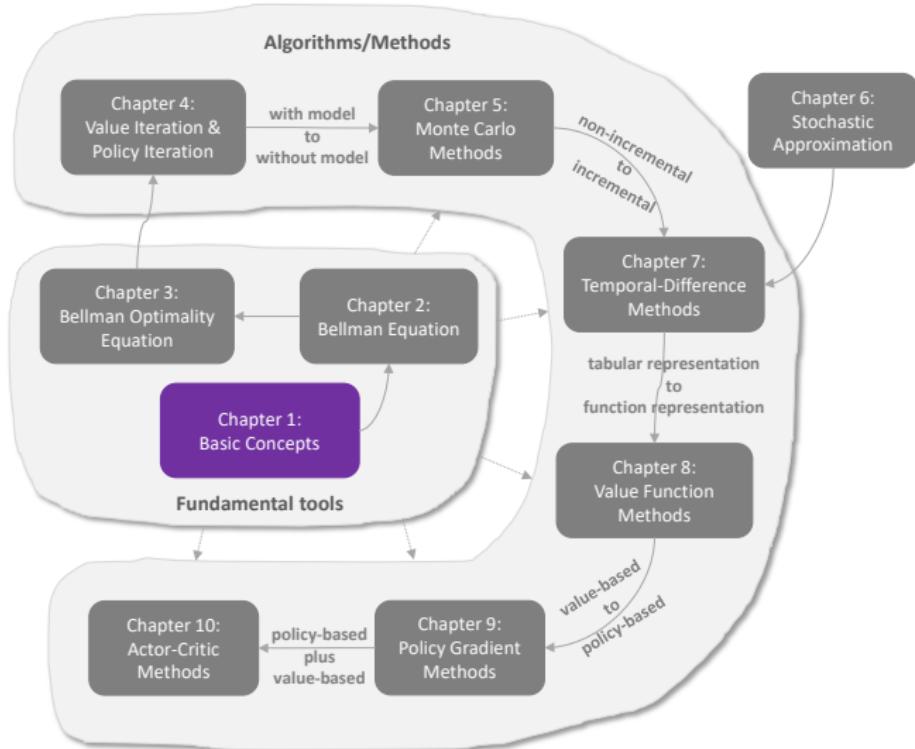


Lecture 1: Basic Concepts

Shiyu Zhao

Department of Artificial Intelligence
Westlake University

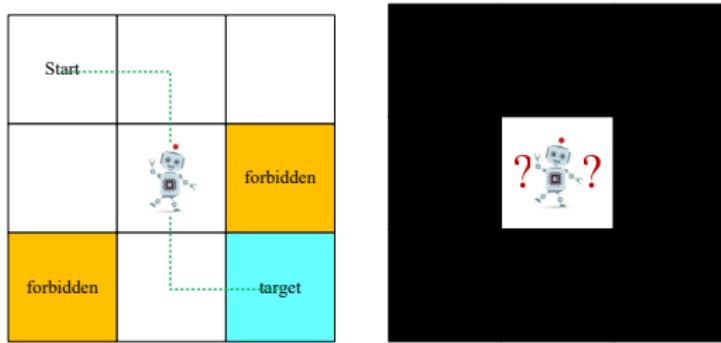
Outline



Contents

- First, introduce fundamental concepts in reinforcement learning (RL) by examples.
- Second, formalize the concepts in the context of Markov decision processes.

A grid-world example



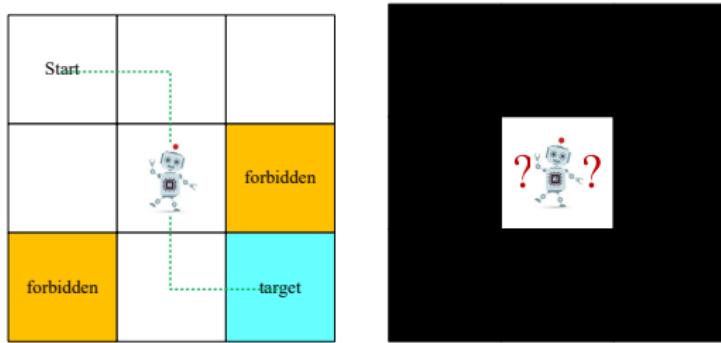
An illustrative example used throughout this course:

- Grid of cells: Accessible/forbidden/target cells, boundary.
- Very easy to understand and useful for illustration

Task:

- Given any starting area, find a “good” way to the target.
- How to define “good”? Avoid forbidden cells, detours, or boundary.

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State

State: The status of the agent with respect to the environment.

- For the grid-world example, the location of the agent is the state. There are nine possible locations and hence nine states: s_1, s_2, \dots, s_9 .

s1	s2	s3
s4	s5	s6
s7	s8	s9

State space: the set of all states $\mathcal{S} = \{s_i\}_{i=1}^9$.

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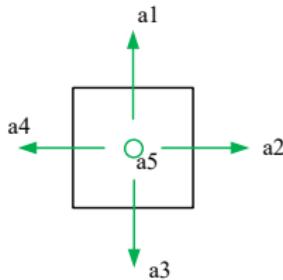
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Action

Action: For each state, there are five possible actions: a_1, \dots, a_5

- a_1 : move upward;
- a_2 : move rightward;
- a_3 : move downward;
- a_4 : move leftward;
- a_5 : stay still;



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Action space of a state: the set of all possible actions of a state.

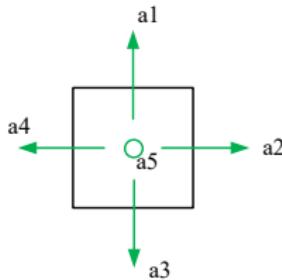
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Question: can different states have different sets of actions?

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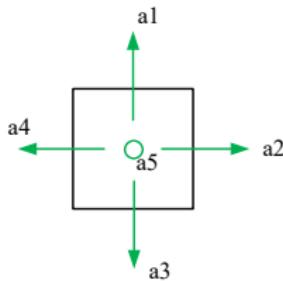
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When taking an action, the agent may move from one state to another. Such a process is called *state transition*.

- Example: In state s_1 , if we choose action a_2 , then what is the next state?

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Pay attention to **forbidden areas**: Example: in state s_5 , if we choose action a_2 , then what is the next state?

- Case 1: the forbidden area is accessible but with penalty. Then,

$$s_5 \xrightarrow{a_2} s_6$$

- Case 2: the forbidden area is inaccessible (e.g., surrounded by a wall)

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- The lectures consider the first case, which is more general and challenging.
- The assignments consider the second case!

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Tabular representation: We can use a table to describe the state transition:

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s_3	s_3	s_3	s_6	s_2	s_3
s_4	s_1	s_5	s_7	s_4	s_4
s_5	s_2	s_6	s_8	s_4	s_5
s_6	s_3	s_6	s_9	s_5	s_6
s_7	s_4	s_8	s_7	s_7	s_7
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Can only represent *deterministic* cases.

State transition

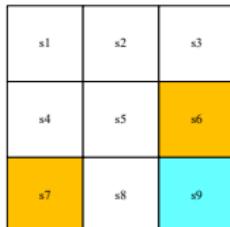
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s_5	s_2	s_6	s_8	s_4	s_5
s_6	s_3	s_6	s_9	s_5	s_6
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s_5	s_2	s_6	s_8	s_4	s_5
s_6	s_3	s_6	s_9	s_5	s_6
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State transition probability: use probability to describe state transition!

- Intuition: In state s_1 , if we choose action a_2 , the next state is s_2 .
- Math:

$$p(s_2|s_1, a_2) = 1$$

$$p(s_i|s_1, a_2) = 0 \quad \forall i \neq 2$$

Here it is a deterministic case. The state transition could be stochastic (for example, wind gust).

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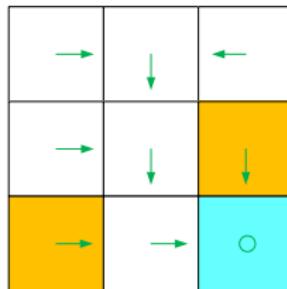
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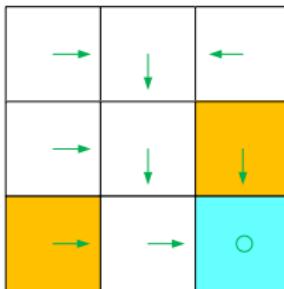
Intuitive representation: We use *arrows* to describe a policy.



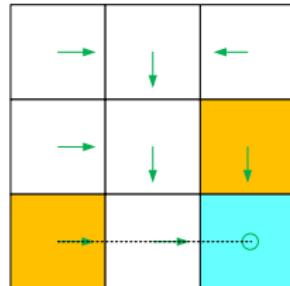
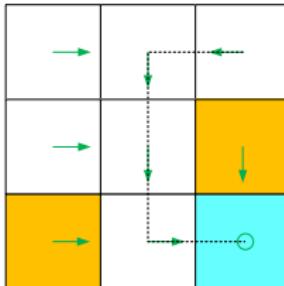
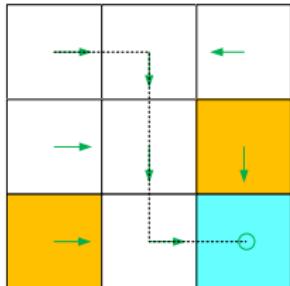
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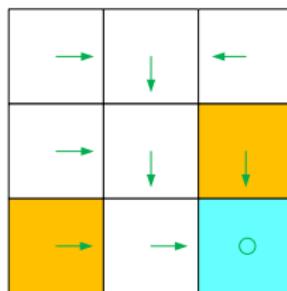
Intuitive representation: We use arrows to describe a policy.



Based on this policy, we get the following trajectories with different starting points.



Policy



Mathematical representation: using conditional probability

For example, for state s_1 :

$$\pi(a_1|s_1) = 0$$

$$\pi(a_2|s_1) = 1$$

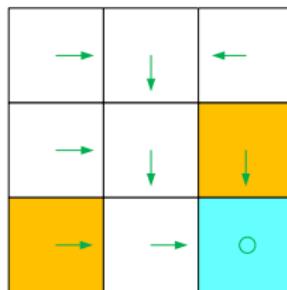
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It is a deterministic policy.

Policy



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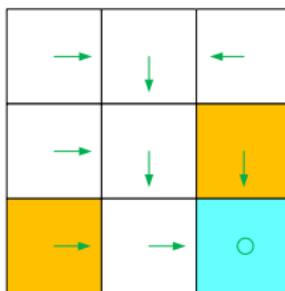
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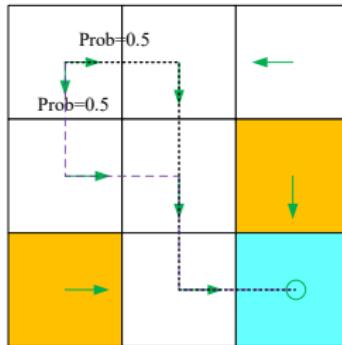
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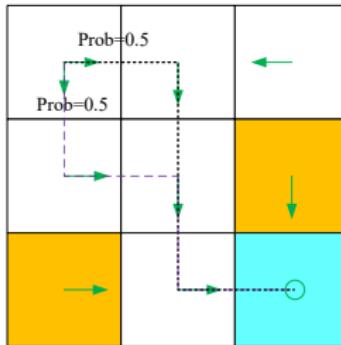
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For example:



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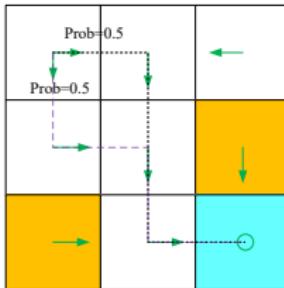
$$\pi(a_2|s_1) = 0.5$$

$$\pi(a_3|s_1) = 0.5$$

$$\pi(a_4|s_1) = 0$$

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Policy



Tabular representation of a policy: how to use this table.

	a_1 (upward)	a_2 (rightward)	a_3 (downward)	a_4 (leftward)	a_5 (still)
s_1	0	0.5	0.5	0	0
s_2	0	0	1	0	0
s_3	0	0	0	1	0
s_4	0	1	0	0	0
s_5	0	0	1	0	0
s_6	0	0	1	0	0
s_7	0	1	0	0	0
s_8	0	1	0	0	0
s_9	0	0	0	0	1

Can represent either *deterministic* or *stochastic* cases.

Reward

Reward is one of the most unique concepts of RL.

Reward: a real number we get after taking an action.

- A **positive** reward represents **encouragement** to take such actions.
- A **negative** reward represents **punishment** to take such actions.

Questions:

- Can positive indicate punishment and negative indicate encouragement?
 - Yes.
 - In this case, reward may be called *cost*.
- What about a zero reward?
 - Relative values matter, not absolute values.
 - Example: $r = \{+1, -1\}$ becomes $r = \{+2, 0\}$ will not change the optimal policy. Details will be shown in the future.

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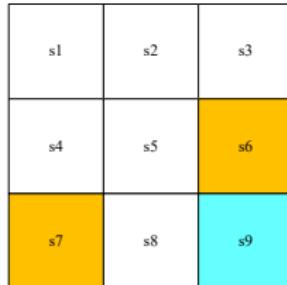
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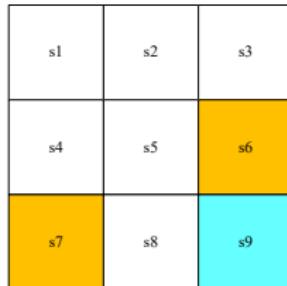
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- If the agent attempts to get out of the boundary, let $r_{\text{bound}} = -1$
- If the agent attempts to enter a forbidden cell, let $r_{\text{forbid}} = -1$
- If the agent reaches the target cell, let $r_{\text{target}} = +1$
- Otherwise, the agent gets a reward of $r = 0$.

Reward can be interpreted as a **human-machine interface**, with which we can guide the agent to behave as what we expect.

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Reward



Tabular representation of reward transition: how to use the table?

	a_1 (upward)	a_2 (rightward)	a_3 (downward)	a_4 (leftward)	a_5 (still)
s_1	r_{bound}	0	0	r_{bound}	0
s_2	r_{bound}	0	0	0	0
s_3	r_{bound}	r_{bound}	r_{forbid}	0	0
s_4	0	0	r_{forbid}	r_{bound}	0
s_5	0	r_{forbid}	0	0	0
s_6	0	r_{bound}	r_{target}	0	r_{forbid}
s_7	0	0	r_{bound}	r_{bound}	r_{forbid}
s_8	0	r_{target}	r_{bound}	r_{forbid}	0
s_9	r_{forbid}	r_{bound}	r_{bound}	0	r_{target}

Can only represent *deterministic* cases.

Reward

s1	s2	s3
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Mathematical description: conditional probability

- Intuition: In state s_1 , if we choose action a_1 , the reward is -1 .
- Math: $p(r = -1|s_1, a_1) = 1$ and $p(r \neq -1|s_1, a_1) = 0$

Remarks:

- Here it is a deterministic case. The reward transition could be stochastic. For example, if you study hard, you will get rewards. But how much is uncertain.

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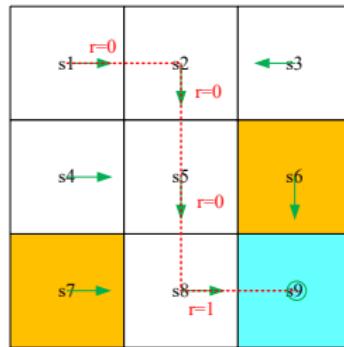
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Trajectory and return



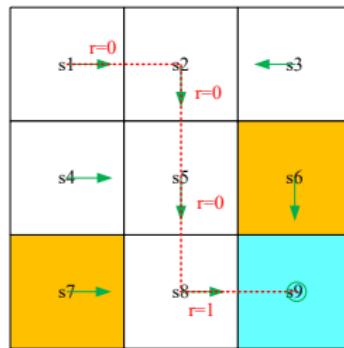
A **trajectory** is a state-action-reward chain:

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The *return* of this trajectory is the sum of all the rewards collected along the trajectory:

$$\text{return} = 0 + 0 + 0 + 1 = 1$$

Trajectory and return



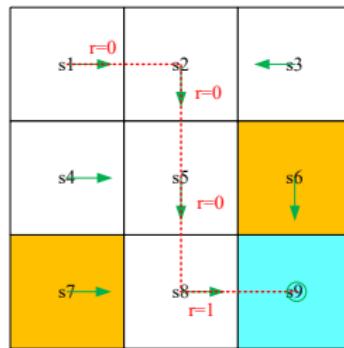
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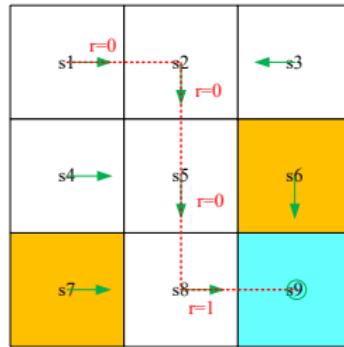
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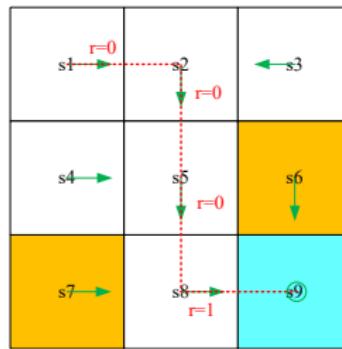
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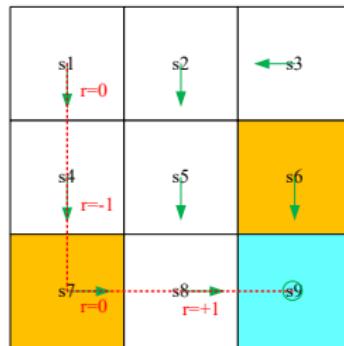
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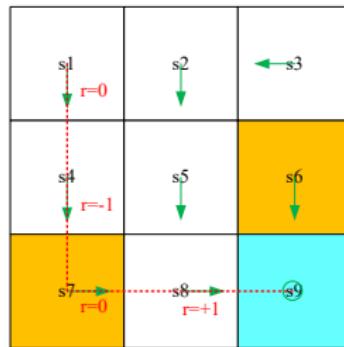
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$$s_1 \xrightarrow[r=0]{a_3} s_4 \xrightarrow[r=-1]{a_3} s_7 \xrightarrow[r=0]{a_2} s_8 \xrightarrow[r=+1]{a_2} s_9$$

The return of this path is:

$$\text{return} = 0 - 1 + 0 + 1 = 0$$

Trajectory and return



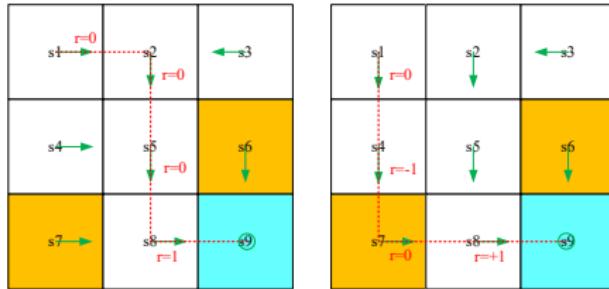
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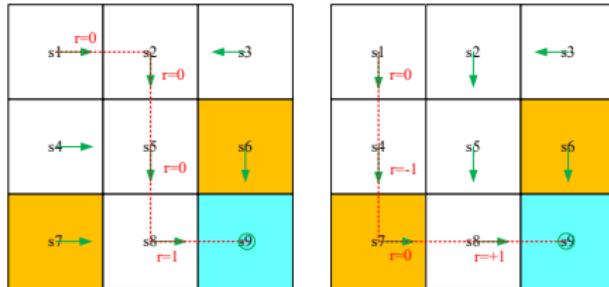
Trajectory and return



Which policy is better?

- **Intuition:** the first is better, because it avoids the forbidden areas.
- **Mathematics:** the first one is better, since it has a greater return!
- Return can be used to evaluate whether a policy is good or not (see details in the next lecture)!

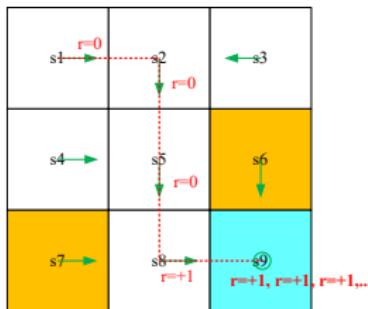
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Discounted return



A trajectory may be infinitely long:

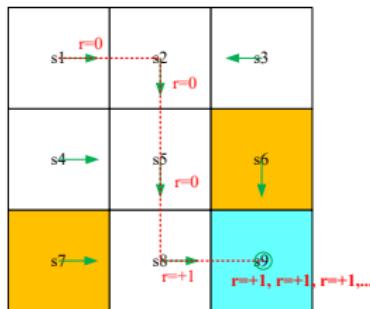
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$$\text{return} = 0 + 0 + 0 + 1 + 1 + 1 + \dots = \infty$$

The definition is invalid since the return diverges!

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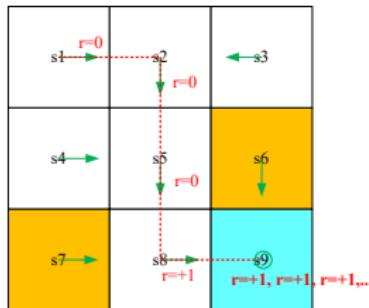
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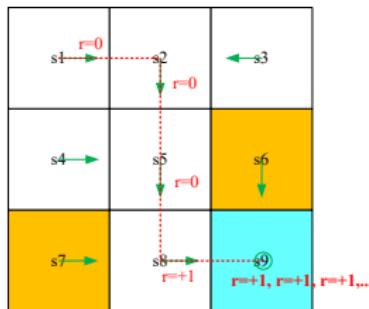
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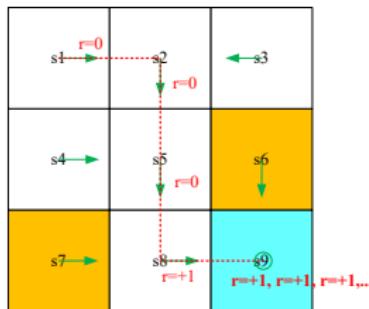
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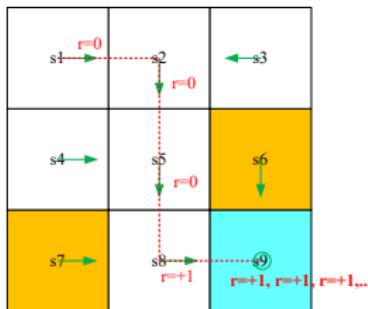
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The return is

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Discounted return



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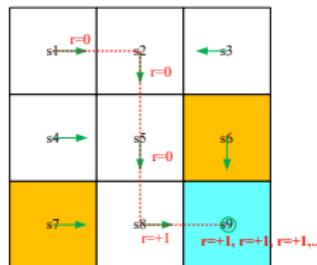
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Discounted return



Need to introduce a *discount rate* $\gamma \in (0, 1)$:

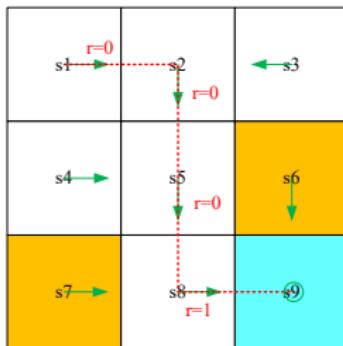
$$\begin{aligned}\text{discounted return} &= 0 + \gamma 0 + \gamma^2 0 + \gamma^3 1 + \gamma^4 1 + \gamma^5 1 + \dots \\ &= \gamma^3 (1 + \gamma + \gamma^2 + \dots) = \gamma^3 \frac{1}{1 - \gamma}.\end{aligned}$$

Roles: 1) the sum becomes finite; 2) balance the far and near future rewards:

- If γ is close to 0, the value of the discounted return is dominated by the rewards obtained in the near future.
- If γ is close to 1, the value of the discounted return is dominated by the rewards obtained in the far future.

Episode

When interacting with the environment following a policy, the agent may stop at some *terminal states*. The resulting trajectory is called an *episode* (or a trial).



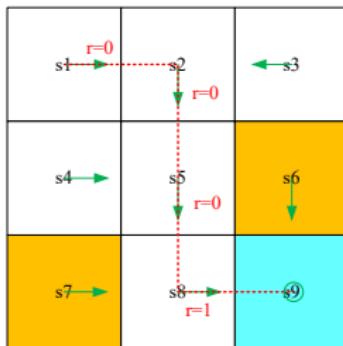
Example: episode

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An episode is usually assumed to be a finite trajectory. Tasks with episodes are called *episodic tasks*.

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Episode

Some tasks may have no terminal states, meaning the interaction with the environment will never end. Such tasks are called *continuing tasks*.

In the grid-world example, should we stop after arriving the target?

- Episodic task + Terminal state
- Continuing task + Absorbing state: Treat the target state as a special *absorbing state*, entering which will lead to staying there forever.
- Continuing task + Normal state: We don't need to distinguish the target state from the others and treat it as a *normal state*. The agent can still leave the target state and gain $r = +1$ when entering the target state.
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Markov decision process (MDP)

Key elements of MDP:

- Sets:
 - State: the set of states \mathcal{S}
 - Action: the set of actions $\mathcal{A}(s)$ is associated for state $s \in \mathcal{S}$
 - Reward: the set of rewards $\mathcal{R}(s, a)$
- Probability distribution (or called system model):
 - State transition probability: in state s , taking action a , the probability to transit to state s' is $p(s'|s, a)$
 - Reward probability: in state s , taking action a , the probability to get reward r is $p(r|s, a)$
- Policy: in state s , the probability to choose action a is $\pi(a|s)$
- *Markov property*: memoryless property

$$p(s_{t+1}|a_t, s_t, \dots, a_0, s_0) = p(s_{t+1}|a_t, s_t),$$
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All the concepts introduced in this lecture can be put in the framework in MDP.

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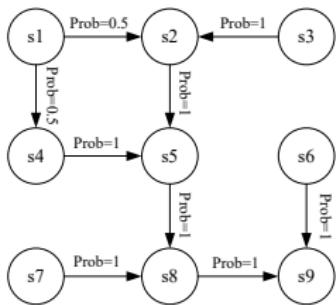
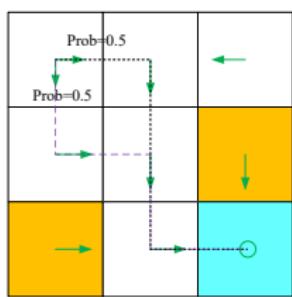
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Markov decision process (MDP)

The grid world could be abstracted as a more general model, *Markov process*.



The circles represent states and the links with arrows represent the state transition.

Summary

By using grid-world examples, we demonstrated the following key concepts:

- State
- Action
- State transition, state transition probability $p(s'|s, a)$
- Reward, reward probability $p(r|s, a)$
- Trajectory, episode, return, discounted return
- Markov decision process