code name……..

The Fiji Code uses the Total Effect Index method for ranking the parameters effects on the overall uncertainty. The ranking of the parameters allows prioritizing improvements to those which will have the biggest impact to reducing the overall uncertainty.

This method has been implemented from guidance provided from the FCPF.

- [] Todo: Reference World Bank UC guidance document received from Rama
- [] Todo: Reference World Bank UC Power Point presentation from Andres

The following description provides a basis for explaining design decisions related to the implementation of the Sensitivity Analysis. More information about the implementation and the algorithm can be found in the package documentation.

Total-effect index:

- [] Todo: Needs more work, this was extracted from wiki pages. But provides the knowledge needed to understand the results!! [Wikipedia_Sensitvity_Analysis]:https://en.wikipedia.org/wiki/Sensitivity_analysis [Wikipedia_Variance_Based_Sensitvity_Analysis]:https://en.wikipedia.org/wiki/Variance-based_sensitivity_analysis [Python_Sensitvity_Analysis]:https://uncertainpy.readthedocs.io/en/latest/theory/sa.html

First-order indices

A direct variance-based measure of sensitivity $\mathrm{S_i}$, called the “first-order sensitivity index”, or “main effect index” is stated as follows,[1]

$$S_i = \frac{V_i}{\mathrm{Var}(Y)}$$

This is the contribution to the output variance of the main effect of $\mathrm{X_i}$, therefore it measures the effect of varying $\mathrm{X_i}$ alone, but averaged over variations in other input parameters. It is standardised by the total variance to provide a fractional contribution. Higher-order interaction indices $\mathrm{S_{ij}}$, $\mathrm{S_{ijk}}$ and so on can be formed by dividing other terms in the variance decomposition by Var(Y). Note that this has the implication that,

$$\sum_{i=1}^{d} S_i + \sum_{i<j}^{d} S_{ij} + \cdots + S_{12\ldots d} = 1$$

Using the S_i, S_{ij} and higher-order indices given above, one can build a picture of the importance of each variable in determining the output variance. However, when the number of variables is large, this requires the evaluation of 2d-1 indices, which can be too computationally demanding. For this reason, a measure known as the "Total Effect index" or "Total Order index", S_{Ti}, is used.[2] This measures the contribution to the output variance of X_i, including all variance caused by its interactions, of any order, with any other input variables. It is given as,

$$S_{Ti} = \frac{E_{\mathbf{X}_{\sim i}}\left(\mathrm{Var}_{X_i}(Y \mid \mathbf{X}_{\sim i})\right)}{\mathrm{Var}(Y)} = 1 - \frac{\mathrm{Var}_{\mathbf{X}_{\sim i}}\left(E_{X_i}(Y \mid \mathbf{X}_{\sim i})\right)}{\mathrm{Var}(Y)}$$

Note that unlike the S_i,

$$\sum_{i=1}^{d} S_{Ti} \geq 1$$

due to the fact that the interaction effect between e.g. X_i and X_j is counted in both S_{Ti} and S_{Tj} In fact, the sum of the S_{Ti} will only be equal to 1 when the model is purely additive.

[1] Sobol', I. (1990). Sensitivity estimates for nonlinear mathematical models. Matematicheskoe Modelirovanie 2, 112–118. in Russian, translated in English in Sobol' , I. (1993). Sensitivity analysis for non-linear mathematical models. Mathematical Modeling & Computational Experiment (Engl. Transl.), 1993, 1, 407–414.

[2] Homma, T. and A. Saltelli (1996). Importance measures in global sensitivity analysis of nonlinear models. Reliability Engineering and System Safety, 52, 1–17.

Fiji Method:

The sensitivity value selected for Fiji is the S_{Ti}, the Total Effect Index of X_i.

This sensitivity value was selected to be reported for variables as it will have larger values for S_{Ti} (approaching 1) being candidates, rather than smaller (approaching 0) values, for reducing overall variance and therefore uncertainty of the estimate calculations.

The total effect index of X_i is given by the following equations:

$$S_{Ti} = \frac{E_{\mathbf{X}_{\sim i}}\left(\mathrm{Var}_{X_i}(Y \mid \mathbf{X}_{\sim i})\right)}{\mathrm{Var}(Y)} = 1 - \frac{\mathrm{Var}_{\mathbf{X}_{\sim i}}\left(E_{X_i}(Y \mid \mathbf{X}_{\sim i})\right)}{\mathrm{Var}(Y)}$$

As the effect of the uncertainty of X_i gets larger the $Var_{X\ \ i}$ gets smaller so the remaining variance approaches 1.

- $Var_{X_{\sim i}}$: Variance of Y without the variance of X_i, X_i is held at its nominal value
- $\mathrm{Var}(Y)$: variance of the Y model including the variance on all variables.
- S_{Ti}: Contribution of the uncertainty of X_i to the total variance

The second form of the equation was used:

$$S_{Ti} = 1 - \frac{\mathrm{Var}_{\mathbf{X}_{\sim i}}\left(E_{X_i}(Y \mid \mathbf{X}_{\sim i})\right)}{\mathrm{Var}(Y)}$$

In other words, the contribution of the variance of X_i standardised by the total variance is equal to 1 minus the variance of the final estimate calculation with the value of X_i fixed divided by the variance of the original estimate calculation including the uncertainty of all variables.

Note: The sum of S_{Ti} for all variable analysed will not equal 1. So a percentage will not be reported, just a fraction without a unit.

Output

- `TEI_Values`: The Total Effect Index values for each assessed parameter.

Produces a table which is used in section 5.3 Sensitivity Analysis of the FCPF Monitoring Report.

Sources of uncertaity with bigger (closer to 1) values have more impact on the overall uncertainty.

	name	v
31	AReforArea	0.0767
8	BiomassConvExpansionARefor	0.2356
9	BiomassConvExpansionHW	0.1019
10	BiomassConvExpansionIncHW	0.0572
16	DeforAreaLow	0.9311
21	DeforAreaUp	0.0943
2	EFDeforLow	-0.0090
1	EFDeforUp	0.1173
41	FDegFellArea	0.1305
17	FRLDeforestation	0.1850
18	FRLForestDegradation	-0.0608
19	FRLRemovalsBySinks	-0.0233
4	MAIBsw	0.1317
5	MAICFell	0.0381
6	MAIVar	0.1388
7	MAIVhw	-0.3514
14	Recovery	0.0446
13	RootToShootDryLandBig	0.0199
12	RootToShootDryLandSmall	-0.0939
11	RootToShootTropRain	0.1952
3	TEF	0.2349
15	WoodDensity	0.0389