

Plotting a Mathematical Function

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Introduction

We will use a 36-inch by 48-inch sheet of paper to answer the box problem in this activity.

We can make an open box by cutting a square with side length x from each corner and folding the four sides up.

We want to use the R language and the ggplot2 function to make a graph of this box's volume function and determine the x value that makes the box's volume the biggest.

Code for the Box Problem

```
library(tidyverse)
library(ggplot2)

W <- 36; L <- 48

vol_box <- function(x, L, W) x * (L - 2*x) * (W - 2*x)

x_star <- ((L + W) - sqrt((L + W)^2 - 3*L*W)) / 6
V_max <- vol_box(x_star, L, W)
base_L <- L - 2*x_star
base_W <- W - 2*x_star

#| fig-alt: "Volume versus cut size x for an open-top box from a 36 by 48 inch sheet. The curve
#| fig-width: 7
#| fig-height: 4.5
#| fig-dpi: 300
ggplot(data.frame(x = c(0, min(L, W)/2)), aes(x = x)) +
  stat_function(fun = function(x) vol_box(x, L, W), linewidth = 1) +
  geom_vline(xintercept = x_star, linetype = 2) +
  labs(
    title    = "Box Problem: Volume vs. Cut Size (x)",
    subtitle = "Sheet = 36×48 in; V(x) = x (48-2x) (36-2x)",
    x = "Cut size x (inches)",
```

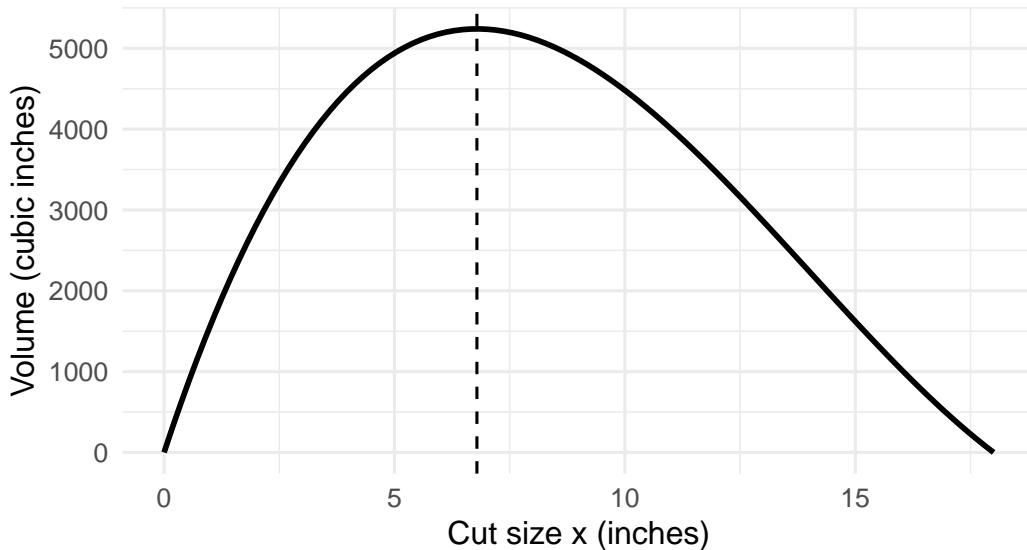
```

y = "Volume (cubic inches)"
) +
theme_minimal(base_size = 12) +
theme(plot.title.position = "plot")

```

Box Problem: Volume vs. Cut Size (x)

Sheet = 36×48 in; $V(x) = x(48-2x)(36-2x)$



Visualization

```

library(ggplot2)

# Function: volume as a function of cut size x
vol_box <- function(x, L, W) x * (L - 2*x) * (W - 2*x)

# Paper dimensions
L <- 48
W <- 36

# Compute maximum point analytically
x_star <- ((L + W) - sqrt((L + W)^2 - 3*L*W)) / 6

# Create the plot using ggplot2
ggplot(data.frame(x = c(0, min(L, W)/2)), aes(x = x)) +
  stat_function(fun = function(x) vol_box(x, L, W),
               linewidth = 1, color = "steelblue") +
  geom_vline(xintercept = x_star, linetype = 2, color = "red") +
  labs(
    title = "Volume vs. Cut Size (x)",
    subtitle = "Sheet = 36x48 in; V(x) = x(48-2x)(36-2x)"
  )

```

```

title      = "Box Problem: Volume vs. Cut Size (x)",
subtitle   = "Sheet = 36 × 48 inches; V(x) = x(48-2x)(36-2x)",
x         = "Cut size x (inches)",
y         = "Volume (cubic inches)"
) +
theme_minimal(base_size = 12) +
theme(plot.title.position = "plot")

```

Box Problem: Volume vs. Cut Size (x)
Sheet = 36×48 inches; $V(x) = x(48-2x)(36-2x)$

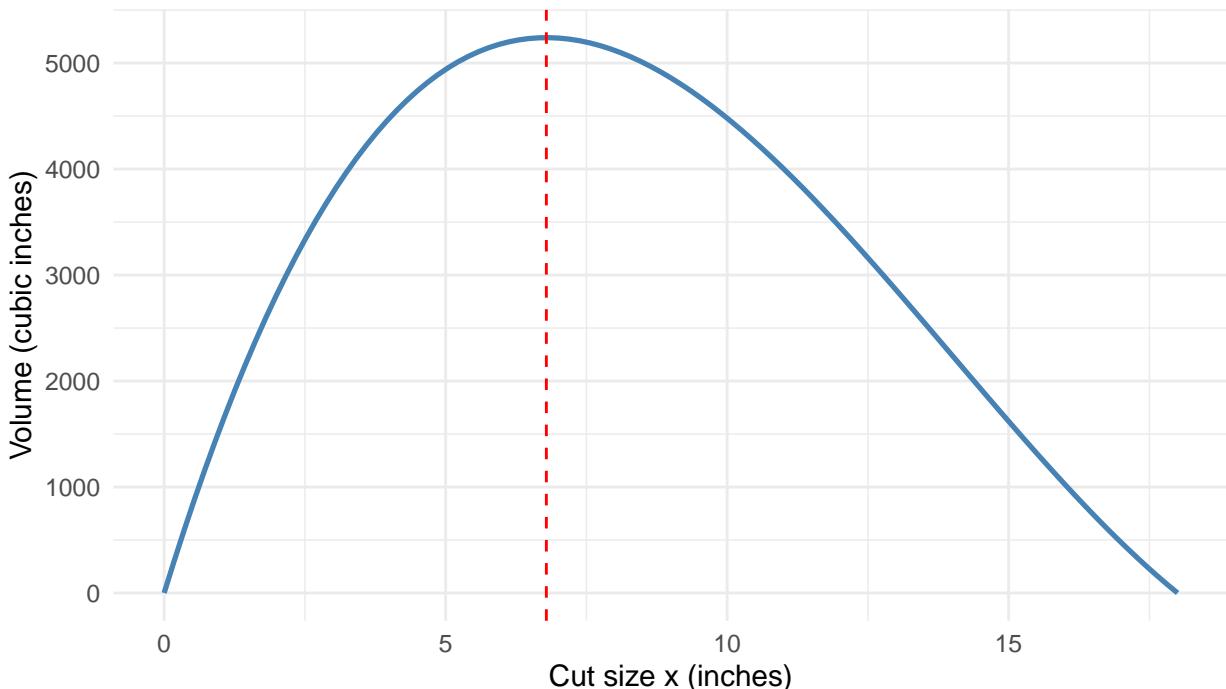


Figure 1: Volume function for the 36×48 inch sheet. The dashed line marks the cut size x that gives the maximum volume.

Narrative Text for the Box Problem

The graph shows that the volume increases rapidly as x grows from zero, reaches a single peak, and then declines back to zero when the sides of the box collapse.

At this cut size, the base of the box measures 34.4 inches by 24.4 inches, and the height of the box is 6.8 inches.

This combination of dimensions creates the largest possible box volume for a sheet that is 36 inches by 48 inches.

The charts and results demonstrate that mathematical modeling and visualization can clearly determine the optimal design parameters.

Reflection

In this workshop, I learned how to create clear and practical charts using R and ggplot2. Through completing the “Box Problem” and “Baby Names Project,” I learned how to clean data, build functions, and create charts that are easier to read and suitable for colorblind individuals. I also learned that using titles, labels, and scale options is crucial for making numbers come alive.