1 Test sufficiency of angular sampling

1. Create a $L \times L \times L$ cube of zeros, $\rho(\vec{r})$. Here, L matches the image length of your 2D projections.

2. Fill $\rho(\vec{r})$ with a centered ball of random values. This ball should have radius R_N, the approximate radius of the nanoparticle in the experimental data. Plot the central sections of $\rho(\vec{r})$: $\rho(x, y, 0)$, $\rho(x, 0, z)$, $\rho(0, y, z)$.

3. Apply a low-pass filter to the densities $\rho(\vec{r})$. To do this, Fourier transform $\rho(\vec{r}) \to \rho(\vec{k})$ and apply the low-pass filter:

$$\rho_B(\vec{k}) = \rho(\vec{k}) \exp\left(-\frac{|\vec{k}|^2}{2k_0^2}\right), \tag{1}$$

   where k_0 is approximately $R_N/2$.

4. Inverse Fourier transform $\rho_B(\vec{k}) \to \rho_B(\vec{r})$. Plot the central sections of this blurred object. Check that you get a blurred versions of the central sections in step 2.

5. We are ready to expand our densities. Do `expand`$(\rho_B(\vec{k}),\ \text{quat}_n) \to \widetilde{\rho}_B(j, i)$ to obtain the tomograms sampled by the list of quaternions quat_n. I will give you a number of quaternions $\{n = 4, \ldots, 10\}$. **Time how long this takes, and how much memory is used.**

6. Now, `compress`$(\widetilde{\rho}_B(j, i),\ \text{quat}_n) \to \widetilde{\rho}_B(\vec{k})$. **Time how long this takes, and how much memory is used.**

7. Compute the resolution-resolved error

$$\Delta(k) = \sqrt{\left\langle \left(|\widetilde{\rho}_B(\vec{k})| - |\rho_B(\vec{k})|\right)^2 \right\rangle_{|\vec{k}|=k}}. \tag{2}$$

   You should modify your algorithm for computing angular averages to do this.

8. Plot how $\Delta(k)$ varies with n, for the $\rho_{L\times L\times L}$ that is relevant to our problem.